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USE OF REDUNDANCY TO DETERMINE PHASE ERRORS

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Use of Redundancy to Determine Phase Errors

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When obsevations with an array include repeated obsevations with the same baseline we term those obsevations redundant. The repeated observations may be made at the same time (simultaneous redundancy) or at different times (day-to-day redundancy). In both cases, if the choice of redundant spacings is suitable, the redundant measurements can be used to determine phase errors in the array. Here we take as examples, configurations relevant to the compact array of the AT, i.e. a 5 or 6 element array, with very little freedom of movement for the sixth element.

Simultaneous Redundancy.

(a) Equi-spaced triplets.

Since almost all the configurations of which we are aware can be completely analysed in terms of equi-spaced triplets, we shall first analyse that configuration in some detail. From observations with an equi-spaced triplet, given the phase errors of any 2 components, we can determine the third phase error. Consider the following configuration:

(1) (2) (3)

Since the spacing between aerials 1 & 2 is the same as the spacing between aerials 2 & 3, the phase of the measured visibilities should be the same, but because of phase errors, this will generally not be the case. Denote the phase of the measured visibility for aerials i & j by v(i,j), the true phase of this visibilty by t(i,j), and the phase error in aerial j by e(j), then:

$$v(i,j) = t(i,j) + e(i) - e(j)$$

and since $t(1,2)=t(2,3)$

$$v(1,2) = t(1,2) + e(1) - e(2)$$

$$v(2,3) = t(1,2) + e(2) - e(3)$$

Suppose e(1) & e(3) are known, then we can rewrite these equations as

$$t(1,2) - e(2) = v(1,2) - e(1)$$

$$t(1,2) + e(2) = v(2,3) + e(3)$$

where the quantities on the left hand side are to be determined, and those on the right hand side are known or measured. The solution of these equations is:

$$t(1,2) = 0.5 * (v(1,2) + v(2,3) + e(3) - e(1))$$

$$e(2) = t(1,2) - v(1,2) + e(1)$$

The only other case that we need consider is when the errors for the central aerial and one other are known, e.g. if e(1) and e(2) are known:

t(1,2) = v(1,2) - e(1) + e(2)

t(1,2) - e(3) = v(2,3) - e(2)

which yields t(1,2) directly and e(3) from:

e(3) = t(1,2) - v(2,3) + e(2)

This completes the demonstration that if any 2 of the errors are known, the third can be determined.

(b) Five element configurations

In determining the phase errors for an array of aerials we must recognise the following restrictions:

* Since phases must be measured relative to something, we can always take the phase error of 1 reference aerial to be zero.

* Since we cannot from internal consistency alone expect to determine both the absolute positions of the sources in the sky, and the pointing error of the array, we cannot eliminate a linear phase gradient from our results. For a linear array, we can take account of this by determining all the phase errors in terms of one unknown phase error, or we can arbitrarily assume that the phase error of a second aerial is also zero, but bear in mind that the observed distribution may have to be shifted. For errors which vary with time, such as those due to atmospheric changes, the shift will be different for different hour angles.

The following are believed to be all the linear configurations of 5 aerials for which the redundancy is sufficient to determine all but 2 phase errors. With 5 aerials we measure 10 spacings, and wish to determine 3 errors, therefore at most 7 of the spacings can be different. The number in parentheses is the number of different spacings measured. Where they are different mirror images are also shown.

	0 1 2 3 4	(4)
5 . 3 2 1 0	0 1 2 3 . 5	(5)
5 4 . 2 1 0	0 1 2 . 4 5	(5)
	0 . 2 3 4 . 6	(5)
6 . . 3 2 1 0	0 1 2 3 . . 6	(6)
6 . 4 . 2 1 0	0 1 2 . 4 . 6	(6)
7 . . 4 3 2 . 0	0 . 2 3 4 . . 7	(6)
7 . . 4 . 2 1 0	0 1 2 . 4 . . 7	(7)
8 . . . 4 . 2 1 0	0 1 2 . 4 . . . 8	(7)
8 . . . 4 3 2 . 0	0 . 2 3 4 . . . 8	(7)
8 . . 5 4 . 2 . 0	0 . 2 . 4 5 . . 8	(7)

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Only the last of the 6-component configurations can not be analysed completely by decomposition into triplets.

To illustrate how the others can be decomposed into triplets, we analyse the last of these examples, assuming at the outset that the 2 end aerials have zero phase errors. We denote by * aerials for which the phase error is known, by ? the aerial which is determined at this step, and by x aerials for which the error is known, but which are irrelevent for this step:

```
*  .  2  .  ?  5  .  .  *
*  .  ?  .  *  5  .  .  x
x  .  *  .  x  ?  .  .  *
x  .  x  .  x  x  .  .  x
```

Clearly the optimum configurations are the last 4 of the table which all give the maximum amount of new information for each day's observations.

