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AT Compact Array : Effect of Deviations from a Linear "East-West" Array

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Summary

For a 2D Fourier transform relationship between the visibility and map planes, the antenna phase centres must lie on a plane perpendicular to the earth's rotation axis. For the AC Compact Array a reasonable limit on elevation differences along an east-west line is $\sim$ 5 m.

More stringent limits are placed on deviations by the requirement that antennas be colinear so all baselines are at the same hour angle and redundancies are preserved. Maximum deviations are $\sim$ 0.2 m.

Elevation differences and the earth's curvature may make it difficult to site all compact array antenna stations along a straight east-west line. There are two effects to consider; deviations from a straight line, and deviations from a plane parallel to the equator.

If the visibility measurements are taken on a d-dimensional surface rather than a plane, one must either use a 3-D Fourier transform, or project the surface onto a plane and use a 2-D transform. The effect of projecting the 3-D measurements is to produce phase errors proportional to distance squared from the phase centre. Letting (u,v,w) be the coordinates of a station with u,v in the equatorial plane and w towards the pole, the error is

$$\phi_{err} \cong \pi r^2 w \text{ radians}$$

where r is the distance from the phase centre in radians and w is measured in wavelengths. Note that (u,v) may be anywhere within a plane of constant w, not necessarily colinear, and the earth's rotation will sweep out a planar surface.

Following the earth's curvature ($\sim$ 3 m drop over 6 km) along a line of constant latitude (east-west) produces a curved baseline; elevation differences along that curve bring one out of the plane. For an elevation difference of h metres, a polar component $w = \frac{h}{\lambda} \sin$ (latitude) results.

A sloping linear array can be placed in the plane of constant w by rotating it out of the true east-west direction. For a gradient g (positive for east high), the baseline must be oriented at an angle ψ south of east where

$$\psi = g \tan \text{(latitude)}$$

(Note: the latitude of Culgoora is negative!).

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The maximum tolerable elevation difference may be expressed in terms of ϕ_{err}, r and λ. r may be expressed in terms of the fraction F of the primary beam response out to the first null: $r = F\lambda/D$ radians, where D is the diameter of the antenna element. Then for Culgoora (latitude = -33°) and D = 22 m we have

$$h \cong \frac{280\ \phi_{err} \text{ metres}}{F^2\ \lambda}$$

This is the elevation at which an error ϕ_{err} radians occurs for a point F primary beams from the field centre at wavelength λ m. The worst case is at the lowest frequency, so take F = 1 and λ = 1 m (300 MHz), and letting ϕ_{err} = 0.1 rad (5.7 degrees), the tolerable elevation difference is 28 metres. However, since the map errors are nearly equal to $\phi_{err}/2\pi$, this specification limits the dynamic range at F = 1 and 300 MHz to $\sim$ 18 dB. Since this type of error does not self-calibrate a tighter constraint could be justified. In order to reach 25 dB the maximum phase error is $\sim$ 0.02 rad, which gives a limit of $h < 5.4$ m.

If all antennas are in the same w-plane but not colinear, then baselines are not all at the same hour angle and possible redundancies may be lost. These effects may result in undesirable complications in image processing systems, particularly if special purpose (e.g. back-projection) processors are used. The magnitude of these errors can be estimated as follows:

For a field size of r radians, the distance between independent points in the U,V plane is r^{-1}. Clearly, baseline differences must be small compared to the inverse source sizes.

We want to determine the error in assuming two baselines are equal for redundancy when in fact one baseline has an error (Δu, Δv, Δw). Consider the comparison of two baselines, one correct (u_o, v_o, w_o) and the other with errors (u, v, w).

Now we require an error model of the form,

$$\phi_m = \phi_t + \theta_i - \theta j$$

so if an error in positioning a telescope is equivalent to a telescope based phase error compared to the assumed redundant position, then the redundancy method will extract that phase as a telescope error.

