Implementation of the IAU 2000 definition of UT1 in astronomy

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Outline

Review of the:

- concepts, definitions and nomenclature associated with the IAU 2000 definition of the ERA
- conventional numerical values associated with the IAU 2000 definition of UT1
- IAU 2006 expressions for the position of the celestial intermediate origin and the equation of the origins
- CIO and equinox-based procedures for transforming between celestial and terrestrial coordinates (SOFA and IERS implementation)

IAU 2000/2006 and IUGG 2003/2007 resolutions regarding Earth's rotation

1) Definition of the celestial and terrestrial coordinate systems in the framework of GR

IAU 2000 Resolution B1.3
Definition of BCRS and GCRS

IUGG 2007 Resolution 2
Definition of GTRS and ITRS

2) Adoption of precession-nutation model with a microarcsecond precision

IAU 2000 Resolution B1.6
IAU 2000 Precession-Nutation Model

IAU 2006 Resolution B1
Adoption of the P03 Precession and definition of the ecliptic

3) Realization of the pole with microarcsecond accuracy

IAU 2000 Resolution B1.7
Definition of Celestial Intermediate Pole

4) Definition of the ERA and UT1 for an accurate estimation of the EOPs

IAU 2000 Resolution B1.8
Definition and use of CEO and TEO

IAU 2006 Resolution B2, Rec 1
Harmonizing « intermediate » to the pole and the origin
→ Celestial/Terrestrial intermediate origin: CIO, TIO

The Earth Rotation Angle (ERA) and the IAU 2000 conventional definition of UT1 (IAU 2000 Resolution B1.8)

- The *Earth Rotation Angle (ERA)* is the angle between the CIO and the TIO
- The conventional linear relationship that defines UT1 from ERA is :

$$\text{ERA}(\mathbf{UT1}) = 2\pi\ [\,0.7790572732640 + 1.00273781191135448 \times (\text{Julian UT1date} - 2451545.0)\,]$$

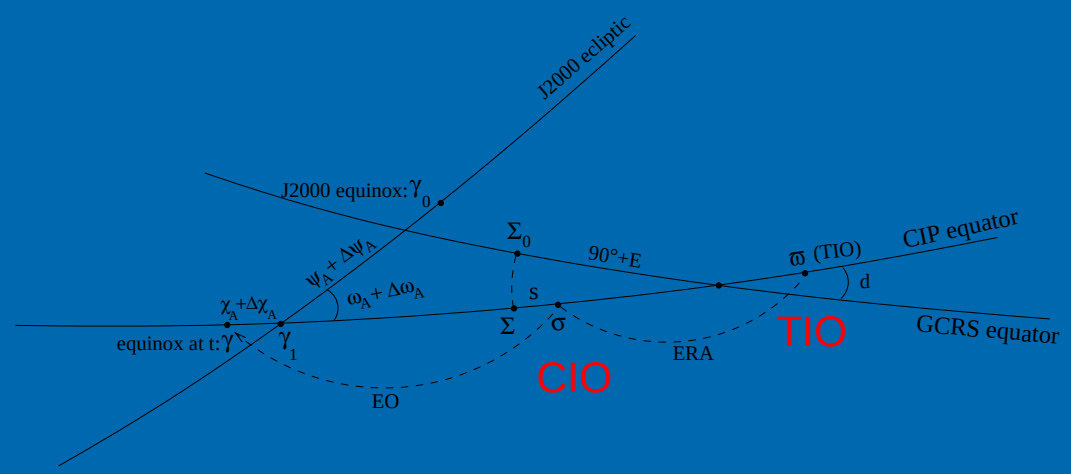
- This expression from (Capitaine, Guinot, McCarthy, 2000) was derived from (i) the $GMST_{1982}$(UT1) expression based on the IAU 1976 precession and (ii) the expression for the GCRS position of the CIO

- 0.7790572732640 and 1.00273781191135448 rev/day are defining constants (conventional values) which have replaced the factor between GMST and UT1

