A Guide to the *Duchamp* Source Finding Software

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Contents

1 Introduction and getting going quickly

This document gives details on the use of the program Duchamp. This has been written to provide a source-detection facility for spectral-line data cubes. The basic execution of Duchamp is to read in a FITS data cube, find sources in the cube, and produce a text file of positions, velocities and fluxes of the detections, as well as a postscript file of the spectra of each detection.

So, you have a FITS cube, and you want to find the sources in it. What do you do? The first step is to make an input file that contains the list of parameters. Brief and detailed examples are shown in Appendix B. This provides the input file name, the various output files, and defines various parameters that control the execution.

The program is run by the command

```
Duchamp -p [parameter file]
```

replacing `[parameter file]` with the name of the file you have just created/copied. The program will then work away and give you the list of detections and their spectra. The program execution is summarise below, and detailed in §3. Information on inputs is in §2 and Appendix A, and descriptions of the output is in §4.

1.1 A summary of the execution steps

The basic flow of the program is summarised here. All these steps are discussed in more detail in the following sections, so read on if you have questions!

1. The parameter file given on the command line is read in, and the parameters absorbed.
2. From the parameter file, the FITS image is located and read in to memory.
3. If requested, blank pixels are trimmed from the edges, and channels corresponding to bright (e.g. Galactic) emission are excised.
4. If requested, the baseline of each spectrum is removed.
5. If the reconstruction method is requested, the cube is reconstructed using the *á trous* wavelet method.
6. Searching for objects then takes place, using the requested thresholding method.
7. The list of objects is trimmed by merging neighbouring objects and removing those deemed unacceptable.
8. The baselines and trimmed pixels are replaced prior to output.
9. The details on the detections are written to screen and to the requested output file.
10. Maps showing the spatial location of the detections are written.
11. The integrated spectra of each detection are written to a postscript file.
12. If requested, the reconstructed array can be written to a new FITS file.

1.2 Guide to terminology

First, a brief note on the use of terminology in this guide. Duchamp is designed to work on FITS "cubes". These are FITS[1] image arrays with three dimensions – they are assumed to have the following form: the first two dimensions (referred to as x and y) are spatial directions (that is, relating to the position on the sky), while the third dimension, z, is the spectral direction, which can correspond to frequency, wavelength, or velocity.

Each spatial pixel (a given (x, y) coordinate) can be said to be a single spectrum, while a slice through the cube perpendicular to the spectral direction at a given z-value is a single channel (the 2-D image is a channel map).

Features that are detected are assumed to be positive. If one wants to search for absorption (negative) features, try multiplying your cube by -1 before running Duchamp.

Note that it is possible to run Duchamp on a two-dimensional image (i.e. one with no frequency or velocity information), or indeed a one-dimensional array, and many of the features of the program will work fine. The focus, however, is on object detection in three dimensions.

1.3 Why "Duchamp"?

Well, it's important for a program to have a name, and it certainly beats the initial version of "cubefind". I had planned to call it "Picasso" (as in the father of cubism), but sadly this had already been used before (Minchin 1999). So I settled on naming it after Marcel Duchamp, another cubist, but also one of the first artists to work with "found objects".

2 User Inputs

Input to the program is provided by means of a parameter file. Parameters are listed in the file, followed by the value that should be assigned to them. The syntax used is `paramName value`. The file is not case-sensitive, and lines in the input file that start with `#` are ignored. If a parameter is listed more than once, the latter value is used, but otherwise the order in which the parameters are listed in the input file is arbitrary.

If a parameter is not listed, the default value is assumed. The defaults are chosen to provide a good result (using the reconstruction method), so the user doesn't need to specify many new parameters in the input file. Note that the image file **must** be specified! The parameters that can be set are listed in Appendix A, with their default values in parentheses.

The 'flag' parameters are stored as `bool` variables, and so are either `true = 1` or `false = 0`. Currently the program only reads them from the file as integers, and so they should be entered in the file as 0 or 1 (see example file in Appendix B).

3 What the program is doing

The execution flow of the program is detailed here, indicating the main algorithmic steps that are used. The program is written in C/C++ and makes use of the CFITSIO, WCSLIB

[1]FITS is the Flexible Image Transport System – see Hanisch et al. (2001) or websites such as http://fits.cv.nrao.edu/FITS.html for details.

and PGPLOT libraries.

3.1 Image input

The cube is read in using basic CFITSIO commands, and stored as an array in a special C++ class structure. This class keeps track of the list of detected objects, as well as any reconstructed arrays that are made (see §3.3). The World Coordinate System (WCS) information for the cube is also obtained from the FITS header by WCSLIB functions (Calabretta & Greisen 2002; Greisen & Calabretta 2002), and this information, in the form of a `wcsprm` structure, is also stored in the same class.

A sub-section of an image can be requested. This is done via the `subsection` parameter in the parameter file. The generalised form of the subsection that is used by CFITSIO is `[x1:x2:dx,y1:y2:dy,z1:z2:dz]`, such that the x-coordinates run from `x1` to `x2` (inclusive), with steps of `dx`. The step value can be omitted (so a subsection of the form `[2:50,2:50,10:1000]` is still valid). Duchamp does not at this stage deal with the presence of steps in the subsection string, and any that are present are removed before the file is opened.

If one wants the full range of a value then replace the range with an asterisk, e.g. `[2:50,2:50,*]`. If one wants to use just a subsection, one must set `flagSubsection = 1`. A complete description of the section syntax can be found at the FITSIO web site [2].

3.2 Image modification

Several modifications to the cube can be made that improve the execution and efficiency of Duchamp (these are optional – their use is indicated by the relevant flags set in the input parameter file).

3.2.1 Milky-Way removal

First, a single set of contiguous channels can be removed – these may exhibit very strong emission, such as that from the Milky Way as seen in extragalactic HI cubes (hence the references to "Milky Way" in relation to this task – apologies to Galactic astronomers!). Such dominant channels will both produce many unnecessary, uninteresting and large (in size and hence in memory usage) detections, and will also affect any reconstruction that is performed (see next section). The use of this feature is controlled by the `flagMW` parameter, and the exact channels concerned are able to be set by the user (using `maxMW` and `minMW`). When employed, the flux in these channels is set to zero. The information in those channels is not kept.

3.2.2 Blank pixel removal

Second, the cube is trimmed of any BLANK pixels that pad the image out to a rectangular shape. This is also optional, being determined by the `flagBlankPix` parameter. The value for these pixels is read from the FITS header (using the BLANK, BSCALE and BZERO keywords), but if these are not present then the value can be specified by the user in the parameter file. If these blank pixels are stored as NaNs, then a normal number will

[2] http://heasarc.gsfc.nasa.gov/docs/software/fitsio/c/c_user/node90.html

be substituted (allowing these pixels to be accurately removed without adverse effects). [NOTE: this appears not to be working correctly at time of writing. If your data has unspecified BLANKs, be wary...]

