AT/17.3.1/011

FRINGE AND DELAY TRACKING AT ANTENNAS WITH INDEPENDENT LOCAL OSCILLATORS

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SUMMARY

With independent frequency standards meeeting the present specifications, if the frequency differences between the local frequency standards are measured once per day by semi real-time fringe checks, then the fringe rate at the output of the correlator can be kept below 0.01 Hz. This allows reasonable integration intervals. Post-correlation fringe fitting to calibrator observations made at intervals < 3000 s / (observing frequency in GHz) will then allow phase to be calibrated to a few degrees, apart from atmospheric effects. The computed LO phase tracking will be sufficiently accurate provided UT is available at each station to an accuracy of a few ms.

Variable phase sampling derived fror the local frequency standards will allow integrations of 3 hours without adjusting the delays at playback. The strongest constraint on clock errors relative to UTC is the computing time needed to find fringes at the semi real-time fringe checks.

NOTE: By the time of the completion of the final draft of this note the decision had been made to transmit the LBA local oscillator reference via satellite. Much of the material in the note is therefore not relevant to the A.T.

BACKGROUND

The rf signals arriving at the mixers on antennas #1 and #2 are written

$$\int df_r \quad A(f_r)\ \cos[2\pi f_r t + \psi(f_r)] \tag{1}$$

and

$$\int df_r \quad A(f_r)\ \cos[2\pi f_r(t-\tau_r) + \psi(f_r)] \tag{2}$$

where τ_r is the extra travel time to antenna #2's mixer. τ_r is assumed independent of frequency but is a function of time because of the rotation of the earth and because of any changes in atmospheric or instrumental rf delays.

The LO signals at the two mixers are

$$\cos(2\pi f_L t + \phi_1), \qquad \text{and} \tag{3}$$

$$\cos(2\pi f_L t + \phi_2). \tag{4}$$

ϕ_1 is taken as constant, but ϕ_2

(i) will vary if the two LO's are not phase locked, and
(ii) will be varied by any scheme for real-time phase tracking.

The IF signals are (rf above LO)

$$\int df_{IF} \quad A\ \cos[2\pi f_{IF} t + \psi - \phi_1] \tag{5}$$

$$\int df_{IF} \quad A\ \cos[2\pi f_{IF} t + \psi - 2\pi f_r \tau_r - \phi_2] \tag{6}$$

Rewrite (6) as

$$\int df_{IF} \quad A\ \cos[2\pi f_{IF} (t-\tau_r) + \psi + 2\pi f_L \tau_r - \phi_2]. \tag{7}$$

Equation (7) shows that the rf delay, τ, produces two different phase effects at IF. The first, $2\pi f_{IF}\tau_r$, is proportional to frequency and can be compensated by an IF delay; the second, $2\pi f_L \tau_r$, is independent of IF frequency and cannot be compensated by a delay at IF. With 'phase tracking at antennas' the LO phase ϕ_2 is changed to compensate this frequency independent phase term.

After the signals (5) and (7) are correlated, and the transform as a function of delay is calculated, any part of the term $2\pi f_{IF} \tau_{r}$ which has not been compensated by an IF delay (i.e. any 'delay tracking error') will cause a phase which is a linear gradient of frequency to be added to the visibilities. On the other hand any part of the term $2\pi f_{L} \tau_{r}$ which has not been compensated by a frequency independent phase correction (i.e. any 'LO phase tracking error') will add a frequency independent phase to the visibilities.

[Note that this separation assumes that all propagation, whether in space, in the atmosphere, or in the instrument, is non-dispersive.]

PHASE TRACKING WITH INDEPENDENT LOCAL OSCILLATORS

The system definitions for the Long Baseline Array call for both phase and delay tracking at the antennas. First consider phase tracking.

In the Compact Array, as a result of monitoring the round-trip phase delay, there are co-phased signals available at all antennas. The phase-rotated local oscillator signals required for fringe tracking are derived from these co-phased signals by adding to them signals whose phase is continiually being shifted under computer control. In addition to having co-phased signals available, this requires that time (UTC) be available at each antenna with sufficient accuracy to allow the calculation of this added signal to the required precision.

To apply this technique to the Long Baseline Array

(i) we must ensure that UTC to the required accuracy is available at each antenna, and

(ii) if local frequency standards are used at each antenna we must overcome the problem that these signals will not be co-phased

In most traditional VLBI no attempt is made to preserve phase - only closure phase. But for the AT LBA we wish to preserve phase, at least over the 300 km baselines originally planned to be connected by microwave links.

If the LO is supplied via satellite, and the round-trip phase path is monitored, then the problem becomes very like that for the Compact Array. For the CA it is envisaged that the LO phase in each 5 sec interval will be adjusted for the round-trip phase measured in the previous 5 sec interval. A similar procedure can be used for the LBA if the control data is transmitted via satellite. However, if the control data is sent ahead of time via a slower communication link, the corrections for round-trip phase must be made after correlation, whether the correlation is done in the Culgoora computer (for semi real-time fringe checks) or in the LBA correlator.