(The ERA/UT1 relationship involves not only the Earth angular velocity but also what UT1 is intended to represent, i.e. the hour angle of the « fictitious mean » Sun)

concepts

The Earth rotation angle



ERA: *Earth rotation angle* = hour angle of the CIO from the TIO meridian

GST: *Greenwich sidereal time*= hour angle of the equinox from the TIO meridian

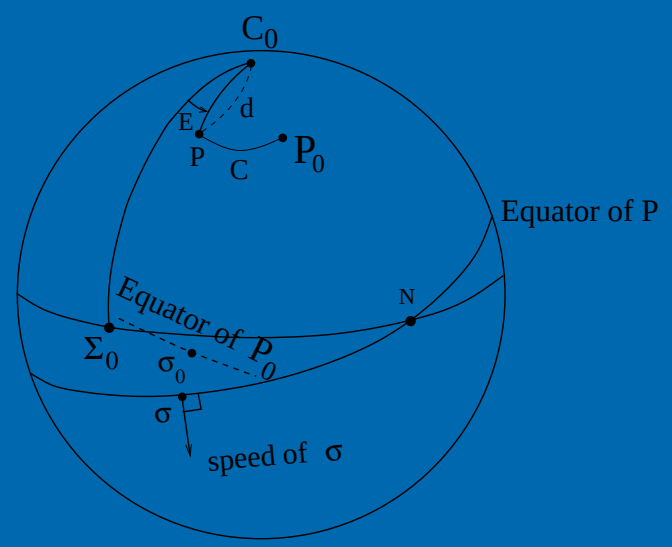
GST = ERA – EO

EO: *equation of the origins:* distance from the CIO to the equinox

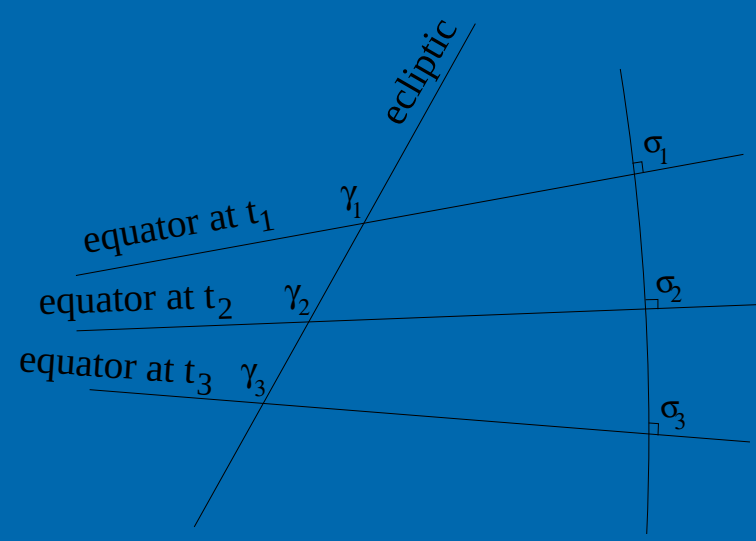
s: CIO locator

Concepts associated with the IAU 2000 resolutions

concept of the NRO



equinox versus the NRO



kinematical definition of the NRO σ ,

dependent on the CIP (i.e. equator) motion

CIO: *celestial intermediate origin* : NRO in the GCRS
TIO: *terrestrial intermediate origin* : NRO in the ITRS

geometrical definition of the equinox, γ ,

dependent on both the equator and ecliptic motions

Complexities:

- *definition of the ecliptic in the GCRS*
- *definition of the mean equinox*

ERA based expression for Greenwich sidereal time

GST : ERA (UT1)

+ accumulated precession in RA (in TT) + $s_{\text{cross x nutation}}$
(*GMST)*

+ accumulated nutation in RA (in TT) + precession x nutation (in TT)
(*kinematical equation of equinoxes)*

(GST= GMST (UT1, TT) + "equation of the equinoxes")