(c) Six-element configurations

The only 6-element configurations which interest us have 5 elements on the three-kilometre track and one at the 6-kilometre point. Therefore we need only consider the following type of configuration:

```
1  2  3  .  5  .  .  .  8  .  .  .  .  .  .  .  16
```

where the spacing 1-5 must equal the spacing 5-6. The configuration of the first 5 aerials might be any of those enumerated above. Clearly this does not give us very many, significantly different configurations for the 6-element array. It therefore seems that the aerial at the 6 kilometre point will often have to be calibrated using day-to-day redundancy.

(d) Two-day sequences for six elements.

The following two day sequence is based on a unit spacing between stations of 13.39 m, giving a total of 224 stations on the 3 km of track. (224 is a multiple of 7 & 8). The numbers indicate aerial positions from 0 to 224 for the 3km track, or 448 for the 6 km point.

```
*    *    *         *                *                              *
0    28   56        112              224                            448

                                                                              (A)

*                   *       *    *   *                              *
0                   96      160  192 224                            448
```

This observing sequence gives the following spacings on the two days:

28*2	56*2	84	112*2	168	196	224*2	336	392	420	448
32*2	64*2	96*2	128	160	192	224*2	256	288	352	448

Except for 224*2 and 448 all spacings are different from one day to the next and both configurations taken separately are optimally redundent. Further more, because spacings 224 & 448 are observed on both days the pointing error for the second day can be matched to the pointing error on the first day. This can be done for each hour angle step by comparing the measured visibilty phases for the 224 spacing. Any discrepancy is removed by applying a suitable linear phase gradient to the second day to match the first.

This two day sequence is slightly less than optimum since two spacings are repeated. The following two-day sequence avoids this wasted redundancy. It is based on a unit spacing of 14.29 m, giving aerial positions from 0 to 210 on the 3 km track, and 416 to 420 at the 6 km "point" (total movement 60 km).

```
*   *    *       *              *                             *
0   26   52      104            208                           416
                                                                         (B)

*    *     *          *             *                              *
0    30    60         120           210                            420
```

These give the following spacings for the 2 successive days:

26*2	52*2	78	104*2	156	182	208*2	312	364	390	416
30*2	60*2	90*2	120	150	180	210*2	300	360	390	420

This 2-day sequence can be fully calibrated, including matching the phase gradient from one day to the next (because spacing 390 is repeated) but the distribution of spacings is less uniform than for the previous pair. This may not matter if the sequence is only the starting point for a longer series of observations, using the principles discussed in the next section, but would be unfortunate if the observations were to be used alone.

A third similar sequence is the following, based on a unit spacing of 3km/216=13.89m and requiring movement of 167m at the 6km point. This sequence requires much larger movement at the 6km point but gives more even coverage if the two days are taken on their own.

```
*              *        *    *    *                                 *
0              108      162  189  216                               432
                                                                              (C)
*             *        *     *    *                               *
0             90       150   180  210                             420
```

This configuration yields the following spacings:-

27*2	54*2	81	108*2	162	189	216*2	233	270	324	432
30*2	60*2	90*2	120	150	180	210*2	240	270	330	420

Appendix A gives details of how these 2-day sequences may be selected.

Day-to-day Redundancy.

Suppose that we have made two day's observations with the sequence (A) above. Then all the spacings observed on those two days are fully calibrated. If we then observe with the following configuration:-

```
  <---- 96 -----> <--56---> <-32> <28> <--------------224-------------->
  *               *        *     *    *                                  *
  12              108      164   196  224                                448
```

which yields spacings:-

28	32	56	60	88	96	116	152	184	212
224	252	284	340	436					

This configuration has been chosen with 5 spacings (4 for a 5-element array) which duplicate spacings measured on previous days. Since the phases for these 5 (4) spacings are already known they can be used to determine the phase errors of 5 (4) aerials relative to 1 reference aerial. There are then 10 (6) fully callibrated new spacings. Note that when this method is used there is only one set of pointing errors (which still vary with hour angle) common to all observing days. Consequently, after d observing days, there are 10d (6d) quantities measured for each hour angle, and only one unknown quantity, the pointing error, to be determined by self cal or some other technique such as matching centroids.

Why use redundancy?

