Aperture illumination in the Compact Array antennas across the 12-mm band.

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Introduction:

The Compact array antennas are Cassegrain systems with shaped optics and have compact corrugated feed horns in cone housing at the vertex of the main reflectors. The 12-mm feed horns cover the 16-26 GHz band. The antenna radiation patterns have been measured at a set of frequencies across the 12-mm band and these have been used to infer the variations in aperture illumination across the band. Any variations in illumination would result in changes in antenna gain across the band.

The design of the Compact array antenna optics describes the aperture plane illumination by a function which is dependent on two parameters. Therefore, any variation in the aperture illumination may be gauged by the frequency dependence of these parameters across the band covered by the horn. The parameters describing the illumination at any frequency may be estimated by a comparison of the measured far-field antenna radiation pattern and the Fourier transform of the aperture distribution function or illumination. The method used here is to compare the derived and observed beam patterns and utilise a minimisation algorithm to find the best fit parameters of the model illumination at each of the frequencies at which observations were made.

Observations:

The source 2251+158 was observed at elevations 30-40 degrees and using Compact Array interferometers between antennas ca01, ca02, ca03 & ca05 with ca04. Antenna ca04 observed the source throughout along its bore-sight where as the other four antennas executed scan patterns to measure their voltage radiation patterns. Patterns were measured at 16, 17.5, 19, 20.5, 22, 23.5, & 25 GHz using dual polarization and 128-MHz bandwidth. Cross scans were made at P.A. 45 and -45 degrees, with offsets in steps of $0.0064\sqrt{2}$ degrees and covering a range $\pm 0.16\sqrt{2}$ degree. The pointing model parameters were updated, using a reference pointing scan, before the cross scans at each frequency.
Visibility data taken in bore-sight scans before and after each cross scan were used to calibrate the scan data in amplitude and phase.

Modeling:

The modeling was done using C++ code in a UNIX environment using Numerical Recipes simplex algorithm for optimization of parameters.

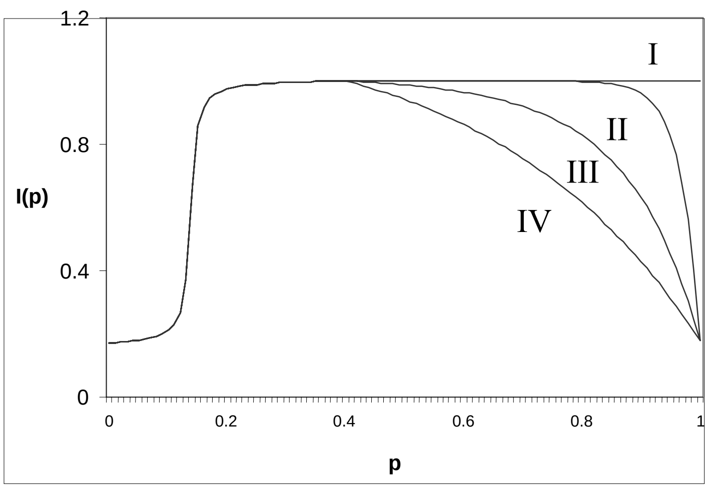
The first stage of the program was to compute the Fourier transform of the model aperture illumination. The illumination was based on a model[1] for a Cassegrain antenna fed by a compact corrugated feed horn; the functional form F(p) is given below. The illumination taper (see Fig. 1) is dependent on two parameters a1 and a2 which may be varied to change the distribution function. This function was scaled to fit the dimensions of the Compact array antenna and the blockage due to the cone housing was accounted for by forcing the response to zero within the central region.

$$F(p) = f_1(p)/f_1(p_o); \quad p < p_o$$
$$F(p) = f_2(p)/f_2(p_o); \quad p > p_o$$

where

$$f_1(p) = 0.58 + 0.27\tan^{-1}[130(p - 1.1\frac{d}{D})]$$
$$f_2(p) = 1 - a_2\exp[a_1(p^2 - 1)]$$

Fig 1. Examples of the model illumination distribution for the parameters given in the table alongside.



