Stability of pulsar rotational and orbital periods

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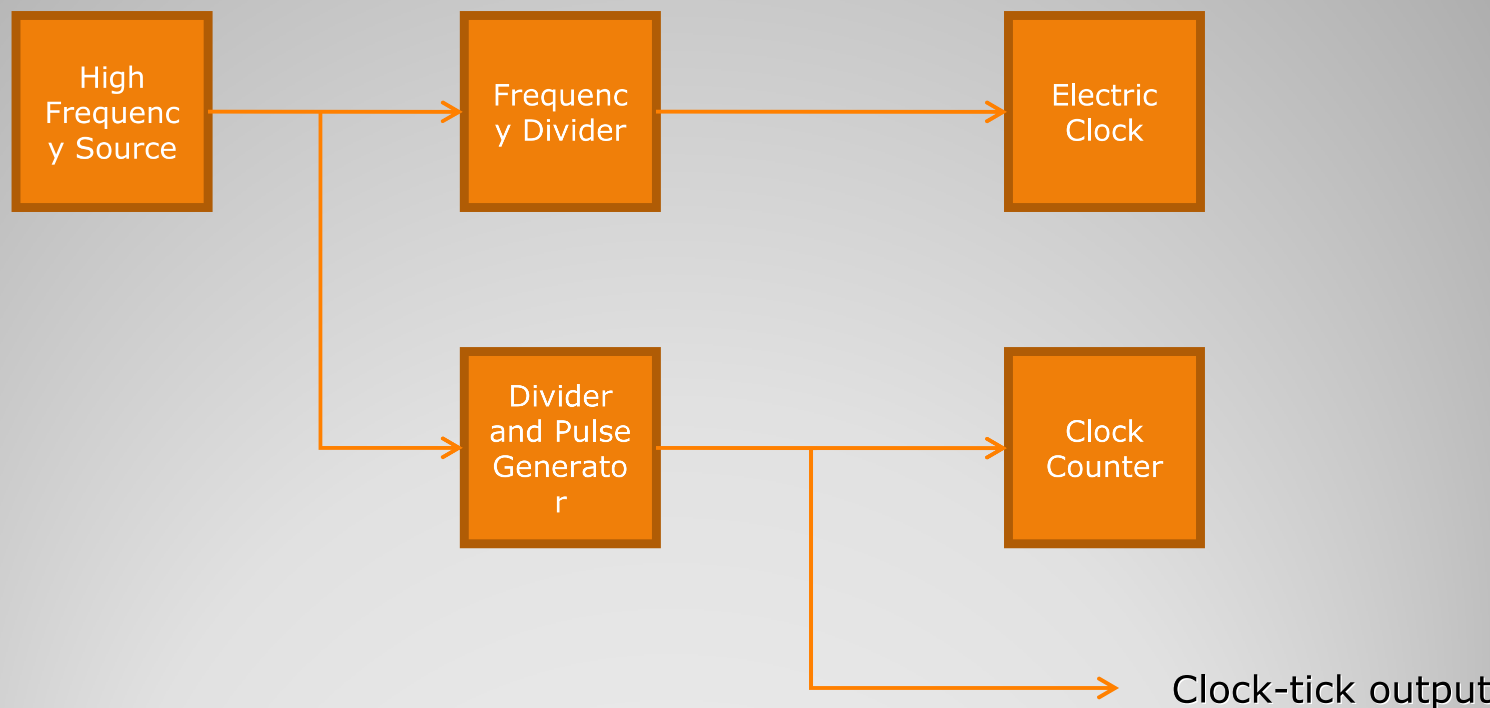
- Clock systems
- Clock Performance
- Clock Stability
- Pulsar Clock
- Timing Noise
- Timing Model
- Timing Residuals and Parameter’ Estimates
- Stability of Pulsar Clock

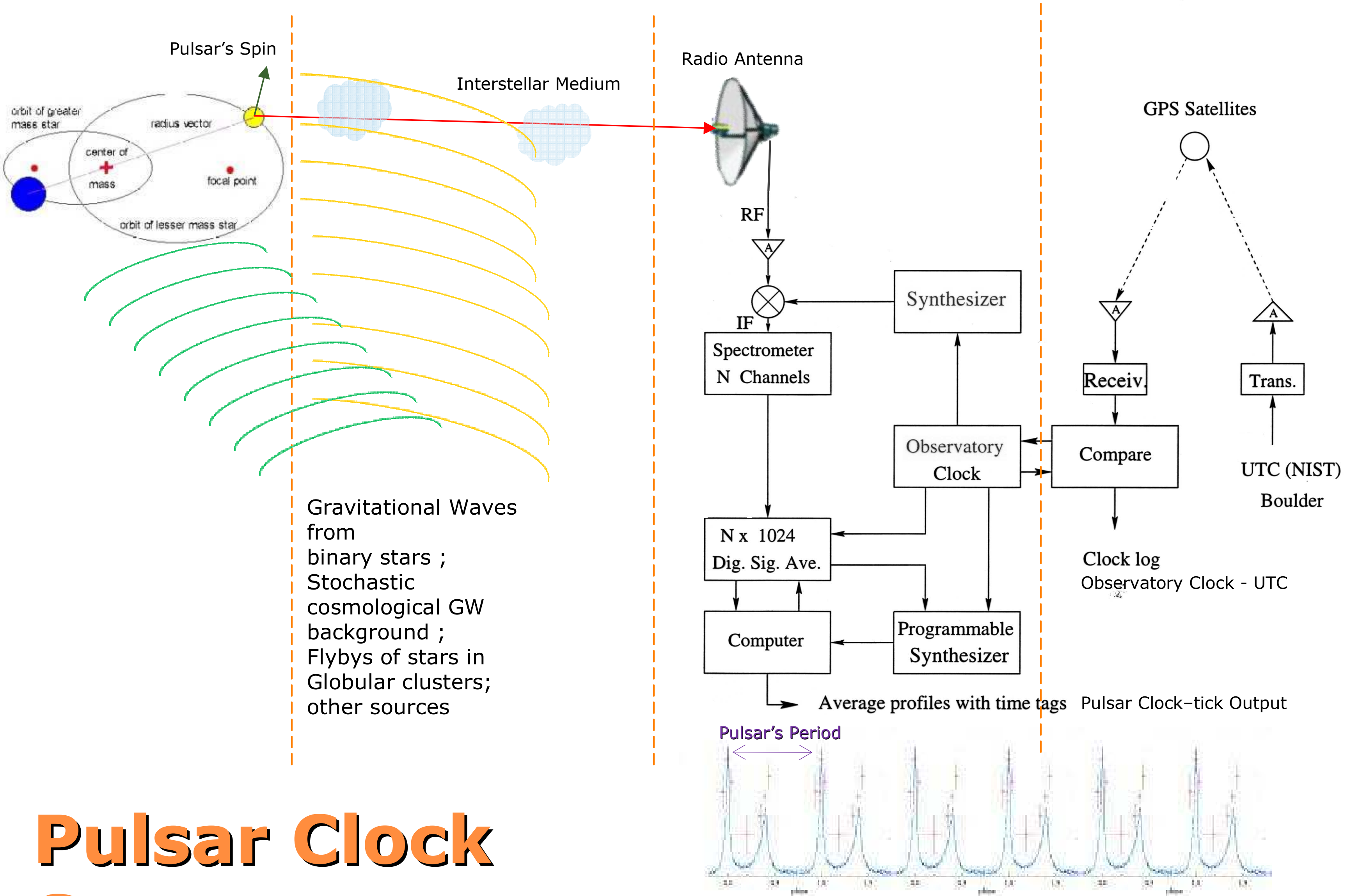
We are discussing:

What is Clock?

A **clock** is an instrument used for indicating and maintaining the time and passage thereof.



A Typical Man-made Clock System

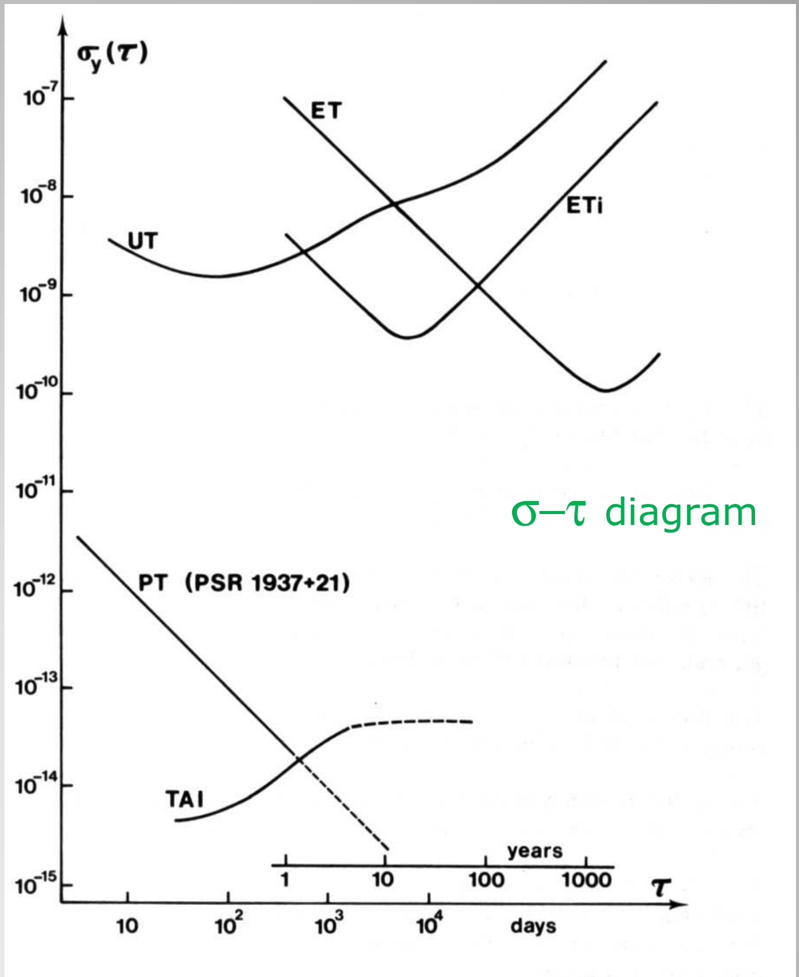


Pulsar Clock System

- **The quality of a clock is not dependent upon its error or its rate**
- **It is the rate variations from interval to interval which determine the quality**
- **If these variations are irregular then the clock's behavior can only be described statistically**
- **If the rate changes systematically, then we talk about a drift of this clock.**

Clock Performance

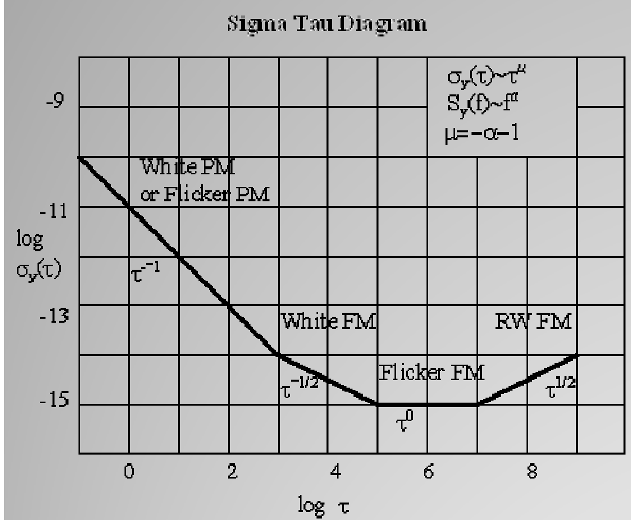
- Ideal Clock
- Laboratory Clock
  - Quartz
  - Cesium UTC
  - Others
- Astronomical Clock
  - Earth UT
  - Solar System ET
  - Pulsar PT
  - Binary pulsar BPT



σ–τ diagram

Clock Stability

Pulsar Clock Performance



Allan Variance versus time

If phase connection could be maintained (we could count the pulses unambiguously), then the precision in the measurement of the rotational period would increases linearly with time.

