

Modelling sources in broad-band ATCA visibility data

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Abstract

A new task, *uvsf*, has been added to MIRIAD that allows source fitting to visibility data over broad frequency ranges. The new task is an adaptation of *uvfit* that fits visibility to a model for sources in the field. Typical models are points, gaussians, rings and disks and the procedure fits the data for their flux-density, position and angular size. *Uvsf* provides new parameters that allow the flux-density of each source to be a function of frequency.

1 Introduction

In radioastronomy it is often useful to determine source characteristics by directly fitting the visibility data to a model for the source. Typically source models are constructed with a modest number of parameters; examples the point source, an elliptical gaussian and a filled disk (having three, six and six parameters respectively). This approach is used when visibility data sparsely samples the uv-plane, and when the brightness distribution in the observed field is known to be simple. Depending on the quantity and distribution of the visibility data, they can be fitted successfully to models comprising several components.

The major radioastronomy software packages have some form of model-fitting task:

- MIRIAD has task *uvfit*;
- AIPS has *UVFIT* and *OMFIT*;
- CASA has task *uvmodelfit* (limited to a single component, although the user can probably script a less constrained version);
- DIFMAP (generally used for VLBI data) has task *modelfit*.

All of these assume no variation of flux-density with observing frequency. Provided the fractional bandwidth is small, this assumption has little effect on the results. Increasingly, wide-bandwidth data are available so that the frequency dependance of flux-density cannot be ignored.

Here I describe a new miriad task *uvsfit*, adapted from *uvfit*, that allows the user to include the spectral shape in the model for each source.

2 New MIRIAD task *uvsfit*

A new task *uvsfit* has been introduced to MIRIAD¹. It allows the source spectral shape to be modelled with an order-2 polynomial in spectral index:

$$\alpha = \alpha_0 + \alpha_1 x + \alpha_2 x^2 \quad (1)$$

where $x = \log(f/f_0)$, f is the frequency and f_0 is a reference frequency. Then the source flux-density as a function of frequency is

$$S(f) = S_0 \left(\frac{f}{f_0} \right)^\alpha \quad (2)$$

where $S_0 = S(f_0)$. Note that only the total flux-density of each model source is frequency dependent. Any frequency dependence of source position or shape will degrade the fit.

¹The ATNF version.

The *uvsfrit* inputs² are:

```
Task:    uvsfit
vis      = 0001-4630_pppp_c.4800,0001-4630_pppp_c.8640
stokes   = i
line     =
select   =
object   = point
spar     = 0.1,47.3,-5.3
fix      =
sspar    = -1.0,0.0,0.0
sfix     = 12
freqref  = 4.8
out      =
options  = residual
log      = a.log
```

The *uvsfrit* inputs differ from those of *uvfit* by the addition of the following:

- **sspar** gives the spectral parameters α_0 , α_1 and α_2 ;
- **sfix** indicates which spectral parameters, if any, are to be held fixed;
- **freqref** gives the reference frequency in GHz. If this is left unset, the frequency of channel 1 is used;
- **log** accepts the name of an optional text file to be written with the best-fit parameter values and their errors. For each fitted source, four lines are written to the log with the following information:
 - line 1 has source number, object type and three or six parameter values, depending on the object type;
 - line 2 has the formal errors in each (0.0 if not a free parameter);
 - line 3 has the values of all three spectral terms;
 - line 4 has the formal errors in those terms (again 0.0 if not included in the fit).

The new task differs from *uvfit* in one other respects: the allowed number of free parameters in *uvsfrit* has been increased to 100, allowing up to 11 sources to be fit for all possible source and spectral parameters (or more sources with some parameters held fixed). In practice such a large fit is slow, and the results should be viewed with caution: the visibility data may not be sufficient to adequately constrain the fit.

The results, as for *uvfit*, written to the user's screen:

²Described in full at <http://www.atnf.csiro.au/computing/software/miriad/doc/uvfit.html>

```
uvsfrit: Revision 1.1, 2012/10/18 22:00:33 UTC

Reading the data ...
Applying gain corrections to 0001-4630_pppp_c_4800
Applying gain corrections to 0001-4630_pppp_c_8640
Total number of correlations: 2340
Performing the fitting process: 4 params...
-----
Reference frequency (GHz): 4.800
RMS residual is 2.875E-02

Source 1, Object type: point
Flux: 9.4560E-02 +/- 1.05E-03
Offset Position (arcsec): 47.4069 -5.4334
Positional errors (arcsec): 1.39E-02 1.27E-02
Alpha0: -1.395 +/- 1.43E+00
Alpha1: 0.000
Alpha2: 0.000
-----
```

