On the Relativistic Theory of Earth Rotation

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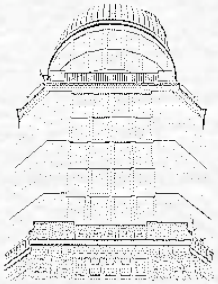
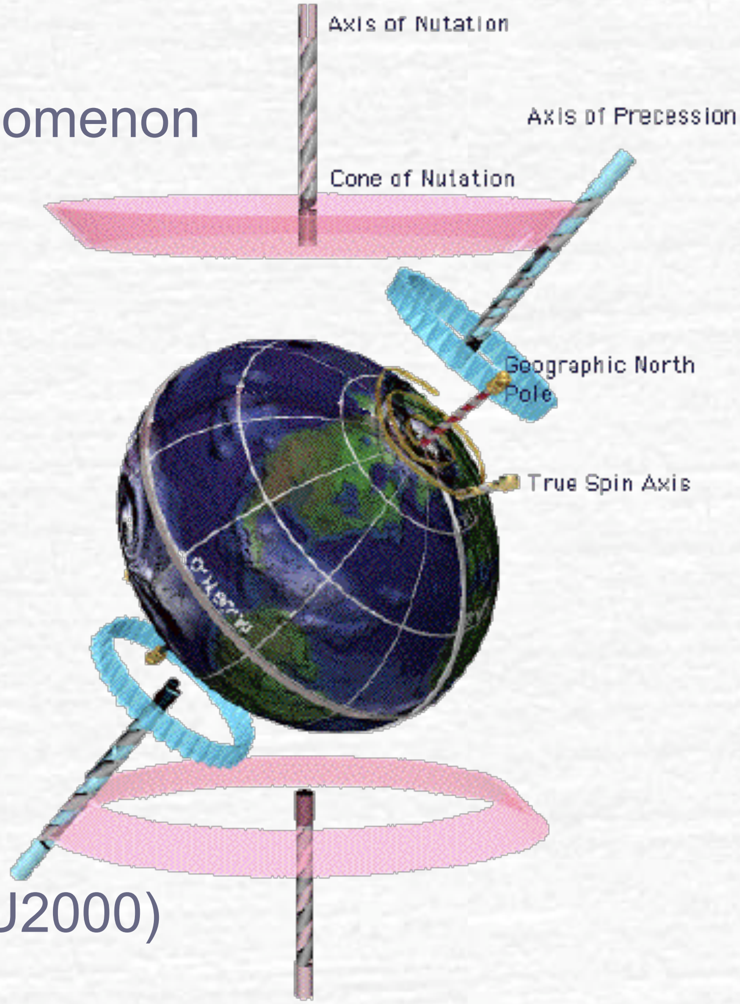


JD6, XXVII GA of the IAU, Rio de Janeiro, Brasil, 6 August 2009

Relativity and Earth rotation: why to bother?



- Earth rotation is the only astronomical phenomenon

  - which is observed with a high accuracy

and

  - which has no widely-used consistent relativistic model

- Modern theories of precession/nutation (IAU2000) are based on purely Newtonian theories with geodetic precession and nutation added in an inconsistent way

- Modern theories of rigid Earth nutation are intended to attain formal accuracy of 1 μas (expected relativistic effects are much larger)

How to model?

- Early attempts (- 1986)
  - One single reference system BCRS for both translational motion of solar system and for rotational motion of all the bodies…
  - The results were clearly physically inadequate coming from bad choice of coordinates:

    E.g. spurious annual variations in LOD with an amplitude of 75 µs…

Reason: “bad” coordinates that provide
no analogy of Newtonian tidal forces at the post-Newtonian level

“Better” coordinates are clearly needed: GCRS

How to model?

- More sophisticated way (1986 -)