= ERA (UT1) – EO

$$\mathrm{EO} = -\,dT_0 - \int_{t_0}^{t} (\widehat{\psi_A + \Delta\psi_1})^{\cdot} \cos(\omega_A + \Delta\epsilon_1)dt + \chi_A - \Delta\psi\cos\epsilon_A + \Delta\psi_1\cos\omega_A$$

EO can also be derived from the expression for s (from Wallace & Capitaine 2006):

$$\mathrm{EO} = s - \tan^{-1}\frac{\mathbf{y}\cdot\mathbf{\Sigma}}{\mathbf{\Upsilon}\cdot\mathbf{\Sigma}}.$$

nomenclature

IAU 2000/2006 Nomenclature

The Nomenclature in Fundamental Astronomy IAU WG provided an IAU 2006 Glossary including a set of detailed definitions (compliant with GR) that best explain all the terms required for implementing the IAU 2000 and 2006 resolutions

http://syrte.obspm.fr/iauWGnfa/NFA_Glossary.html

- Barycentric Celestial Reference System (BCRS)
- Barycentric Dynamical Time (TDB)
- **Celestial Intermediate Origin (CIO)**
- **Celestial Intermediate Reference System (CIRS)**
- **CIO locator**
- CIO right ascension and declination
- epoch
- **equation of the origins (EO)**
- equinox right ascension
- **Geocentric Celestial Reference System (GCRS)**
- **Geocentric Terrestrial Reference System (GTRS)**
- ICRS place
- intermediate equator
- intermediate place
- intermediate right ascension and declination
- International Celestial Reference Frame (ICRF)
- International Celestial Reference System (ICRS)
- International Terrestrial Reference Frame (ITRF)
- International Terrestrial Reference System (ITRS)
- Julian century
- Julian date
- Julian year
- right ascension
- Teph
- Terrestrial Intermediate Origin (TIO)
- Terrestrial Intermediate Reference System (TIRS)
- Terrestrial Time (TT)
- TIO locator

The IAU 2006 NFA Glossary
a few examples of newly proposed terms

http://syrte.obspm.fr/iauWGnfa/NFA_Glossary.html

non-rotating origin: in the context of the GCRS or the ITRS, the point on the intermediate equator such that its instantaneous motion with respect to the system (GCRS or ITRS as appropriate) has no component along the intermediate equator (i.e. its instantaneous motion is perpendicular to the intermediate equator). It is called the CIO and TIO in the GCRS and ITRS, respectively.

Earth Rotation Angle (ERA): angle measured along the intermediate equator of the Celestial Intermediate Pole (CIP) between the Terrestrial Intermediate Origin (TIO) and the Celestial Intermediate Origin (CIO), positively in the retrograde direction. It is related to UT1 by a conventionally adopted expression in which ERA is a linear function of UT1 (see IAU 2000 Resolution B1.8). Its time derivative is the Earth's angular velocity. Previously, it has been referred to as the stellar angle.

Celestial Intermediate Origin (CIO): origin for right ascension on the intermediate equator in the celestial intermediate reference system. It is the non-rotating origin in the GCRS that is recommended by the IAU 2000 Resolution B 1.8, where it was designated the Celestial Ephemeris Origin. The CIO was originally set close to the GCRS meridian and throughout 1900-2100 stays within 0.1 arcseconds of this alignment.

Terrestrial Intermediate Origin (TIO): origin of longitude in the Intermediate Terrestrial Reference System. It is the non-rotating origin in the ITRS that is recommended by the IAU 2000 Resolution B1.8, where it was designated Terrestrial Ephemeris Origin. The name Terrestrial Intermediate Origin was adopted by IAU 2006 Resolution B2. The TIO was originally set at the ITRF origin of longitude and throughout 1900-2100 stays within 0.1 mas of the ITRF prime meridian.

The IAU 2006 NFA Glossary
a few examples of newly proposed terms

http://syrte.obspm.fr/iauWGnfa/NFA_Glossary.html

CIO locator (denoted s) : the difference between the GCRS right ascension and the intermediate right ascension of the intersection of the GCRS and intermediate equators. The CIO was originally set close to the mean equinox at J2000.0. As a consequence of precession-nutation the CIO moves according to the kinematical property of the non-rotating origin. The CIO is currently located by using the quantity *s*.

