CALIBRATION OF THE AUSTRALIA TELESCOPE.

July 26 1985. AT/10.1/039.

Max Komesaroff.

SCHEME.

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I The problem.

Now that it has been decided that the AT is to have linearly polarized feeds, it seemed desirable to set out in some detail how such a system might be calibrated. The following discussion goes over some of the material which has previously been dealt with in AT/21.3.1/011 AT/21.3.1/011and AT/20.1.1/005 but goes into considerably greater detail on the questions of the "primary" and "secondary" calibration, particularly the former.

The purpose of synthesis interferometry is to map the distribution of the four Stokes parameters over a radio source. At any one time, any two-element interferometer of an array provides an estimate of one spatial Fourier component, measured with respect to the interferometer baseline, of each of the four Stokes parameters. We designate these Fourier components by the symbols $\underline{I}$,$\underline{Q}$,$\underline{U}$, and $\underline{V}$. In general $\underline{I}$,$\underline{Q}$,$\underline{U}$, and $\underline{V}$ are complex. The main effects which make accurate estimates of $\underline{I}$, $\underline{Q}$, $\underline{U}$ and $\underline{V}$ very difficult are variable receiver gains, cross coupling between feeds and variable differential atmospheric phase delays between antennas. The following discussion shows that provided the feed systems are made mechanically rigid so that cross-couopling effects are stable in time, it is possible to use a simple calibrating system, based on a set of noise tubes, to make extremely accurate estimates of

<u>q</u> ≅ <u>Q</u>/<u>I</u> <u>u</u> ≅ <u>U</u>/<u>I</u> and <u>v</u> ≅ <u>V</u>/<u>I</u>, in the presence of variable receiver gains and variable atmospheric phase delays. Accurate determination of <u>I</u> may still require a technique a technique such as SELFCAL.

It is also shown that accurate measurements of cross-coupling and of receiver gains may be made by means of a "primary" calibration which involves observing an unresolved calibrator source of known flux density and polarization over a wide range of hour angles. This calibration should be quite effective even in the presence of large atmospheric phase fluctuations.

II. Measuring the "polarization matrix."

(a) An "ideal" system.

(i) Equatorial mounts.

The AT will have alt-az mounted antennas, and feeds which are fixed with respect to the telescope structure, but for the moment we consider the simpler case of equatorial mounts.

Consider an "ideal" two-element interferometer employing equatorial mounts and orthogonal linearly polarized feeds. If the feeds are oriented along the Y and X axes, which point to celestial North and East respectively, the system produces four outputs proportional to the correlations $\langle e_{AX}.e^*_{BX}\rangle$, $\langle e_{AX}.e^*_{BY}\rangle$, $\langle e_{AY}.e^*_{BX}\rangle$, and $\langle e_{AY}.e^*_{BY}\rangle$, given by

$$\langle e_{AX}.e^*_{BX}\rangle = 1/2[\underline{I} - \underline{Q}]$$

$$\langle e_{AX}.e^*_{BY}\rangle = 1/2[\underline{U} - i\underline{V}]$$

$$\langle e_{AY}.e^*_{BX}\rangle = 1/2[\underline{U} + i\underline{V}]$$

$$\langle e_{AY}.e^*_{BY}\rangle = 1/2[\underline{I} + \underline{Q}]$$

Here e_{AY} and e_{AX} are proportional to the Y and X components of the electric vector from a distant radio source as received at antenna a; e_{BY} and e_{BX} are proportional to the components received at antenna b. The angle brackets denote time averages and the asterisk denotes a complex conjugate. These results may be written in matrix notation thus

$$\left\langle \begin{pmatrix} e_{AX} \\ e_{AY} \end{pmatrix} (e^*_{BX} \ e^*_{BY}) \right\rangle = \langle e_A . e^\dagger_B \rangle$$

$$= \begin{pmatrix} \langle e_{AX} . e^*_{BX} \rangle & \langle e_{AX} \rangle . \langle e^*_{BY} \rangle \\ \langle e_{AY} . e^*_{BX} \rangle & \langle e_{AY} \rangle . \langle e^*_{BY} \rangle \end{pmatrix} \qquad \text{II 1a}$$

$$= P_{AB}$$

where the matrix P_{AB} is defined by

$$P_{AB} = 1/4 \begin{pmatrix} \underline{I}-\underline{Q} & \underline{U}-i\underline{V} \\ \underline{U}+i\underline{V} & \underline{I}+\underline{Q} \end{pmatrix} \qquad \text{II 1b}$$

Clearly it is the matrix P_{AB} which we wish to recover from our observations.

The following should be noted:

1. If each element of a 2x2 matrix is multiplied by 1/2, the matrix itself (and hence its determinant) is multiplied by $(1/2)^2 = 1/4$. Hence the factor of 1/4 in the definition of P_{AB}.

2. The character † denotes the conjugate transpose.

3. Here and subsequently a square matrix is denoted by a bold-face upper case character and a vector (a row or column matrix)by a bold-face lower-case character.

(ii) Alt-azimuth mounts.

The AT antennas will have alt-az mounts and the feeds will be fixed with respect to the telescope structure. One feed (the y) will be oriented at position angle $\pi/4$ radian with respect to the local vertical , and the other (the x) at $-\pi/4$ (see AT/01.13/004e and AT/10.1/037). That is, the x y coordinate system will be rotated through an angle $\chi + \pi/4$ with respect to the X Y system , χ being the parallactic angle. The parallactic angle depends on the position of the source being observed and also on the antenna location.

Denoting the field components for equatorial and alt-az mounts by upper and lower case subscripts respectively, the relation between the electric field components in the two coordinate systems is given by

$$\begin{pmatrix} e_{ax} \\ \\ e_{ay} \end{pmatrix} = \mathbf{R}_a \begin{pmatrix} e_{AX} \\ \\ e_{AY} \end{pmatrix} \qquad \begin{pmatrix} e_{bx} \\ \\ e_{by} \end{pmatrix} = \mathbf{R}_b \begin{pmatrix} e_{BX} \\ \\ e_{BY} \end{pmatrix} \qquad \text{II 2}$$

The rotation matrices $\mathbf{R}_a$ and $\mathbf{R}_b$ are defined by

$$\mathbf{R}_a = \begin{pmatrix} \cos(\chi_a+\pi/4) & -\sin(\chi_a+\pi/4) \\ \sin((\chi_a+\pi/4) & \cos(\chi_a+\pi/4) \end{pmatrix} \qquad \text{II } 3$$

$$\mathbf{R}_b = \begin{pmatrix} \cos(\chi_b+\pi/4) & -\sin(\chi_b+\pi/4) \\ \sin((\chi_b+\pi/4) & \cos(\chi_b+\pi/4) \end{pmatrix} \qquad \text{II } 4$$

The set of measured quantities is given by the correlation matrix

$$\mathbf{S}_{ab} = \langle \mathbf{e}_a . \mathbf{e}^{\dagger}{}_b \rangle$$
$$= \begin{pmatrix} \langle e_{ax} . e^*{}_{bx} \rangle & \langle e_{ax} . e^*{}_{by} \rangle \\ \langle e_{ax} . e^*{}_{by} \rangle & \langle e_{ay} . e^*{}_{by} \rangle \end{pmatrix} \qquad \text{II } 5$$

Taking account of eqs.(II 1(a), 1(b), 2, and II 5), we can write

$$S_{ab} = R_a \begin{pmatrix} \langle e_{AX} \cdot e^*_{BX} \rangle & \langle e_{AX} \cdot e^*_{BY} \rangle \\ \langle e_{AY} \cdot e^*_{BX} \rangle & \langle e_{AY} \cdot e^*_{BY} \rangle \end{pmatrix} R_b^{-1}$$

$$= R_a \begin{pmatrix} \frac{\underline{I} - \underline{Q}}{2} & \frac{\underline{U} - i\underline{V}}{2} \\ \frac{\underline{U} + i\underline{V}}{2} & \frac{\underline{I} + \underline{Q}}{2} \end{pmatrix} R_b^{-1}$$

$$= R_a P_{ab} R_b^{-1}$$

II 6

(For a rotation matrix such as R_b, the transpose and reciprocal are identical.)