  - A physically adequate local GCRS

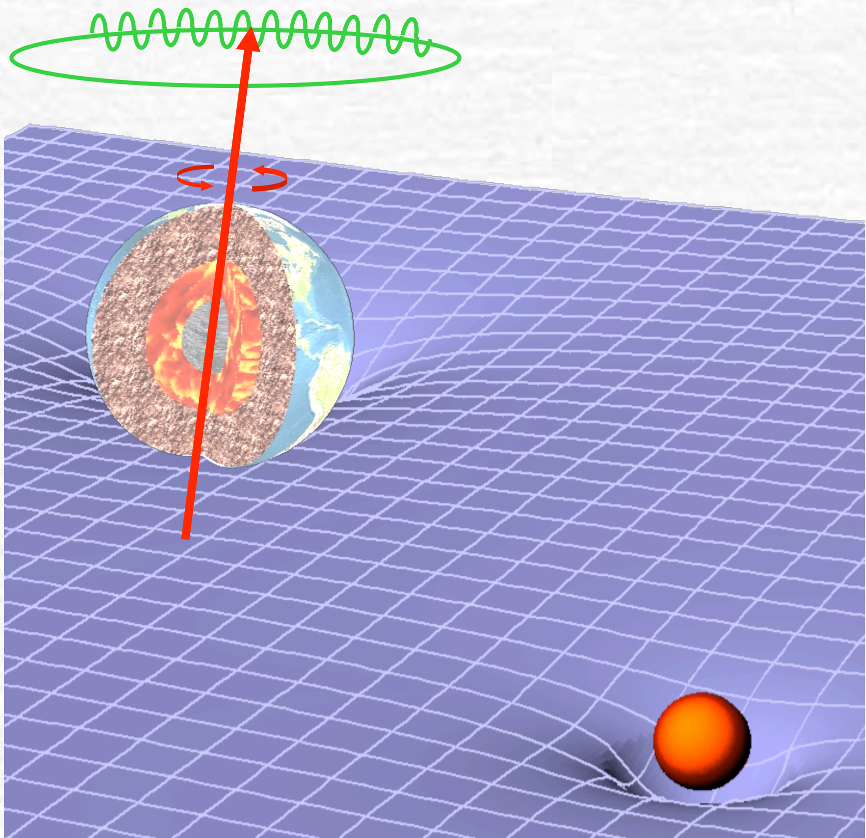
  - Still some coordinates, but chosen in such a way that
    the influence of external gravitational fields is as small as possible:

    full analogy of Newtonian tidal forces at the post-Newtonian level

Main goal of the project

- Derivation of a new consistent and improved precession/nutation series for a rigidly rotating multipole model of the Earth in the post-Newtonian approximation of general relativity

  - using post-Newtonian definitions of :
    - potential coefficients
    - moment of inertia tensor
  - dynamical equations in the GCRS
  - correct relativistic time scales
  - rigorous treatment of the geodetic precession and nutation



Equations of rotational motion in the GCRS

- Post-Newtonian equations of rotational motion in the GCRS (Damour, Soffel, Xu, 1993, Klioner, Soffel et al 1996-)

$$\frac{d}{dTCG}\left(C^{ab}\boldsymbol{\omega}^{b}\right)=\sum_{l=1}^{\infty}\frac{1}{l!}\boldsymbol{\varepsilon}_{abc}M_{bL}G_{cL}+L^{a}(\mathbf{C},\boldsymbol{\omega},\boldsymbol{\Omega}_{\text{iner}})+\ldots$$

The last terms is the Coriolis torque from the relativistic precessions:

$$\Omega^{a}_{\text{iner}}=-\frac{3}{2c^{2}}\varepsilon_{aij}v^{i}_{E}\frac{\partial}{\partial x^{j}}w_{\text{ext}}(\mathbf{x}_{E})+\frac{2}{c^{2}}\varepsilon_{aij}\frac{\partial}{\partial x^{j}}w^{i}_{\text{ext}}(\mathbf{x}_{E})-\frac{1}{2c^{2}}\varepsilon_{aib}v^{i}_{E}G_{b}$$

geodetic precession

Lense-Thirring precession

$$\boxed{w_{\text{ext}}(\mathbf{x})=\sum_{A\neq E}\frac{GM_{A}}{\left|\mathbf{x}-\mathbf{x}_{A}\right|},\quad w^{i}_{\text{ext}}(\mathbf{x})=\sum_{A\neq E}\frac{GM_{A}}{\left|\mathbf{x}-\mathbf{x}_{A}\right|}v^{i}_{A}}$$

Thomas precession (negligible)

Rigidly rotating multipoles in the GCRS

- Klioner, Soffel, Xu, Wu, 2001 (based on many previous results):

  - Post-Newtonian equations of rotational motion in the GCRS

$$\frac{d}{dTCG}\left(C^{ab}\omega^{b}\right)=\sum_{l=1}^{\infty}\frac{1}{l!}\varepsilon_{abc}M_{bL}G_{cL}+L^{a}(\mathbf{C},\omega,\mathbf{\Omega}_{\rm iner})+\dots$$