This stage is particularly important for the reconstruction step, as lots of BLANK pixels on the edges will smooth out features in the wavelet calculation stage. The trimming will also reduce the size of the cube's array, speeding up the execution. The amount of trimming is recorded, and these pixels are added back in once the source-detection is completed (so that quoted pixel positions are applicable to the original cube).

Rows and columns are trimmed one at a time until the first non-BLANK pixel is reached, so that the image remains rectangular. In practice, this means that there will be BLANK pixels left in the trimmed image (if the non-BLANK region is non-rectangular). However, these are ignored in all further calculations done on the cube.

3.2.3 Baseline removal

Finally, the user may request the removal of baselines from the spectra, via the parameter `flagBaseline`. This may be necessary if there is a strong baseline ripple present, which can result in spurious detections on the high points of the ripple. The baseline is calculated from a wavelet reconstruction procedure (see §3.3) that keeps only the two largest scales. This is done separately for each spatial pixel (i.e. for each spectrum in the cube), and the baselines are stored and added back in before any output is done. In this way the quoted fluxes and displayed spectra are as one would see from the input cube itself – even though the detection (and reconstruction if applicable) is done on the baseline-removed cube.

3.3 Image reconstruction

This is an optional step. The user can direct Duchamp to reconstruct the data cube using the *à trous* wavelet procedure. A good description of the procedure can be found in Starck & Murtagh (2002). This is an effective way of removing a lot of the noise in the image, but at this stage is relatively time- and memory-intensive. The steps in the procedure are as follows:

1. Set the reconstructed array to 0 everywhere.
2. The cube is discretely convolved with a given filter function. This is determined from the parameter file via the `filterCode` parameter – see Appendix A for details on the filters available.
3. The wavelet coefficients are calculated by taking the difference between the convolved array and the input array.
4. If the wavelet coefficients at a given point are above the threshold requested (given by `snrRecon` as the number of σ above the mean and adjusted to the current scale), add these to the reconstructed array.
5. The separation of the filter coefficients is doubled.
6. The procedure is repeated from step 2, using the convolved array as the input array.
7. Continue until the required maximum number of scales is reached.

8. Add the final smoothed (i.e. convolved) array to the reconstructed array. This provides the "DC offset", as each of the wavelet coefficient arrays will have zero mean.

It is important to note that the *à trous* decomposition is an example of a "redundant" transformation. If no thresholding is performed, the sum of all the wavelet coefficient arrays and the final smoothed array is identical to the input array. The thresholding thus removes only the unwanted structure in the array.

The statistics of the cube are estimated using robust methods, to avoid corruption by strong outlying points. The mean is actually estimated by the median, while the median absolute deviation from the median (MADFM) is calculated and corrected assuming Gaussianity to estimate the standard deviation σ. The Gaussianity (or Normality) assumption is critical, as the MADFM does not give the same value as the usual rms or standard deviation value – for a normal distribution $N(\mu,\sigma)$ we find MADFM$= 0.6744888\sigma$. The difference between the MADFM and σ is corrected for, so the user need only think in the usual multiples of σ when setting `snrRecon`. See Appendix E for a derivation of this value.

When thresholding the different wavelet scales, the value of σ as measured from the input array needs to be scaled to account for the increased amount of correlation between neighbouring pixels (due to the convolution). See Appendix E for details on this scaling.

The user can also select the minimum scale to be used in the reconstruction – the first scale exhibits the highest frequency variations, and so ignoring this one can sometimes be beneficial in removing excess noise. The default, however, is to use all scales (`minscale = 1`).

The reconstruction has at least two iterations. The first iteration makes a first pass at the wavelet reconstruction (the process outlined in the 8 stages above), but the residual array will inevitably have some structure still in it, so the wavelet filtering is done on the residual, and any significant wavelet terms are added to the final reconstruction. This step is repeated until the change in the σ of the background is less than some fiducial amount.

The user can optionally select to save the reconstructed image as a FITS file – at the moment this is just saved in the same directory as the input file, so it won't work if the user does not have write permissions on that directory. See Appendix A for details on the naming of the output image. The residual image, which is the difference between the input image and the reconstructed image, can also be saved in the same manner.

Finally, note that any BLANK pixels that are still in the cube will not be altered by the reconstruction – they will be left as BLANK so that the shape of the valid part of the cube is preserved.

3.4 Searching the image

The image is searched for detections in two ways: spectrally (a 1-dimensional search in the spectrum in each spatial pixel), and spatially (a 2-dimensional search in the spatial image in each channel). In both cases, the algorithm finds connected pixels that are above the user-specified threshold. In the case of the spatial image search, the algorithm of Lutz (1980) is used to raster scan through the image and connect groups of pixels on neighbouring rows.

Note that this algorithm cannot be applied directly to a 3-dimensional case, as it requires that objects are completely nested in a row: that is, if you are scanning along a row, and one object finishes and another starts, you know that you will not get back to the first one (if at all) until the second is finished for that row. Three-dimensional data does not have this property, which is why we break up the searching into 1- and 2-dimensional cases.

The determination of the threshold is done in one of two ways. The first way is a simple sigma-clipping, where a threshold defined as $n\sigma$ above the mean is set and pixels above this threshold are flagged as detected. As before, the value for σ is estimated by the MADFM, and corrected by the ratio derived in Appendix E.

The second method uses the False Discovery Rate (FDR) technique (Hopkins et al. 2002; Miller et al. 2001), whose basis we briefly detail here. The false discovery rate (given by the number of false detections divided by the total number of detections) is fixed at a certain value α (e.g. $\alpha = 0.05$ implies 5% of detections are false positives). In practice, an α value is chosen, and the ensemble average FDR (i.e. $< FDR >$) when the method is used will be less than α. One calculates p – the probability, assuming the null hypothesis is true, of obtaining a test statistic as extreme as the pixel value (the observed test statistic) – for each pixel, and sorts them in increasing order. One then calculates d where

$$d = \max_j \left\{ j : P_j < \frac{j\alpha}{c_N N} \right\},$$

and then rejects all hypotheses whose p-values are less than or equal to P_d. (So a $P_i < P_d$ will be rejected even if $P_i \geq j\alpha/c_N N$.) Note that "reject hypothesis" here means "accept the pixel as an object pixel" (i.e. we are rejecting the null hypothesis that the pixel belongs to the background).

The c_N values here are normalisation constants that depend on the correlated nature of the pixel values. If all the pixels are uncorrelated, then $c_N = 1$. If N pixels are correlated, then their tests will be dependent on each other, and so $c_N = \sum_{i=1}^{N} i^{-1}$. Hopkins et al. (2002) consider real radio data, where the pixels are correlated over the beam. In this case the sum is made over the N pixels that make up the beam. The value of N is calculated from the FITS header (if the correct keywords – BMAJ, BMIN – are not present, a default value of 10 pixels is assumed).

If a reconstruction has been made, the residuals (defined as original − reconstruction) are used to estimate the noise parameters of the cube. Otherwise they are estimated directly from the cube itself. In both cases, the median is used as a robust estimator of the mean value, although the σ is estimated by the standard deviation (of the residual array, in the case of the reconstruction, but of the original array otherwise).