The elements of the matrix Σ_{ab} are given in Appendix I.

In this "ideal" case then we can recover the required matrix P_{ab} by the simple matrix operation represented by

$$P_{ab} = R_a S_{ab} R_b^{-1}$$

(b) The practical case.

There are three effects which were neglected in the above simplified discussion and which must be taken into account in practice if the highest measuring accuracy is to be acheived. these are

1. Gain differences between the channels ax, ay bx and

by.

2. Cross-coupling.

3. Phase shifts imposed on the incoming signals by the ionosphere and the atmosphere; in general these will be different for the two antennas.

As a result of cross-coupling the voltage v_x to the correlator from the x feed will contain, in addition to the main component proportional to e_x, a smaller component proportional to e_y. Similarly the voltage v_y from the y feed will contain a small component proportional to e_x. Taking into account all of these effects we can therefore write

$$v_{ax} = g_{ax} (e_{ax} + a_{ax}e_{ay}) \exp i\psi_a$$

$$v_{ay} = g_{ay} (e_{ay} + a_{ay}e_{ax}) \exp i\psi_a$$

$$v_{bx} = g_{bx} (e_{bx} + a_{bx}e_{by}) \exp i\psi_b$$

$$v_{by} = g_{by} (e_{by} + a_{by}e_{ax}) \exp i\psi_b$$

The cross-coupling coefficients a_{ax}, a_{ay}, a_{bx}, a_{by} will in general be complex. It is expected that the major contribution to these terms will be from the feeds, resulting from slight misorientations and from the fact that the accepted polarizations are not pure linears, but rather highly elongated ellipticals. A very much smaller contribution, due to coupling between receivers, may also be expected. From AT/21.3.1.1/003 the cross-polarization response of the feeds is expected to be lower by 26 or more db than the main response.

Thus $|a_{ax}|, |a_{ay}|, |a_{bx}|, |a_{by}| \lesssim 1/20$.

The complex gain terms g_{ax}, g_{by}, etc. represent the combined antenna and receiver gains.

The terms ψ_a and ψ_b are phase delays due to the ionosphere and lower atmosphere. These phase delays are expected to be the same, to very high accuracy, for the two orthogonal polarizations.

To take all of these effects into account we represent the four measured complex correlations by the matrix Σ_{ab} rather than S_{ab} (given by equations II 6). Σ_{ab} is defined by

$$\begin{aligned}
\Sigma_{ab} &= \langle v_a . v^{\dagger}_b \rangle \\
&= G_a A_a S_{ab} A^{\dagger}_b G^{*}_b \exp 2i\delta\psi \\
&= G_a A_a \langle e_a . e^{\dagger}_b \rangle A^{\dagger}_b G^{*}_b \exp 2i\delta\psi \\
&= G_a A_a R_a P_{ab} R_b^{-1} A^{\dagger}_b G^{*}_b \exp 2i\delta\psi \qquad \text{II 7}
\end{aligned}$$

where $\delta\psi = \psi_a - \psi_b$ and

$$\mathbf{A}_a = \begin{pmatrix} 1 & a_{ax} \\ a_{ay} & 1 \end{pmatrix} \quad ; \quad \mathbf{A}^{\dagger}_b = \begin{pmatrix} 1 & a_{by}^* \\ a_{bx}^* & 1 \end{pmatrix} \qquad \text{II 8}$$

$$\mathbf{G}_a = \begin{pmatrix} g_{ax} & 0 \\ 0 & g_{ay} \end{pmatrix} \quad ; \quad \mathbf{G}^*_b = \begin{pmatrix} g^*_{bx} & 0 \\ 0 & g^*_{by} \end{pmatrix} \qquad \text{II 9}$$

The formal solution of eq (II 7) is of course

$$\mathbf{P}_{ab} = \mathbf{R}_a^{-1}\ \mathbf{A}_a^{-1}\ \mathbf{G}_a^{-1}\ \boldsymbol{\Sigma}_{ab}\ (\mathbf{G}^*_b)^{-1}\ (\mathbf{A}^{\dagger}_b)^{-1}\ \mathbf{R}_b\ \exp{-2i\delta\psi}.$$

Here the rotation matrices $\mathbf{R}_a$ and $\mathbf{R}_b$ depend only on the parallactic angles χ_a and χ_b, (see Eqs II 3, 4) and thus are easily calculated each time they are required. The cross-coupling matrices $\mathbf{A}_a$ and $\mathbf{A}_b$ will have been calculated during a previous "primary" calibration, (see Section III b) and are expected to remain stable during an observing session.

However the elements of the diagonal gain matrices $\mathbf{G}_a$ and $\mathbf{G}_b$ will almost certainly vary during an observing session. Their values must be monitored during a "secondary" calibration which takes place concurrently with the observations.

III Calibrating and correcting the measurements.

(a) The "secondary" calibration.

(i) Calibrator sources.

In principle the gains of the system may be calibrated by interspersing each observing session with observations of an unresolved calibrator source of known position and for which the four Stokes parameters (and hence the matrix $\mathbf{P}_{ab}$) are known. This can be seen as follows. We define the matrix $\mathbf{M}_{ab}$ by

$$\mathbf{M}_{ab} = \mathbf{A}_a \mathbf{R}_a \mathbf{P}_{ab} (\mathbf{R}_b)^{-1} \mathbf{A}^{\dagger}{}_b$$

From our previous discussion it follows that for a known calibrator source all four elements of the matrix $\mathbf{M}_{ab}$ are known. Eq(II 7) may be written

$$\mathbf{\Sigma}_{ab} = \mathbf{G}_a \mathbf{M}_{ab} \mathbf{G}^{*}{}_b \exp 2i\delta\psi$$

Since $\mathbf{\Sigma}_{ab}$ represents our measurements it follows that for a known calibrator source the only unknowns in thos equation are the scalar term $\exp 2i\delta\psi$ and the two $\mathbf{G}$ matrices.

Remembering that these latter are both diagonal we may rewrite this equation thus

$$\Sigma_{ab} = \begin{pmatrix} g_{ax}g^*_{bx} M_{xx} & g_{ax}g^*_{by} M_{xy} \\ g_{ay}g^*_{bx} M_{yx} & g_{ay}g^*_{by} M_{yy} \end{pmatrix} \exp i\delta\psi$$

The four elements of the **M** matrix being known, we can calculate the four required gain terms

$g_{ax}g^*_{bx} \exp i\delta\psi$, $g_{ax}g^*_{by} \exp i\delta\psi$,
$g_{ay}g^*_{bx} \exp i\delta\psi$, and $g_{ay}g^*_{by} \exp i\delta\psi$.

It should be noted that we cannot separate the phase component due to the complex gain product terms such as $g_{ax}g^*_{bx}$ from the component ($\delta\psi$) due to the ionosphere and the atmosphere, but that in any case this separation is not required for calibration purposes. However we can estimate the complex gain ratios

$$\left.\begin{aligned} \Gamma_a &= g_{ay}/g_{ax} \\ \Gamma_b &= g^*_{by}/g^*_{bx} \end{aligned}\right\} \text{II } 10$$

These ratios are independent of the differential atmospheric phase term $\exp i\delta\psi$. Furthermore they can be monitored with extremely high accuracy using a system based on noise tubes and described in Section IIIa(ii) below. In this way it is possible to achieve a highly accurate "polarization calibration" as the following argument shows.

