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CSIRO – Australia Telescope National Facility

Wide-Band Circular Polarizers Made with Quarter-Wave and Half-Wave Plates

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1. Introduction

An ortho-mode transducer ("OMT") for circular polarization can be made by preceding a linear OMT with a 90-degree differential phase shifter. By analogy to its optical counterpart, this phase shifter is called a Quarter-Wave Plate ("QWP"). Likewise, a 180-degree phase shifter is called a Half-Wave Plate.

The bandwidth of this polarizer is governed by the bandwidth of the linear OMT and by the bandwidth over which the QWP phase shift remains acceptably close to 90 degrees. Here we will discuss a method of making QWPs with wide bandwidth.

A simple phase shifter can be made by distorting the cross section of a circular or square waveguide so that the phase velocity is different for the two orthogonal polarizations. For example, a circular waveguide could be made elliptical or flat facets could be formed on opposite sides. The flat-facet form[1, 2] would be convenient to manufacture by electroforming. For these simple phase shifters, the phase velocity of each orthogonal mode is determined by its cutoff frequency in the usual way.

Other phase shifters can be made using dielectric slabs or complex surfaces. For cryogenic or mm-wave applications, these become difficult to manufacture. We will not consider such phase shifters here.

2. Axial Ratio

A measure of elliptical polarization is the axial ratio, which is the ratio of the major axis to the minor axis of the projected ellipse traced out by the tip of the E-vector. This might be called the "voltage" axial ratio. The square of this is called the "power" axial ratio. The power axial ratio expressed in dB is called the "ellipticity" in this paper. For circular polarization, the axial ratio is 1 and the ellipticity is 0 dB.

The following relations are useful. For two orthogonal linearly polarized waves of amplitudes E and F and relative phase D, the (voltage) axial ratio is:

$$\frac{a}{b} = \frac{E^2 + F^2 \pm \sqrt{E^4 + F^4 + 2E^2F^2(1 - 2Sin^2(D))}}{2EFSin(D)}$$

Choice of " + " in the above equation gives an axial ratio greater than or equal to 1, while " - " gives its reciprocal. The ellipticity is Abs(20*Log(a/b)).

The major axis makes an angle α with the horizontal, where

$$Tan(2a) = \frac{2EFCos(D)}{E^2 - F^2}.$$

3. *Single- and Multi-Section QWPs*

Single-Section:

A simple QWP and an ortho-mode transducer (rotated axially 45 degrees) make a polarizer which is suitable for narrow-band operation. The QWP phase error is 0 (i.e., the phase shift is 90 degrees) at only one frequency and varies monotonically across the band. .

Two-Section (QWP + HWP):

Wider band operation can be obtained by cascading HWPs with the QWP. A communication from Jeff Peterson describes a QWP suggested by John Carlstrom and John Kovac. Their device uses a QWP followed by one HWP. The HWP is rotated axially with respect to the QWP. The OMT which follows is in turn rotated relative to the HWP. The combination of QWP + HWP gives 90 degree phase shift at two frequencies in the band, creating a wider band over which the error is acceptably small.

Three-Section (QWP + HWP + HWP):

Adding a second HWP gives 3 frequencies with zero phase error, and even wider bandwidth. The optical version of this is described in reference 3.

N-Section:

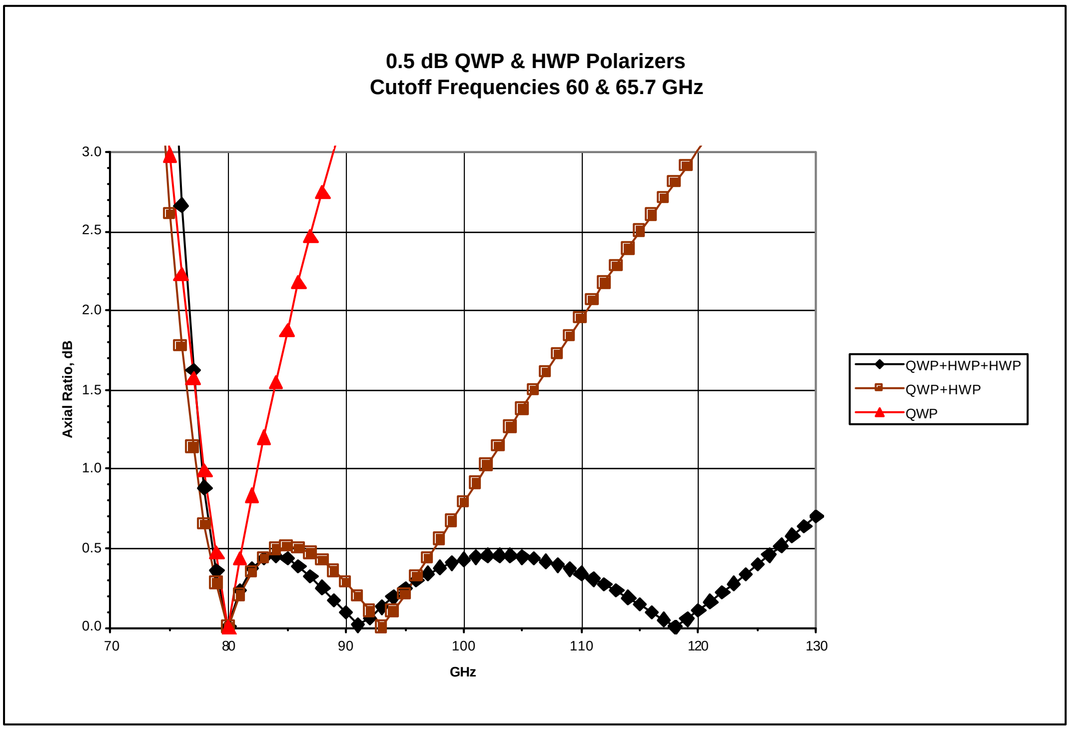
It seems clear that increased performance could be obtained by cascading more HWPs, and four-section (or more) devices might be attractive where extreme flatness is required.

However, diminishing returns set in at some point. The length, and therefore the insertion loss, of the overall QWP increases with the number of sections.

Also, as more sections are added, the manufacturing tolerances required to maintain the improved performance would rapidly become impractical.

4. Comparison of 1-, 2-, and 3-Section Polarizers

This plot shows the calculated axial ratios of polarizers with one-, two-, and three-section QWPs. The two- and three-section polarizers are designed to give less than about 0.5 dB ellipticity.



The plotted results are idealized. They do not take into account the effects of mismatches and reflections at junctions between sections. The deeper the intrusion of the flattened sections, the greater these reflections will be. (But the physical length needed to achieve the required phase shift becomes smaller.)