TIO locator (denoted s'): the difference between the ITRS longitude and the instantaneous longitude of the intersection of the ITRS and intermediate equators. The TIO was originally set at the ITRF origin of longitude. As a consequence of polar motion the TIO moves according to the kinematical property of the non-rotating origin. The TIO is currently located using the quantity s′, whose rate is of the order of 50 mas/cy which is due to the current polar motion.

- **equation of the origins:** distance between the CIO and the equinox along the intermediate equator; it is the CIO right ascension of the equinox; alternatively the difference between the Earth rotation angle and Greenwich apparent sidereal time (ERA – GAST).

- **Greenwich mean sidereal time (GMST):** Greenwich hour angle of the mean equinox defined by a conventional relationship to Earth rotation angle or equivalently to UT1.

- **Greenwich sidereal time (GST):** Greenwich apparent sidereal time (GAST), the hour angle of the true equinox from the Terrestrial Intermediate Origin (TIO) meridian (Greenwich or International meridian).

expressions

IAU 2000/2006 expressions for the fundamental precession parameters

(Capitaine, Wallace, Chapront, 2003)

	Source		mas t^0	mas/cy t	mas/cy^2 t^2	mas/cy^3 t^3	mas/cy^4 t^4	mas/cy^5 t^5
Ecliptic	IAU 2000	P_A		4197.6	194.47	−0.179		
	P03			4199.094	193.9873	−0.22466	−0.000912	0.0000120
	IAU	Q_A		−46815.0	50.59	0.344		
	P03			−46811.015	51.0283	0.52413	−0.000646	−0.0000172
Equator	IAU 2000	ψ_A		5038478.750	−1072.59	−1.147		
	P03			5038481.507	−1079.0069	−1.14045	0.132851	−0.0000951
	IAU 2000	ω_A	84381448.0	−25.240	51.27	−7.726		
	P03		84381406.0	−25.754	51.2623	−7.72503	−0.000467	0.0003337

	Source	t^0	t	t^2	t^3	t^4	t^5
CIO based precession quantities	X	− 16.617	2004191.898	− 429.7829	−198.61834	0.007578	0.0059285
	Y	− 6.951	−25.896	− 22407.2747	1.90059	1.112526	0.0001358
	$s + XY/2$	0.094	3.80865	− 0.12268	− 72.57411	0.02798	0.01562

GMST(IAU2000)	**14.506**	**ERA+4612157.399**	**1396.6784**	**– 0. 09344**	**0.0188**	
GMST(P03)	**14.506**	**ERA+4612156.534**	**1391.5817**	**– 0.00044**	**0.029956**	**–0.0000368**

expression dependent on the precession-nutation model

(P03 ≡ IAU 2006)

IAU 2000 expression for the CIO locator

(Capitaine, Wallace, McCarthy, 2003; IERS Conventions 2003, Ch 5)

and IAU 2006 update

(Capitaine, Wallace, Chapront 2003; IERS Conventions, Ch 5, revised 2009)

$$s(t) = -\int_{t_0}^{t} \frac{X(t)\dot{Y}(t) - Y(t)\dot{X}(t)}{1 + Z(t)} dt - (\sigma_0 N_0 - \Sigma_0 N_0)$$

Table 5.2d Development of $s(t)$ compatible with the IAU 2006/2000A precession-nutation model: all terms exceeding 0.5 μas during the interval 1975–2025 (unit μas). The expressions for the fundamental arguments appearing in column 1 are given by Eq. (43).

$$\begin{aligned} s(t) = & -XY/2 + 94 + 3808.65 - 122.68t^2 - 72574.11t^3 + \textstyle\sum_k C_k \sin \alpha_k \\ & +1.73t \sin\Omega + 3.57t \cos 2\Omega + 743.52t^2 \sin\Omega + 56.91t^2 \sin(2F - 2D + 2\Omega) \\ & +9.84t^2 \sin(2F + 2\Omega) - 8.85t^2 \sin 2\Omega \end{aligned}$$