From the definitions of $\mathbf{G}_a$ and $\mathbf{G}_b$ (eqs II 9) and of Γ_a and Γ_b (eqs II 10) we may rewrite eqs(II 7) in the form

$$\Sigma_{ab} = (g_{ax} g^*_{bx})^2 \, \mathbf{\Gamma}_a \, \mathbf{A}_a \, \mathbf{R}_a \, \mathbf{P}_{ab} \, \mathbf{R}_b^{-1} \, \mathbf{A}^\dagger_b \, \mathbf{\Gamma}^*_b \exp i\delta\psi$$

where

$$\mathbf{\Gamma}_a = \begin{pmatrix} 1 & 0 \\ 1 & \Gamma_a \end{pmatrix} \quad \mathbf{\Gamma}^*_b = \begin{pmatrix} 1 & 0 \\ 1 & \Gamma^*_b \end{pmatrix} \qquad \text{II 11}$$

The solution of this equation may be written

$$(g_{ax} \, g^*_{bx})^2 \, \mathbf{P}_{ab} \exp 2i\delta\psi =$$
$$\mathbf{\Gamma}_a^{-1} \, \mathbf{A}_a^{-1} \, \mathbf{R}_a^{-1} \, \Sigma_{ab} \, \mathbf{R}_b \, (\mathbf{A}^\dagger_b)\text{-}1 \, (\mathbf{\Gamma}^*_b)^{-1}$$

For a source whose polarization is being measured, all the matrices on the right hand side are known, and thus we can calculate

$$(g_{ax} \, g^*_{bx})^2 \, \mathbf{P}_{ab} \exp 2i\delta\psi,$$

which yields

$g_{ax} \, g^*_{bx} \underline{I} \exp i\delta\psi$, $\quad g_{ax} \, g^*_{bx} \underline{Q} \exp i\delta\psi$,

$g_{ax} \, g^*_{bx} \underline{U} \exp i\delta\psi$, and $g_{ax} \, g^*_{bx} \underline{V} \exp i\delta\psi$, and from these we can calculate the complex ratios $\underline{q} = \underline{Q}/\underline{I}$, $\underline{u} = \underline{U}/\underline{I}$, and $\underline{v} = \underline{V}/\underline{I}$ which are independent of the term exp $i\delta\psi$.

Note: It was stated at the outset that this method works <u>in principle</u>. The method was outlined mainly for purposes of illustration. Most convenient calibrator sources which can be repeatedly observed during an observing session will probably not be strong enough to accurately determine the complex gain products $g_{ax}g^*_{by}$ and $g_{ay}g^*_{bx}$, which are estimated from the off-diagonal terms of the matrices, which themselves are determined primarily by the polarized components of intensity. Certain "scaling factors" relating the noise tube measurements to the above gain products (see Sec.IIIa(ii) below) will probably have to be determined during a "primary calibration". (see Sec.IIIb below). However the above method will yield estimates of the products. $g_{ax}\ g^*_{bx}$ and $g_{ay}\ g^*_{by}$, from which the value of $\underline{I}$ may be determined. Refining this estimate of $\underline{I}$ is a separate problem, not dealt with here, relying on a technique such as SELFCAL.

(ii) <u>The noise tube calibration</u>

It will be necessary to maintain the calibration from one observation of a calibrator source to the next. For this purpose it has been suggested (AT/21.3.1/011 and AT/21.3.1/010) that a noise tube calibration system be used. Three sets of components are required for this system.

A. For each antenna a continuously switched noise tube. The output of the noise tube is divided and coupled into the input of each of the two receivers (x and y) thus providing them with phase-coherent noise signals.

B. Synchronous demodulators for measuring the magnitudes of the switched components of total power output from each of the IFs just before these are digitized.

C. One "complex correlator" for each antenna. The inputs to these are the (digitized) IF signals from the x and y receivers of that antenna, just before these go to the main correlators. Each such correlator will require a synchronous demodulator, to separate the switched noise tube output from the correlated output due to any radio source being observed.

The assumption on which the use of this system is based is that the ratio of the complex coupling factors of each noise tube to the x and y receivers of its antenna will remain stable over very long periods of time. It should be possible to achieve this by making the systems very mechanically rigid.

If all the noise tube outputs were completely time-stable, then by monitoring the switched total power outputs (B above) and comparing them with their values at the time the various products $g_{ax}g^{*}_{bx}$ exp $i\delta\phi$ etc., were measured by reference to a calibrator source, as described in III(a)(i) above and IIIb below, we should be able to keep track of all the products.

$|g_{ax}\ g_{bx}|$, $|g_{ax}\ g_{by}|$, $|g_{ay}\ g_{bx}|$, and $|g_{ay}\ g_{by}|$ as well as the ratios $|g_{ay}/g_{ax}|$ and $|g_{by}/g_{bx}|$.

In practice the ratios $|g_{ax}/g_{ax}|$ and $|g_{by}/g_{bx}|$ will be determined with very much greater accuracy than the products $|g_{ax}\ g_{bx}|$, etc., since the estimates of the ratios depend only on the stability of the coupling between the noise tubes and the receivers, but not on the stability of the noise tubes themselves. The estimated values of the products, on the other hand do depend on the stabilities of the noise tubes.

From the switched components of the correlator (C above) outputs we can estimate the angles $\phi_a+\phi_{ca}$ and $\phi_b+\phi_{cb}$.

Here ϕ_{ca} and ϕ_{cb} are the phase differences associated with the coupling of the noise tubes into the x and y amplifiers of antennas a and b respectively; ϕ_a and ϕ_b are the phases of the complex products $g_{ax}g^*_{ay}$ and $g_{bx}g^*_{by}$ respectively.

Now $-\phi_a$ is the phase of the ratio g_{ay}/g_{ax} and ϕ_b is the phase of the ratio g^*_{by}/g^*_{bx}. Having directly measured these ratios during our observations of the primary calibrator source (see Sec.3b below) we can estimate ϕ_{ca} and ϕ_{cb} and thus keep track of $-\phi_a$ and ϕ_b in the intervals between observations of primary calibrators.

With stable coupling between the noise tubes and the receivers it should therefore be possible to measure the complex ratios g_{ay}/g_{ax} and g^*_{by}/g^*_{bx} and thus the ratios Q/I, U/I and VI with an accuracy limited only by the sensitivity of the receivers.

III(a)(iii)Taking account of digitization.

It seems that the IF voltages from the various receivers will be digitized before being correlated, but that the level of digitization will depend on the particular type of observation. The following discussion refers to one-bit digitization. (For a comparison of the effects of various levels of digitization see D'Addario 1985 VLB Array Memo No. 428)

In general a "complex correlator measures two quantities - an "in-phase" component and a "quadrature phase" component of correlation. For a one-bit digital complex correlator, these have the form

$$R_{xx} = \frac{2}{\pi} \sin^{-1}[R(\Sigma_{xx})] \qquad \text{eq. III 1}$$

$$I_{xx} = \frac{2}{\pi} \sin^{-1}[I(\Sigma_{xx})] \qquad \text{eq. III 2}$$

for the in-phase" and the "quadrature phase" components respectively of the x-x correlation. Here $R(\Sigma_{xx})$ and $I(\Sigma_{xx})$ are the real and imaginary parts of the complex quantity

$$\Sigma_{xx} = \frac{\langle v_{ax} v^*_{bx} \rangle}{[\langle V_{ax} V^*_{ax} \rangle \langle V_{bx} V^*_{bx} \rangle]^{1/2}} \exp i(\phi_{ax} - \phi_{bx}) \qquad \text{eq. III 3}$$

where v_{ax}, v_{bx} are the correlated voltage components at the inputs of receivers ax and bx, V_{ax}, V_{bx} are the total voltage components, and ϕ_{ax}, ϕ_{bx} are the phase delays associated with the two antenna-receiver combinations.