Detections must have a minimum number of pixels to be counted. This minimum number is given by the input parameters `minPix` (for 2-dimensional searches) and `minChannels` (for 1-dimensional searches). Note again that only positive thresholding is done – negative features are not searched for.

3.5 Merging detected objects

The searching step produces a list of detections that will have many repeated detections of a given object – for instance, spectral detections in adjacent pixels of the same object and/or spatial detections in neighbouring channels. These are then combined in an algorithm that matches all objects judged to be "close". This determination is made in one of two ways.

One way is to define two thresholds – one spatial and one in velocity – and say that two objects should be merged if there is at least one pair of pixels that lie within these threshold distances of each other. These thresholds are specified by the parameters `threshSpatial` and `threshVelocity` (in units of pixels and channels respectively).

Alternatively, the spatial requirement can be changed to say that there must be a pair of pixels that are *adjacent* – a stricter, but more realistic requirement, particularly when the spatial pixels have a large angular size (as is the case for HI surveys). This method can be selected by setting the parameter `flagAdjacent` to 1 (i.e. `true`) in the parameter file. The velocity thresholding is done in the same way as the first option.

Once the detections have been merged, they may be "grown". This is a process of increasing the size of the detection by adding adjacent pixels that are above some secondary threshold. This threshold is lower than the one used for the initial detection, but above the noise level, so that faint pixels are only detected when they are close to a bright pixel. The value of this threshold is a possible input parameter (`growthCut`), with a default value of 1.5σ. The use of the growth algorithm is controlled by the `flagGrowth` parameter – the default value of which is `false`. If the detections are grown, they are sent through the merging algorithm a second time, to pick up any detections that now overlap or have grown over each other.

Finally, to be accepted, the detections must span *both* a minimum number of channels (to remove any spurious single-channel spikes that may be present), and a minimum number of spatial pixels. These numbers, as for the original detection step, are set with the `minChannels` and `minPix` parameters. The channel requirement means there must be at least one set of this many consecutive channels in the source for it to be accepted.

4 Outputs

4.1 During execution

Duchamp provides the user with feedback whilst it is running, to keep the user informed on the progress of the analysis. Most of this consists of self-explanatory messages about the particular stage the program is up to. The relevant parameters are printed to the screen at the start (once the file has been successfully read in), so the user is able to make a quick check that the setup is correct.

If the cube is being trimmed (§3.2), the resulting dimensions are printed to indicate how much has been trimmed. If a reconstruction is being done, a continually updating message shows the current iteration and scale (compared to the maximum scale).

During the searching algorithms, the progress through the 1D and 2D searches are shown. When the searches have completed, the number of objects found in both the 1D and 2D searches are reported (see §3.4 for details).

In the merging process (where multiple detections of the same object are combined – see §3.5), two stages of output occur. The first is when each object in the list is compared with all others. The output shows two numbers: the first being how far through the list we are, and the second being the length of the list. As the algorithm proceeds, the first number should increase and the second should decrease (as objects are being combined). When the numbers meet, the second phase begins, of removing multiply-appearing pixels in each object and removing objects not meeting the minimum channels requirement. During this phase, the total number of accepted objects is shown, which should steadily increase until all have been accepted or rejected. Note that these steps can be very quick for small numbers of detections.

Since this continual printing to screen has some overhead of time and CPU involved, the user can elect to not print this information by setting the parameter `verbose = 0`. In this case, the user is still informed as to the steps being undertaken, but the details of the progress are not shown.

4.2 Results

Finally, we get to the results – the reason for running Duchamp in the first place. Once the detection list is finalised, it is sorted by the mean velocity of the detections (or, if there is no good WCS with the cube, by the mean Z-pixel position). The results are then printed to the screen and to the output file, denoted by the `OutFile` parameter. The results list, an example of which can be seen in Appendix C, contains the following columns:

Obj#:	The ID number of the detection (simply the sequential count for the list, which is ordered by increasing velocity).
Name:	The IAU-format name of the detection (based on the RA & Dec).
X:	The average X-pixel position.
Y:	The average Y-pixel position.
Z:	The average Z-pixel position.
RA:	The Right Ascension of the centre of the object.
DEC:	The Declination of the centre of the object.
w_RA:	The width of the object in Right Ascension [arcmin].
w_DEC:	The width of the object in Declination [arcmin].
VEL:	The mean velocity of the object [km/s].
w_VEL:	The full velocity width of the detection (max channel − min channel, in velocity units [km/s]).
X1, X2:	The minimum and maximum X-pixel coordinates.
Y1, Y2:	The minimum and maximum Y-pixel coordinates.
Z1, Z2:	The minimum and maximum Z-pixel coordinates.
Npix:	The number of pixels & channels (i.e. distinct (x, y, z) coordinates) in the detection.
F_tot:	The integrated flux over the object, in the units of the FITS file.
F_peak:	The peak flux over the object, in the units of the FITS file.

If the WCS is not valid (i.e. is not present or does not have all the necessary information), the Name, RA, DEC, VEL and related columns are not printed, but the pixel coordinates are still provided.

Two alternative results files can also be requested. One option is a VOTable-format XML file, containing just the RA, Dec, Velocity and the corresponding widths of the detections, as well as the fluxes. The user should set `flagVOT = 1`, and put the desired filename in the parameter `votFile` – note that the default is for it not to be produced.

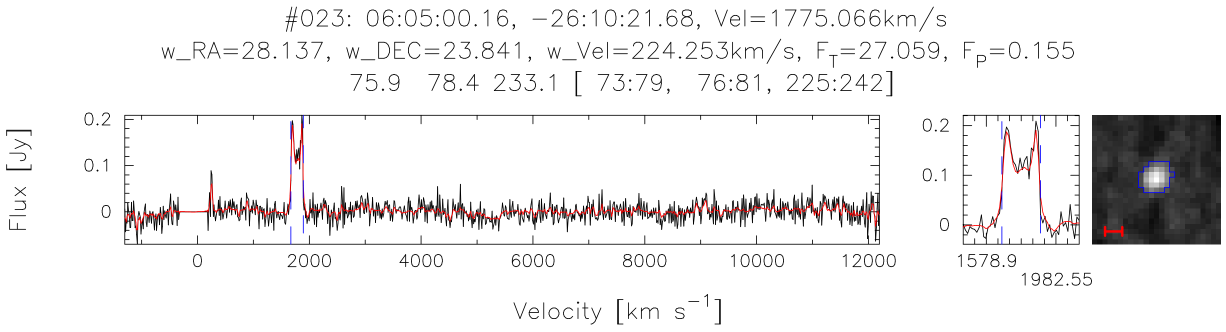

Figure 1: An example of the spectrum output. Note several of the features discussed in the text: the removal of the Milky Way emission around 0 km/s; the red lines indicating the reconstructed spectrum; the blue dashed lines indicating the spectral extent of the detection; and the blue border showing its spatial extent on the 0th moment map.