- Rigidly rotating multipoles:

several assumptions on the multipole moments and the tensor of inertia

$$C^{ab}=P^{ac}P^{bd}\bar{C}^{ab},\ \bar{C}^{ab}=\mathrm{const},$$

$$\mathrm{M}_{a_1a_2\dots a_l}=P^{a_1b_1}P^{a_2b_2}\dots P^{a_lb_l}\,\bar{\mathrm{M}}_{b_1b_2\dots b_l},\ \bar{\mathrm{M}}_{b_1b_2\dots b_l}=\mathrm{const},\quad l\ge 2,$$

$$\omega^{a}=\frac{1}{2}\varepsilon_{abc}P^{db}\frac{d}{dTCG}P^{dc}$$

$P^{ab}(TCG)$ is an orthogonal matrix defining the orientation of the ITRS in GCRS

Numerical code: an overview

- Fortran 95, about 20000 lines
- careful coding to avoid excessive numerical errors
- two numerical integrators: ODEX and ABM with dense output
- automatic accuracy check: forth and back integrations
- any available arithmetic: 64 bit, 80 bit, 128 bit
- extended-precision arithmetic for precision-critical operations (switchable)
- the STF code has been automatically generated by *Mathematica*
- baseline:

  ODEX with 80 bits on Intel architecture gives errors <0.001 μas for 150 years
- in the Newtonian limit reproduces SMART within the errors of the latter
- performance:

Newtonian case:	2.2 sec per yr
all the relativity on:	8.8 sec per yr

Long-term numerical integrations

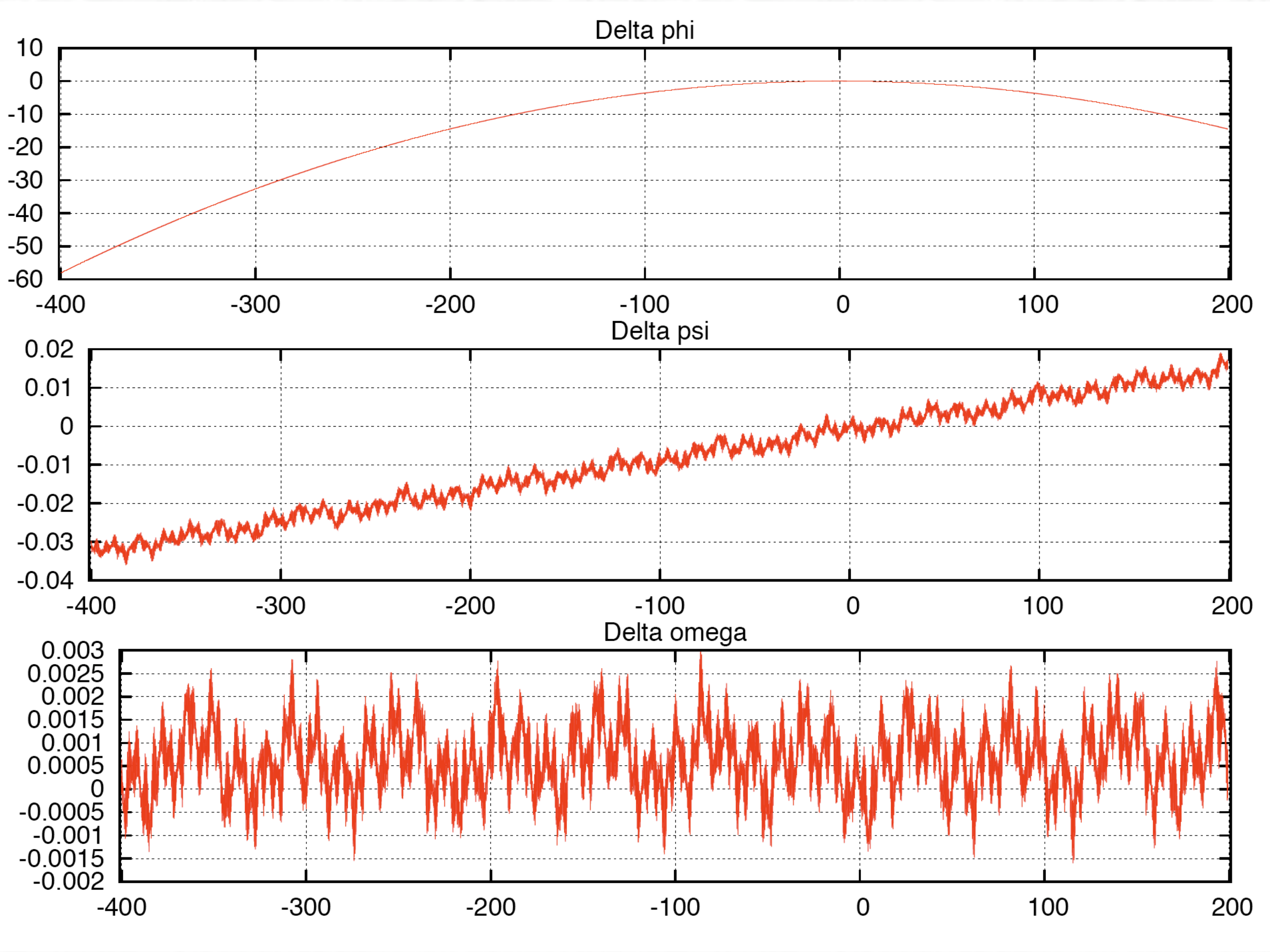
A first step to a relativistic theory of precession …