If the system is observing an unresolved unpolarized calibrator source for which the first Stokes parameter is I Jansky, neglecting cross-coupling we can rewrite this expression for Σ_{xx} as

$$\Sigma_{xx} = \left(\frac{\gamma_{ax}}{T_{sax}}\right)^{1/2} \left(\frac{\gamma_{bx}}{T_{sbx}}\right)^{1/2} I \exp i(\phi_{ax}-\phi_{bx}+\delta\psi) \qquad \text{eq. III 4}$$

Here γ_{ax}, γ_{bx} are the gains of the a and b antennas, expressed in Kelvins/Jansky, and T_{sax}, T_{sbx} are the system temperatures, and, as previously, any differential phase delay due to the atmosphere is represented by $\delta\psi$.

From Eq. III 4 it follows that the discussion in the previous Sections may be understood as referring to a one-bit digitization system if the following substitutions are made for the gain terms

$$g_{ax} = \left(\frac{\gamma_{ax}}{T_{sax}}\right)^{1/2} \exp i\phi_{ax} \qquad \text{eq. III 5}$$

$$g_{bx} = \left(\frac{\gamma_{bx}}{T_{sbx}}\right)^{1/2} \exp i\phi_{bx} \qquad \text{eq. III 6}$$

By measuring the ratio of noise tube step to system temperature for all four receivers we can refer the current values of T_{sax}, T_{say}, T_{sbx}, and T_{sby} and hence of $|g_{ax}$, $|g_{ay}|$.

$|g_{bx}|$ and $|g_{by}|$ to their values at the time a calibrator source was observed, and hence keep track of the effective gains of the system. The relative values of $|g_{ay}|/|g_{ax}|$ and of $|g_{by}|/|g_{bx}|$ will be very accurately determined, since they do not depend on noise tube stability, but $|g_{ay}g_{bx}|$ etc. will be less accurately determined.

If $|\Sigma_{xx}| << 1$, then from Eqs. III 1, III 2 and III 4 the complex quantity measured by the x-x correlator may be approximated by ρ_{xx}, given by

$$\rho_{xx} = \frac{2}{\pi} \left(\frac{\gamma_{ax}}{T_{sax}}\right)^{1/2} \left(\frac{\gamma_{bx}}{T_{sbx}}\right)^{1/2} I \exp i(\phi_{ax}-\phi_{bx}+\delta\psi) \qquad \text{eq. III 7}$$

Thus for a one-bit digitization system, item B of the calibration scheme described in III(a)(ii) above will comprise

1. A very accurate square-law amplifier, the input to which is the IF signal before digitization.

followed by

2. A synchronous demodulator to measure the amplified switched noise power

and also followed by

3. An "average level" detector.

If T_n is the temperature component due to the switched noise tube, and T_s is the system temperature (which will contain a contribution of $1/2(T_n)$ if the noise tube is switched continuously with a duty cycle of 1/2), then the outputs of 2 and 3 above are KT_n and KT_s, where K is an overall amplification factor. Thus with this arrangement we can monitor the ratio T_n/T_s.

III a (iv) Accuracy of the calibration.

1. Using the noise tubes.

The quantities we wish to estimate using the noise tube system are the magnitudes of the gain product terms

$|g_{ax}g_{bx}|$, $|g_{ax}g_{by}|$ etc.,

and the magnitudes of the gain ratios

$$|\Gamma_a| = \frac{|g_{ay}|}{|g_{ax}|} ; \qquad |\Gamma_b| = \frac{|g_{by}|}{|g_{bx}|}.$$

If $\Delta|g_{ax}g_{bx}|$, $\Delta|g_{ax}g_{by}|$, $\Delta|g_{ay}g_{bx}|$, $\Delta|g_{ay}g_{by}|$ are the RMS estimation uncertainties of the quantities

$|g_{ax}g_{bx}|$, $|g_{ax}g_{by}|$, $|g_{ay}g_{bx}|$, $|g_{ay}g_{by}|$,

and $\Delta|\Gamma_a|$, $\Delta|\Gamma_b|$ are the RMS uncertainties in $\Delta|\Gamma_a$ and $\Delta|\Gamma_b|$ then, after some algebra, it may be shown that

$$\frac{\Delta|g_{ax}g_{bx}|}{|g_{ax}g_{bx}|} \simeq \frac{[(\frac{T_{sax}}{T_{nax}})^2 + (\frac{T_{sbx}}{T_{nbx}})^2]^{1/2}}{(\tau\ \Delta f)^{1/2}}$$

with corresponding expressions for the other gain-product terms, and

$$\frac{\Delta|\Gamma_a|}{|\Gamma_a|} \simeq \frac{[(\frac{T_{sax}}{T_{nax}})^2 + (\frac{T_{say}}{T_{nay}})^2]^{1/2}}{(\tau\ \Delta f)^{1/2}}$$

etc.

Here T_{sax}, T_{sbx}, etc are the system temperatures of the various channels, T_{nax}, T_{nbx} etc are the components of temperature at the inputs of the various receivers due to the (switched) noise tube, τ sec is the integration time, and δf Hz. is the IF bandwidth.

If $T_{sax} \simeq T_{say} \simeq T_{sbx} \simeq T_{sby} \simeq 30K$,
and $T_{nax} \simeq T_{nay} \simeq T_{nbx} \simeq T_{nby} \simeq 3K$,
the integration time τ = 10 sec., and $\Delta f = 4x10^7$ Hz., then,

$$\frac{\Delta|g_{ax}g_{bx}|}{|g_{ax}g_{by}|} \simeq \frac{\Delta|g_{ax}g_{by}|}{|g_{ax}g_{bx}|} \text{ etc.,}$$

$$\simeq \frac{\Delta|\Gamma_a|}{|\Gamma_a|} \simeq \frac{\Delta|\Gamma_b|}{|\Gamma_b|}$$

$$\simeq 7x10^{-4}.$$

In making this estimate it was assumed that the noise-tube outputs are all time-stable. A stability of 7 parts in 10^4 is hardly to be expected, and therefore we cannot realistically expect this accuracy in the gain <u>product</u> terms $|g_{ax}g_{bx}|$. However in the case of the gain <u>ratio</u> terms - the same noise tube is used in our estimates of $|g_{ax}|$ and $|g_{ay}|$, and the same noise tube is used in estimating $|g_{bx}|$ and $|g_{by}|$. Therefore these terms are affected in the same ratio by instabilities in the noise tube outputs, and thus it would seem that the above estimates of the ratios $|\Gamma_a|$, $|\Gamma_b|$ should be realistic.

2. Using an unresolved calibrator.

In the following discussion it will be assumed that the IF signals have "white" spectra, are band-limited to identical flat pass-bands of Δf Hz. and are integrated for τ sec.

For the case of an unresolved unpolarized calibrator source, the magnitude of the quantity measured by the x-x correlator is (from Eq. III 7)

$$|\rho_{xx}| \simeq \frac{2}{\pi} \left(\frac{|\gamma_{ax}|}{T_{sax}}\right)^{1/2} \left(\frac{\gamma_{bx}}{T_{sbx}}\right)^{1/2} I \qquad \text{III 8}$$

Here it has been assumed that $|\rho_{xx}| << 1$ (which will be the case for most unresolved sources observed by the AT). Cross-coupling effects have been neglected.

Knowing the value of I we may estimate the value of

$$|g_{ax}g_{bx}| = \left(\frac{\gamma_{ax}}{T_{sax}}\right)^{1/2} \left(\frac{\gamma_{sbx}}{T_{sbx}}\right)^{1/2}$$

(from IIIa(iii)).