This file should be compatible with all Virtual Observatory tools (such as Aladin[3]). The second option is an annotation file for use with the Karma toolkit of visualisation tools (in particular, with `kvis`). This will draw a circle at the position of each detection, and number it according to the Obj# given above. To use, the user should set `flagKarma = 1`, and put the desired filename in the parameter `karmaFile` – again, the default is for it not to be produced.

As the program is running, it also (optionally) records the detections made in each individual spectrum or channel (see §3.4 for details on this process). This is recorded in the file denoted by the parameter `LogFile`. This file does not include the columns `Name, RA, DEC, w_RA, w_DEC, VEL, w_VEL`. This file is designed primarily for diagnostic purposes: e.g. to see if a given set of pixels is detected in, say, one channel image, but does not survive the merging process. The list of pixels (and their fluxes) in the final detection list are also printed to this file, again for diagnostic purposes. This feature can be turned off by setting `flagLog = false`. (This may be a good idea if you are not interested in its contents, as it can be a large file.)

As well as the output data file, a postscript file is created that shows the integrated spectra of each detection, together with a small cutout image (0th moment) and basic information of the detection. If the cube was reconstructed, the spectrum from the reconstruction is shown in red, over the top of the original spectrum. The spectral extent of the detection is indicated with green lines, and a zoom is shown in a separate window. The cutout image can optionally include a border around the spatial pixels that are in the detection (turned on and off by the parameter `drawBorders`). It also includes a scale bar in the bottom left corner to indicate size – it is 30 arcmin long. An example detection can be seen below in Fig. 1.

Finally, a couple of images are optionally produced: a 0th moment map of the cube, combining just the detected channels in each object, showing the integrated flux in grey-scale; and a "detection image", a grey-scale image where the pixel values are the number of channels that spatial pixel is detected in. In both cases, if `drawBorders = true`, a border is drawn around the spatial extent of each detection. An example moment map is shown in Fig. 2. The production or otherwise of these images is governed by the `flagMaps`

[3] Aladin can be found on the web at http://aladin.u-strasbg.fr/

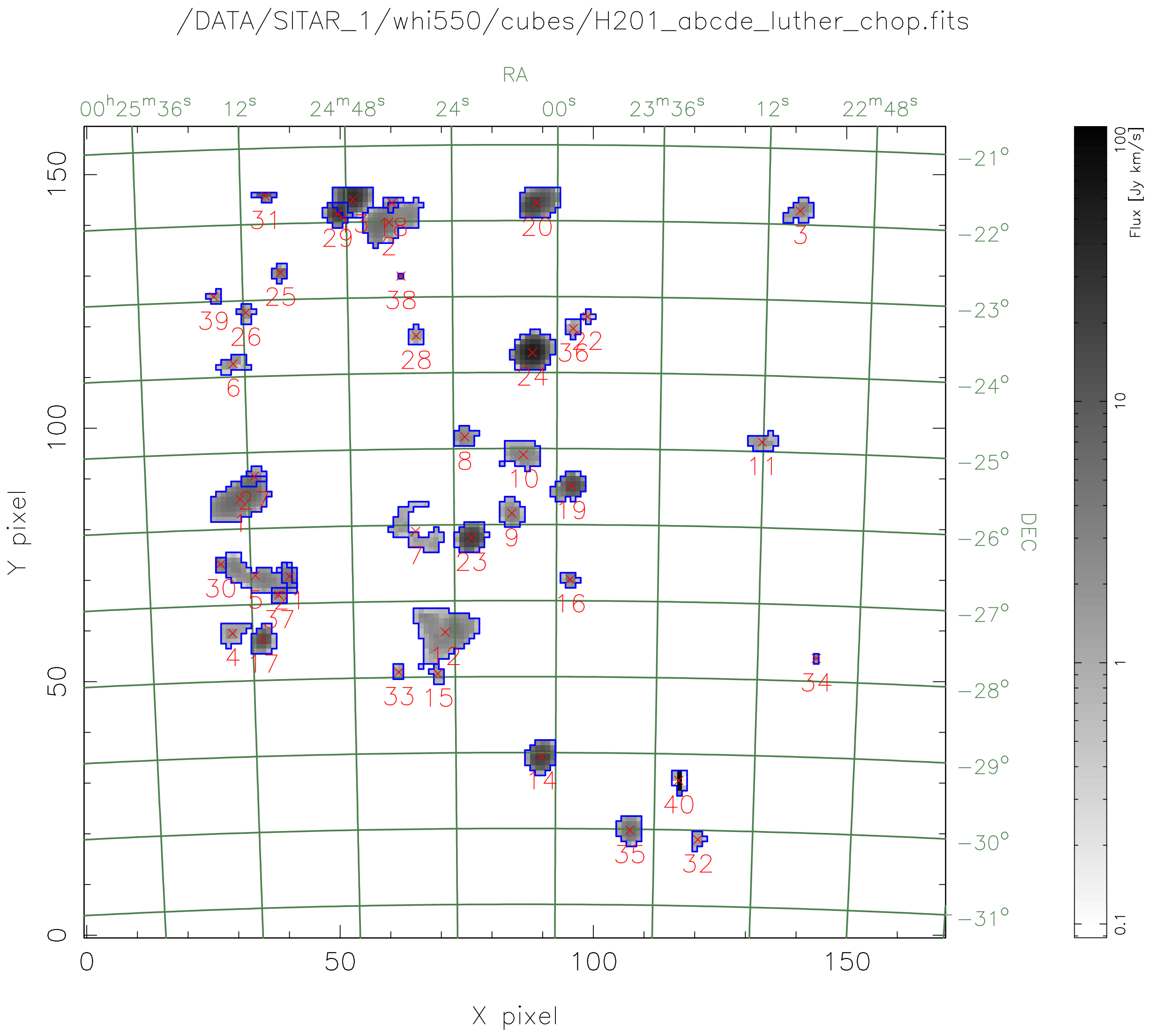

Figure 2: An example of the moment map created by Duchamp. The full extent of the cube is covered, and the 0th moment of each object is shown (integrated individually over all the detected channels).

parameter.

The purpose of these images are to provide a visual guide to where the detections have been made, and, particularly in the case of the moment map, to provide an indication of the strength of the source. In both cases, the detections are numbered (in the same way as the output list), and the spatial borders are marked out as for the cutout images in the spectra file. Both these images are saved as postscript files (given by the parameters `momentMap` and `detectionMap` respectively), with the latter also displayed in a PGPLOT window (regardless of the state of `flagMaps`).

5 Notes and hints on the use of Duchamp

In using Duchamp, the user has to make a number of decisions about the way the program runs. This section is designed to give the user some idea about what the various selections do...

The main choice is whether or not to use the wavelet reconstruction. The main benefits of this are the marked reduction in the noise level, leading to regularly-shaped detections, and good reliability for faint sources. The main drawback with its use is the long execution time: to reconstruct a $170 \times 160 \times 1024$ (HIPASS) cube often requires three iterations and takes about 20-25 minutes. The searching part of the procedure is much quicker (although see the note on merging, below), so if one uses the FDR method on the un-reconstructed cube, the execution time is only a couple of minutes.