Consider a source at (x, y, z) not necessarily at the field centre (x_o, y_o, z_o). Then we observe a phase,

$$\phi_{m_o} = (x - x_o)\ u_o + (y - y_o)\ v_o + (z - z_o)\ w_o$$

for the accurate baseline (assume receiver components of θ_i are zero).

While for a baseline u, v, w we obtain:

$$\phi_m = xu + yv + zw - (x_o u_o + y_o v_o + z\, w_o)$$

$$\text{and} \quad \Delta\phi = \phi_m - \phi_{m_o} = x\ (u-u_o) + y\ (v - v_o) + z(w - w_o)$$

$$= x\ \Delta u + y\ \Delta v + z\ \Delta w$$

In other words $\Delta\phi$ is consistent with a phase error in the telescope with the baseline error. This phase error is variable since either (x, y, z) or (Δu, Δv, Δw) are variable depending on one's viewpoint.

Consider then two sources at (x_1, y_1, z_1), (x_2, y_2, z_2). Now we obtain

$$\Delta\phi_1 = X_1\ \Delta u + y_1\ \Delta v + \Delta z_1\ \Delta w$$

$$\Delta\phi_2 = x_2\ \Delta u + y_2\ \Delta v + \Delta z_2\ \Delta w$$

and two phases are required to correct the data.

Worst Case:

Two sources either side of field centre such that

$$\phi_{m_1} = + \phi_o$$

$$\phi_{m_2} = - \phi_o$$

The amplitude (if both sources are of equal flux S_o) is given by

$$S = 2S_o \cos\ \phi_o$$

Now consider the effects of $\Delta\phi_1$, $\Delta\phi_2$ on the "redundant" baseline,

$$\phi_{m_1} + \Delta\phi_{m_1} = \phi_o + x_1\ \Delta u + y_1\ \Delta v + z_1\ \Delta w$$

$$\phi_{m_2} + \Delta\phi_{m_2} - \phi_o + x_2\ \Delta u + y_2\ \Delta v + z_2\ \Delta w$$

and we get a measured fring amplitude,

$$S = 2S_o \cos \tfrac{1}{2}\ (2\phi_o - (x_1 - x_2)\Delta u = (y_1 - y_2)\Delta v + (z_1 - z_2)\Delta w)$$

$$= S_o\ [(\underline{x}_1 - \underline{x}_2).\Delta U]\ \sin\ \phi_o$$

$$+ \cos\ \phi_o$$

and when $\phi_o \div \frac{\pi}{2}$, the error is 1st order.

That is we get a confusion of amplitude and phase effects of order $\underset{\sim}{x}.\Delta\underset{\sim}{u}$ where $\underset{\sim}{x}$ is source extent and Δu is coordinate error.

For a given level of accuracy E, the baseline error $|\Delta u|$ must be less than

$$|\Delta\underset{\sim}{u}| < \frac{ED}{F} \text{ metres}$$

where D is the telescope diameter and F is the fraction of a primary beam spanned by the sources. For E = .003 (- 25 dB), D = 22 m and F = 1 we get

$$|\Delta\underset{\sim}{u}| < 0.066 \text{ m}$$

This requirement may be relaxed by a factor ~ 4 in practical cases for the following reasons:

1. If only one telescope is out of position, not all baselines will be affected, although errors may propagate in an unfortunate manner in the redundancy solution. Perhaps a factor two relaxation is possible.

2. The worst case described will only be present for one particular hour angle. A factor of approximately $\sqrt{2}$ reduction in ultimate effect will occur for a full synthesis.

3. The source chosen is a pathological case. Again perhaps a factor $\sqrt{2}$ reduction is relevant.

Furthermore, for single compact sources of small extent, $|\Delta y|$ is determined by the source extent and errors significantly greater than those above may be tolerated.

Finally, therefore, in order to exploit redundancy, we require offsets in antenna centre positions, both along and perpendicular to the array line, to be $\lesssim$ 0.2 m.