In this case, an error in the period of 10^{-12} s after one hour translates into an error of 10^{-17} s after ten years.

This uncertainty is further divided by the square root of the number of rotational phase measurements.

This means that some pulsar periods can be determined to 14 significant decimal places or more!

Pulsar's Slow Down

- However, rotational periods are not constant.
  For every single pulsar in the Galactic disk, the pulse period is observed to increase with time, $dP/dt > 0$
- This increase is normally linear with time
- Therefore, the uncertainty of dP/dt is proportional to $1/T^2$
- For most pulsars, this effect is relatively small. Measurements have to be made for more than one year to decouple the annual (nearly sinusoidal) effect of the Earth's motion on the TOAs from the quadratic effect of dP/dt on the TOAs.

- Most pulsars show significant departures from simple, uniformly slowing rotation. The two main departures are:
  - Glitches. This corresponds to star quakes, caused by a sudden change in the configuration of the magnetic field or a sudden unpinning of quantum vortices from pulsar's crust.
  - Intrinsic noise. The origin of this phenomenon is mostly related to the stochastic behavior of the magnetic field torque.

Glitches and Intrinsic Noise

- Having a pulsar clock in a binary system allows us to measure the range relative to the center of mass of the binary, normally with a precision of the order of 1 km or better!
- This makes pulsar timing thousands of times more precise for measuring orbital parameters of the binary than any other astronomical technique.
- Binary pulsars are in many cases “clean” in the sense that their orbital dynamics is driven primarily by equations of general relativity
- This is the fundamental reason why the binary pulsars are superior astrophysical and metrological tools.

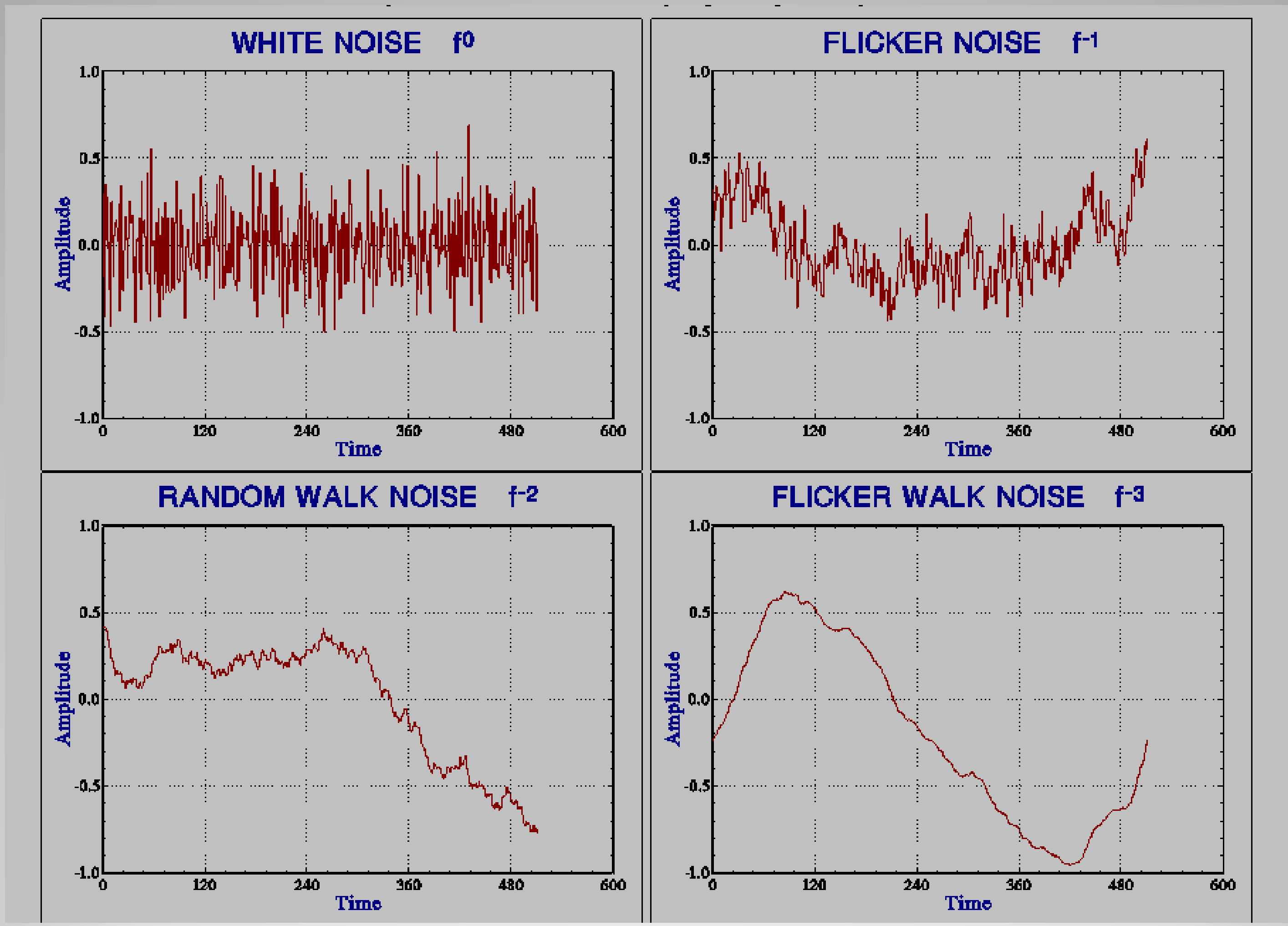
Binary Pulsar Timing

Timing Noise Model

- Stability and accuracy of pulsar's clock depend on the noise presents in
  - Environment and lab clock
  - Pulsar's interior and magnetic field
  - Signal propagation delay

- Timing noise is generally red with spectrum S(f) ~ $f^{-\alpha}$ $(\alpha > 0)$, so coherence times are generally long.

