Some Implications of Self Calibration for Synthesis Array Configurations

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1. Many vs few telescopes?

The main question here is whether a simultaneous observation with N telescopes will reach a lower noise level than observations with fewer telescopes spread over a longer time. Of course, selfcalibration introduces such a difference as we show.

For an observation of an unresolved source the scatter in the estimated gains after selfcalibration is:

$$\sigma_g^2 = \sigma^2/S^2(N-3)$$

where σ is the noise in the selfcalibration integration time and S is the source flux (see workshop on synthesis Telescopes, chap. 13). Roughly speaking, for the selfcalibration to work we require that this be less than the scatter that we wish to eliminate. If not then the errors in the data are increased and a poorer map results. If this limit is obeyed then the final noise level in the map of the point source is increased over the coherent level by a factor $[(N-1)/(N-3)]^{\frac{1}{2}}$. However this latter effect could always be overcome by simply integrating longer. To summarize, for a given level of gain instability more telescopes allow selfcalibration on weaker sources; this cannot be made up by integrating longer with fewer telescopes.

If total collecting area of the array is held fixed and the number of elements varied, ie

$$A_n = A_{tot}/N$$

then we obtain the interesting result that

$$\sigma_g^2 = N^2/A_{tot}^2(N-3)$$

which has a minimum for N = 6 (see Figure 1).

This analysis is only applicable for a point source. When an extended source is used for selfcalibration more constraints are necessary and these have to be obtained by better uv coverage (or by use of redundant spacings). In this case, the above analysis will not apply and it is most likely that more telescopes will produce a better result. The acquisition of the additional coverage by multiple observation will also help constrain the source model even though it does not enhance the selfcal S/N directly.

2. Does redundancy help selfcalibration?

The WSRT map of 3C84 at λ6 cm (Noordam and deBruyn, Nature 299, 597) seems to indicate that redundancy considerably aids the (self) calibration of data. That the VLA has not achieved comparable dynamic range may seem to support this conclusion. However the following counter-arguments can be made.

1. Although a fully redundant array produces a model independent method of calibration, aside from position and amplitude uncertainties, this is not really an important consideration. The requirement for a good initial model in selfcalibration first arose in VLBI hybrid mapping where the coverage and data quality were both poor. Experience with MERLIN shows that with good quality data, by which is meant low closure errors, and moderate numbers of telescopes (≤4) convergence of a selfcalibration algorithm is nearby always possible starting from a point source model. Thus, the model in an algorithmic convenience for introducing image plane constraints. One might still argue that in some cases, particularly when an array "fully samples" a region of the sky at the maximum resolution the use of a deconvolution algorithm such as CLEAN or MEM is unnecessary. Such an attitude reflects an outmoded view of data analysis in which one makes an all purpose representation of the sky, using, for example, the direct transform, and thence deduces other parameters. The success of CLEAN and selfcalibration supports the contrary view that very great advantages arise when the problem is specified more completely. For example, the CLEAN algorithm allowed a great advance by specifying that large regions of the required image of the sky should be of zero brightness. We believe that this trend of specifying problems more completely should be continued.

2. On a more practical point a redundant array is very wasteful of telescope pairs. Let us consider the numerology of high dynamic range mapping. Suppose that the correlator gains are constant for a full track and that the antenna gains "switch" to new values T times. Then to calibrate the data we require $N(N-1)/2 + N.T$ complex numbers. These numbers must "come" from somewhere; in a redundant array they come from the redundancy whereas in the case of the VLA or MERLIN they come from the CLEAN algorithm's restrictions on allowed brightness distributions. One subtle and relevant point which should not go un-noticed is that any deconvolution algorithm implicitly recognizes redundancy in the sense that, within the limitations imposed by noise and gridding, an excess of sample points in one region of the u,v plane is used to correct for a deficiency elsewhere. The clinching argument is that if redundancy is used to get high dynamic range then even in low dynamic range mapping the maximum field of view is less than that of the non-redundant array for which CLEAN or MEM must be used to obtain small field, high dynamic range.

3. It seems probable that WSRT has achieved higher dynamic range than the VLA because of the superior stability of closure errors, not because of the redundancy. Tests with the VLA indicate that the maximum dynamic range of about 35dB is consistent with the short term amplitude and phase closure stability of about ¼%.

Self calibration gain error for array's with constant collecting area.