A long solar system ephemeris is needed:

DE404 is used to check the situation: 6000 yr

Noise from the downgrade: DE404 vs DE403

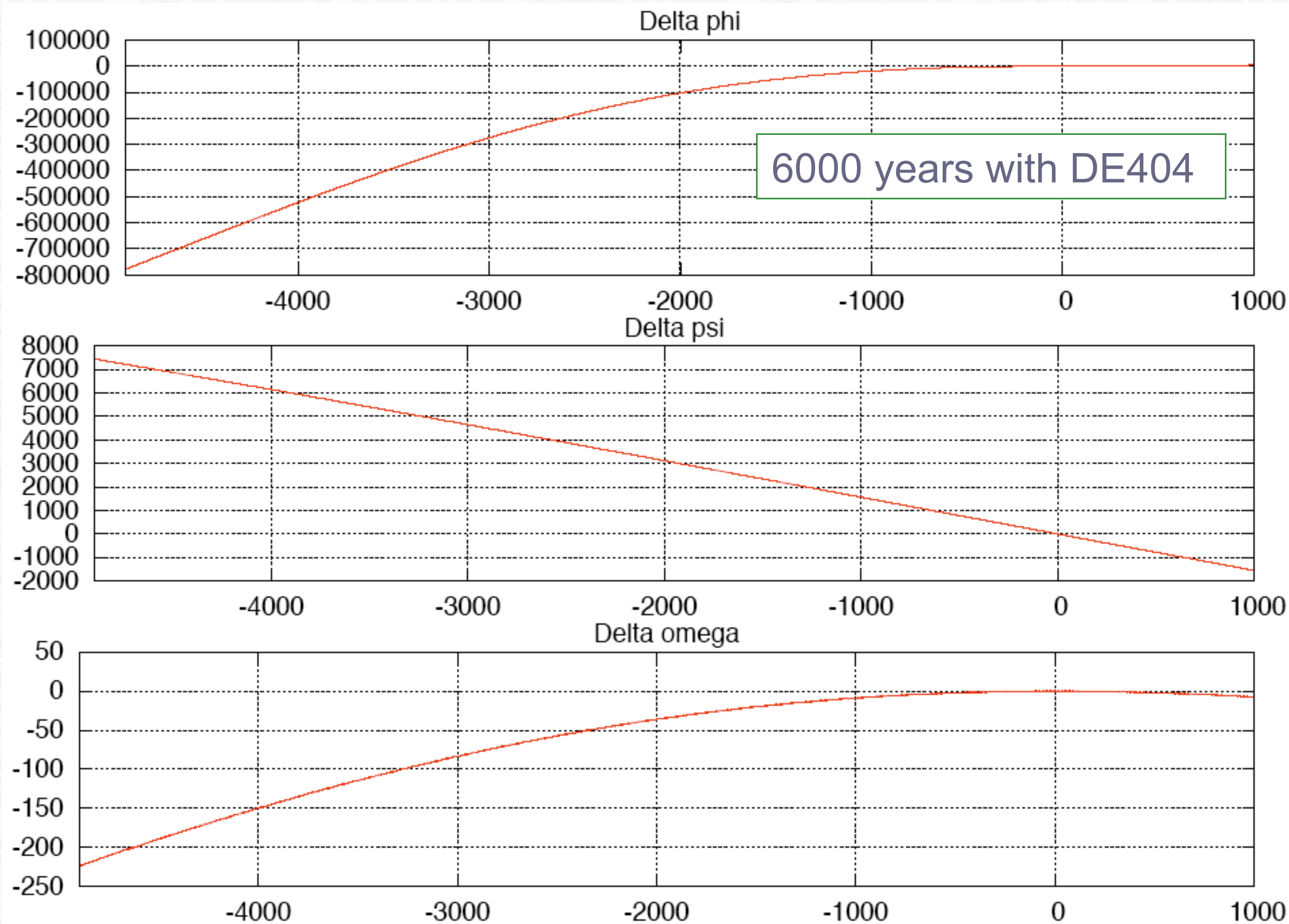
μas



years from J2000

Effects of the post-Newtonian torque