	a1	a2
I	-	0
II	16	0.822
III	4	0.825
IV	1	0.89

The Fourier transform of the aperture illumination was performed using a Hankel transform due to the inherent circular symmetry in the optics:

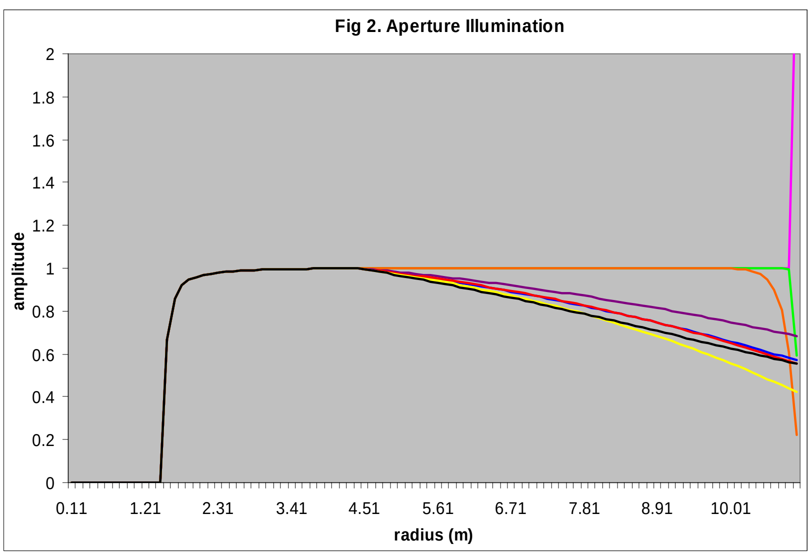
$$F(q) = 2\pi\int E(\frac{r}{\lambda})J_o(2\pi\frac{r}{\lambda}q)\frac{r}{\lambda}d(\frac{r}{\lambda})$$

[1] Radiation-pattern control of Cassegrain antennae.
G.L. James, IEE PROCEEDINGS, Vol. 134, Pt. H, No. 2, April 1987.

The computed far-field voltage radiation pattern for each frequency was compared with the observed data using the Multidimensional Downhill Simplex Method by Nelder and Mead[2]. The method requires function evaluations only and outputs values of the parameters a1 and a2 which correspond to a weighted least-squares best fit at each frequency.

Because the location of the phase centre of the feed horn moves along the axis of the feed horn with frequency, and the optics are focused only at a single frequency within the 12-mm band, the optics will not be focused at all the frequencies at which the beam pattern measurements have been made. Consequently, the beam patterns will include effects owing to radial phase gradients in the aperture illumination. For a realistic analysis, an allowance has to be made for such radial phase gradients and the aperture illumination has been modeled as a complex voltage distribution with a radial quadratic phase gradient.

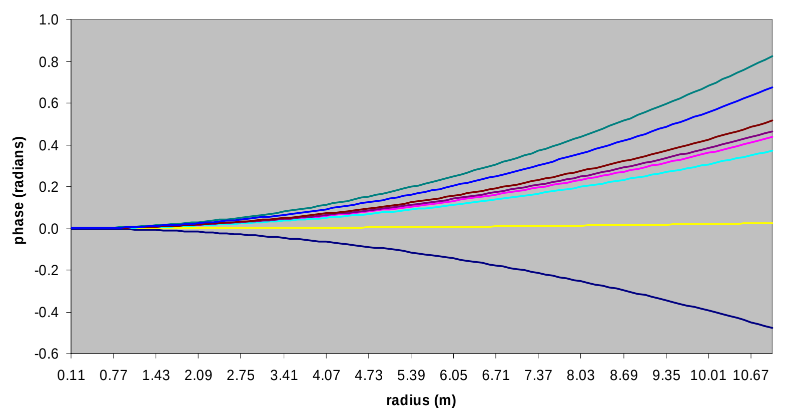
Using this theoretical model the following distribution function and phase curves were computed as a function of frequency (see Figs 2 + 3). The aperture illumination significantly changes across the 12-mm band. At the lower frequencies the fit indicates a fairly uniform distribution; however, at the higher frequencies the illumination is tapered.