A further drawback with the reconstruction is that it is susceptible to edge effects. If the valid area in the cube (i.e. the part that is not BLANK) has very curved edges (such as the HIPASS polar cap cube, H001, which has a roughly circular shape after gridding), the convolution can produce artefacts in the reconstruction that mimic the edges and lead to some spurious sources. Caution is advised with such data – the user is advised to check carefully the reconstructed cube for the presence of such artefacts.

If one chooses the reconstruction method, a further decision is required on the signal-to-noise cutoff used in determining acceptable wavelet coefficients. A larger value will remove more noise from the cube, at the expense of losing fainter sources, while a smaller value will include more noise, which may produce spurious detections, but will be more sensitive to faint sources. Values of less than about 3σ tend to not reduce the noise a great deal and can lead to many spurious sources.

The FDR method certainly produces more reliable results than a simple sigma-clipping (i.e. thresholding at some number of σ above the mean). However, at this point it does not seem to be giving the sensitivity expected for the supplied value of `alpha` (i.e. it is not finding as many sources as expected). Work is being done to assess this, and to judge whether there is a real problem (such as with the determination of the statistics), or simply a result of working in 3 dimensions as opposed to 2.

A further point to bear in mind is that the shape of the detections in a cube that has been reconstructed will be much more regular and smooth – the ragged edges that objects in the raw cube possess are smoothed by the removal of most of the noise.

Finally, as Duchamp is still undergoing development, there are some elements that are not fully developed. In particular, it is not as clever as I would like at avoiding interference. The ability to place requirements on the minimum number of channels and pixels partially circumvents this problem, but work is being done to make Duchamp smarter at rejecting signals that are clearly (to a human eye at least) interference. See the following section for further improvements that are planned.

6 Future Developments

This is both a list of planned improvements and a wish-list of features that would be nice to include (but are not planned in the immediate future):

- Ability to invert cube to search for absorption features. **Planned.**
- More varied output formats. **Planned.**
- Better determination of the noise characteristics of spectral-line cubes, including understanding how the noise is generated and developing a model for it. **Planned.**

- Include more source analysis. Examples could be: shape information; measurements of HI mass; better measurements of velocity width and profile... **Some planned.**
- Provide some indication of the significance of the detection (i.e. some S/N-like value). **Planned.**
- Improved ability to reject interference, possibly on the spectral shape of features. **Planned.**
- Link to lists of possible counterparts (e.g. via NED/SIMBAD/other VO tools?). **Wishlist.**
- Add ability to read in a reconstructed cube that has been saved. In this case the residual array will also need to be read in. The idea of this will be to avoid the extended time required for the reconstruction if the same cube is being analysed multiple times. **Wishlist.**
- At this point, the "Milky Way" channels are discarded and set to zero. It may be that users would like to have those put back in the final cube after the source detection is done, so at some point this option may be added. **Wishlist – if needed.**

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A Available parameters

The full list of parameters that can be listed in the input file are given here. If not listed, they take the default value given in parentheses. Since the order of the parameters in the input file does not matter, they are grouped here in logical sections.

Input-output related

ImageFile (no default assumed): The filename of the data cube to be analysed.

OutFile `[./duchamp-Results]`**:** The file where the final detections are to be recorded. This also records the list of input parameters.

SpectraFile `[./duchamp-Spectra.ps]`**:** The postscript file containing the resulting integrated spectra and images of the detections.

flagLog `[true]`**:** A flag to indicate whether intermediate detections should be logged.

LogFile `[./duchamp-Logfile]`**:** The file in which intermediate detections are logged. These are detections that have not been merged. This is primarily for use in debugging and diagnostic purposes – normal use of the program will probably not require this.

flagSubsection `[false]`**:** A flag to indicate whether one wants a subsection of the requested image.

Subsection `[ [*,*,*] ]`**:** The requested subsection, which should be specified in the format `[x1:x2,y1:y2,z1:z2]`, where the limits are inclusive. If the full range of a dimension is required, use a `*`, e.g. if you want the full spectral range of a subsection of the image, use `[30:140,30:140,*]`.

flagOutputRecon `[false]`**:** A flag to say whether or not to save the reconstructed cube as a FITS file. The filename will be derived from the ImageFile – the reconstruction of `image.fits` will be saved as `image.RECON?.fits`, where `?` stands for the value of `snrRecon` (see below).

flagOutputResid `[false]`**:** As for `flagOutputRecon`, but for the residual array – the difference between the original cube and the reconstructed cube. The filename will be `image.RESID?.fits`.

flagVOT `[false]`**:** A flag to say whether to create a VOTable file corresponding to the information in `outfile`. This will be an XML file in the Virtual Observatory VOTable format.

votFile `[./duchamp-Results.xml]`**:** The VOTable file with the list of final detections. Some input parameters are also recorded.

flagKarma `[false]`**:** A flag to say whether to create a Karma annotation file corresponding to the information in `outfile`. This can be used as an overlay for the Karma programs such as `kvis`.

karmaFile `[./duchamp-Results.ann]`**:** The Karma annotation file showing the list of final detections.

flagMaps `[true]`**:** A flag to say whether to save postscript files showing the 0th moment map of the whole cube (parameter `momentMap`) and the detection image (`detectionMap`).

momentMap `[./latest-moment-map.ps]`**:** A postscript file containing a map of the 0th moment of the detected sources, as well as pixel and WCS coordinates.

detectionMap `[./latest-detection-map.ps]`**:** A postscript file showing each of the detected objects, coloured in greyscale by the number of channels they span. Also shows pixel and WCS coordinates.

Modifying the cube

flagBlankPix `[true]`**:** A flag to say whether to remove BLANK pixels from the analysis – these are pixels set to some particular value because they fall outside the imaged area.

blankPixValue `[-8.00061]`**:** The value of the BLANK pixels, if this information is not contained in the FITS header (the usual procedure is to obtain this value from the header information – in which case the value set by this parameter is ignored).

flagMW `[true]`**:** A flag to say whether to remove channels contaminated by Milky Way (or other) emission – the flux in these channels is currently just set to 0.

maxMW `[112]`**:** The maximum channel for the Milky Way emission.

minMW `[75]`**:** The minimum channel for the Milky Way emission. Note that the channels specified by `maxMW` and `minMW` are assumed to be Milky Way channels (i.e. the range is inclusive).

flagBaseline `[false]`**:** A flag to say whether to remove the baseline from each spectrum in the cube for the purposes of reconstruction and detection.

Detection related

General detection

snrCut `[3.]`**:** The cut-off value for thresholding, in terms of number of σ above the mean.

flagGrowth `[true]`**:** A flag indicating whether or not to grow the detected objects to a smaller threshold.

growthCut `[1.5]`**:** The smaller threshold using in growing detections. In units of σ above the mean.