The main point of this note is to consider the case where the LO's are derived from local frequency standards at each antenna. These frequency standards will have high stability but not necessarily high accuracy. In other words they will all have slightly different, albeit very stable, frequencies. This is the problem addressed by Ray Norris in AT/17.3.1/006. In traditional VLBI these frequency offsets between stations are determined at playback and corrected by 'fringe rotation at the correlator'. To fringe track at the LBA antennas we would need to know the frequency offsets at the time of the observations with sufficient accuracy to be able to calculate and apply LO phase corrections which would ensure zero fringe frequency at the phase centre. The March LBA workshop suggested using the semi real-time fringe check facility to achieve this. What follows attempts to assess the possibility of achieving the required accuracy in this way.

If the standards were absolutely stable in frequency, but at different frequencies, one measurement of the frequency would be sufficient to determine the phase (referred to the desired frequency) for all time, and the LO phase rotators could be programmed to correct for this. In fact the frequency standards are to be accepted provided the square root of the two-sample Allan variance (Vessot,1976, p201 et seq.) - hereafter Allan standard deviation - of the fractional frequency variations lies below 2E-12 s / t(s) for averaging times t < 300s, reaches a minimum of about 5E-15 near t = 1000 s, and for longer times lies below 1E-14 at one hour and below 1E-12 at one month (ACY LBA Workshop - see figure). Interpolating yields an upper limit of 9E-14 at one day.

We assume that the rms phase deviation over a time t is t * the Allan standard deviation of the fractional frequency for an averaging time t (see e.g. Kartaschoff, 1979). Hence the above specification is equivalent to specifying that the phase errors (relative to a fixed, but possibly unknown, frequency) are to be

< 2E-12 s	(i.e. 0.7 degrees / GHz)	for t < 300 s,
about 5E-12 s	(i.e. 1.8 degrees / GHz)	at t = 1000 s,
< 4E-11 s	(i.e. 15 degrees / GHz)	at t = 1 hour, &
< 8E-9 s	(i.e. 8 rotations / GHz)	at t = 1 day.

Since 2E-12 s correspomds to 1 radian at 80 GHz such standards would hardly be useful for 115 GHz observations.

These specifications show that the notion of a fixed frquency with relatively small phase variations is correct only for short intervals of time. Over a day phase variations can amount to tens of rotations. The basic frequency can change by almost a part in E13 in a day, i.e. by as much as 0.01 Hz at 115 GHz. Thus it is not a case of establishing the frequency differences once per day and using calibrators to monitor small phase changes throughout the day (as will be done e.g. for the Compact Array to monitor the unpredictable propagation phase-delay variations). In this sense 'fringe tracking at antennas' is not possible with such independent frequency standards.

However, if

(a) the long term linear frequency trends are known for each standard, and
(b) the frequency differences between the standards are measured once per day, and
(c) the LO phase rotation at the antennas is adjusted to allow for these measured frequency differences,

then the fringe rates at the correlator will remain below 0.01 Hz. This will allow integrations of tens of seconds with no degradation. If a calibrator source is observed every (3000 s / observing freqency in GHz) then, from the above figures, it should be possible to correct the phase to an accuracy of a few degrees - apart from atmospheric effects. Even with calibrations every (1E4 s / observing frequency in GHz) i.e. every 20 min at 8 GHz, the rms phase deviations between calibrations would be only about 70 degrees so that interpolated values should be accurate to better than 10 degrees.

MEASUREMENT OF THE FREQUENCY DIFFERENCES BETWEEN STANDARDS

Now consider what semi real-time fringe checks are required each day to establish the frequency differences to the required accuracy. Assume that the baselines have already been determined to the needed accuracy, and consider determining the frequency difference from two fringe checks on a calibrator source, each of which determines a phase difference. For integration times < 300 s each phase measurement will have an error of 2E-12 s arising from the instabilities of the frequency standards themselves. This contributes a phase error of 0.1 * (freq / 10 GHz) radians.

The US VLB Array tape recorders allow for a block of 4 Mbits of data to be transmitted to the central site for the real-time phase check. For a system temperature of 50 K and two 22 m diameter antennas (giving 0.06 K/Jy each) the rms noise in the average of 4 Mbits of 1-bit data will be

(1.57/1.41) * (50/0.06) * SQRT (2/4E6) Jy

or 0.7 Jy. If we assume that we can use sources with correlated flux densities .GE. 7 Jy then the rms phase error per measurement should be

.LE. 0.1 radians	for freqs below 10 GHz
0.1 * (freq / 10 GHz) radianss	for freqs above 10 GHz

[With antennas of 64 m diameter and 22 m diameter the rms error would be reduced by a factor of 3].

In addition we must consider the effects of the atmosphere. I am not aware of detailed information on the effect of the atmosphere on baselines of hundreds of km. For the VLA baselines (see e.g. AT/10.3/004) the rms phase difference on time scales from minutes to hours at points D km apart is

360 * (fGhz/75) * (Dkm/10)**0.7 degrees.

