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Timing Requirements for the AT Delay and Fringe Tracking System

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*And thus the whirligig of time
brings in his revenge.*

Summary In this note the Australia Telescope delay and fringe rotation system is described by identifying the sources of physical delay in the system and the condition for maximum correlation. The effect of an error in delay and phase on the correlator output is discussed. The effect of baseline and position offsets, instrumental and atmospheric delays, and errors in timing are derived. Finally the accuracy and stability requirements for AT time and frequency standards are considered for both the compact (CA) and long baseline (LBA) arrays. The main conclusions reached are summarized below.

An error in absolute time is of little consequence for normally calibrated CA observations. The main effect is a position error proportional to the source-calibrator separation. A time error is equivalent to a rotation of the array about the earth's pole and limits the accuracy of geodetic and astrometric observations.

Relative time at the CA antennas must be kept to an accuracy of 0.4 ns to avoid decorrelation due to delay errors. If repeated instrumental delay calibration is to be avoided this accuracy must be maintained over the time between antenna moves (weeks).

The relative phase variation of the LOs at the CA antennas should be less than 10° on the timescale of an integration period (< 20 sec) at all observing frequencies in order to avoid instrumental decorrelation. On longer timescales (10-20 min) phase variations due to the atmosphere will limit the CA phase stability provided the LO phase variation is less than about 1°/GHz.

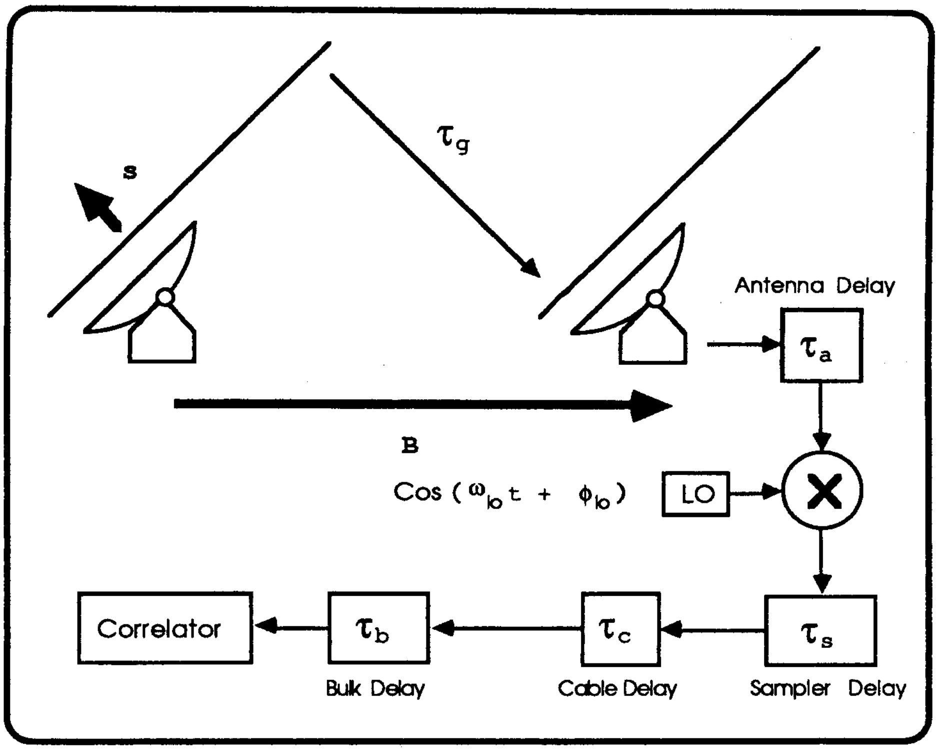
If a secondary frequency standard is used (*eg*. rubidium oscillator) its frequency should be calibrated to a percentage accuracy of 10^{-9} to avoid scaling errors in baseline, position and velocity.

For the LBA the short-term (< 1000 sec) stability of the AT local oscillators will probably limit the coherent integration time possible at a given frequency, and therefore the weakest sources observable with the LBA. For a 100 sec coherent integration time at 116 GHz the stability required is 10^{-15}.

The long-term stability of the LBA oscillators determines the accumulated time error and therefore the lag range which must be searched for fringes. An accumulated frequency error will cause decorrelation if fringe rotation is done at the antennas. In order to reduce the lag range and residual fringe rates to acceptable values for the LBA a long-term frequency stability better than 10^{-13} is required.

The GPS receivers currently on the market offer long-term calibration of both time and frequency to the accuracy required for all AT clocks and oscillators.

The AT Delay & Fringe-Rotation System. The Figure below shows the path followed by a compact array (CA) signal arriving at an antenna with baseline vector **B** from an astronomical source with position vector s.