à trous reconstruction

flagATrous `[true]`**:** A flag indicating whether or not to reconstruct the cube using the *à trous* wavelet reconstruction. Currently does this in 3-dimensions. See §3.3 for details.

scaleMin `[1]`**:** The minimum wavelet scale to be used in the reconstruction. A value of 1 means "use all scales".

snrRecon `[4]`: The thresholding cutoff used in the reconstruction – only wavelet coefficients this many σ above the mean (or greater) are included in the reconstruction.

filterCode `[2]`: The code number of the filter to use in the reconstruction. The options are:

- **1:** B_3-spline filter: coefficients = $(\frac{1}{16}, \frac{1}{4}, \frac{3}{8}, \frac{1}{4}, \frac{1}{16})$
- **2:** Triangle filter: coefficients = $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$
- **3:** Haar wavelet: coefficients = $(0, \frac{1}{2}, \frac{1}{2})$

FDR method

flagFDR `[false]`: A flag indicating whether or not to use the False Discovery Rate method in thresholding the pixels.

alphaFDR `[0.01]`: The α parameter used in the FDR analysis. The average number of false detections, as a fraction of the total number, will be less than α (see §3.4).

Merging detections

flagAdjacent `[true]`: A flag indicating whether to use the "adjacent pixel" criterion to decide whether to merge objects. If not, the next two parameters are used to determine whether objects are within the necessary thresholds.

minPix `[2]`: The minimum number of spatial pixels for a single detection to be counted.

minChannels `[3]`: The minimum number of consecutive channels that must be present in the detection for it to be accepted by the Merging algorithm.

threshSpatial `[3.]`: The maximum allowed minimum spatial separation (in pixels) between two detections for them to be merged into one. Only used if `flagAdjacent = false`.

threshVelocity `[7.]`: The maximum allowed minimum channel separation between two detections for them to be merged into one.

Other parameters

drawBorders `[true]`: A flag indicating whether borders are to be drawn around the detected objects in the moment maps included in the output (see for example Fig. 1).

verbose `[true]`: A flag indicating whether to print the progress of computationally-intensive algorithms (such as the searching and merging) to screen.

B Example parameter files

This is what a typical parameter file would look like.

```
imageFile       /DATA/SITAR_1/whi550/cubes/H201_abcde_luther_chop.fits
logFile         temp-Logfile
outFile         temp-Results
spectraFile     spectra.ps
flagSubsection  0
flagOutputRecon 0
flagOutputResid 0
flagBlankPix    1
blankPixValue   -8.00061
flagMW          1
minMW           75
maxMW           112
minPix          3
flagGrowth      1
growthCut       1.5
flagATrous      0
scaleMin        1
snrRecon        4
flagFDR         1
alphaFDR        0.1
numPixPSF       20
snrCut          3
threshSpatial   3
threshVelocity  7
minChannels     4
```

Note that it is not necessary to include all these parameters in the file, only those that need to be changed from the defaults (as listed in Appendix A), which in this case would be very few. A minimal parameter file might look like:

```
imageFile       /DATA/SITAR_1/whi550/cubes/H201_abcde_luther_chop.fits
flagLog         0
snrRecon        3
snrCut          2.5
minChannels     3
```

This will reconstruct the cube with a lower SNR value than the default, select objects at a lower threshold, with a looser minimum channel requirement, and not keep a log of the intermediate detections.

The following page demonstrates how the parameters are presented to the user, both on the screen at execution time and in the output and log files:

Presentation of parameters in output and log files:

```
---- Parameters ----
Image to be analysed                     = /DATA/SITAR_1/whi550/cubes/H201_abcde_luther_chop.fits
Intermediate Logfile                     = logfile.txt
Final Results file                       = results.txt
Spectrum file                            = spectra.ps
VOTable file                             = results.xml
0th Moment Map                           = latest-moment-map.ps
Detection Map                            = latest-detection-map.ps
Saving reconstructed cube?               = false
Saving residuals from reconstruction?    = false
------
Fixing Blank Pixels?                     = true
Blank Pixel Value                        = -8.00061
Removing Milky Way channels?             = true
Milky Way Channels                       = 75-112
Beam Size (pixels)                       = 10.1788
Removing baselines before search?        = false
Minimum # Pixels in a detection          = 2
Growing objects after detection?         = false
Using A Trous reconstruction?            = true
Minimum scale in reconstruction          = 1
SNR Threshold within reconstruction      = 4
Filter being used for reconstruction     = B3 spline function
Using FDR analysis?                      = false
SNR Threshold                            = 2.5
Using Adjacent-pixel criterion?          = true
Min. # channels for merging              = 4
--------------------
```

C Example output file

This the typical content of an output file, after running Duchamp with the parameters illustrated on the previous page.

```
Results of the Duchamp source finder: Tue Mar 21 16:28:50 EST 2006
---- Parameters ----

(... omitted for clarity -- see previous page for examples...)

--------------------
Total number of detections = 23
--------------------
 Obj#         Name     X     Y       Z           RA          DEC     w_RA    w_DEC       VEL     w_VEL  X1  X2  Y1  Y2   Z1   Z2 Npix     F_tot F_peak
-------------------------------------------------------------------------------------------------------------------------------------------------------------
    1   J0609-2200  59.4 140.6   114.7  06:09:38.50 -22:00:48.20    48.50    39.42   213.061    65.957  55  66 136 145  113  118  185   17.5725 0.2125
    2   J0608-2605  65.2  79.6   116.2  06:08:10.23 -26:05:06.57    44.47    39.47   233.119    39.574  60  70  76  85  115  118   50    4.1441 0.1002
    3   J0606-2724  70.8  59.8   121.4  06:06:33.08 -27:24:43.28    52.48    47.57   302.213    39.574  65  77  53  64  120  123  213   17.0659 0.1497
    4   J0611-2142  52.5 145.1   162.5  06:11:36.34 -21:42:00.01    32.40    23.47   843.727   118.722  49  56 142 147  158  167  303   44.3940 0.4103
    5   J0600-2903  89.7  35.3   202.4  06:00:51.38 -29:03:02.51    23.94    28.09  1370.285   184.679  87  92  32  38  195  209  319   26.5725 0.1729
    6   J0559-2643  95.5  70.2   222.6  05:59:10.59 -26:43:05.94    15.94    12.09  1637.316   105.531  94  97  69  71  219  227   35    1.9253 0.0630
    7   J0617-2727  34.8  58.3   227.5  06:17:24.45 -27:27:53.89    20.77    23.41  1701.802   303.400  33  37  56  61  215  238  176   11.4138 0.0929
    8   J0609-2145  60.3 144.4   229.6  06:09:23.17 -21:45:36.06    16.15    11.81  1729.279   105.531  59  62 143 145  225  233   25    1.4760 0.0679
    9   J0559-2529  95.7  88.6   231.1  05:59:08.81 -25:29:34.50    27.88    24.14  1749.440   250.635  92  98  86  91  220  239  257   16.9297 0.1155
   10   J0601-2145  88.9 144.4   232.3  06:01:10.14 -21:45:58.59    31.96    24.13  1764.657   224.253  86  93 142 147  222  239  415   34.0304 0.1655
   11   J0615-2638  40.0  70.8   232.6  06:15:44.32 -26:38:29.42    16.56    19.57  1769.033    52.765  38  41  69  73  231  235   44    2.7565 0.0685
   12   J0605-2610  75.9  78.4   233.1  06:05:00.16 -26:10:21.68    28.14    23.84  1775.066   224.253  73  79  76  81  225  242  352   27.0587 0.1545
   13   J0601-2344  88.0 114.9   235.7  06:01:25.72 -23:44:18.18    35.96    32.07  1809.749   263.826  84  92 112 119  226  246  724   85.1317 0.2968
   14   J0615-2238  38.2 130.6   253.6  06:15:48.32 -22:38:45.75    12.39    15.70  2046.530   118.722  37  39 129 132  248  257   40    2.3169 0.0697
   15   J0617-2309  31.4 122.8   258.0  06:17:51.07 -23:09:29.22    16.46    15.53  2103.912    39.574  30  33 121 124  256  259   23    1.4243 0.0624
   16   J0612-2153  49.5 142.3   271.1  06:12:29.32 -21:53:16.05    24.36    19.56  2276.976   395.740  47  52 140 144  257  287  318   20.7117 0.1008
   17   J0616-2137  35.2 145.9   300.0  06:16:34.07 -21:37:30.95    20.22     7.46  2658.607   224.252  33  37 145 146  294  311   40    3.8507 0.1271
   18   J0544-2740 144.0  54.9   325.4  05:44:31.07 -27:40:42.30     3.58    12.13  2993.384    39.574 144 144  54  56  324  327    7    0.4362 0.0569
   19   J0555-3000 107.2  20.7   367.5  05:55:28.58 -30:00:46.76    19.67    24.31  3547.812    39.574 105 109  18  23  366  369   72    6.4819 0.1692
   20   J0559-2325  96.0 119.6   532.1  05:59:04.98 -23:25:19.04    11.92    16.08  5720.289    52.765  95  97 118 121  530  534   27    1.2865 0.0508
   21   J0616-2653  37.9  67.0   547.0  06:16:23.08 -26:53:11.73    12.36    11.67  5916.731    39.574  37  39  66  68  546  549   25    1.6374 0.0642
   22   J0619-2256  25.1 125.9   724.2  06:19:39.49 -22:56:06.15    12.38    11.60  8254.112    39.573  24  26 125 127  723  726   13    0.6982 0.0593
   23   J0552-2920 116.9  30.5   727.0  05:52:33.03 -29:20:54.43    11.60    20.25  8290.842   303.400 116 118  28  32  716  739  132   35.8343 0.4787
```