It may be shown that the RMS uncertainty in $|\rho_{xx}|$ is

$$\Delta\rho_{xx} = (\tau\ \Delta f)^{-1/2}.$$

The fractional uncertainty in our estimate of

$|g_{ax}g_{bx}|$ is then given by

$$\frac{\Delta|g_{ax}g_{bx}|}{|g_{ax}g_{bx}|} = \frac{\Delta|\rho_{xx}|}{|\rho_{xx}|}$$

$$\simeq \frac{\pi}{2I} \left\{\frac{T_{sax}T_{sbx}}{\tau\Delta f\gamma_{ax}\gamma_{bx}}\right\}^{1/2}$$

(from Eq)

For the Parkes 64 m. the antenna gain is about 0.6 Kelvin/Jansky. For the AT 22 m. Antennas we may expect values of about

$(22/64)^2 \times 0.6 = 0.07$ K/Jy for γ_{ax} and γ_{bx}.

Assuming $T_{sax} \simeq T_{sbx} = 30$ K, then for a source of 1 Jy observed for τ=300sec., with $\Delta f = 4\times10^7$ Hz.,

$$\frac{\Delta|g_{ax}g_{bx}|}{|g_{ax}g_{bx}|} = \frac{\Delta|\rho_{xx}|}{|\rho_{xx}|} \simeq 6\times10^{-3}.$$

III(b) The "primary" calibration.

As a primary calibrator we choose a strong source which is unresolved at all baselines, and for which the Stokes Parameters I, Q,U, and V (as measured with a single antenna) are known very accurately. A southern declination is desirable so that a wide range of parallactic angles may be observed.

It is assumed that the four elements of the correlation matrix Σ_{ab} are integrated over short time intervals,say 10 sec.,and recorded. Using the noise tube calibrating system described in SecIIIa(ii) above, we can estimate the values of

$$\Gamma_a = g_{ay}/g_{ax} \text{ and } \Gamma_b = g^*_{by}/g^*_{bx},$$

which ideally are constant, as ratios with respect to their values at some "starting time", and correct the measurements accordingly. In the following discussion we treat these parameters as constant. The gain product magnitudes $|g_{ax} g_{bx}|$, $|g_{ax} g_{by}|$ etc. can also be corrected back to some "starting time", but this is less accurate, depending as it does on the stability of the noise tubes.

For most sources, at least as observed with the Compact Array, and except very close to transit $\chi_a \simeq \chi_b$. We therefore approximate with $\chi_a = \chi_b = \chi$. This is done merely to simplify the rather cumbersome algebra. The method outlined here will work for $\chi_a \neq \chi_b$ provided $|\chi_a - \chi_b| \ll$ one radian, but the algebra is a little more awkward.

Since the source is unresolved,

$$\underline{I} = I,\ \underline{Q} = Q,\ \underline{U} = U, \text{ and } \underline{V} = V.$$

We define p, the degree of linear polarization, by

$$p = (Q^2 + U^2)^{1/2}/I ,$$

and the position angle θ, of the linear polarization is given by

$$\tan 2\theta = U/Q.$$

The degree of circular polarization

$$v = V/I.$$

We assume that $0.05 < p < 0.1$ and $v \ll p$.

From Appendix I the elements of the correlation matrix $\Sigma_{ab}(\chi)$ as a function of parallactic angle χ are given in all their glory by

$$\Sigma_{xx}(\chi) = g_{ax}g^*_{bx}\, I\{1-p \sin 2(\theta-\chi)$$

$$-a^*_{bx}[p \cos 2(\theta-\chi)+iv]$$

$$-a_{ax}[p \cos 2(\theta-\chi)-iv]$$

$$+a_{ax}\, a^*_{bx}\, [1+p \cos 2(\theta-\chi)]\} \exp i\delta\phi$$

$$\Sigma_{xy}(\chi) = -g_{ax}\, g^*_{by}\, I\{p \cos 2(\theta-\chi)+iv$$

$$-a^*_{by}[1-p \sin 2(\theta-\chi)]$$

$$-a_{ax}[1+p \sin 2(\theta-\chi)]$$

$$+a_{ax}\, a^*_{by}[1+p \cos 2(\theta-\chi)-iv]\} \exp i\delta\phi$$

$$\Sigma_{yx}(\chi) = -g_{ay}\, g^*_{bx}\, I\{p \cos 2(\theta-\chi)-iv$$

$$-a^*_{bx}[1+p \sin 2(\theta-\chi)]$$

$$-a_{ay}[1-p \sin 2(\theta-\chi)]$$

$$+a_{ax}\, a^*_{by}[1+p \cos 2(\theta-\chi)+iv]\} \exp i\delta\phi$$

$$\Sigma_{yy}(\chi) = g_{ay}\, g^*_{by}\, I\{1+p \sin 2(\theta-\chi)$$

$$-a_{ay}[p \cos 2(\theta-\chi)+iv]$$

$$-a^*_{by}[p \cos 2(\theta-\chi)-iv]$$

$$+a_{ay}\, a^*_{by}\, [1-p \cos 2(\theta-\chi)]\} \exp i\delta\phi$$

For the complex cross-coupling terms we write

$$a_{ax} = \alpha_{ax} + i\beta_{ax}$$

$$a_{ay} = \alpha_{ay} + i\beta_{ay}$$

$$a_{bx} = \alpha_{bx} + i\beta_{bx}$$

$$a_{by} = \alpha_{by} + i\beta_{by}$$

where the αs and βs are real.

First Approximation

We calculate the modulus of $\Sigma_{xx}(\chi)$ for each integration. This may be approximated with very great accuracy by

$$|\Sigma_{xx}(\chi)| = |g_{ax}\ g_{bx}|\ I\ \{1-p[\sin 2(\theta-\chi) - (\alpha_{ax}+\alpha_{bx})\cos 2(\theta-\chi)]\}$$

By a linear least squares technique we can fit this with a function of the form

$$F(\chi) = A + B\cos(2\chi) + C\sin(2\chi)$$

From this we can calculate

$$\alpha_{ax}+\alpha_{bx},\ |g_{ax}\ g_{bx}|\ I \text{ and } |g_{ax}\ g_{bx}|p.$$

Going back to the original complex values of $\Sigma_{xx}(\chi)$, we can then estimate the values of the complex quantity

$g_{ax}\ g^{*}_{bx}\ I\ \exp i\ \delta\psi$. We divide this into $\Sigma_{xy}(\chi)$ to yield

$$s_{xy}(\chi) = \Sigma_{xy}(\chi)/(g_{ax}\ g^{*}_{bx}\ I\ \exp i\delta\psi)$$

To good approximation

$$s_{xy}(\chi) = \Gamma^{*}_{y}\{\alpha_{ax}+\alpha_{by}$$
$$+p[\cos 2(\theta-\chi)+(\alpha_{ax}-\alpha_{by})\sin 2(\theta-\chi)]$$
$$+i[\beta_{ax} - \beta_{by} - v + (\beta_{ax} + \beta_{by})\sin 2(\theta-\chi)]\}$$

From this it follows that

$$[(s_{xy}(\theta) - s_{xy}(\theta+\pi/2)]/2 = p\Gamma^{*}_{b}$$

and thus, knowing p we can calculate Γ^{*}_{b}.

Now

$$[(s_{xy}(\theta)+s_{xy}(\theta+\pi/2)]/(2\Gamma^{*}_{b}) = (\alpha_{ax}+\alpha_{by})+i(\beta_{ax} - \beta_{by} - v) \qquad \text{III 9}$$

and

$$[s_{xy}(\theta)-s_{xy}(\theta+\pi/2)] = (\alpha_{ax}-\alpha_{by})+i(\beta_{ax}+\beta_{by}). \qquad \text{III 10}$$

Remembering that the αs and βs are real numbers and that v is a known real number, we can combine Eqs (III 9 and III 10) to yield

α_{ax}, α_{by}, β_{ax} and β_{by}.