We identify five sources of physical delay in the system:

(1) τ_g - the geometrical delay

(2) τ_a - the rf antenna delay

(3) τ_s - the sampler delay

(4) τ_c - the IF cable delay

(5) τ_b - the bulk delay

The geometrical delay is determined by the source position and baseline vector. It is the basic quantity measured by an interferometer. The combined delay from all sources of radiation within the primary beam of the interferometer forms the complex visibility function. The geometric delay is given by

$$\tau_g = \mathbf{B} \cdot \mathbf{s} / c$$

where c is the speed of light. The antenna delay is due to waveguide and other components in the rf system before the first mixing stage. This delay should be constant or slowly varying for each antenna. If not accounted for it produces an instrumental phase proportional to the observing frequency. After mixing and digitizing, the sampler freezes the incoming wavetrain at some instant given by its start time. From that point on only delays in units of one sample are possible. This is an advantage since variations in the IF cable delay and sample period are easily removed in a FIFO at the other end of the line. In order to align the incoming bit-streams to within a single sample period the geometric, antenna and cable delay is removed with the bulk delay in steps of one sample. Fine delay tracking in units smaller than a sample period is done by varying the start time of the sampler. The sampler delay tracks smoothly between discrete steps of the bulk delay. It should be understood that these delays (and the baseline vector) refer to *differences* between two antennas. If, for example, the rf delay is identical for both antennas then τ_a is zero.

The signal arriving at the input to the correlator is proportional to:

$$\cos [\, \omega_o (t + \tau_g + \tau_a + \tau_s + \tau_c + \tau_b) - \omega_{lo} (t + \tau_s + \tau_c + \tau_b) - \phi_{lo} \,]$$

where $\omega_o = 2\pi f_o$ is the observing frequency in radians per second. The condition for zero delay at the correlator is: $\tau_g + \tau_a + \tau_s + \tau_c + \tau_b = 0$. Fringe rotation is accomplished by setting $\phi_{lo} = \omega_{lo}\tau_g$. By adding a term $\omega_{lo}\tau_a$ to ϕ_{lo} the instrumental phase due to an rf path delay is removed. Setting the bulk plus sampler delay to cancel the geometric, antenna and cable

delays: $$\tau_s + \tau_b = -(\tau_g + \tau_a + \tau_c)$$

and setting the LO phase: $$\phi_{lo} = \omega_{lo} (\tau_g + \tau_a)$$

the signal at the input to the correlator is:

$$\cos (\omega_o - \omega_{lo}) t = \cos (\omega_{if} t).$$

This is the same as the signal at the reference position and is the condition for maximum correlation provided the time t is identical at both ends of the baseline. In practice there will be a constant phase difference between the signals from the two antennas due to random startup phases on individual oscillators at the antennas. This can be removed by adding a constant offset to the LO phase.

In order to reach this condition the instrumental delays τ_a and τ_c must be found. This is usually done astronomically using a calibration source. In normal observations the geometric delay τ_g is calculated for some time t and the variable delay $(\tau_s + \tau_b)$ and LO phase ϕ_{lo} are applied at that time. Since τ_g does not change very quickly for most connected interferometers (about 1 ns s^{-1} for the 6 km baseline) it is general practice to calculate it at intervals and linearly interpolate between them. With the delay and phase tracking correctly the interferometer fringes are stopped and the signals are properly delayed at the input to the correlator. Our ability to track accurately depends on the precision of our knowledge of the geometric and instrumental delays.

Note that the LO phase has a rate of change ϕ'_{lo} corresponding to a frequency offset $d\phi_{lo}/dt = \omega_{lo}\tau'_g$. The phase of the sampler is $\phi_s = \omega_s \tau_s$ where ω_s is the sampler frequency (set by the bandwidth of the IF signal) and τ_s is the fractional part of τ_g not removed by the bulk delay. Since τ_s has the same rate of change as τ_g during an integration period (τ_b is only stepped between integrations) the sampler runs at a frequency offset $\phi'_s = \omega_s \tau'_g$. Therefore both the local oscillator and the sampler run at a frequency modulated by $(1+\tau'_g)$ in order to track the changing geometrical delay. In essence the telescope oscillators are adjusted to remove the relative Doppler shift between the antenna and the reference position. The bulk Doppler shift important for spectral-line observations is removed by modulating all oscillator frequencies in the array by the same amount. Since this cannot be done easily in the AT master oscillator due to tuning range limitations the required offsets are applied by a combination of oscillators in the IF conversion chain, the fringe rotator, and in software.

The AT conversion system employs more than one LO and fringe rotation need not be realized with a single oscillator. In the above expressions ω_{lo} is the sum of *all* LO frequencies in the chain. ϕ_{lo} may likewise be split up among the various oscillators if desired.