D Example VOTable output

This is part of the VOTable, in XML format, corresponding to the output file in Appendix C (the indentation has been removed to make it fit on the page!).

```
<?xml version="1.0"?>
<VOTABLE version="1.1" xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
 xsi:noNamespaceSchemaLocation="http://www.ivoa.net/xml/VOTable/VOTable/v1.1">
<COOSYS ID="J2000" equinox="J2000." epoch="J2000." system="eq_FK5"/>
<RESOURCE name="Duchamp Output">
<TABLE name="Detections">
<DESCRIPTION>Detected sources and parameters from running the Duchamp source finder.</DESCRIPTION>
<PARAM name="FITS file" datatype="char" ucd="meta.file;meta.fits" value="/DATA/SITAR_1/whi550/cubes/H201_abcde_luther_chop.fits"/>
<FIELD name="ID" ID="col1" ucd="meta.id" datatype="int" width="4"/>
<FIELD name="Name" ID="col2" ucd="meta.id;meta.main" datatype="char" arraysize="14"/>
<FIELD name="RA" ID="col3" ucd="pos.eq.ra;meta.main" ref="J2000" datatype="float" width="10" precision="6" unit="deg"/>
<FIELD name="Dec" ID="col4" ucd="pos.eq.dec;meta.main" ref="J2000" datatype="float" width="10" precision="6" unit="deg"/>
<FIELD name="w_RA" ID="col3" ucd="phys.angSize;pos.eq.ra" ref="J2000" datatype="float" width="7" precision="2" unit="arcmin"/>
<FIELD name="w_Dec" ID="col4" ucd="phys.angSize;pos.eq.dec" ref="J2000" datatype="float" width="7" precision="2" unit="arcmin"/>
<FIELD name="Vel" ID="col4" ucd="phys.veloc;src.dopplerVeloc" datatype="float" width="9" precision="3" unit="km/s"/>
<FIELD name="w_Vel" ID="col4" ucd="phys.veloc;src.dopplerVeloc;spect.line.width" datatype="float" width="8" precision="3" unit="km/s"/>
<FIELD name="Integrated_Flux" ID="col4" ucd="phys.flux;spect.line.intensity" datatype="float" width="10" precision="3" unit="km/s"/>
<DATA>
<TABLEDATA>
<TR>
<TD>   1</TD><TD>    J0609-2200</TD><TD> 92.410416</TD><TD>-22.013390</TD><TD>  48.50</TD><TD>  39.42</TD><TD>  213.061</TD><TD>  65.957</TD><TD>    17.572</TD>
</TR>
<TR>
<TD>   2</TD><TD>    J0608-2605</TD><TD> 92.042633</TD><TD>-26.085157</TD><TD>  44.47</TD><TD>  39.47</TD><TD>  233.119</TD><TD>  39.574</TD><TD>     4.144</TD>
</TR>
<TR>
<TD>   3</TD><TD>    J0606-2724</TD><TD> 91.637840</TD><TD>-27.412022</TD><TD>  52.48</TD><TD>  47.57</TD><TD>  302.213</TD><TD>  39.574</TD><TD>    17.066</TD>
</TR>
<TR>
<TD>   4</TD><TD>    J0611-2142</TD><TD> 92.901421</TD><TD>-21.700003</TD><TD>  32.40</TD><TD>  23.47</TD><TD>  843.727</TD><TD> 118.722</TD><TD>    44.394</TD>
</TR>
<TR>
<TD>   5</TD><TD>    J0600-2903</TD><TD> 90.214081</TD><TD>-29.050697</TD><TD>  23.94</TD><TD>  28.09</TD><TD> 1370.285</TD><TD> 184.679</TD><TD>    26.573</TD>
</TR>
(... table truncated for clarity ...)
</TABLEDATA>
</DATA>
</TABLE>
</RESOURCE>
</VOTABLE>
```

E Robust statistics for a Normal distribution

The Normal, or Gaussian, distribution for mean μ and standard deviation σ can be written as

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(x-\mu)^2/2\sigma^2}.$$

When one has a purely Gaussian signal, it is straightforward to estimate σ by calculating the standard deviation (or rms) of the data. However, if there is a small amount of signal present on top of Gaussian noise, and one wants to estimate the σ for the noise, the presence of the large values from the signal can bias the estimator to higher values.

An alternative way is to use the median (m) and median absolute deviation from the median (s) to estimate μ and σ. The median is the middle of the distribution, defined for a continuous distribution by

$$\int_{-\infty}^{m} f(x)\mathrm{d}x = \int_{m}^{\infty} f(x)\mathrm{d}x.$$

From symmetry, we quickly see that for the continuous Normal distribution, $m = \mu$. We consider the case henceforth of $\mu = 0$, without loss of generality.