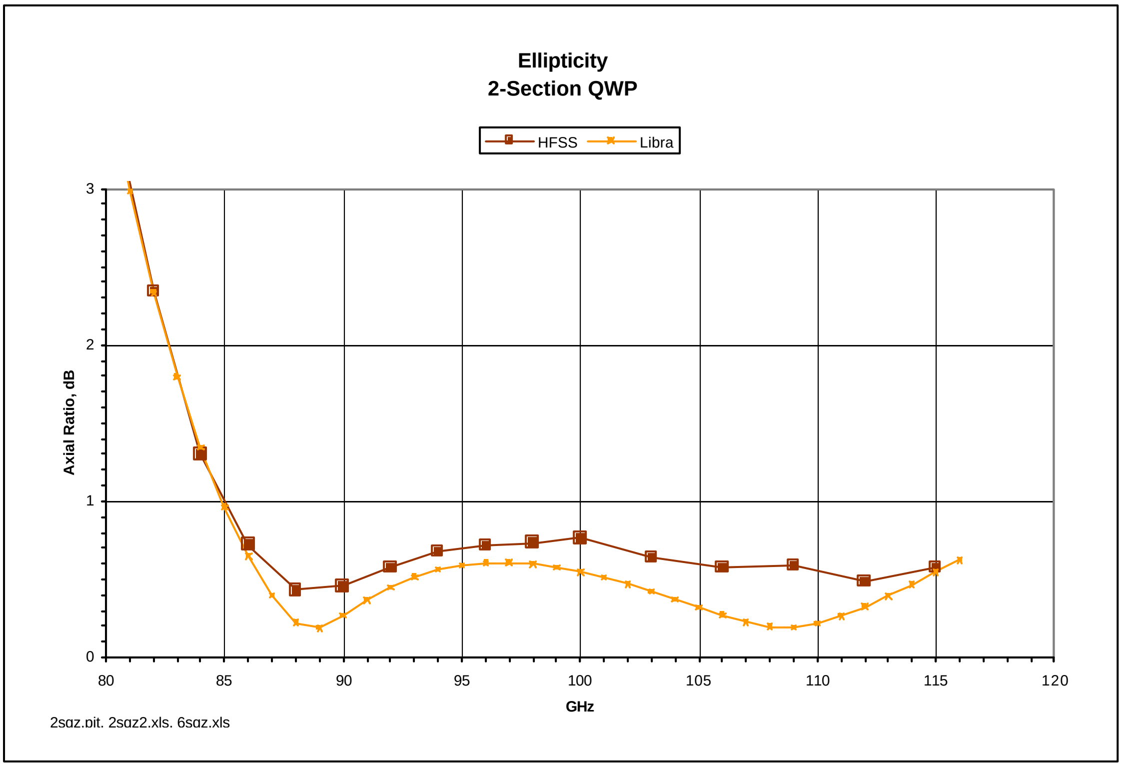
5. Simulation of a 2-Section polarizer.

To evaluate these effects, an example of a 2-section QWP was modeled in the electromagnetic field simulator HFSS. (This QWP differs slightly from the one plotted in the figure above.) The phase-shifting sections were modeled as having a cross section which is a circle of radius of 1.435 mm, with flats on opposite sides intruding into the circle to a depth of 0.172 mm (h/r = 0.12).

To improve the impedance match, each section was preceded and followed by a quarter-wavelength matching section. For this matching section h/r = 0.07 was chosen to give impedances near the geometric mean of the impedances of the adjoining sections. Each matching section contributes about 2 degrees differential phase shift at center frequency.

The following plot compares the HFSS simulation with a Libra analysis. The HFSS simulation is less accurate above 100 GHz as the adaptive simulation was not as well “converged” in that frequency range. The Libra model uses ideal components and ignores all “waveguide” effects except the phase shift.

The good agreement indicates that simulation by simple circuit analogs will probably be accurate enough without and large electromagnetic simulations. (This HFSS simulation ran to a mesh of more than 50,000 tetrahedra).



The HFSS-simulated return loss of the QWP was 25 dB or better across the frequency range.

6. Practical Considerations

The roll-off of ellipticity is quite sharp at the low end of the band. Lowering the cutoff frequencies of the phase-shifting sections gives some improvement, but this may allow propagation of higher modes in the passband, with attendant difficulties.

A precise knowledge of the cutoff frequencies of the phase-shift sections is required for design. This requires prototyping or careful electromagnetic simulation to determine. The cross-section modeled here, made up of circular arcs and straight lines, seems to be difficult for the simulator to handle.

It would be prudent to build test lengths of the phase shift section to verify the cutoff frequencies before going into production. Errors in this parameter will show up as a shift in the frequencies flow, fmid, and fmax but will not affect the axial angles.

For prototype work, the sections could be made separately, and deliberately overlong so that they could be trimmed to length. Round, un-pinned flanges with a circumferential clamp would allow adjustment of the axial angles.

7. Calculations

Formulas are given below for configurations which offer 0 dB ellipticity at one, two, or three points in the band. While these give good starting values, they do not consider impedance matching-sections or other details. It will be necessary to model and optimize the proposed polarizer in a circuit simulator such as Libra or by some other scheme.

Selecting the frequencies:

Choose the cutoff frequencies f_{co1} and f_{co2} of the orthogonal modes of the phase-shifting sections.