A similar procedure involving s_{yx} yields Γ_a, α_{ay}, α_{bx}, β_{ay} and β_{bx}.

We can use these estimates as a starting point for a second approximation which should yield more accurate values.

We have thus evaluated all the elements of the required matrices

$\mathbf{\Gamma}_a$, Γ_b, $\mathbf{\Lambda}_a$ and $\mathbf{\Lambda}_b$. (see Eqs II 8 and II 11).

The above discussion indicates that it is feasible to evaluate all the required matrix elements. In practice a non-linear least squares fitting routine may be a better approach.

IV. The Long Baseline Array: Circular or Linear Polarization?

The following general considerations should be taken into account when deciding on the feed system for the LBA.

1. For an interferometer of arbitrary baseline employing alt-az mounts, accepting orthogonal linear polarizations, and with no provision for feed rotation, in order to recover any one of $\underline{I}$, $\underline{Q}$, $\underline{U}$ or $\underline{V}$ with full sensitivity, it is necessary to measure all four correlations (see equation A1 Appendix 1). If the x-x and y-y correlations alone are measured, then (neglecting cross-coupling) adding them yields

$$\underline{I}\cos(\chi_a-\chi_b) - i\underline{V}\sin(\chi_a-\chi_b)$$

Thus if $|\chi_a-\chi_b| > 1$ radian, there will be a loss of sensitivity in estimating $\underline{I}$. (for continuum sources the term involving $\underline{V}$ will usually be negligible). However in the particular case of the LBA, for any interferometer pair

$$|\chi_a-\chi_b| << 1 \text{ radian}$$

except for sources very close to transit, and the loss of sensitivity in estimatng $\underline{I}$ from the sum of the x-x and y-y correlations will be negligible. This can be seen from Table I in which the parallactic angle χ is shown as a function of hour angle at Culgoora for the various stations of the LBA and for various declinations. If only one correlation, say the x-x, is recorded, then on baselines such that

$$|\chi_a - \chi_b| << 1 \text{ radian},$$

the quantity recovered is

$$\simeq \underline{I} - \underline{Q}\cos 2(\chi+\alpha) - \underline{U}\sin 2(\chi+\alpha)$$

where the angle α depends on the orientation of the feeds relative to the local vertical. Clearly, unless we have some a priori reason for believing that $\underline{Q}$ and $\underline{U}$ are negligibly small, this is an unsatisfactory result.

2. If each antenna accepts opposite circularly polarizations, then once again we must measure all four correlations in order to estimate $\underline{I}$, $\underline{Q}$, $\underline{U}$ and $\underline{V}$. The R-R and L-L correlations are proportional to $\underline{I} + \underline{V}$ and $\underline{I} - \underline{V}$ respectively. Taking their sum and difference we can, of course, estimate $\underline{I}$ and $\underline{V}$ independently. One of the reasons that it is thought acceptable to measure say, R-R alone, is that for continuum sources at low resolution we know that

$$|\underline{I}| >> |\underline{V}|$$

However we do not know that this is true at sub-arcsec. resolution.

3. If one antenna of an interferometer pair accepts opposite circular polarizations, and the other accepts orthogonal linears, by measuring all four correlations we can again

estimate

$$\underline{I},\ \underline{Q},\ \underline{U} \text{ and } \underline{V}$$

with full sensitivity. (see for example Schwab VLB Array Memo 337). In the more restricted case where one antenna accepts only one circular polarization (say R) and the other accepts two orthogonal linears, this permits an estimate of

$$\underline{I} + \underline{V}\ ,$$

but at the cost of two correlations.

If the accepted polarizationsof the remote elements of the LBA (except of course Tidbinbilla) were to be orthogonal linears, this would have the advantage of minimal cross-coupling and relatively uncomplicated design. However there are two difficulties. The first is , that as currently envisaged, the tape recorders for the LBA will be capable of recording only two IF channels. Thus where two frequency bands are required simultaneously, only one will be available for each band. As indicated above, with linearly polarized feeds, this is unsatisfactory. In principle this difficulty could be overcome by employing a time-sharing system based on frequency switching, provided this is acceptable in all applications. A second difficulty is that current VLBI experiments usually involve recording one sense of circular polarization at each observing site. If the LBA elements have linearly polarized outputs, then, as indicated above, two correlations will be required per frequency channel.

These comments require some qualification however. It seems reasonably likely that between now and when the AT becomes operative there will be greater demand for measuring polarization as well as total intensity on VLBI baselines. Presumably there will also have been further technological advance, making four IFs per tape a less formidable problem than it is at present. In that case, in view of comment (3) above, there should be no disadvantage in the use of linear polarization, regardless of the feed systems used at other VLBI sites around the world. A further consideration is that, with only slight modification present plans for the LBA appear to give the possibility of providing two opposite circular polarizations or two orthogonal linears, as circumstances of particular experiments require. It is currently envisaged that each remote antenna other than Tidbinbilla will have a pair of orthogonal dipoles and two low noise amplifier-IF systems per frequency band. In principle, to adapt such a system to provide opposite circularly polarized outputs would require only

(i) An additional line length of $\lambda/4$ in one local oscillator line.

(ii) A network for producing the sum and difference of the two IF outputs.

V VLBI Considerations.

The calibration procedure described in Section III should be applicable to VLBI. This would provide very accurate polarimetry on VLBI baselines. The calibration procedure can be carried out provided there are at least two antennas reasonably close together at each site. This requirement is imposed by the need to use unresolved calibrator sources. The discussion in Section IV and in VLB Array Memo No 337 indicates that if some of the antennas in a VLBI network accept opposite circular polarizations and others accept orthogonal linears, this poses no problem of principle to VLBI polarimetry.

VI Zenith Angle Effects.

Single dish measurements with the Parkes 64 m. antenna have indicated a spurious component of linear polarization having a maximum magnitude of about 0.5% and with an approximately parabolic zenith angle dependence. Presumably this reflects e deformation of the reflector under its own weight as it is tilted away from the zenith. The AT reflectors, being both smaller and stiffer, one would expect any such effect in their case to be much smaller. If the effect is large enough to be measurable, it will appear as a zenith angle dependence of the cross-coupling terms discussed in Section III.

A zenith angle dependent effect which will be present, especially at the shorter wavelengths, is atmospheric attenuation. This will of course appear as a variation of apparent gain with zenith angle.

These effects, if not taken into account, could confuse the "primary" calibration procedure. A refinement of the procedure described in Section III(b) would involve fitting additional parameters, designed to allow both the "effective gain", and (perhaps) the cross-coupling terms to be zenith-angle dependent.

Appendix I The Matrices $\mathbf{S}_{ab}$ and Σ_{ab}.