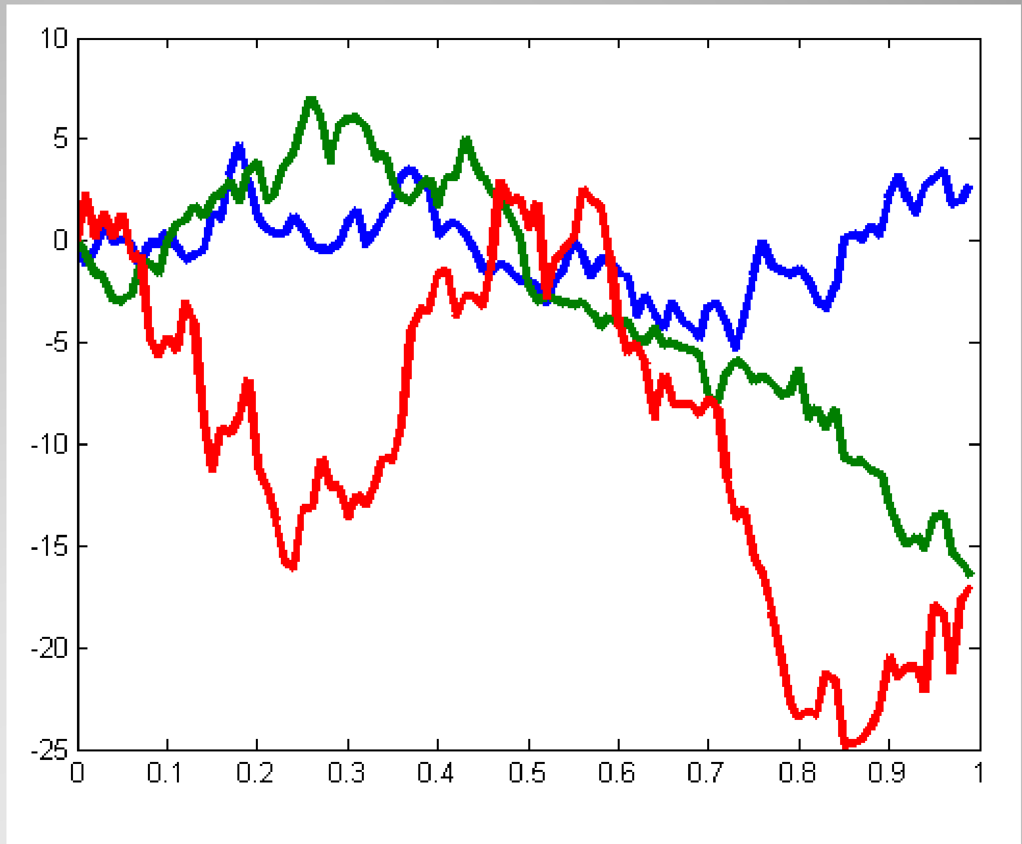
- Normally, the noise consists of stationary and non-stationary components. Both must be analyzed!

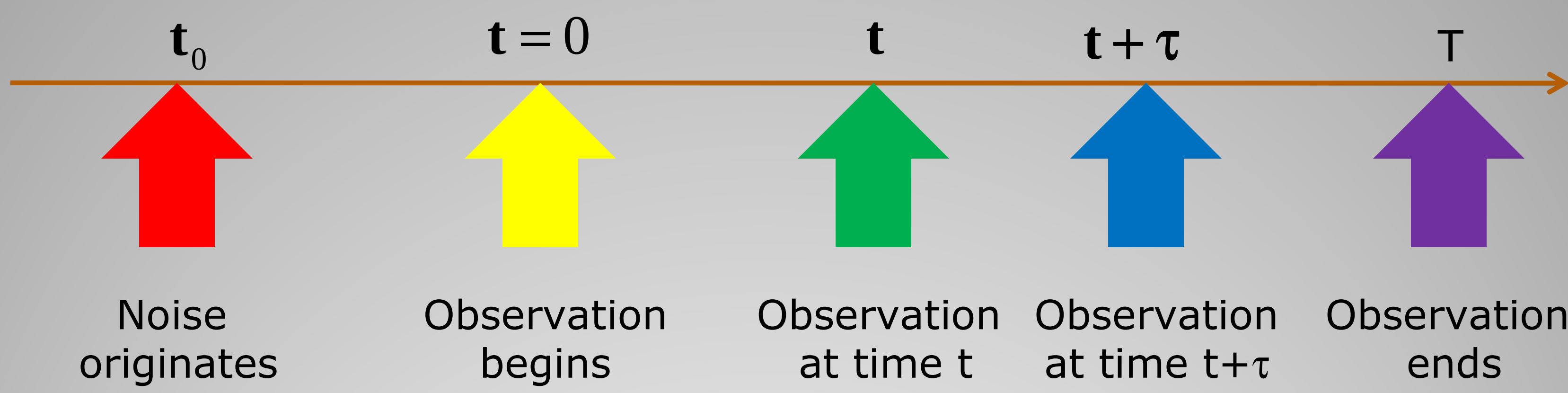


Power-Law Noise Spectra

Ergodicity Problem

Pulsar timing have access to only one realization of stochastic process. How to evaluate an ensemble average? Is it equivalent to the time average?