Argument α_k	Amplitude C_k
Ω	-2640.73
2Ω	-63.53
$2F - 2D + 3\Omega$	-11.75
$2F - 2D + \Omega$	-11.21
$2F - 2D + 2\Omega$	$+4.57$
$2F + 3\Omega$	-2.02
$2F + \Omega$	-1.98
3Ω	$+1.72$
$l' + \Omega$	$+1.41$
$l' - \Omega$	$+1.26$
$l + \Omega$	$+0.63$
$l - \Omega$	$+0.63$

largest change w.r.t. the IAU 2000 expression

< 3 µas

unit : microarcsecond

IAU 2000 expression for GMST

(Capitaine, Wallace, McCarthy, 2003; IERS Conventions 2003, Ch 5)

Greenwich sidereal time: GST= GMST (UT1, TT) + "equation of the equinoxes"
= ERA (UT1) – "equation of the origins" (i.e. accumulated precession/nutation in RA)

- using IAU 2000 A precession-nutation model

- such that there is equivalence between CIO-based and equinox-based z-rotation : *link between s(t) and the equation of the equinoxes* EE_{2000}

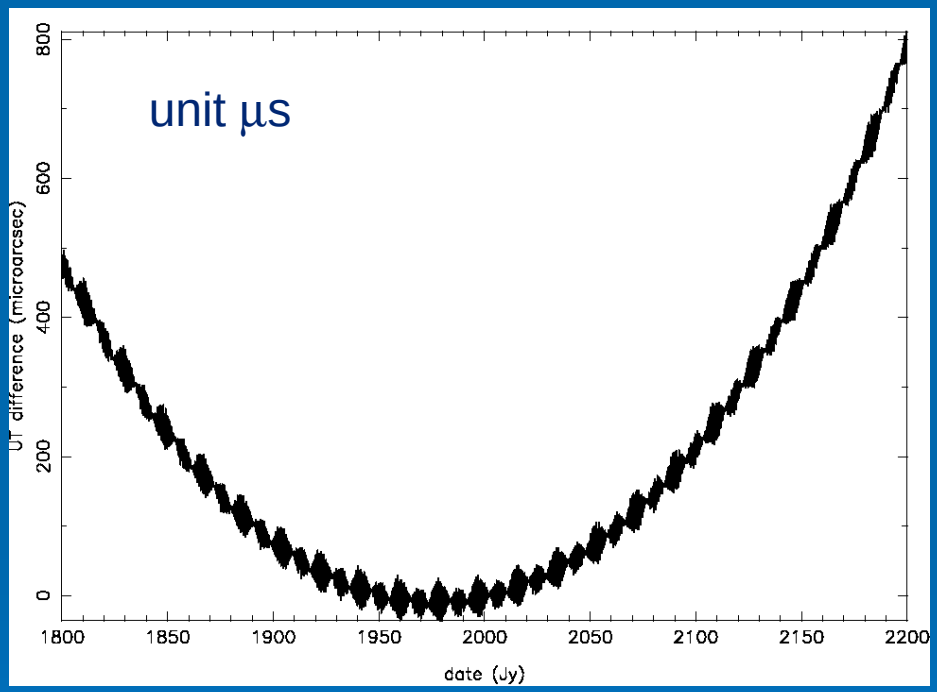
- such that there is continuity in UT1 on 01/01/2003 0h TT with the previous relationship $GMST_{1982}$(UT1) and current UT1 estimate

$$GMST_{2000} = 0''.014506 + \theta + 4612''.15739966\, t + 1''.39667842\, t^2 - 0''.00009344\, t^3 + 0''.0000188\, t^4$$

$$EE_{2000} = + \Delta\psi \cos \varepsilon_A - \Sigma_k C_k \sin \alpha_k - 0.87\ \mu as\ t \sin \Omega$$

