Better redundant configurations for the AT.

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Because of other pressures the original redundant configurations for the AT were chosen without a full investigation of error propagation. It now turns out that after a few days observations with these configurations, error propagation becomes a serious problem. This is illustrated by Table 1. On the left side of this table are listed the aerial positions for each of 10 days, for the original 3km redundant array. Below each aerial position is the variance of the estimate of the phase error for that aerial. On the right side of the table are listed the lengths of the spacings observed, and below each the variance of the estimate of the spacing phase. A * beside a spacing indicates that the same spacing was measured twice on this day. A ^ beside a spacing indicates that that spacing was also measured on a previous day.

The variances are estimated by assuming that all measured phases have equal, unit variance. The mathematical details of this calculation are given in the Appendix. A variance of 9 means an error 3 times the measurement error, and a variance of 43 means that the error estimate is nearly 7 times the measurement error. Table 1 shows that after 5 days the errors are quite large and the situation continues to deteriorate as more days are added.

The solution is to allow a little more redundancy. Table 2 illustrates what can be achieved with the existing station locations, for a 3-km 6-day sequence. After a least-squares fit to the observational data (see Appendix), the variance of all deduced quantities is now less than 4, i.e. the errors are less than double the measurement errors. This significant improvement has been achieved at a very modest cost; the new 6-day scheme measures 33 different spacings, whereas the original scheme measured 37 different spacings in 6 days. (A non-redundant sequence measures 60 uncalibrated spacings in the same time).

This particular solution is not optimum in any sense; it has been chosen to give acceptable error levels and reasonably uniform coverage. It seems probable that longer sequences can be found which retain these two essential properties, and that similar 1.5-km and 6-km sequences can be found, using the existing station locations. However significant computing effort will be required to do this. In the mean time the solution already found can be used to compare the performance of the redundant scheme with a non-redundant scheme, calibrated by using self-cal.

These results do not modify the requirements for the station locations. The station locations were selected to include the positions needed for the first three days of a redundant sequence, but the requirement of a redundant observing sequence did not otherwise affect the selection of stations. The first two days of the original redundant sequence are retained in the present solution, and indeed are essential to any redundant configuration. Therefore the reasons for selecting the present set of stations are still valid.

Appendix
Evaluation of phases using over-determined, redundant arrays.

Notation:

From observations on day d with a pair of aerials i, and j at east-west positions x_{di}, and x_{dj}, we determine the measured phase ϕ_{dij}, which is related to the aerial phase errors e_{di}, e_{dj}, and the true phase due to the sky $\psi(s_{dij})$ where the spacing $s_{dij}=x_{di}-x_{dj}$

$$\phi_{dij}=\psi(s_{dij})+e_{di}-e_{dj} \qquad . \quad . \quad . \quad (1)$$

Constraints

From a number of such observations we obtain a set of linear equations relating the observations, $\{\phi_{dij}\}$ to the quantities that we need to know, $\{\psi(s_{dij})\}$ and the errors, $\{e_{di}\}$.

This set of equations will be singular unles we add constraints which serve to define the zero phase for each day, and fix the slope of the errors for a series of days. For example, for each d, for the zero of phase:

$$\Sigma \; e_{di}=0 \qquad . \quad . \quad . \quad (2)$$

and for all d's taken to-gether, for the slope:

$$\Sigma \; \Sigma \; x_{di} \; e_{di} \; = 0 \qquad . \quad . \quad . \quad (3)$$

Solution - minimal redundancy

Writing equations (1), (2), and (3) to-gether in matrix notation, we have

$$A \; x = b \qquad . \quad . \quad . \quad (4)$$

where the vector x is defined as:

$$x^T=(\psi(s_{112}),\psi(s_{113}),\ldots,e_{11},e_{12},\ldots)$$

and b by:

$$b^T = (\phi_{112}, \phi_{113}, \ldots)$$

and A is a matrix which contains only +1, -1, or 0, except in the row which represents equation (3).

For the original redundant configuration the redundancy was chosen carefully to ensure that A was square and non-singular, i.e. A^{-1} exists, and

$$x = A^{-1} b \quad . \quad . \quad . \quad (5)$$

Solution - extra redundancy

When extra redundancy is introduced, equations (4) are overdetermined, provided the configurations are chosen sensibly. We then seek a least squares solution for x:

$$(A x - b)^T (A x - b) = \text{minimum}$$

$$A^T A x = A^T b$$

$$x = (A^T A)^{-1} A^T b \quad . \quad . \quad . \quad (6)$$

Here $(A^T A)$ is a square matrix, and should be non-singular.