µas



years from J2000

Effects of the post-Newtonian torque

6000 years with DE404:

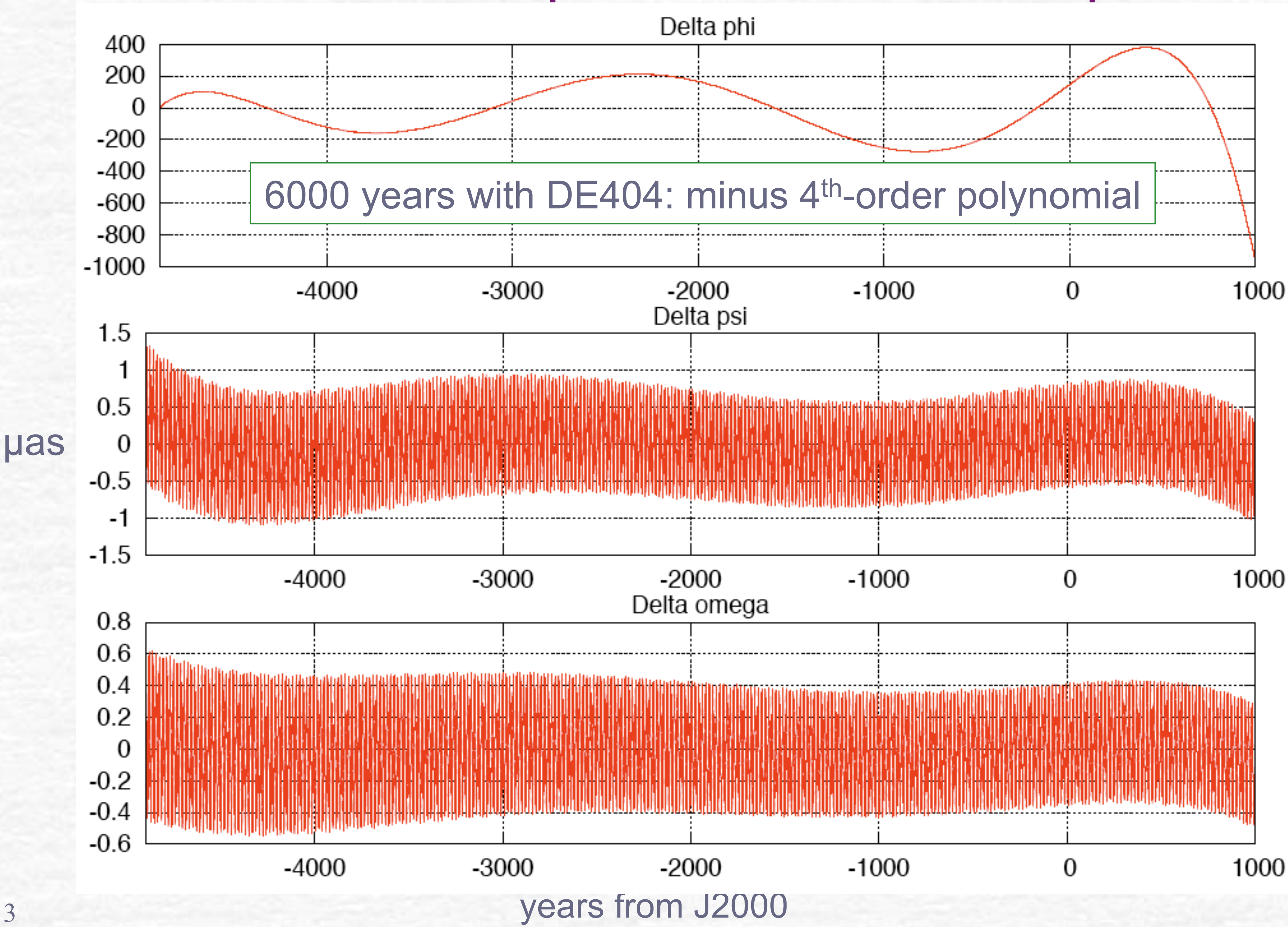
$$\Delta\varphi = -146.67 + 640.60t - 7921.70t^2 + 11375.50t^3 + 1308.23t^4$$