If self calibration works so well for other observatories, why should we use redundancy for the Australia Telescope? The answer, I believe, is that with our small number of aerials, we cannot expect self cal. to work as well as it does for 27 aerials at the VLA, or 12 aerials at Westerbork. Self cal., like phase closure, works much better with >10 aerials than <10 aerials. We can see this from the following argument. In general, from observations with n aerials we can measure n(n-1)/2 spacings, but we need to determine (n-1) phase errors plus n(n-1)/2 visibilities. Hence we require self cal. to determine (n-1)(n+2)/2 quantities from n(n-1)/2 measurements. The ratio of these two numbers is (n+2)/n, which is close to 1 for large n, but significantly larger for n=5 (1.4) or n=6 (1.33). With careful use of redundancy the ratio of quantities determined to quantities measured can be reduced to something very close to 1, at the expense of extra observing time. This will certainly make much more accurate mapping possible.

Penalties_for_Designing_Redundancy_into_the_Array?

1) It seems desirable to adjust the unit spacing to suit one of the two-day sequences such as (A), (B), or (C). (See also Appendix A). This requires that the unit spacing should be 13.39, 14.29, or 13.89m respectively, giving 224, 210, or 216 steps in 3km. Further investigation may remove this restriction, although, particularly for case (B), the adjustment is very small.

Decreasing the unit spacing, increases the number of days needed to form a full synthesis from 20 to 21 or 22 days. However it should be recalled that the only reason for performing a full synthesis is to obtain a higher dynamic range, which may fail without the calibration afforded by redundancy.

2) The array must be a "grating" array, with the consequent grating lobes. Again it is my private belief that redundancy will do more for improving dynamic range than randomising the array.

3) In redundancy mode the observing time is increased by a factor ~10/6. However it should be possible to find a set of station locations which permit both redundant and non-redundant observations. If this is the case then having the option of redundant observations available does not cause any penalty at all.

The basis for this optimism is the folowing: It is essential, for observations with the 6km array that there be a sufficient number of stations. For example 2 stations at the 6km point and 35 on the 3km track mean that only 2x35=70 spacings in the range 3 to 6km can ever be observed. With 35 stations we can form 595 spacings in the range 0 to 3km, but only 200 of these can be different (assuming a 15m unit spacing). Therefore every spacing occurs an average of 3 times in the configuration, giving enormous scope for designing observing sequences using day to day redundancy. The only special consideration would therefore seem to be the need to provide a few days of simultanoeous redundancy, to calibrate a basic set of spacings on which to build day-to-day redundant configurations.

Appendix A

Method of selecting 2-day redundant sequences.

To build up sequences such as (A), (B), and (C) above we are limited by the following constraints.

The 3km array should be close to 3km long, because the spacing 1-5 must equal spacing 5-6; this is the only way that the aerial 6 can be calibrated.

The 3km array should be one of the configurations which gives 7 different spacings, listed in section (b). The length of the first of those is 7 times the shortest spacing and so we must choose this shortest spacing about 200/7=28.6 unit spacings long. The other configurations all have lengths equal to 8 times the shortest spacing, and so must be built up of multiples of about 200/8=25 unit spacings.

Consider now the following, generalised 2-day sequence:

```
*    *    *        *                  *                                   *
0    n   2n       4n                 8n                                  16n

*                  *        *     *     *                                   *
0                 3m       5m    6m    7m                                  14m
```

Possible values for n are close to 25, and possible values for m are close to 29:

n=	23	24	25	26	27	28	
8n=	184	192	200	208	216	224	
16n=	368	384	400	416	432	448	
m=	26	27	28	29	30	31	32
7m=	182	189	196	203	210	217	224
14m=	364	378	392	406	420	434	448

To minimise the length of track needed at the 6km point we must choose values of n and m such that 16n~14m. Also to relate the calibration from the first day to the calibration from the second day we need at least one common baseline. Since all the baselines on the first day are multiples of n and all the baselines on the second day are multiples of m, we require that the least common multiple of m and n be less than 400. For a given pair m & n, we set the larger of 7m or 8n equal to 3km - this fixes the unit spacing. |16n-14m|*unit is then the travel needed at the 6km point.

The following table lists some possible combinations:

n	m spacing	unit spacing	common travel	6km	example	comments
23	26	16.30	-	65.22		No common baseline.
23	27	15.87	-	158.7		No common baseline.
24	27	15.63	216	93.75		Large travel at 6km.
24	28	15.31	168,336	122.45		Large travel at 6km.
25	28	15.00	-	90.0		No common baseline.
25	29	14.78	-	88.6		No common baseline.
26	29	14.42	-	144.2		No common baseline
26	30	14.28	390	57.1	B	
27	30	13.89	270	111.1	C	Large movement at 6km.
27	31	13.82	-	27.6		No common baseline.
28	32	13.39	224,448	0	A	Redundant redundancy.
29	32	12.93	-	206.9		No common baseline