Time Ephemeris and Relativistic Scaling of Ephemerides

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Time Ephemeris

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1. Introduction

- 4D Coordinate Transformation $x^{\mu} = f^{\mu}\left(X^{\alpha}\right)$
  - Expansion around New Spatial Origin $X^{k} = 0$
  - Time Component $t = f\left(T\right) + f_{k}\left(T\right)X^{k} + \cdots$
- Time Ephemeris (TE) $\Delta\tau\left(t\right) \equiv t - f^{-1}\left(t\right)$
- TE of the Earth: BCRS (x) and GCRS (X)
  - TCB-TCG Relation

$$\mathrm{TCB} - \mathrm{TCG} = \Delta\tau_{E}\left(\mathrm{TCB}\right) + \frac{\mathbf{v}_{E} \cdot \mathbf{X}}{c} + \cdots$$

Relativistic Scaling

- Space-Time Unit Conversion
  - BCRS (m_B, s_B) vs GCRS (m_G, s_G)
- Same Number of Speed of Light
  - C = 299792458 m_B/s_B = 299792458 m_G/s_G
- Non-Unity Scaling Factor (SF)
  - m_G/m_B = s_G/s_B = <dTCG/dTCB> = $1 - L_C$
  - TE Expression $\Delta\tau_E = L_C\left(t - t_0\right) + \text{Periodic Terms}$

Effect of Scaling

- [A] = "SI Unit of Quantity A"
  - [GM] = m^3s^{-2}, [R] = m, [e] = 1, [P] = s, …
- Same c => Same SF for m and s
- + Same G => Same SF for m and kg
- + Same k => Same SF for AU and M_{Sun}
- Conversion of Numerical Values
  - $M_G/M_B = R_G/R_B = P_G/P_B = 1 - L_C$

2. Time Ephemeris

- From Lunar/Planetary Ephemerides
  - DE405 => TE405, etc.
- Equation of Proper Time: Case of GTR

$$\frac{\mathrm{d}}{\mathrm{dt}}(\Delta\tau_E) \equiv 1 - \left(\sqrt{g_{00} + \frac{g_{0j}v^j}{c} + \frac{g_{jk}v^j v^k}{c^2}} \right)_E$$

$$= \frac{\mathbf{v}_E^2 + 2U_E}{2c^2} + \frac{\mathbf{v}_E^4 + 12\mathbf{v}_E^2 U_E - 4U_E^2 - 32\mathbf{v}_E \cdot \mathbf{w}_E - 4W_E}{8c^4} + \cdots$$

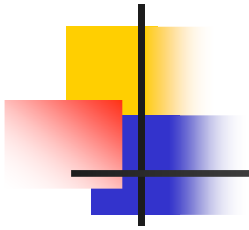
Quantities in Integrand

- EIH Metric Assumed

$$U_E \equiv \sum_{J \neq E} U_{EJ}, \qquad \mathbf{w}_E \equiv \sum_{J \neq E} U_{EJ} \mathbf{v}_J$$

$$W_E \equiv \sum_{J \neq E} U_{EJ} \left[4\mathbf{v}_J^2 - \left(\frac{\mathbf{r}_{EJ} \cdot \mathbf{v}_J}{r_{EJ}} \right)^2 + \sum_{K \neq J} U_{JK} \left(2 + \frac{\mathbf{r}_{EJ} \cdot \mathbf{r}_{JK}}{r_{JK}^2} \right) \right]$$

$$U_{EJ} \equiv \frac{GM_J}{r_{EJ}}, \qquad \mathbf{r}_{JK} \equiv \mathbf{x}_J - \mathbf{x}_K, \qquad r_{JK} \equiv \left| \mathbf{r}_{JK} \right|$$



Time Ephemerides

- Analytical: Integral of Fourier Series
  - Moyer (1981a, b): Keplerian Approx.
  - Hirayama et al. (1988), Fairhead & Bretagnon (1990): from VSOP82 + ELP2000
- Numerical: Romberg Quadrature
  - Fukushima (1995): TE102, TE200, TE245
  - Irwin & Fukushima (1999): TE405

Series Expression

- Harada & Fukushima (2003): HF2002.f
  - 463 Fourier + 36 Mixed Secular Terms

N	S_N (ns)	C_N (ns)	P_N (day)
1	505079.2018	-1551857.1407	365.2652622182
2	21856.7326	-23134.7679	365.22102337
3	20733.1083	-8526.5271	398.88401884
4	-11108.6620	-8369.7220	182.62982594
5	-3405.2830	-3354.5797	4333.21415