$$\Delta\psi = -0.61 - 1560.31t + 3.22t^2 + 2.13t^3 - 0.06t^4$$

$$\Delta\omega = -0.23 + 0.014t - 7.99t^2 + 0.64t^3 + 0.08t^4$$

in μas, *t* is in thousand years

Effects of the post-Newtonian torque

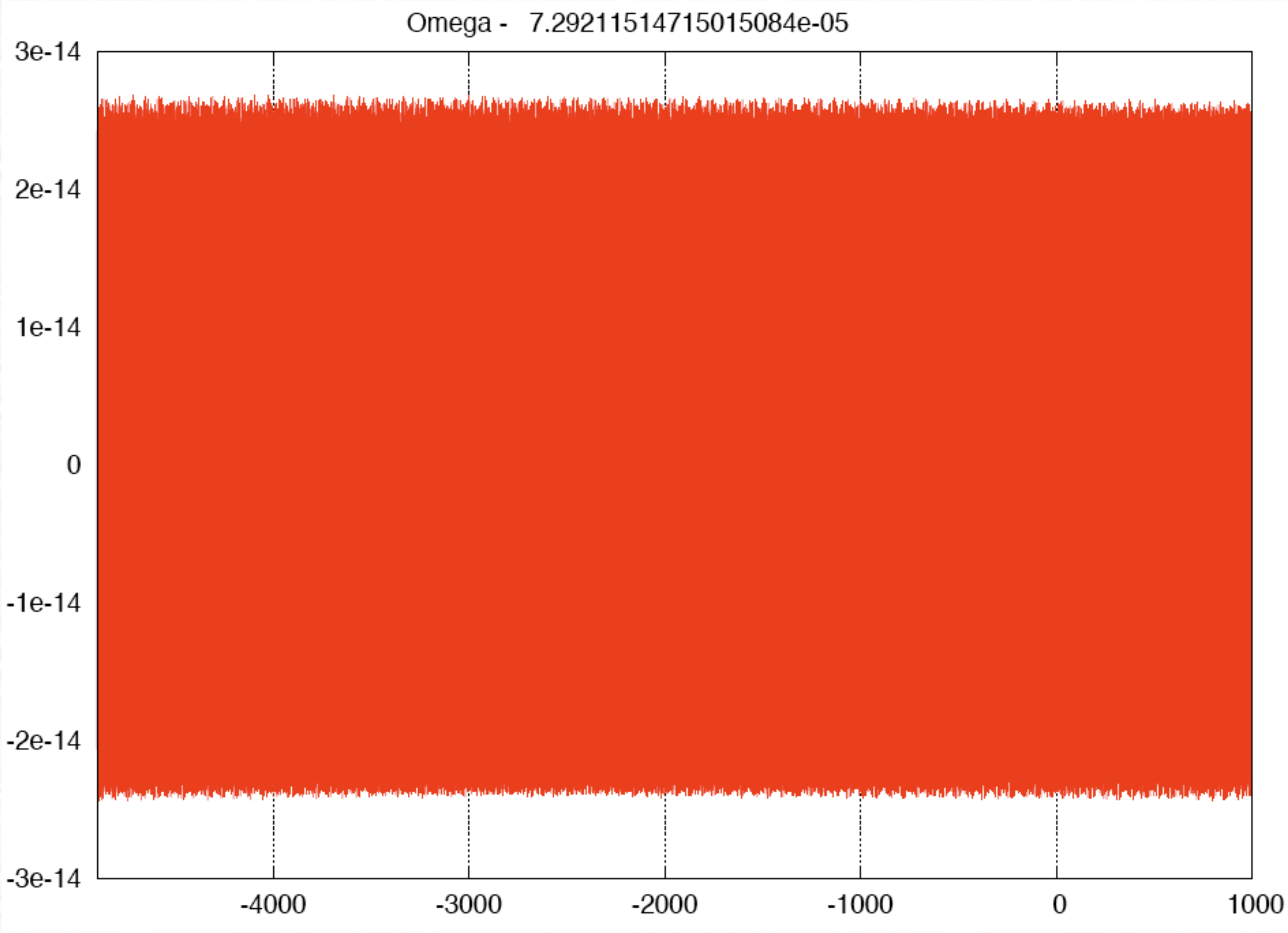


Effects on the LOD

6000 years with DE404:

$$\dot{\varphi} + \dot{\psi}\cos\omega = \Omega$$

rad/s



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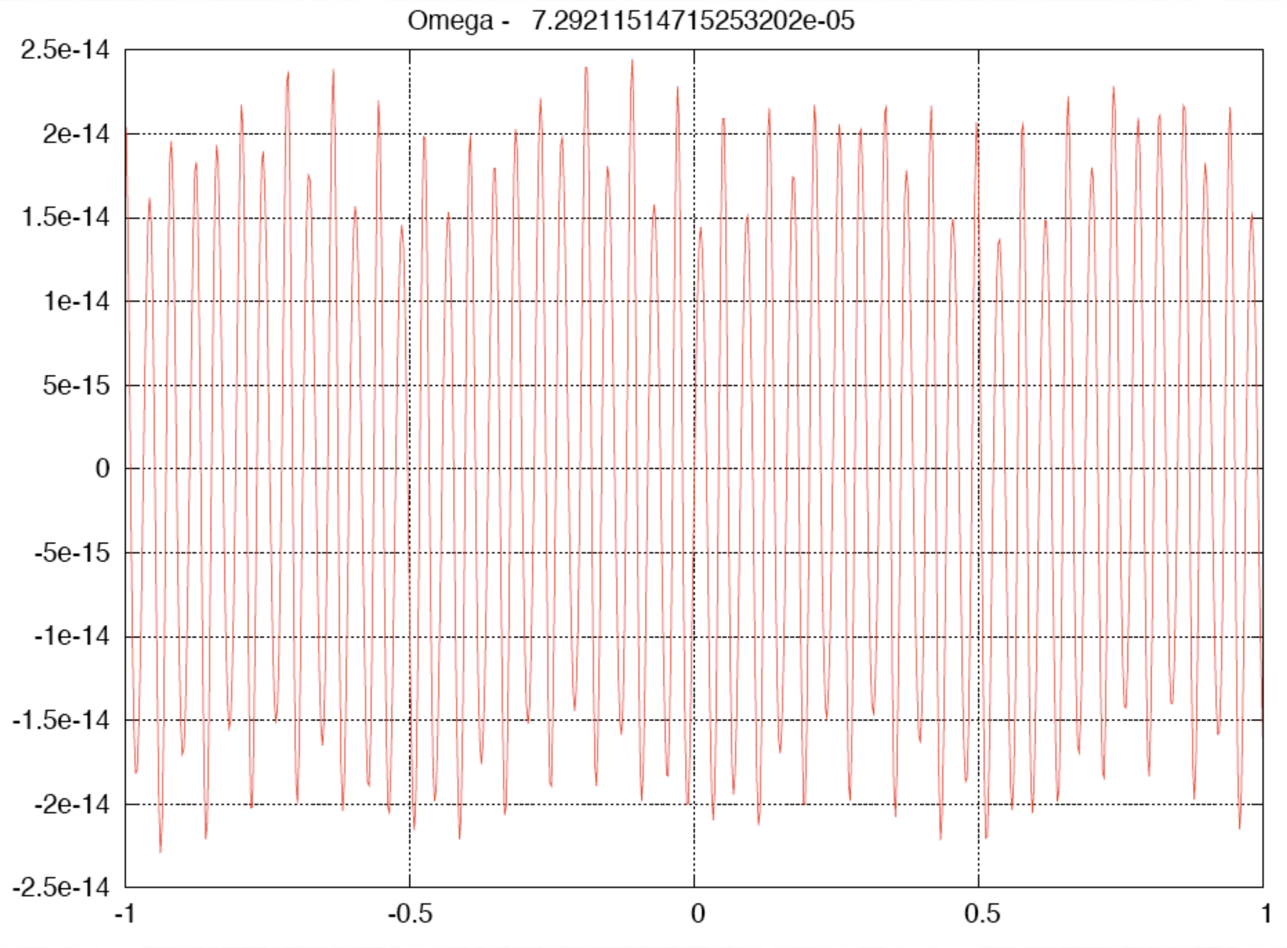