For a system employing orthogonal linearly polarized feeds at $\pi/4$ and $-\pi/4$ radian to the vertical (as will be the case for the AT) the matrix $\mathbf{S}_{ab}$ (see Eqs II 1(6),3 4 and II 6) is given by

$$S_{ab} = \begin{bmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{bmatrix}$$

where

$$S_{xx}=1/2\{\ \underline{I}\cos(\chi_a-\chi_b) + \underline{Q}\sin(\chi_a+\chi_b) - \underline{U}\cos(\chi_a+\chi_b) - i\underline{V}\sin(\chi_a-\chi_b)\ \}$$

$$S_{xy}=1/2\{-\underline{I}\sin(\chi_a-\chi_b) - \underline{Q}\cos(\chi_a+\chi_b) - \underline{U}\sin(\chi_a+\chi_b) - i\underline{V}\cos(\chi_a-\chi_b)\ \}$$

$$S_{yx}=1/2\{\ \underline{I}\sin(\chi_a-\chi_b) - \underline{Q}\cos(\chi_a+\chi_b) - \underline{U}\sin(\chi_a+\chi_b) + i\underline{V}\cos(\chi_a-\chi_b)\ \}$$

$$S_{yy}=1/2\{\ \underline{I}\cos(\chi_a-\chi_b) - \underline{Q}\sin(\chi_a+\chi_b) + \underline{U}\cos(\chi_a+\chi_b) - i\underline{V}\sin(\chi_a-\chi_b)\ \}$$

A1

The matrix Σ_{ab} (see Eqs II7,) is given by

$$\Sigma_{ab} = \begin{matrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{matrix}$$

where, from Eqs. II7, II8 and II9,

$$\left.\begin{aligned}
\Sigma_{xx} &= g_{ax}g^*_{bx}(S_{xx}+a^*_{bx}S_{xy}+a_{ax}S_{yx}+a_{ax}a^*_{bx}S_{yy}) \\
\Sigma_{xy} &= g_{ax}g^*_{by}(S_{xy}+a^*_{by}S_{xx}+a_{ax}S_{yy}+a_{ax}a^*_{by}S_{yx}) \\
\Sigma_{yx} &= g_{ay}g^*_{bx}(S_{yx}+a^*_{bx}S_{yy}+a_{ay}S_{xx}+a_{ay}a^*_{bx}S_{xy}) \\
\Sigma_{yy} &= g_{ay}g^*_{by}(S_{yy}+a^*_{by}S_{yx}+a_{ay}S_{xy}+a_{ay}a^*_{by}S_{xx})
\end{aligned}\right\} \quad A2$$

TABLE I

Parallactic Angle vs. (Culgoora) Hour Angle

for the LBA stations.

DECLINATION= 4.00

PARKES PARALL ANG. (DEG.)	SID.SPR. PARALL ANG. (DEG.)	- - -CULGOORA- - - H.A. (H M S)			PARALL ANG. (DEG.)	TBB. PARALL ANG. (DEG.)	HOBART PARALL ANG. (DEG.)
-158.6	-158.4	- 1	0	0	-158.3	-160.9	-165.1
-160.0	-159.9	- 0	55	0	-159.9	-162.3	-166.3
-161.6	-161.5	- 0	50	0	-161.6	-163.8	-167.4
-163.1	-163.2	- 0	45	0	-163.3	-165.2	-168.6
-164.7	-164.9	- 0	40	0	-165.1	-166.7	-169.8
-166.3	-166.6	- 0	35	0	-166.9	-168.2	-171.0
-168.0	-168.4	- 0	30	0	-168.7	-169.8	-172.2
-169.6	-170.1	- 0	25	0	-170.5	-171.3	-173.5
-171.3	-171.9	- 0	20	0	-172.4	-172.9	-174.7
-173.0	-173.7	- 0	15	0	-174.3	-174.5	-176.0
-174.7	-175.6	- 0	10	0	-176.2	-176.1	-177.2
-176.5	-177.4	- 0	5	0	-178.1	-177.7	-178.5
-178.2	-179.3	0	0	0	180.0	-179.3	-179.7
-179.9	178.9	0	5	0	178.1	179.1	179.0
178.3	177.0	0	10	0	176.2	177.5	177.8
176.6	175.2	0	15	0	174.3	175.9	176.5
174.9	173.4	0	20	0	172.4	174.3	175.3
173.1	171.6	0	25	0	170.5	172.8	174.0
171.4	169.8	0	30	0	168.7	171.2	172.8
169.7	168.0	0	35	0	166.9	169.6	171.6
168.1	166.2	0	40	0	165.1	168.1	170.4
166.4	164.5	0	45	0	163.3	166.6	169.2
164.8	162.9	0	50	0	161.6	165.1	168.0
163.2	161.2	0	55	0	159.9	163.7	166.8

DECLINATION= 0.00

PARKES PARALL ANG. (DEG.)	SID.SPR. PARALL ANG. (DEG.)	- - -CULGOORA- - - H.A. (H M S)			PARALL ANG. (DEG.)	TBB. PARALL ANG. (DEG.)	HOBART PARALL ANG. (DEG.)
-156.6	-156.3	- 1	0	0	-156.1	-159.3	-164.1
-158.2	-157.9	- 0	55	0	-157.9	-160.8	-165.3
-159.8	-159.7	- 0	50	0	-159.7	-162.3	-166.6
-161.5	-161.5	- 0	45	0	-161.5	-163.9	-167.8
-163.2	-163.3	- 0	40	0	-163.5	-165.5	-169.1
-165.0	-165.2	- 0	35	0	-165.4	-167.2	-170.4
-166.7	-167.1	- 0	30	0	-167.4	-168.8	-171.7
-168.6	-169.1	- 0	25	0	-169.5	-170.5	-173.0
-170.4	-171.0	- 0	20	0	-171.5	-172.2	-174.3
-172.3	-173.1	- 0	15	0	-173.6	-173.9	-175.7
-174.2	-175.1	- 0	10	0	-175.7	-175.7	-177.0
-176.1	-177.1	- 0	5	0	-177.9	-177.4	-178.4
-178.0	-179.2	0	0	0	180.0	-179.2	-179.7
-179.9	178.8	0	5	0	177.9	179.1	179.0
178.2	176.7	0	10	0	175.7	177.3	177.6
176.2	174.7	0	15	0	173.6	175.5	176.3
174.3	172.6	0	20	0	171.5	173.8	174.9
172.4	170.6	0	25	0	169.5	172.1	173.6
170.6	168.6	0	30	0	167.4	170.4	172.3
168.7	166.7	0	35	0	165.4	168.7	171.0
166.9	164.8	0	40	0	163.5	167.0	169.7
165.1	162.9	0	45	0	161.5	165.4	168.4
163.3	161.1	0	50	0	159.7	163.8	167.1
161.6	159.3	0	55	0	157.9	162.2	165.9

DECLINATION= -10.00

PARKES PARALL ANG. (DEG.)	SID.SPR. PARALL ANG. (DEG.)	- - -CULGOORA- - - H.A. (H M S)	PARALL ANG. (DEG.)	TBB. PARALL ANG. (DEG.)	HOBART PARALL ANG. (DEG.)
-149.3	-148.2	- 1 0 0	-147.6	-153.2	-160.5
-151.2	-150.2	- 0 55 0	-149.7	-155.0	-162.0
-153.1	-152.3	- 0 50 0	-152.0	-156.9	-163.4
-155.2	-154.6	- 0 45 0	-154.3	-158.9	-164.9
-157.3	-156.9	- 0 40 0	-156.8	-160.9	-166.5
-159.6	-159.4	- 0 35 0	-159.4	-162.9	-168.0
-161.9	-161.9	- 0 30 0	-162.1	-165.1	-169.6
-164.3	-164.6	- 0 25 0	-164.9	-167.3	-171.3
-166.8	-167.3	- 0 20 0	-167.8	-169.5	-172.9
-169.3	-170.1	- 0 15 0	-170.8	-171.8	-174.6
-171.9	-173.0	- 0 10 0	-173.8	-174.2	-176.2
-174.5	-175.9	- 0 5 0	-176.9	-176.5	-177.9
-177.2	-178.8	0 0 0	180.0	-178.9	-179.6
-179.9	178.2	0 5 0	176.9	178.7	178.7
177.4	175.3	0 10 0	173.8	176.4	177.0
174.8	172.4	0 15 0	170.8	174.0	175.3
172.1	169.5	0 20 0	167.8	171.7	173.7
169.5	166.7	0 25 0	164.9	169.4	172.0
167.0	164.0	0 30 0	162.1	167.1	170.4
164.5	161.4	0 35 0	159.4	164.9	168.8
162.1	158.9	0 40 0	156.8	162.8	167.2
159.8	156.4	0 45 0	154.3	160.7	165.6
157.5	154.1	0 50 0	152.0	158.7	164.1
155.4	151.9	0 55 0	149.7	156.8	162.6