For this example, the log file has format:

```
#
# Output generated by task uvfit
# Executed on: 12OCT22:16:25:15.0
# Command line inputs follow:
#   vis=0001-4630_pppp_c.4800,0001-4630_pppp_c.8640
#   stokes=i
#   object=point
#   spar=0.1,47.3,-5.3
#   sfix=12
#   freqref=4.8
#   options=residual
#   log=a.log
#
Reference frequency (GHz):  4.800
RMS residual is 3.004E-02
  1 point      9.452E-02    47.397    -5.423
                1.055E-03     0.013     0.010
                -1.141       0.000       0.000
                3.4644E-02    0.000       0.000
```

3 Usage examples

3.1 1934–638

This source is the ATCA flux-density calibrator. The flux-density scale of the telescope is set by observing 1934–638 and adjusting antenna gains so that the observed flux-density agrees with its accepted flux-density spectrum (Reynolds, 1994). 1934–638 is a “GPS” —Gigahertz-Peaked-spectrum— source and its spectrum is noticeably curved over the 1–10 GHz range. In this example, three visibility datasets for 1934–638 were fit for a point source at the phase centre; all three spectral terms were allowed to vary. The resulting *uvfit* output was:

```
Total number of correlations: 1416709
Performing the fitting process:  6 params...
-----
Reference frequency (GHz):  2.100
RMS residual is 6.117E-01

Source  1, Object type: point
Flux:                12.57      +/- 7.52E-04
Offset Position (arcsec):  -0.0087    0.0015
Positional errors (arcsec):  5.13E-04  1.15E-04
Alpha0:              -0.6262      +/- 2.26E-04
Alpha1:              -0.4626      +/- 6.05E-04
Alpha2:               0.1145      +/- 5.43E-04
-----
```

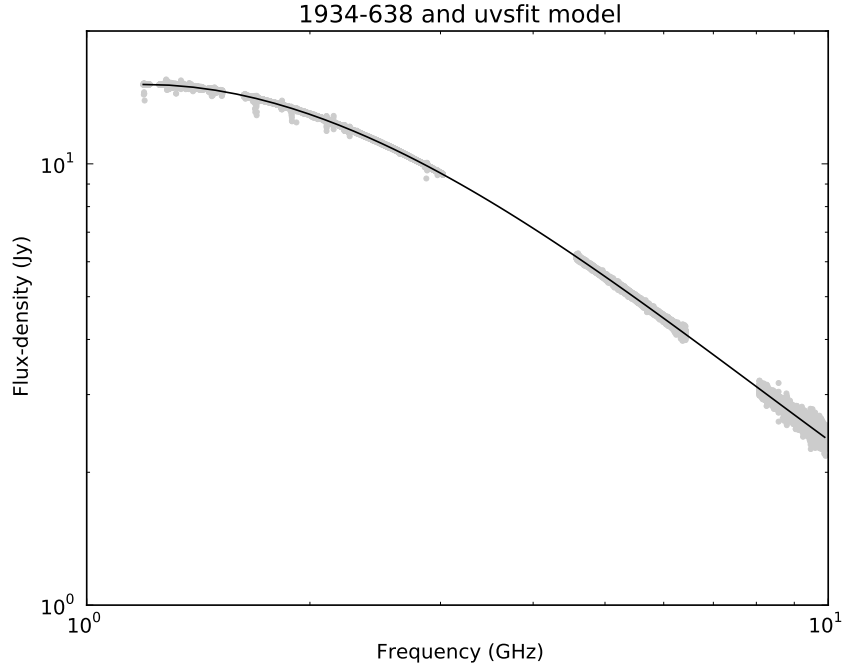


Figure 1: Flux-density spectrum of 1934–638. The grey points are measured values, after bandpass calibration (MIRIAD task *mfc*al). The solid line is calculated from equations 1 and 2 with the values for S_0 , α_0 , α_1 and α_2 returned by *uvsfit*.

3.2 Multi-band data from the AT correlator

In addition to its use for wide-bandwidth data, the spectral capability of *uvsfit* is useful for joint fitting of data with several widely-spaced narrow bands. Archival ATCA data from the original AT correlator often has this form with several 128MHz bands over the full ATCA frequency range.