Choose an upper frequency, f_{high}, and a lower frequency, f_{low}. The phase error, e, through the nominally 90° section will have same magnitude at these two frequencies. Between them is the third frequency, f_{mid}, at which the phase shift will be exactly 90°. The frequencies will not be uniformly spaced.

90-degree phase shifter length L and f_{mid}:

$$L = \frac{c/2}{\sqrt{f_{high}^2 - f_{co2}^2} - \sqrt{f_{high}^2 - f_{co1}^2} + \sqrt{f_{low}^2 - f_{co2}^2} - \sqrt{f_{low}^2 - f_{co1}^2}}$$

$$f_{mid} = \sqrt{\frac{\left(f_{co1}{}^2 - f_{co2}{}^2 - a^2\right)^2}{4a^2} + f_{co1}{}^2} \qquad \text{where} \quad a = \frac{c}{4L}$$

and c is the speed of light.

Single-Section:

The single-section polarizer is now completely determined The absolute value of the phase error at f_{high} or f_{low} is

$$e = \frac{p}{2}\,\frac{\sqrt{f_{high}{}^2 - f_{co2}{}^2} - \sqrt{f_{high}{}^2 - f_{co1}{}^2} - \sqrt{f_{low}{}^2 - f_{co2}{}^2} + \sqrt{f_{low}{}^2 - f_{co2}{}^2}}{\sqrt{f_{high}{}^2 - f_{co2}{}^2} - \sqrt{f_{high}{}^2 - f_{co1}{}^2} + \sqrt{f_{low}{}^2 - f_{co2}{}^2} - \sqrt{f_{low}{}^2 - f_{co1}{}^2}}$$

The ellipticity is zero dB at f_{mid} and reaches

$$20Log_{10}\left(\frac{1+Cos(e)}{Sin(e)}\right) \qquad \text{dB at } f_{low} \text{ and } f_{high}.$$

Two-Section:

At f_{high} and f_{low}, the phase error *of the 90-degree section only* will be e as given above. (The net phase error of the combined 90-degree and 180-degree sections is zero at these frequencies.)

The axial angles q_1 and q_2 can be found from

$$Cos(2q_1) = \frac{-Tan(e)}{Tan(2e)} \quad \text{and} \quad Tan(2q_2) = \frac{-Cot(2q_1)}{Cos(2e)}$$

.

The Ellipticity is zero dB at f_{low} and f_{high}. At f_{mid}, it is

$$20Log\left(\frac{Sin(q_1 - q_2)}{Cos(q_1 - q_2)}\right)$$

Three-Section:

These equations for the three-section polarizer are from ref. 3

With e as defied above, calculate the following:

$$A = 4(1 - Cos(e))Cos^3(e)$$
$$B = 4(Cos(2e) - Cos(e))Cos^2(e)$$
$$C = 2(1 - Cos(e))Cot^2(e) - Cos^2(2e)$$

Solve $ASin^2(a) + BSin(a) + C = 0$ for a.

Solve $Sin(e) = \dfrac{2Cos^2(e)Sin(a) - Cos(2e)}{2Cos(e)}$ for e.

The axial angles in radians are:

$$q_1 = (p/2 + a)/2$$
$$q_2 = (a + e)/2$$
$$q_3 = e/2$$

The ellipticity will be zero dB at f_{low}, f_{mid}, and f_{high}.

8. References

1. J. R. Pyle and R. J. Angley, “Cutoff Wavelengths of Waveguides with Unusual Cross Sections”, IEEE Transactions on Microwave Theory and Techniques, Vol. 12 , No. 5, September 1964 pp. 556-557

2. C. Y. Wang, “Frequencies of a Truncated Circular Waveguide—Method of Internal Matching”, IEEE Transactions on Microwave Theory and Techniques, Vol. 48, No. 10, October 2000 pp. 1763-1765

3. S. Pancharatnam, “Achromatic Combinations of Birefringent Plates”, Proceedings of the Indian Academy of Sciences, Vol. XLI, No. 4, Sec. A, 1955 pp. 130-144.