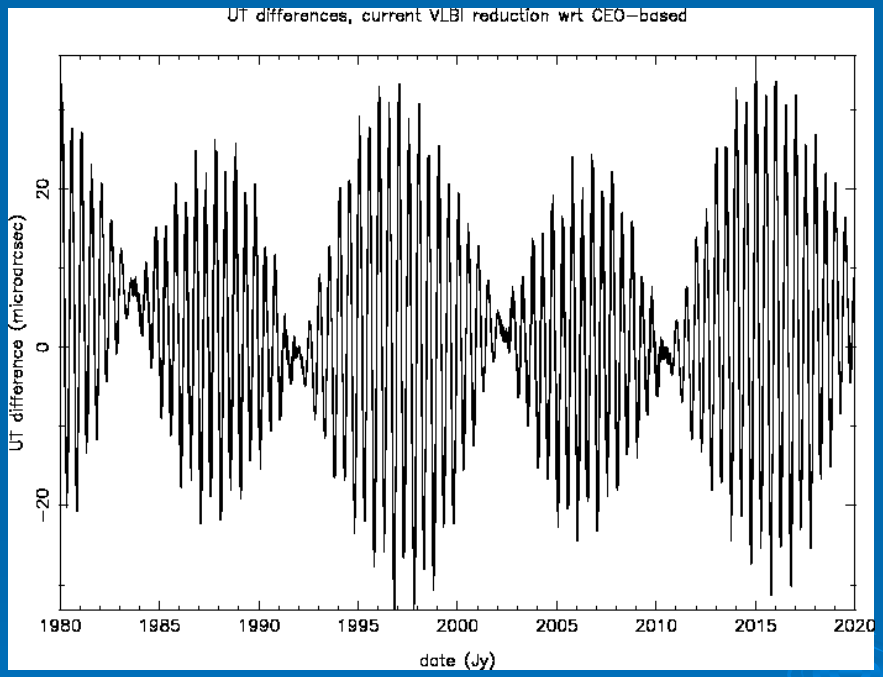
Differences in UT1

over 2 centuries around J2000



post-2003 minus pre-2003 procedures
(quadratic term: due to the precession rate in obliquity; secular term: due to the variation in TT – UT1)

period 1980-2020



equinox based VLBI reductions w.r.t CIO-based

(due to the incompleteness of the IAU 1994 expression for the equation of equinoxes)

IAU 2006 expression for the equation of the origins

(Capitaine, Wallace, Chapront, 2003; IERS Conventions, Ch 5, revised 2009)

$$\mathrm{EO} = -\,dT_0 - \int_{t_0}^{t} (\widehat{\psi_A + \Delta\psi_1})^{\cdot} \cos(\omega_A + \Delta\epsilon_1)dt + \chi_A - \Delta\psi\cos\epsilon_A + \Delta\psi_1\cos\omega_A$$