To find s, we find the distribution of the absolute deviation from the median, and then find the median of that distribution. This distribution is given by

$$\begin{aligned} g(x) &= \text{distribution of } |x| \\ &= f(x) + f(-x),\ \ x \geq 0 \\ &= \sqrt{\frac{2}{\pi\sigma^2}}\, e^{-x^2/2\sigma^2},\ \ x \geq 0. \end{aligned}$$

So, the median absolute deviation from the median, s, is given by

$$\int_{0}^{s} g(x)\mathrm{d}x = \int_{s}^{\infty} g(x)\mathrm{d}x.$$

Now, $\int_0^\infty e^{-x^2/2\sigma^2}\mathrm{d}x = \sqrt{\pi\sigma^2/2}$, and so $\int_s^\infty e^{-x^2/2\sigma^2}\mathrm{d}x = \sqrt{\pi\sigma^2/2} - \int_0^s e^{-\frac{x^2}{2\sigma^2}}\mathrm{d}x$. Hence, to find s we simply solve the following equation (setting $\sigma = 1$ for simplicity – equivalent to stating x and s in units of σ):

$$\int_{0}^{s} e^{-x^2/2}\mathrm{d}x - \sqrt{\pi/8} = 0.$$

This is hard to solve analytically (no nice analytic solution exists for the finite integral that I'm aware of), but straightforward to solve numerically, yielding the value of $s = 0.6744888$. Thus, to estimate σ for a Normally distributed data set, one can calculate s, then divide by 0.6744888 (or multiply by 1.4826042) to obtain the correct estimator.

Note that this is different to solutions quoted elsewhere, specifically in Meyer et al. (2004), where the same robust estimator is used but with an incorrect conversion to standard deviation – they assume $\sigma = s\sqrt{\pi/2}$. This, in fact, is the conversion used to convert the *mean* absolute deviation from the mean to the standard deviation. This means that the cube noise in the HIPASS catalogue (parameter $\mathrm{Rms}_{\mathrm{cube}}$) should be 18% larger than quoted.

F How Gaussian noise changes with wavelet scale.

The key element in the wavelet reconstruction of an array is the thresholding of the individual wavelet coefficient arrays. This is usually done by choosing a level to be some number of standard deviations above the mean value.

However, since the wavelet arrays are produced by convolving the input array by an increasingly large filter, the pixels in the coefficient arrays become increasingly correlated as the scale of the filter increases. This results in the measured standard deviation from a given coefficient array decreasing with increasing scale. To calculate this, we need to take into account how many other pixels each pixel in the convolved array depends on.

To demonstrate, suppose we have a 1-D array with N pixel values given by $F_i,\ i = 1, ..., N$, and we convolve it with the B$_3$-spline filter with coefficients $\{1/16, 1/4, 3/8, 1/4, 1/16\}$. The flux of the ith pixel in the convolved array will be

$$F'_i = \frac{1}{16}F_{i-2} + \frac{1}{16}F_{i-2} + \frac{3}{8}F_i + \frac{1}{4}F_{i-1} + \frac{1}{16}F_{i+2}$$

and the flux of the corresponding pixel in the wavelet array will be

$$W'_i = F_i - F'_i = \frac{1}{16}F_{i-2} + \frac{1}{16}F_{i-2} + \frac{5}{8}F_i + \frac{1}{4}F_{i-1} + \frac{1}{16}F_{i+2}$$

Now, assuming each pixel has the same standard deviation $\sigma_i = \sigma$, we can work out the standard deviation for the coefficient array:

$$\sigma'_i = \sigma\sqrt{\left(\frac{1}{16}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{5}{8}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{16}\right)^2} = 0.72349\ \sigma$$

Thus, the first scale wavelet coefficient array will have a standard deviation of 72.3% of the input array. This procedure can be followed to calculate the necessary values for all scales, dimensions and filters used by Duchamp.

Calculating these values is, therefore, a critical step in performing the reconstruction. Starck & Murtagh (2002) did so by simulating data sets with Gaussian noise, taking the wavelet transform, and measuring the value of σ for each scale. We take a different approach, by calculating the scaling factors directly from the filter coefficients by taking the wavelet transform of an array made up of a 1 in the central pixel and 0s everywhere else. The scaling value is then derived by adding in quadrature all the wavelet coefficient values at each scale. We give the scaling factors for the three filters available to Duchamp on the following page. These values are hard-coded into Duchamp, so no on-the-fly calculation of them is necessary.

Memory limitations prevent us from calculating factors for large scales, particularly for the three-dimensional case (hence the – symbols in the tables). To calculate factors for higher scales than those available, we note the following relationships apply for large scales to a sufficient level of precision:

- 1-D: factor(scale i) = factor(scale $i-1$)/$\sqrt{2}$.
- 2-D: factor(scale i) = factor(scale $i-1$)/2.
- 1-D: factor(scale i) = factor(scale $i-1$)/$\sqrt{8}$.

- **B_3-Spline Function:** $\{1/16, 1/4, 3/8, 1/4, 1/16\}$

Scale	1 dimension	2 dimension	3 dimension
1	0.723489806	0.890796310	0.956543592
2	0.285450405	0.200663851	0.120336499
3	0.177947535	0.0855075048	0.0349500154
4	0.122223156	0.0412474444	0.0118164242
5	0.0858113122	0.0204249666	0.00413233507
6	0.0605703043	0.0101897592	0.00145703714
7	0.0428107206	0.00509204670	0.000514791120
8	0.0302684024	0.00254566946	–
9	0.0214024008	0.00127279050	–
10	0.0151336781	0.000636389722	–
11	0.0107011079	0.000318194170	–
12	0.00756682272	–	–
13	0.00535055108	–	–

- **Triangle Function:** $\{1/4, 1/2, 1/4\}$

Scale	1 dimension	2 dimension	3 dimension
1	0.612372436	0.800390530	0.895954449
2	0.330718914	0.272878894	0.192033014
3	0.211947812	0.119779282	0.0576484078
4	0.145740298	0.0577664785	0.0194912393
5	0.102310944	0.0286163283	0.00681278387
6	0.0722128185	0.0142747506	0.00240175885
7	0.0510388224	0.00713319703	0.000848538128
8	0.0360857673	0.00356607618	0.000299949455
9	0.0255157615	0.00178297280	–
10	0.0180422389	0.000891478237	–
11	0.0127577667	0.000445738098	–
12	0.00902109930	0.000222868922	–
13	0.00637887978	–	–

- **Haar Wavelet:** $\{0, 1/2, 1/2\}$

Scale	1 dimension	2 dimension	3 dimension
1	0.707167810	0.433012702	0.935414347
2	0.500000000	0.216506351	0.330718914
3	0.353553391	0.108253175	0.116926793
4	0.250000000	0.0541265877	0.0413398642
5	0.176776695	0.0270632939	0.0146158492
6	0.125000000	0.0135316469	0.00516748303