The ATCA follow-up observations (project C142) of the PMN 5GHz survey provide an example in the field centred on PMN J0000-3923. *uvsfit* allows a joint fit over both bands and returns the apparent spectral index for both sources. No plausible spectral curvature can be deduced from the two widely-spaced narrow bands, so here we fit only the zeroth spectral term. The field contains two sources; the *uvsfit* parameters are:

```

vis      = 0000-3923_pp_c.4800,0000-3923_pp_c.8640
object   = point,gaussian
spar     = 0.046,-61.3,-125.2,0.047,-8.5,77.6,0.5,0.25,56
sspar    = -1,0,0,-1,0,0
sfix     = 12,12
freqref  = 4.8

```

The results are:

```

Total number of correlations: 1105
Performing the fitting process: 11 params...
-----
Reference frequency (GHz):  4.800
RMS residual is 2.553E-02

Source 1, Object type: point
Flux:                        3.9934E-02 +/- 1.37E-03
Offset Position (arcsec):    -61.2960 -125.1175
Positional errors (arcsec):  4.97E-02 9.59E-02
Alpha0:                     -1.350    +/- 1.23E-01
Alpha1:                      0.000
Alpha2:                      0.000

Source 2, Object type: gaussian
Flux:                        4.5471E-02 +/- 1.25E-03
Offset Position (arcsec):    -8.4917  77.5862
Positional errors (arcsec):  1.97E-02 5.63E-02
Major,minor axes (arcsec):   0.4887  0.0373
Axes errors (arcsec):        1.57E-01 1.79E+00
Position angle (degrees):    47.7
Pos angle error (degrees): 2.64E+01
Alpha0:                     -0.9062    +/- 1.00E-01
Alpha1:                      0.000
Alpha2:                      0.000
-----

```

To determine the flux-density of the sources at each observing frequency it is necessary to correct for the primary beam attenuation at their positions. Table 1 illustrates the process, described in the caption. Note that the actual spectral indices of the two sources as calculated from the corrected flux-densities (-0.86 , -0.74) differ from the values returned by *uvsfitt* (-1.35 , -0.91). The effect of the frequency-dependent primary beam attenuation is to steepen the apparent spectrum of sources that are displaced from the field centre.

3.3 Adding artificial sources to visibility data

Mark Wieringa (private communication) has used *uvsfitt* to add artificial sources to visibility data, using the task's ability to produce a residual

Table 1: Flux-density determination for the two example sources ATPMN J000041.5-392557 and J000046.0-392234. *uvsfits* returns the flux-density (S_{f_0} , given below in mJy) at the reference frequency and the apparent spectral index α_0 . In the example $f_0 = 4.8$ GHz. From these we determine the apparent flux-densities at the two band centres $S_{4.8}$ and $S_{8.64}$. $\Delta\theta$ is the distance of each source from the field centre. We use MIRIAD task *pbplot* to find the primary beam attenuation at those positions ($P_{4.8}$ and $P_{8.64}$) and so estimate the true flux-densities of each source at both band frequencies.

Source	S_{f_0}	α_0	Apparent		$\Delta\theta$	Primary beam		Corrected	
			$S_{4.8}$	$S_{8.64}$		$P_{4.8}$	$P_{8.64}$	$S_{4.8}$	$S_{8.64}$
1	39.9	-1.35	39.9	18.0	2'32	0.87	0.65	45.9	27.7
2	45.5	-0.91	45.5	26.7	1'30	0.96	0.87	47.4	30.7

dataset. The residual R is calculated as

$$R(u, v, f) = V(u, v, f) - M(u, v, f)$$

where V and M are the visibility data and model data respectively, both functions of the baseline vector (u, v) and frequency f . If the model visibilities are formed with a negative flux-density, the new dataset R will be equal to the input data with the addition of a positive source.

To run *uvsfits* in this way, fix all parameters so that no fitting is performed. Below are example inputs that will produce a new visibility file for each input band, both containing a new point source whose flux-density at 4.8GHz is 80 mJy and apparent spectral index is $\alpha = -0.1$.

```

Task:    uvsfit
vis      = 0000-3923_pp_c.4800,0000-3923_pp_c.8640
stokes   = i
object   = point
spar     = -0.08,-1.3,-25.2
fix      = fxy
sspar    = -0.1,0.0,0.0
sfix     = 012
freqref  = 4.8
out      = plusfake.4800,plusfake.8640
options  = residual

```


A Algorithm

In both *uvfit* and *uvsfit*, estimates of source parameters are made through the minimisation of the least-squared differences between the measured visibilities and those computed from the source model.