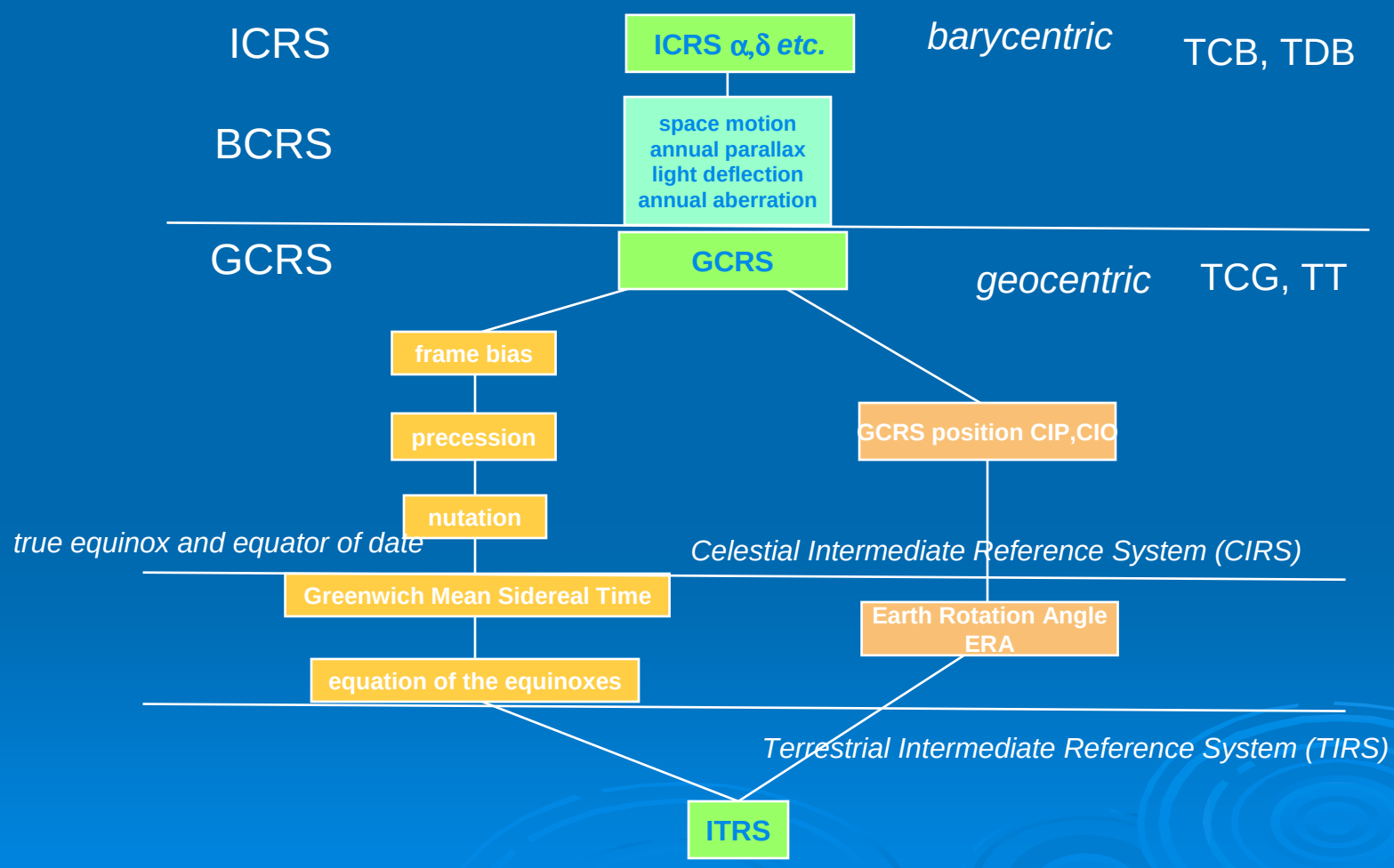
Table 5.2e Development of EO compatible with IAU 2006/2000A precession-nutation model: all terms exceeding 0.5 μas during the interval 1975–2025 (unit μas).

$$\mathrm{EO} = -0.014506'' - 4612.156534''t - 1.3915817''t^2 + 0.00000044''t^3 - \Delta\psi\cos\epsilon_A - \sum_k C'_k \sin\alpha_k$$

Argument α_k	Amplitude C'_k
Ω	+2640.96
2Ω	+63.52
$2F - 2D + 3\Omega$	+11.75
$2F - 2D + \Omega$	+11.21
$2F - 2D + 2\Omega$	−4.55
$2F + 3\Omega$	+2.02
$2F + \Omega$	+1.98
3Ω	−1.72
$l' + \Omega$	−1.41
$l' - \Omega$	−1.26
$l + \Omega$	−0.63
$l - \Omega$	−0.63

transformation

Transformation from ICRS to observed places of stars



Forming the celestial to terrestrial matrix transformation based on the IAU 2006 precession

$$ {}_{GCRS}\mathbf{R}_{TIRS} = \mathrm{R}_3(\mathrm{ERA}) \times \begin{pmatrix} \mathbf{v}_{CIO} \\ \mathbf{v}_{CIP} \times \mathbf{v}_{CIO} \\ \mathbf{v}_{CIP} \end{pmatrix} = \mathrm{R}_3(\mathrm{GST}) \times \begin{pmatrix} \mathbf{v}_{equinox} \\ \mathbf{v}_{CIP} \times \mathbf{v}_{equinox} \\ \mathbf{v}_{CIP} \end{pmatrix} $$

$$ = \mathrm{ERA} - \mathrm{EO} $$

Capitaine N., Wallace P.T., (2006, A&A 450, 855-872)
"High precision methods for locating the celestial intermediate pole and origin"