For 2.3 GHz, extrapolating this formula to longer baselines yields

D	50 km	100 km	275 km
rms	35 deg	55 deg	110 deg

For Merlin Ray Norris reports that the rms does not increase appreciably from 50 km to 100 km, while for the PKS-TBB interferometer (d=275 km) he says the variations are all attributable to the frequency standards and are less than 360 deg/hour at 2.3 GHz.

If we have to contend with atmospheric phase differences of 100 degrees then we need observations separated by 300 sec (5 min) to measure the frequency difference with an error of 0.001 Hz. To transmit the 4 Mbits of data over a 9600 baud line will take about 500 sec, so a spacing of 10 minutes is reasonable. Three or four blocks of 4 Mbits should be sent at 10 minute intervals.

TIME ACCURACY REQUIRED FOR PHASE TRACKING

The computer program controlling the phase rotators on the local oscillators will compute the phase shift required for a certain UT1. Consider now the phase errors introduced if the local clock at the antenna has an error relative to UT1 of e (sec) at the start of an integration of duration T (sec). We assume that the change in the error, e, during the integration is insignificant for the present, and justify this later.

At the start of the observation the error in calculating the delay is

$$\tau(t+e) - \tau(t) = e\ \tau'(t) \qquad (8)$$

and at the end is

$$e\ \tau'(t+T). \qquad (9)$$

We now detemine the limit on the clock error, e. so that the change in the error of the calculated delay over the integration interval, namely

$$e\ [\tau'(t+T) - \tau'(t)] = e\ T\ \tau'' \qquad (10)$$

will not appreciably degrade the signal.

[The phase error itself will have to be determined at playback from observations of calibrators, and corrections applied after correlation, in the same way as for phase errors determined by the atmosphere etc.]

The differential error (10) has a maximum value (e.g. see AT/20.1.1/024 Page 6)

$$1.8\ E{-}14\ B\ e\ T \quad (s) \qquad (11)$$

where B (km) is the component of baseline parallel to the equator. For this to be less than 0.11 periods (2% signal loss) requires at 10 Ghz

$$B\ e\ T\ <\ 600$$

or for B = 200 km, T = 1000 s,

$$e\ <\ 3\ \text{ms}.$$

Establishing local time relative to UTC to this accuracy presents no problems. The discussion of delays (below) suggests that it is desirable to know local time reltive to UTC to a few microseconds. [Rick Forster (AT/20.1.1/024 Page 10 makes the interesting comment that UT1 relative to UTC is predictable to only about 5 ms.]

DELAY TRACKING WITH INDEPENDENT LOCAL OSCILLATORS

In the Compact Array 'delay tracking' is achieved by adjusting the time at which the signal is sampled at each antenna. The aim is to sample the same wavefront as it arrives at each antenna. Clearly this requires time (UTC) to be known at all antennas with a differental error between antennas of less than one sample interval (2 ns). This can be achieved for the Compact array by tying the sampling to the co-phased signals used for generating the local oscillator signals. For the Long Baseline Array with independent frequency standards that is not possible (even though the shortest sampling interval is now 31 ns.)

However if for the LBA the sampling waveforms are derived from the local frequency standards using the Compact Array techniques, then at each antenna the sampling rate will be sufficiently accurate that, over a period of 3 hours, the difference between the actual sampling times and the ideal times will not change by more than 0.1 sample intervals (3 ns). Hence at playback correlations from data spans of 3 hours can be accumulated without adjusting the delays bewteen tape units.

To see the truth of the above first consider changes, during the integration interval, of the differences between the clocks at different antennas. If the daily fringe checks discussed above establish the frequency offsets between the local frequency standards to 0.001 Hz at 10 GHz, i.e. to a part in 1E13, then time slippage due to this cause will be no more than 10 ns per day, comparable with the random variations of 8 ns on a time scale of a day (page 4). Overall the relative time error between pairs of antennas should vary by less than 1 ns / hour.

Next consider the accuracy with which antenna time must be known relative to UTC. From the discusion above for phase tracking [see eqn. (11)] it is clear that quite large errors in antenna time relative to UTC will not cause appreciable errors in the claculation of the sampling rate. The real limit to allowable errors relative to UTC is determined by the time needed to search for fringes at the real time fringe checks. Timing to a few microseconds is desireable from this point of view.

CONCLUSION

Phase and delay tracking at the antennas is possible with independent frequency standards but the tracking is not exact. More post-correlation processing will be needed than with the Compact Array.

A first pass through the tapes will be needed to determine the phase and delay errors of calibrator sources. Corrections for intervening observations will then be derived by interpolation. In a second passs through the tapes all sources can be analysed, applying corrections in the array processor for the phase and delay errors by adding a phase and a phase gradient to the to the Fourier transforms.

SPECIFICATIONS FOR UWA CAVITIES FOR LBA