Given a set of observed visibilities, each measured at a point (u, v) that is the projected baseline vector expressed in wavelengths, and at a frequency f , we write

$$V_i = V(u_i, v_i, f_i)$$

There are $n = n_b \times n_f$ visibility measurements, where n_b and n_f are the number of baselines and the number of frequency channels respectively.

Given a model source, we can compute model visibilities

$$M_i = M(u_i, v_i, f_i)$$

at each point in (u, v, f) space for which we have visibility measurements. The model visibilities can be calculated as the sum of visibilities for several model sources. The fitting process is the adjustment of model source parameters to minimise the quantity $\sum |V_i - M_i|^2$. Both tasks use the Levenberg-Marquardt algorithm for function minimisation. The diagonal terms of the covariance matrix are returned as the variances in fitted parameter values.

B Source models

Two commonly used models defined by *uvfit* and *uvsfit* are the point source and the two-dimensional gaussian source with half-power extent described by the major and minor axes and position angle of an ellipse. Other models supported are the disk, the shell and the ring.

B.1 Point source

For the point source with flux-density S and position (l, m) relative to the phase centre, *uvfit* computes each model visibility as:

$$M(u, v) = S(\cos \theta + i \sin \theta)$$

where

$$\theta = 2\pi(ul + vm)$$

In *uvsfit*, the flux-density is expressed as a function of frequency, and new parameters are introduced to describe the spectral shape. The model source is now a function of S_0 , the flux-density at a reference frequency f_0 , position relative to phase centre (l, m) , and spectral shape parameters $(\alpha_0, \alpha_1, \alpha_2)$.

The model visibilities are then:

$$\begin{aligned}
 M(u, v, f) &= S(f)(\cos \theta + i \sin \theta) \\
 \theta &= 2\pi(ul + vm) \\
 S(f) &= S_0 \left(\frac{f}{f_0} \right)^\alpha \\
 \alpha &= \alpha_0 + \alpha_1(f - f_0) + \alpha_2(f - f_0)^2
 \end{aligned}$$

B.2 Gaussian source

Model visibilities for the two-dimensional Gaussian source with flux-density $S(f)$, position (l, m) , major and minor axes and position angle (a, b, ϕ) , and spectral shape described by $(\alpha_0, \alpha_1, \alpha_2)$ are calculated by *uvsfits* as:

$$\begin{aligned}
 M(u, v, f) &= S(f)(\cos \theta + i \sin \theta) \exp\left(-\frac{\pi^2}{4 \log 2} \beta\right) \\
 \theta &= 2\pi(ul + vm) \\
 S(f) &= S_0 \left(\frac{f}{f_0} \right)^\alpha \\
 \alpha &= \alpha_0 + \alpha_1(f - f_0) + \alpha_2(f - f_0)^2 \\
 \beta &= (b(u \cos \phi - v \sin \phi))^2 + (a(u \sin \phi + v \cos \phi))^2
 \end{aligned}$$

B.3 Disk, ring and shell source models

The expressions for model visibilities for the disk, shell and ring models are similar to those for the two-dimensional gaussian model. Both disk and ring are described by major and minor axes and position angle (a, b, ϕ) in addition to the (frequency dependent) flux-density and position parameters. The shell source models the two-dimensional projection of a thin spherical shell described by its diameter a . The visibility expressions are:

$$\begin{aligned}
 M(u, v, f) &= S(f)(\cos \theta + i \sin \theta) \mathcal{F}(\beta) \\
 \mathcal{F}(\beta) &= 2 \frac{J_1(\sqrt{\beta})}{\sqrt{\beta}} && \text{Disk} \\
 \mathcal{F}(\beta) &= J_0(\pi \sqrt{\beta}) && \text{Ring} \\
 \mathcal{F}(\beta) &= \frac{\sin(\pi \sqrt{\beta})}{\pi \sqrt{\beta}} && \text{Shell} \\
 \beta &= (b(u \cos \phi - v \sin \phi))^2 + (a(u \sin \phi + v \cos \phi))^2 && \text{Disk, ring} \\
 \beta &= (au)^2 + (av)^2 && \text{Shell}
 \end{aligned}$$

Here J_0 and J_1 are Bessel functions of the first kind.

Acknowledgements

The new task *uvsfitt* is a relatively minor variant of *uvfitt*, which was written by Bob Sault (the principle author of MIRIAD) with some contributions by Bryan Gaensler. I thank Bob Sault and Mark Wieringa for advice on the design of the new task.