The noise is a stochastic process $\varepsilon(t)$ that is characterized by a set of multipole moments. The second multipole is the auto-covariance function: $\mathbf{R}_{t_0}(\mathbf{t}, \boldsymbol{\tau}) = \langle \boldsymbol{\varepsilon}(\mathbf{t})\, \boldsymbol{\varepsilon}(\mathbf{t} + \boldsymbol{\tau}) \rangle_{t_0}$

Standard assumption: $\mathbf{R}_{t_0}(\mathbf{t}, \boldsymbol{\tau}) = \mathbf{R}(\boldsymbol{\tau})$ - stationary, no memory

Timeline of noise and observations

Timing Model

(TEMPO; TEMPO2; TIMAPR2)

T – time of emission of a radio pulse

$$N(T) = N_0 + \nu T + \frac{1}{2}\dot{\nu} T^2 + \varepsilon_{\substack{\text{intrinsic}\\ \text{noise}}}(T)$$

- $N(T)$ – Pulse's number
- ν – Pulsar's rotational frequency
- $\dot{\nu}$ – Pulsar's rotational frequency derivative

$$t = D\left[T + \Delta_R + \Delta_\mu + \Delta_\pi + \Delta_E + \Delta_S + \Delta_B\right] + \frac{DM}{f^2} + \varepsilon_{\substack{\text{propagation}\\ \text{noise}}}(t)$$

- t – Coordinate time of arrival of the radio pulse
- Δ_R – Römer delay
- Δ_μ – Proper motion delay
- Δ_π – Parallax delay
- Δ_E – Einstein delay
- Δ_S – Shapiro delay
- Δ_B – Bending Delay
- $\frac{DM}{f^2}$ – Plasma delay

$$\tau = t + \Delta_R + \Delta_\mu + \Delta_\pi + \Delta_E + \Delta_S + \varepsilon_{\substack{\text{clock}\\ \text{noise}}}(\tau)$$

- τ – Atomic time of arrival of the radio pulse
- Δ_R – Römer delay
- Δ_μ – Proper motion delay
- Δ_π – Parallax delay
- Δ_E – Einstein delay
- Δ_S – Shapiro delay

$$ma^i = \underset{0}{F^i} + c^{-2}\underset{2}{F^i} + c^{-4}\underset{4}{F^i} + c^{-5}\underset{5}{F^i} + O(c^{-6})$$

Damour-Deruelle 1982

Kopeikin 1985,1986 PhD

Schäfer 1985

Fokker-Plank force
due to GW background?

Celestial Mechanics of Binary Pulsars

$$\underset{2}{F^i} = -\frac{1}{2}mv^2a^i - m(va)v^i + (Gmm'/R)(\frac{7}{2}a'^i - 3a^i) +$$
$$+ (Gmm'/R^2)\{N^i(-\frac{3}{2}v^2 - 2v'^2 + 4(vv') + \frac{3}{2}(Nv')^2 + \frac{1}{2}R(Na') +$$
$$+ Gm/R + Gm'/R) + 3(Nv)v^i - 3(Nv')v^i +$$
$$+ 3(Nv')v'^i - 4(Nv)v'^i\}$$

$$F_4^i = -\frac{3}{8}mv^4a^i - \frac{3}{2}mv^2(va)v^i +$$
$$+ Gmm'\{\frac{15}{8}R\ddot{a}'^i - \frac{1}{8}R(N\ddot{a}')N^i + 2(Nv)\dot{a}'^i - \frac{11}{2}(Nv')\dot{a}'^i -$$
$$- \frac{1}{2}(Nv')(N\dot{a}')N^i - 2(v\dot{a}')N^i + 2(v'\dot{a}')N^i + \frac{3}{2}(N\dot{a}')v^i -$$
$$- \frac{3}{2}(N\dot{a}')v'^i + \frac{3}{2}(Na')a^i - \frac{21}{4}(Na')a'^i +$$
$$+ \frac{15}{8}a'^2N^i - \frac{3}{8}(Na')^2N^i\} +$$

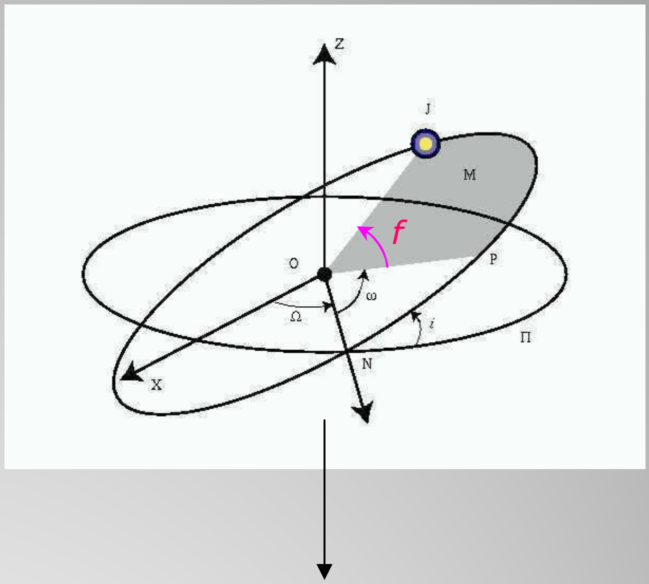
$$\underset{5}{F^i} = (Gm^2m'^2)/(m+m')^2\{-\frac{1}{30}R^2\overset{(5)}{R}_i + ((RR) - \frac{4}{3}(R\ddot{R}))\dddot{R}_i + \frac{11}{3}(\dot{R}\ddot{R})\ddot{R}_i +$$
$$+ (3(R\overset{(5)}{R}) + \frac{17}{2}(\dot{R}\ddddot{R}) + \frac{2}{3}(\ddot{R}\dddot{R}))\dot{R}_i + (\frac{3}{5}(R\overset{(6)}{R}) +$$
$$+ \frac{13}{6}(\dot{R}\overset{(5)}{R}) + \frac{61}{30}(\ddot{R}\ddddot{R}))R_i\} +$$
$$+ (Gm^3m'/(m+m')^2)\{\frac{19}{30}R^2\overset{(5)}{R}_i + 5(R\dot{R})\ddddot{R}_i + (7(\dot{R}\dot{R}) + \frac{22}{3}(R\ddot{R}))\dddot{R}_i +$$
$$+ 3(\dot{R}\dddot{R})\ddot{R}_i + (-\frac{2}{3}(\ddot{R}\ddot{R}) + \frac{1}{2}(\dot{R}\dddot{R}))\dot{R}_i +$$
$$+ (-\frac{61}{30}(\ddot{R}\dddot{R}) - \frac{17}{30}(\dot{R}\ddddot{R}))R_i\}$$