DECLINATION= -20.00

PARKES PARALL ANG. (DEG.)	SID.SPR. PARALL ANG. (DEG.)	- - -CULGOORA- - - H.A. (H M S)			PARALL ANG. (DEG.)	TBB. PARALL ANG. (DEG.)	HOBART PARALL ANG. (DEG.)
-135.1	-132.1	- 1	0	0	-130.2	-141.6	-154.0
-137.2	-134.2	- 0	55	0	-132.4	-143.7	-155.8
-139.4	-136.6	- 0	50	0	-134.9	-145.9	-157.6
-141.8	-139.2	- 0	45	0	-137.6	-148.4	-159.6
-144.5	-142.2	- 0	40	0	-140.8	-151.0	-161.5
-147.5	-145.4	- 0	35	0	-144.3	-153.9	-163.6
-150.7	-149.1	- 0	30	0	-148.2	-156.9	-165.7
-154.1	-153.1	- 0	25	0	-152.5	-160.1	-167.9
-157.9	-157.4	- 0	20	0	-157.3	-163.5	-170.2
-161.9	-162.1	- 0	15	0	-162.6	-167.0	-172.4
-166.2	-167.2	- 0	10	0	-168.1	-170.7	-174.8
-170.6	-172.4	- 0	5	0	-174.0	-174.4	-177.1
-175.2	-177.8	0	0	0	180.0	-178.2	-179.5
-179.8	176.7	0	5	0	174.0	177.9	178.2
175.5	171.3	0	10	0	168.1	174.1	175.8
171.0	166.1	0	15	0	162.6	170.4	173.5
166.5	161.1	0	20	0	157.3	166.7	171.2
162.2	156.5	0	25	0	152.5	163.2	168.9
158.2	152.2	0	30	0	148.2	159.8	166.7
154.4	148.3	0	35	0	144.3	156.7	164.5
150.9	144.7	0	40	0	140.8	153.6	162.5
147.7	141.5	0	45	0	137.6	150.8	160.4
144.7	138.6	0	50	0	134.9	148.2	158.5
142.0	136.1	0	55	0	132.4	145.8	156.6

Erratum to AT/10.1/039

CALIBRATION OF THE AUSTRALIA TELESCOPE

Max Komesaroff.

Note 1. at the bottom of page 4 is incorrect.

If each element of a nxn matrix is multiplied by a scalar m, the <u>determinant</u> is indeed multiplied by m^n, but the matrix is multiplied by m.

This mistake makes only a formal difference to the derived results. Equation II 1b should then obviously be

$$P_{AB} = 1/2 \begin{pmatrix} \underline{I}-\underline{Q} & \underline{U}-i\underline{V} \\ \underline{U}+i\underline{V} & \underline{I}+\underline{Q} \end{pmatrix}$$

The factor $\exp 2i\delta\psi$ in Eq. II 7 and some of the later equations should then be $\exp i\delta\psi$, and the product $(g_{ax}g^{*}_{bx})^2$ in the equations before and after II 11 should be $g_{ax}g^{*}_{bx}$.

DECLINATION= -50.00

PARKES PARALL ANG. (DEG.)	SID.SPR. PARALL ANG. (DEG.)	- - -CULGOORA- - - H.A. (H M S)			PARALL ANG. (DEG.)	TBB. PARALL ANG. (DEG.)	HOBART PARALL ANG. (DEG.)
-41.4	-37.5	- 1	0	0	-35.4	-43.7	-61.5
-38.9	-35.0	- 0	55	0	-32.8	-40.9	-58.7
-36.2	-32.3	- 0	50	0	-30.2	-38.0	-55.7
-33.3	-29.5	- 0	45	0	-27.5	-35.0	-52.3
-30.4	-26.7	- 0	40	0	-24.6	-31.8	-48.5
-27.4	-23.7	- 0	35	0	-21.8	-28.5	-44.3
-24.3	-20.7	- 0	30	0	-18.8	-25.0	-39.7
-21.0	-17.6	- 0	25	0	-15.8	-21.4	-34.5
-17.7	-14.4	- 0	20	0	-12.7	-17.6	-28.9
-14.3	-11.2	- 0	15	0	-9.6	-13.8	-22.7
-10.8	-7.9	- 0	10	0	-6.4	-9.9	-16.0
-7.3	-4.6	- 0	5	0	-3.2	-5.9	-8.9
-3.7	-1.3	0	0	0	0.0	-1.9	-1.6
-0.1	2.0	0	5	0	3.2	2.2	5.7
3.4	5.3	0	10	0	6.4	6.2	12.9
7.0	8.6	0	15	0	9.6	10.2	19.7
10.5	11.9	0	20	0	12.7	14.1	26.2
14.0	15.1	0	25	0	15.8	17.9	32.1
17.4	18.2	0	30	0	18.8	21.6	37.5
20.8	21.3	0	35	0	21.8	25.2	42.3
24.0	24.3	0	40	0	24.6	28.7	46.7
27.1	27.3	0	45	0	27.5	32.0	50.6
30.2	30.1	0	50	0	30.2	35.2	54.2
33.1	32.8	0	55	0	32.8	38.2	57.4

DECLINATION= -30.00

PARKES PARALL ANG. (DEG.)	SID.SPR. PARALL ANG. (DEG.)	- - -CULGOORA- - - H.A. (H M S)			PARALL ANG. (DEG.)	TBB. PARALL ANG. (DEG.)	HOBART PARALL ANG. (DEG.)
-106.4	-99.4	- 1	0	0	-95.2	-116.5	-140.6
-107.0	-99.6	- 0	55	0	-95.0	-117.9	-142.7
-107.9	-99.8	- 0	50	0	-94.8	-119.6	-144.9
-108.9	-100.2	- 0	45	0	-94.7	-121.6	-147.4
-110.2	-100.7	- 0	40	0	-94.6	-124.0	-150.1
-111.9	-101.5	- 0	35	0	-94.6	-127.0	-153.1
-114.1	-102.5	- 0	30	0	-94.7	-130.6	-156.2
-116.9	-104.1	- 0	25	0	-94.9	-135.0	-159.6
-120.8	-106.5	- 0	20	0	-95.4	-140.5	-163.2
-126.2	-110.3	- 0	15	0	-96.5	-147.2	-167.0
-133.8	-117.1	- 0	10	0	-98.9	-155.2	-170.9
-144.7	-130.8	- 0	5	0	-106.6	-164.6	-175.0
-160.1	-161.6	0	0	0	180.0	-175.0	-179.1
-179.2	153.2	0	5	0	106.6	174.2	176.8
161.5	127.0	0	10	0	98.9	163.9	172.7
145.8	115.3	0	15	0	96.5	154.6	168.7
134.5	109.4	0	20	0	95.4	146.6	164.9
126.7	105.9	0	25	0	94.9	140.0	161.2
121.2	103.7	0	30	0	94.7	134.7	157.7
117.2	102.3	0	35	0	94.6	130.3	154.5
114.3	101.3	0	40	0	94.6	126.8	151.4
112.0	100.6	0	45	0	94.7	123.8	148.6
110.3	100.1	0	50	0	94.8	121.4	146.0
109.0	99.7	0	55	0	95.0	119.4	143.7