3. Scaling Factor

- Principle: Long Time Average
  - Question of Very Long Periodic Terms
- Semi-Analytical: Fukushima (1995)
  - Difference from Existing Fourier Series
  - Primitive Harmonic Analysis of Residuals
- Numerical: Harada & Fukushima (2003)
  - Non-Linear Harmonic Analysis

Contribution to L_C

- TE245: Unit=10^{-17} (Fukushima 1995)

Sun	987062583	Uranus	2250
Velocity	493530342	Neptune	1741
Jupiter	182856	Mars	240
Saturn	29647	Mercury	171
Moon	14191	Post-Newton	-11
Venus	2877	Asteroids	1

Determinations of L_C

- Unit: 10^{-17}, Additional Value to 1.4808268×10^{-8}

Approximation	Value	Error	Ephemeris	Reference
Newton	57	5	VSOP82	F 1995
	69.80	0.5	DE102	F 1995
	57.13	0.5	DE200	F 1995
	56.21	0.5	DE245	F 1995
Full Post-Newton	45.7	1.0	DE245	F 1995
	67.41	2.0	DE405	IF 1999
	67.4	0.6	DE405	HF 2003

4. Conclusion

- Latest Time Ephemeris: TE405
  - Core Part of TCB-TCG Transformation
  - Fourier Series: Harada & Fukushima (2003)
  - HF2002.f: 0.446 ns RMS, 502 Terms
- Uncertainty of L_C : 6×10^{-18}
  - Asteroid Contribution: $\sim 4.5 \times 10^{-18}$
  - Question of Very Long Periodic Terms

Appendix: Non-Linear Harmonic Analysis

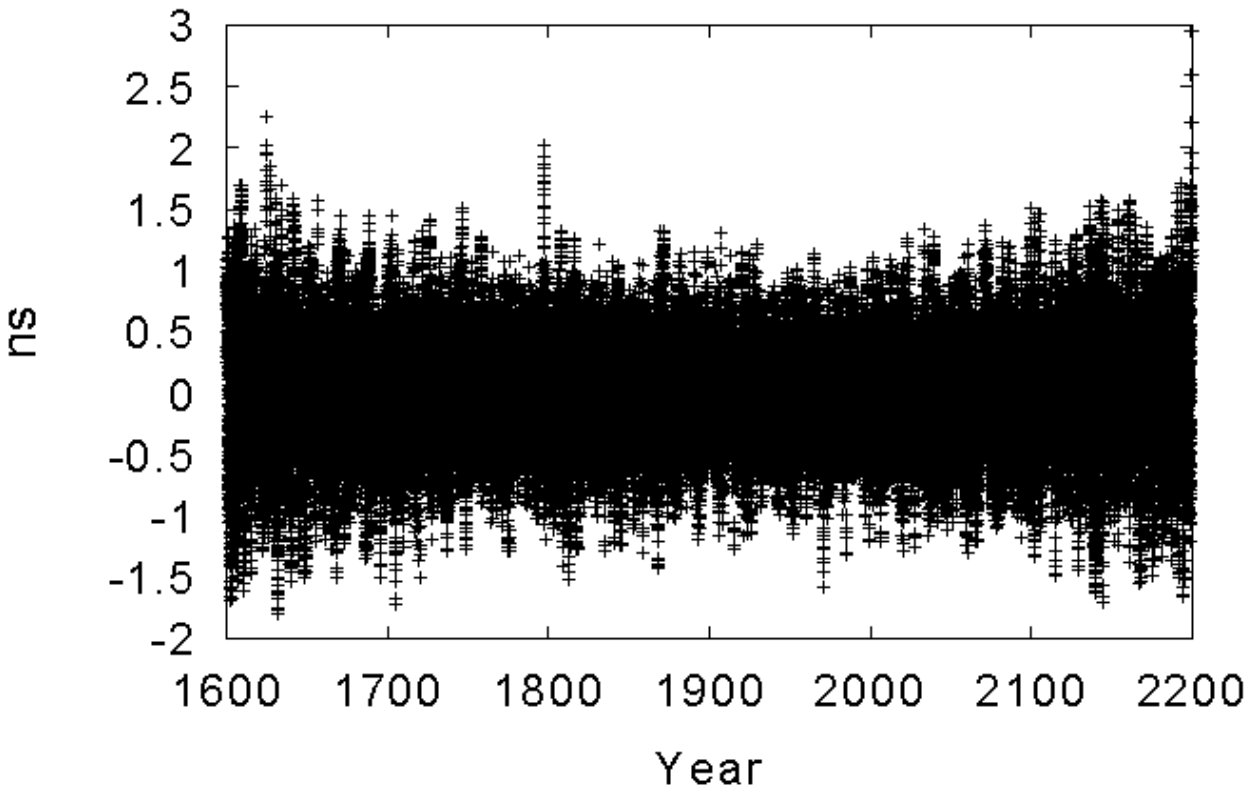
- Least Square: $\underset{c,s,\omega}{\text{Min}} \sum_k \left[f(c,s,\omega;t_k) - f_k \right]^2$
- Solving Frequencies as Unknowns