Orbital Parameterizations

Osculating elements:

$$r(t) = a(t)\left[1 - e(t)\cos E\right],$$
$$n(t - T_0) = E - e(t)\sin E - \Delta l(t),$$
$$\theta = f + \omega(t),$$
$$\tan\frac{f}{2} = \sqrt{\frac{1+e(t)}{1-e(t)}}\tan\frac{E}{2}$$

Gravitational radiation
+
Schklovski's effect
+
other kinematics



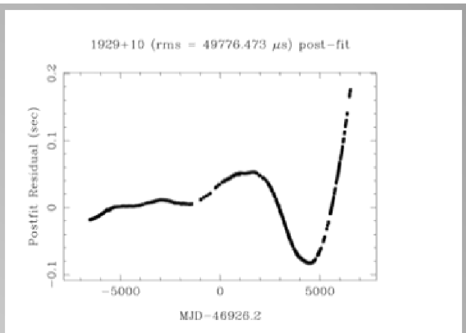
To observer

Other parameterizations:

$$r(t) = a_p\left[1 - e_r\cos U\right],$$
$$n(t - T_0) = U - e\sin U, \qquad n = \frac{2\pi}{P_b}\left[1 + \frac{\dot{P}_b}{2P_b}(t - T_0)\right],$$
$$\theta = \theta_0 + (1+k)A_{e_\theta}, \qquad k = \frac{\dot{\omega}}{n},$$
$$\tan\frac{A_{e_\theta}}{2} = \sqrt{\frac{1+e_\theta}{1-e_\theta}}\tan\frac{U}{2},$$
$$e_r = e(1+\delta_r),$$
$$e_\theta = e(1+\delta_\theta).$$

Klioner & Kopeikin,
ApJ, 427, 951 (1994)

- Blandford-Teukolsky
- Epstein-Haugan
- Brumberg
- Damour-Deruelle



$$r(t, p_1, p_2, \ldots, p_k) = \frac{N^{\text{obs}} - N(t, p_1, p_2, \ldots, p_k)}{\nu}$$

$$\underbrace{r(t, p_1, p_2, \ldots, p_k)}_{\text{True values of fitting parameters}} = \varepsilon(t)$$

-random noise with an ensemble
-averaged value = 0 (assumption)

$$\underbrace{r(t, p_1^*, p_2^*, \ldots, p_k^*)}_{\text{Estimated values of fitting parameters}} = \varepsilon(t) - \sum_{a=1}^{k} \beta_a \underbrace{\psi_a(t, p_1^*, p_2^*, \ldots, p_k^*)}_{\text{Fitting functions}} + O(\beta^2)$$

where $\beta_a = \delta p_a = p_a^* - p_a$, and $\psi_a(t, p_1^*, p_2^*, \ldots, p_k^*) = \left[\frac{\partial N(t, p_1, p_2, \ldots, p_k)}{\partial p_a}\right]_{p_a = p_a^*}$

Timing Residuals

(Kopeikin 1997, 1999; Kopeikin & Potapov 2004)

$$\beta_a = \sum_{a=1}^{k}\sum_{i=1}^{N} L_{ab}^{-1}\psi_b(t_i)\varepsilon(t_i) \Rightarrow \langle \beta_a \rangle = 0 \ \text{ if } \langle \varepsilon(t_i) \rangle = 0$$

an ensemble average

$$L_{ab} = \sum_{i=1}^{N} \psi_a(t_i)\psi_b(t_i)$$

MATRIX OF INFORMATION

$$M_{ab} = \langle \beta_a \beta_b \rangle$$

MATRIX OF PARAMETER'S CORRELATION

$$\sigma_{\beta_a}^2 = \langle \beta_a - \langle \beta_a \rangle \rangle^2 = \langle \beta_a^2 \rangle - \langle \beta_a \rangle^2$$

If the ensemble average of noise is not zero (for example, a polynomial drift) the parameter's mean values are biased

Parameter Estimates

Filter Function and Residuals

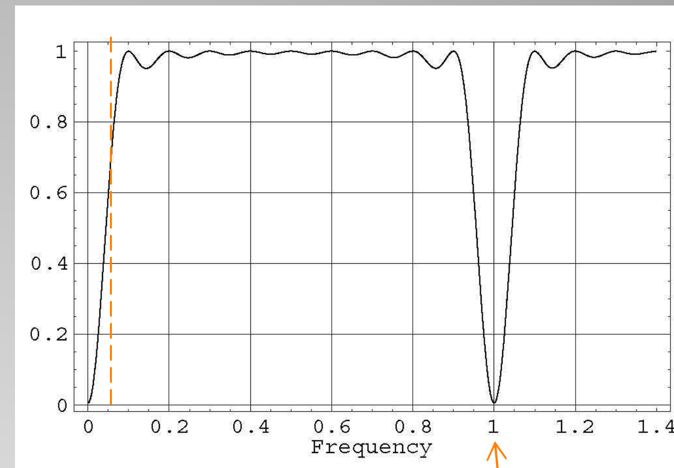
$$\langle r^2 \rangle = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} F(t_i,t_j)\,R(t_i,t_j)$$

$$\underbrace{R(t_i,t_j)}_{\text{Autocovariance function of noise}} = \underbrace{R^{+}(t_i,t_j)}_{non-stationary} + \underbrace{R^{-}(|t_i - t_j|)}_{stationary}$$

$$\underbrace{F(t_i,t_j)}_{\text{Filter function}} = \delta_{ij} - \sum_{a=1}^{k}\sum_{b=1}^{k} L^{-1}_{ab}\,\psi_a(t_i)\,\psi_b(t_j)$$