Frequency	a1	a2	colour
16	1398	-1.53	Pink
17.5	222	0.42	Green
19	34.5	0.78	Orange
20.5	0.007	0.99	Marone
22	0.001	1.00	blue
23.5	0.18	0.85	Red
24.5	0.42	0.82	Yellow
25	-0.7	204	black

[2] Downhill Simplex Method, Numerical Recipes in C, William H. Press et. al, 2nd Ed. P 408.

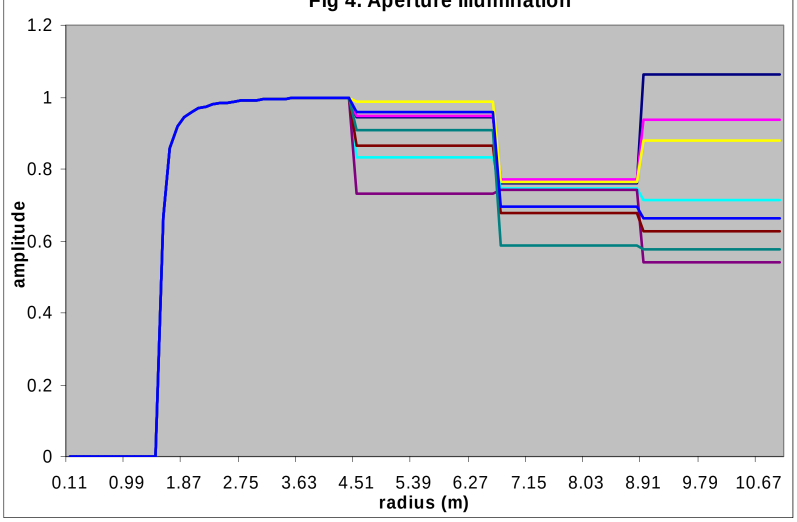
Fig 3. Quadratic Phase change.



Frequency	k	colour
16	-0.48	Navy
17.5	0.44	Pink
19	0.01	Yellow
20.5	0.37	Cyan
22	0.46	Purple
23.5	0.52	red
24.5	0.82	Blue
25	067	Green

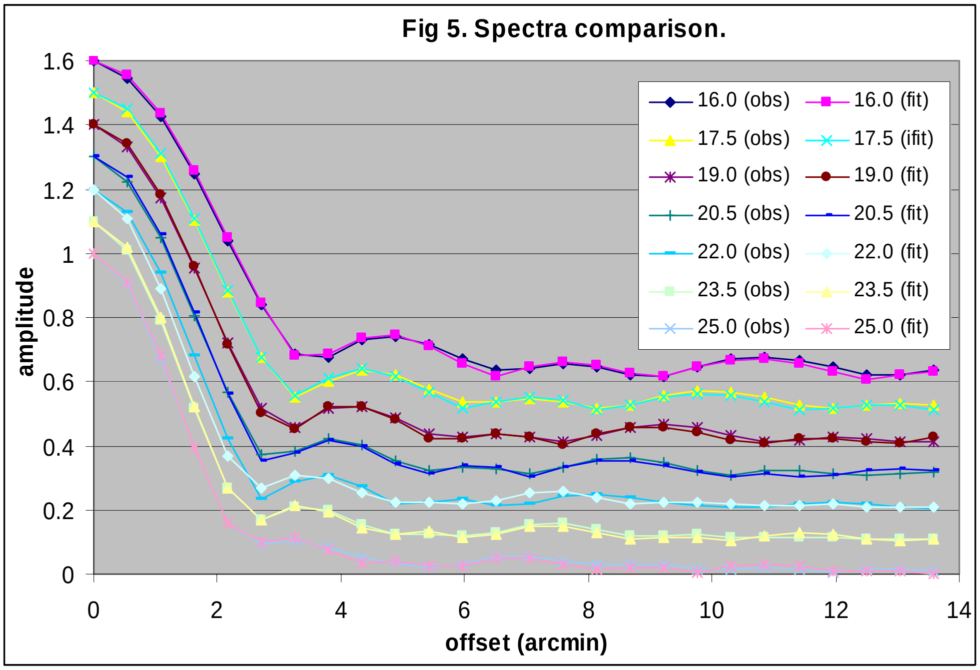
The above theoretical model imposes limitations on the form the aperture illumination may take. Consequently a new model was created which allows greater variability in the functional form and also highlights any systematic trends. Here, the aperture illumination outside 4.5 m was divided into three segments of variable amplitude. Three parameters control these segments which are subject to the minimisation process and return the amplitude of their respective segment (see Fig 4).