$A^{inv} = (A^T A)^{-1} A^T$ is known as the generalised inverse. There is a NAG subroutine, F01BLF, which calculates this generalised inverse, and detects any problems, such as singularity.

Variance of solution values.

The variance of an element x_i of x can be calculated from (6):

$$\sigma_i^2 = \Sigma (A_{ij}^{inv})^2 v_j$$

where σ_i^2 is the variance of x_i, and v_j is the variance of b_j.

TABLE 1

ORIGINAL 3KM 10 DAY REDUNDANT

Sequence:- Day	Aerial positions			Spacings											
0	4	100	148	172	196	24*	48*	72	96*	144	168	192			
	1.98	0.33	0.33	0.42	1.13	0.82	0.98	2.19	1.90	4.16	2.44	6.92			
1	0.	84	140	168	196	28*	56*	84*	112	140	168^	196			
	0.55	0.35	0.28	0.34	0.56	0.81	0.87	0.84	2.50	1.89	2.44	0.74			
2	147	168	182	189	196	7*	14*	21*	28^	35	42	49			
	4.10	0.27	0.48	1.01	1.02	0.74	0.99	2.68	0.81	7.57	9.73	9.39			
3	0	4	14	189	196	4	7^	10	14^	175	182	185	189	192^	196^
	0.83	5.34	1.66	1.53	0.63	9.54	0.74	11.39	0.99	4.87	3.06	10.59	3.46	6.92	0.74
4	0	6	10	172	196	4^	6	10^	24^	162	166	172	186	190	196^
	4.14	19.34	2.80	5.45	3.53	9.54	40.84	11.39	0.82	15.80	44.34	3.32	12.54	39.25	0.74
5	0	16	100	147	196	16	47	49^	84^	96^	100	131	147	180	196^
	1.66	2.95	1.49	6.76	1.24	7.76	13.70	9.39	0.84	1.90	5.13	15.84	10.94	6.48	0.74
6	0	8	113	148	196	8	35^	48^	83	105	113	140^	148	188	196^
	2.17	2.62	5.77	0.84	1.25	8.13	7.57	0.98	11.64	12.27	12.02	1.89	4.74	5.52	0.74
7	0	2	12	172	190	2	10^	12	18	160	170	172^	178	188^	190^
	15.29	16.25	4.38	14.92	8.23	62.67	11.39	30.31	42.93	28.63	60.97	3.32	12.64	5.52	39.25
8	0	2	102	109	196	2^	7^	87	94	100^	102	107	109	194	196^
	27.30	9.26	15.72	16.79	28.22	62.67	0.74	87.29	85.71	5.13	84.33	9.03	86.18	66.77	0.74
9	0	14	32	168	190	14^	18^	22	32	136	154	158	168^	176	190^
	5.65	6.42	19.06	12.76	17.66	0.99	42.93	58.76	45.38	60.47	5.77	10.05	2.44	43.94	39.25

TABLE 2

NEW 3KM 6 DAY REDUNDANT

Sequence:- Day	Aerial positions			Spacings										
0	4	100	148	172	196	24*	48*	72	96*	144	168	192		
	0.64	0.28	0.28	0.26	0.52	0.56	0.73	1.77	0.98	2.31	0.53	2.90		
1	0	84	140	168	196	28*	56*	84*	112	140	168^	196		
	0.27	0.34	0.27	0.20	0.51	0.62	0.85	0.60	2.38	1.80	0.53	0.81		
2	0	8	16	32	4	4*	8*	12	16*	24^	28^	32		
	0.61	0.21	0.29	0.55	0.28	0.75	0.80	1.85	0.99	0.56	0.62	2.98		
3	0	98	147	168	196	21	28^	49*	70	98*	147	168^	196^	
	0.29	0.26	0.42	0.61	0.31	2.62	0.62	0.66	1.71	0.38	2.07	0.53	0.81	
4	112	147	182	14	195	13	35*	48^	70^	83	98^	133	168^	181
	0.64	0.28	0.66	0.41	1.20	3.63	0.96	0.73	1.71	3.59	0.38	1.95	0.53	3.27
5	0	98	182	189	196	7*	14	64^	91	98#	182	189	196^	
	0.51	0.28	0.89	0.39	0.60	1.14	3.54	0.60	1.98	0.38	2.97	2.34	0.81	