years from J2000

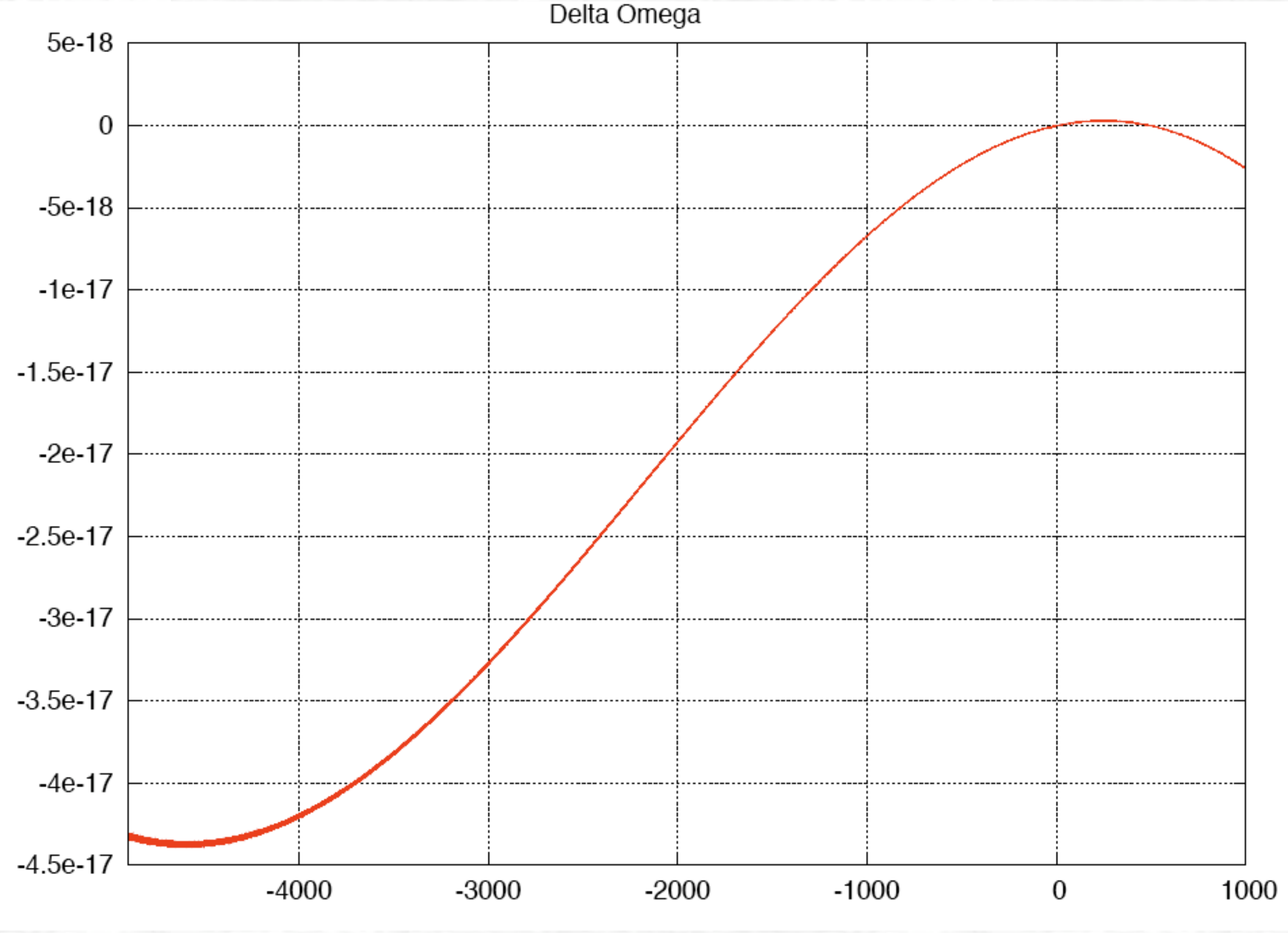
$$\left|\delta\Omega_N\right| < 450\ \mu as / d \quad \Rightarrow \quad \Delta LOD_N < 30\ \mu s$$

Effects on the LOD

6000 years with DE404:

$$\dot{\varphi} + \dot{\psi}\cos\omega = \Omega$$

rad/s



years from J2000

$$\left|\Omega_{pN} - \Omega_{Newt}\right| < 0.8\ \mu as / d \quad \Rightarrow \quad \Delta LOD_{pN} < 0.06\,\mu s$$