Fig 4. Aperture Illumination

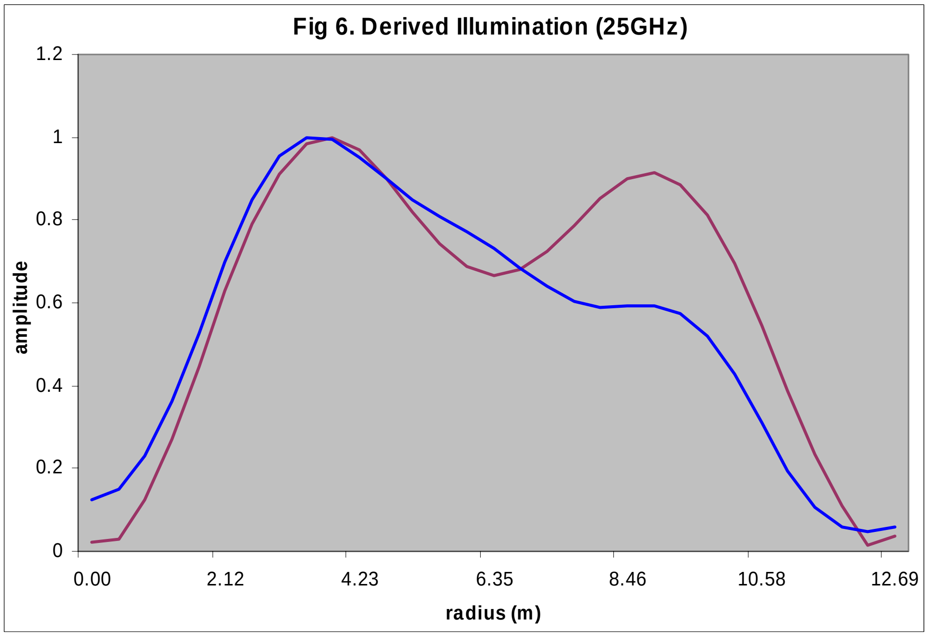


The graph neatly exposes a systematic trend in the aperture illumination with increasing frequency and that where high frequencies have a response which is monotonically decreasing; low frequencies have an inherent second maximum. Additionally, for this parameterization of the aperture illumination the derived beam patterns, corresponding to the best fit illumination, are a closer match to the observed beam patterns.

The corresponding fits between the observed and derived spectra are shown in Fig 5. The voltage radiation patterns at the different frequencies have been offset vertically for clarity. The beam pattern derived from the best-fit model illumination is a very good approximation to the main lobe as well as the first three side lobes. And the fits are good at almost all of the frequencies.



To verify the trend seen in the above illuminations, the inverse Fourier transform of the observed beam pattern was computed to yield the aperture distribution function (see Fig 5). Two examples are shown – at 16GHz (marone) and 25GHz (blue); the curves corresponding to the other frequencies between these two lie between the curves shown in the figure. These derived illumination function suffer from the limitations arising from the limited range in the beam scan angle; however, the trend noticed in the fits is confirmed in these derived illuminations: at the lowest frequency the illumination goes through a minimum at intermediate radii and at the higher frequencies within the 12-mm band the illumination simply tapers off at larger radii without any intermediate minimum.



Conclusions:

1. The aperture illumination changes significantly across the 12-mm band.

2. Simple illumination functions that allow a monotonic taper towards the outer rim do not give a good fit to the observed beam patterns. This is the case even if the aperture phase is allowed a quadratic form variation. Remarkably good fits to the beam pattern including the first three side-lobes are possible with empirical illumination functions that admit a minimum at intermediate radii.

3. Over the 12-mm band, the aperture illumination goes through an intermediate minimum for the lower frequencies. At the higher frequencies within this band the illumination has a monotonic taper towards the outer end.