If $R^{+}(t_i,t_j) = \underbrace{g(t_i)}_{\text{any smooth function}} \underbrace{\left(a_0 + a_1 t_j + a_2 t_j^2 + a_3 t_j^3 + \ldots\right)}_{\text{polynomial of time}}$, then

$$\langle r^2 \rangle = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} F(t_i,t_j)\,\underbrace{R^{-}(|t_i - t_j|)}_{\text{stationary part of autocovariance function}}$$



orbital frequency

Spectral analysis -> next talk by Potapov & Kopeikin

$$\mathbf{y} = \frac{\delta \nu}{\nu} \Rightarrow \sigma_{\mathbf{y}} = \sqrt{\left\langle \mathbf{y}^2 \right\rangle}$$

Idealized Allan variance for rotational frequency

$$\mathbf{z} = \frac{\delta \ddot{\nu}}{\nu} \Rightarrow \sigma_{\mathbf{z}} = \sqrt{\left\langle \mathbf{z}^2 \right\rangle}$$

Modified Allan variance for rotational frequency (Matsakis et al 1997)

$$\mathbf{v} = \frac{\delta n_b}{n_b} \Rightarrow \sigma_{\mathbf{v}} = \sqrt{\left\langle \mathbf{v}^2 \right\rangle}$$

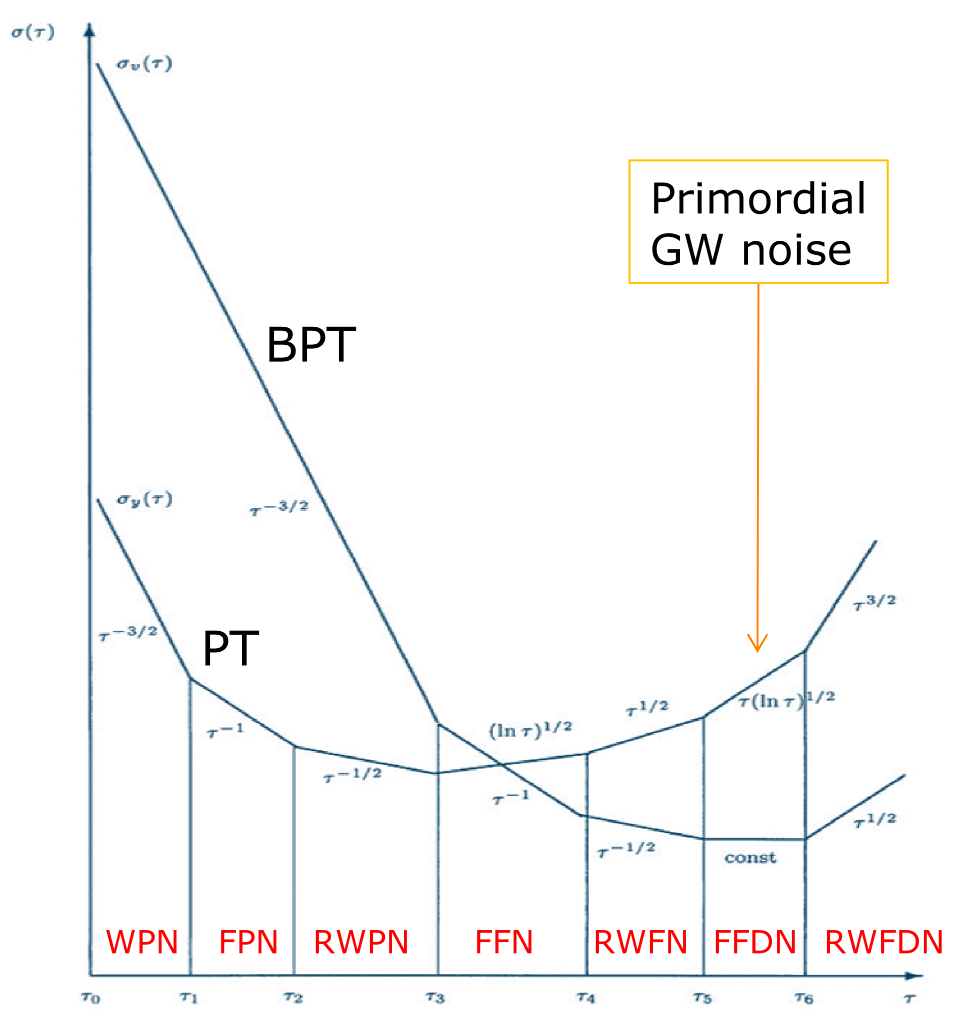
Idealized Allan variance for orbital frequency