Wallace P.T., Capitaine N., (2006, A&A 459, 981-985)
"Precession-nutation procedures consistent with IAU 2006 resolutions"

Software routines to implement the IAU 2000A/2006 transformations

(Wallace 2008; IERS Conventions, Ch 5, revised 2009)

SOFA routines that support the IAU 2006/2000A models include the following (among others):

BP06	celestial-to-true matrix (*etc.*), given $\Delta\psi$ and $\Delta\epsilon$
C2I06A	celestial-to-intermediate matrix, IAU 2006/2000A
C2IXYS	celestial-to-intermediate matrix, given X, Y and s
C2T06A	celestial-to-terrestrial matrix, IAU 2006/2000A
C2TCIO	CIO based celestial-to-terrestrial matrix
EORS	equation of the origins, given celestial-to-true matrix and s
ERA00	Earth Rotation Angle
GMST06	Greenwich Mean Sidereal Time, IAU 2006
GST06A	Greenwich (apparent) Sidereal Time, IAU 2006/2000A
NUM06A	nutation matrix, IAU 2006/2000A
NUT06A	nutation components, IAU 2006/2000A
PNM06A	celestial-to-true matrix, IAU 2006/2000A
POM00	polar motion matrix
SP00	the quantity s'
XY06	X, Y from semi-analytical series, IAU 2006/2000A
XYS06A	X, Y, s, IAU 2006/2000A

angle/time

Complexities of UT1 being considered both as an angle and a time scale

- **Universal Time (UT):** a measure of time that conforms, within a close approximation, to the mean diurnal motion of the Sun and serves as the basis of all civil timekeeping. The term "UT" is used to designate a member of the family of Universal Time scales (e.g. UTC, UT1).

  Universal Time (UT1): angle of the Earth's rotation about the CIP axis defined by its conventional linear relation to the Earth rotation angle (ERA). It is related to Greenwich apparent sidereal time through the ERA (see equation of the origins). It is determined by observations (currently from VLBI observations of the diurnal motions of distant radio sources). UT1 can be regarded as a time determined by the rotation of the Earth. It can be obtained from the uniform time scale UTC by using the quantity $UT1 - UTC$, which is provided by the IERS.

  ***UT1 – UTC*:** difference between the UT1 parameter derived from observation and the uniform time scale UTC, the latter being currently defined as: $UTC = TAI + n$, where n is an integer number of seconds, such that $|UT1 - UTC| < 0.9$ s.

(cf. IAU 2006 NFA Glossary)

GMST expressions in seconds

$$\mathrm{GMST82}(t_u, t) = \mathrm{UT1} + 24\,110.54841 + 8\,640\,184.812866\, t_u + 0.093104\, t^2 - 0.0000062\, t^3.$$

ERA (UT1) = 2π [0.7790572732640
+ 1.00273781191135448
x (Julian UT1date – 2451545.0)]

$$\mathrm{GMST}_{P03}(\mathrm{UT1}, t) = \theta(\mathrm{UT1}) + 0''.014506 + 4612''.156534\, t + 1''.3915817\, t^2 - 0''.00000044\, t^3 - 0''.000029956\, t^4 - 0''.0000000368\, t^5.$$

with 1 s = 15 ‘’

$$\mathrm{GMST}_{P03}(t_u, t) = UT1 + 24110.5493771 + 8640184.79447825\, t_u + 307.4771013\,(t - t_u) + 0.092772110\, t^2 - 0.0000002926\, t^3 - 0.00000199708\, t^4 - 0.000000002454\, t^5$$