$$f(c,s,\omega;t) = c_0 + s_0 t + \sum_j \left(c_j \cos \omega_j t + s_j \sin \omega_j t \right) + \cdots$$

- Method
  - Detection: Extended Periodogram
  - Solution: Quasi-Newton Method (BFGS)
  - Economization: Step-by-Step Increase of Wave Numbers

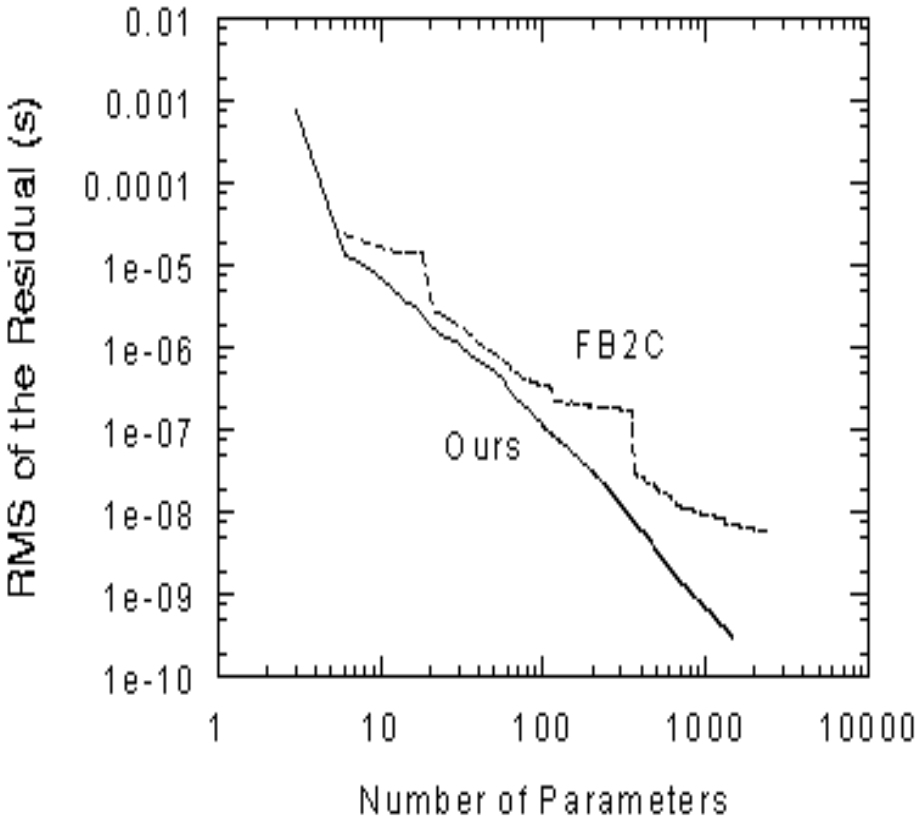
HF2002 from TE405

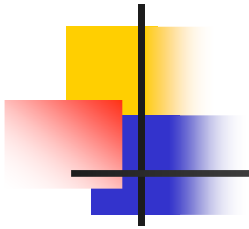
- 463 Fourier
- 36 Mixed Secular Terms
- Quadratic Polynomial
- RMS=0.446ns
- Max=3.0ns



Comparison in Total

Item	HF2002	FB2C
Trig #	463	478
T Trig #	36	205
T^2 Trig #	0	85
T^3 Trig #	0	29
T^4 Trig #	0	4
RMS (ns)	0.45	6.19





Main Terms

- HF2002 (DE405) vs FBL (VSOP82+ELP2000)

HF2002	FBL	Amplitude (us)	Origin
365.265195416(1)	365.2563	1631.9821(4)	E
365.221023366(4)	-	31.8266(4)	?
398.884018838(4)	398.8840	22.417927(2)	E-J
182.629825938(1)	182.6282	13.8908796(5)	2E
4333.21415(4)	4332.5893	4.780079(2)	J