Statistics for evaluation of stability of PT and BPT

$S(f)$	$\sigma_y^2(\tau)$	$\sigma_z^2(\tau)$	$\sigma_v^2(\tau)$
h_0	$\frac{3675}{16}\Delta t\, h_0\, \tau^{3}$	$\frac{2835}{16}\Delta t\, h_0\, \tau^{3}$	$\frac{75}{2\pi^2}\frac{P_b^2}{x^2}\Delta t\, h_0\, \tau^{3}$
h_1/f	$\frac{4851}{64}h_1\tau^{2}$	$\frac{2499}{64}h_1\tau^{2}$	$\frac{75}{4\pi^2}\frac{P_b^3}{x^2}h_1\tau^{3}$
h_2/f^2	$\frac{1575}{416}h_2\tau^{1}$	$\frac{441}{416}h_2\tau^{1}$	$\frac{1275}{88\pi^4}\frac{P_b^4}{x^2}\left(\sin^2\sigma+\frac{11}{17}\cos^2\sigma\right)h_2\tau^{3}$
h_3/f^3	$(C_3+\ln\tau)h_3$	$\frac{819}{2560}h_3$	$\frac{15}{32\pi^4}\frac{P_b^4}{x^2}h_3\tau^{2}$
h_4/f^4	$\left(C_4-\frac{525}{18304}\right)h_4\tau$	$\frac{203}{18304}h_4\tau$	$\frac{25}{2288\pi^4}\frac{P_b^4}{x^2}h_4\tau^{1}$
h_5/f^5	$\frac{1}{4}(C_5+\ln\tau)h_5\tau^2$	$\frac{93}{20480}h_5\tau^2$	$\frac{5}{1792\pi^4}\frac{P_b^4}{x^2}h_5$
h_6/f^6	$\left(C_6-\frac{581}{5601024}\right)h_6\tau^3$	$\frac{21}{77792}h_6\tau^3$	$\frac{5}{64064\pi^4}\frac{P_b^4}{x^2}h_6\tau$

Comparison of various statistics

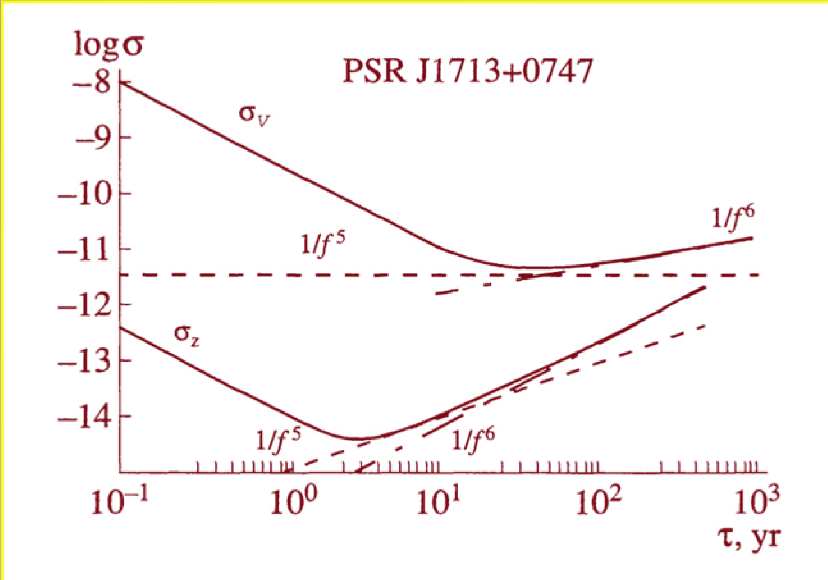
(Ilyasov, Kopeikin, Rodin, Astron. Letters 1997)

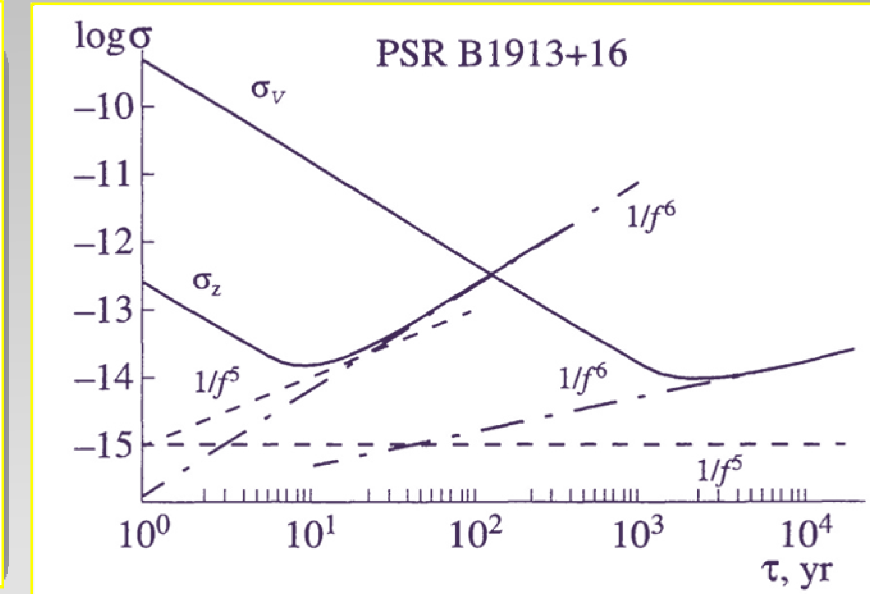


WPN	$S(f) = h_0 = const.$
FPN	$S(f) = h_0 f^{-1}$
RWPN	$S(f) = h_0 f^{-2}$
FFN	$S(f) = h_0 f^{-3}$
RWFN	$S(f) = h_0 f^{-4}$
FFDN	$S(f) = h_0 f^{-5}$
RWFDN	$S(f) = h_0 f^{-6}$

Sigma-Tau Diagram for PT and BPT

Ilyasov, Kopeikin, Rodin 1997





$$\sigma_{\mathbf{v}} = 2.4\times10^{-20}\sqrt{\Omega_g}\,P_b^2 x^{-1} H$$

Floor for GW bacground

Stability of PT versus BPT

- Pulsar timing red noise is highly desirable to include to timing models;
- The most optimal red-noise parameter estimators should be worked out and studied;
- Sigma-z and sigma-v statistics are informative metrological and astrophysical instruments (especially in study of GW);
- Millisecond and binary pulsars are excellent astronomical time-keepers on large time intervals (like a decade and longer);
- Future prospects – pulsar timing array with an ensemble of pulsars uniformly distributed over the sky – look very promising (JD6 talks by Manchester, Rodin, and others)

Conclusions