The Geometric Delay. In equatorial coordinates the geometric delay is given by:

$$\tau_g = \mathbf{B} \cdot \mathbf{s} / c = (B_x \cos\delta \cos h + B_y \cos\delta \sin h + B_z \sin\delta) / c$$

where:

c = speed of light (299 792 458 m s^{-1})

B_x = baseline length in direction of **Local Meridian**

B_y = baseline length in direction of due **West**

B_z = baseline length in direction of the **North** pole

h = hour angle measured positive to the West = $t - \alpha$

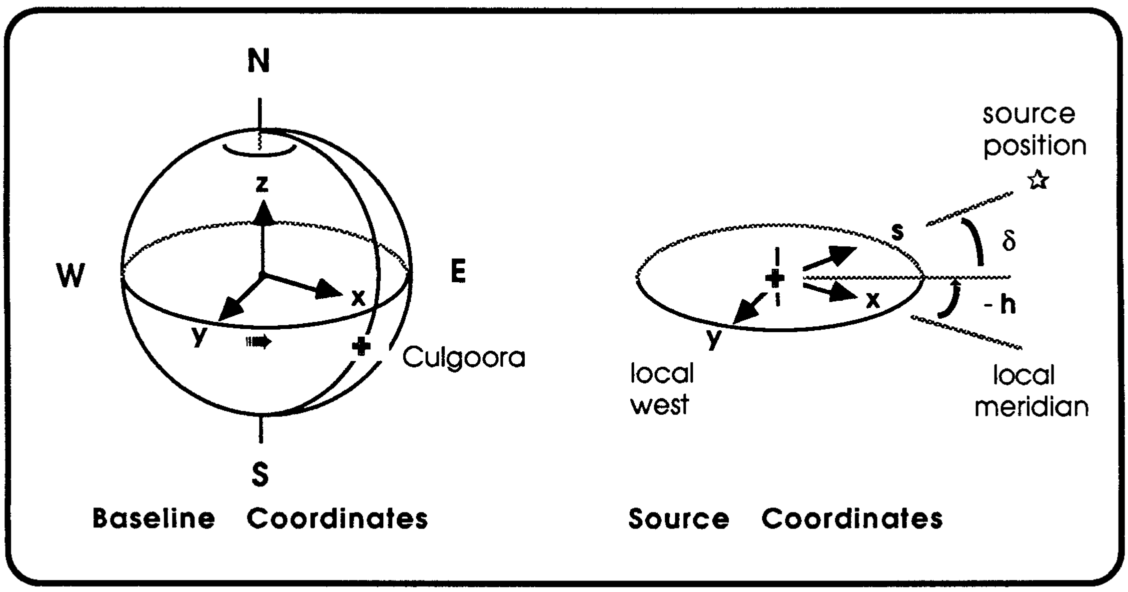
t = local sidereal time

α = right ascention

δ = declination

with angles in radians, baseline lengths in metres, delay in seconds.

In this coordinate system the baseline vector **B** is stationary and the source position vector **s** moves like a telescope. The origin of coordinates is assumed to be at one of the Culgoora antenna stations. For a pure east-west baseline $B_x = B_z = 0$. The equatorial coordinate system is sketched below.



For a point source the phase measured by an interferometer is $\phi = \omega_0 \tau_g$ in the absence of fringe rotation. The natural fringe frequency is $\phi' = \omega_0 \tau'_g$ where

$$\tau'_g = (- B_x \cos\delta \sin h + B_y \cos\delta \cos h) (dh / dt) / c$$

and
$$dh / dt = 2\pi / 8.64\ 10^4 = 7.27\ 10^{-5} \quad (rad / sec).$$

For an east-west interferometer the natural fringe frequency (in Hz or lobes / sec) is

$$f_0 \tau'_g = 0.243\ f_0\ B_y \cos\delta \cos h \quad (Hz / km / GHz)$$

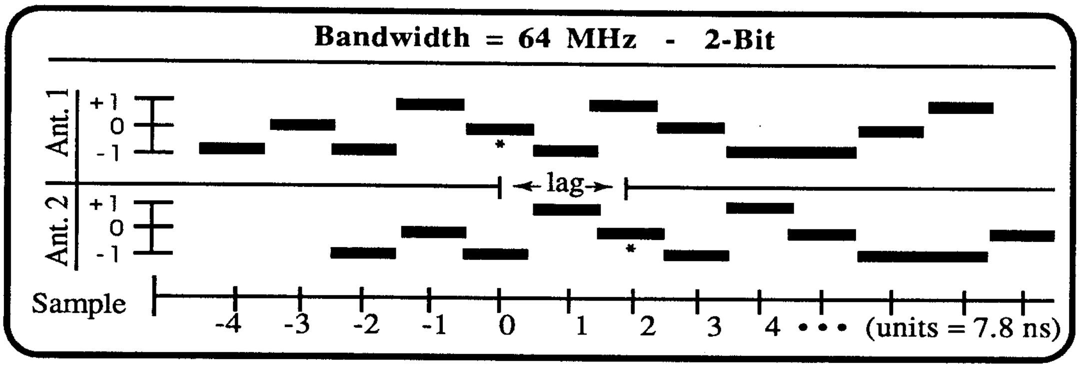
for f_0 in GHz and B_y in km. The highest fringe rate reached by the compact array is about 170 Hz for 116 GHz operation on the 6 km baseline. On any baseline only the equatorial component contributes to the fringe rate; the expression for τ'_g contains no B_z component. For the LBA with mainly north-south baselines only about half of the total baseline length is in the equatorial plane (*cf.* AT/23.4.1/001 - PH "*Doppler Phase Rotator Proposal*"). When the delay and fringe rotation system is operating properly the interferometer phase should be constant for a calibration source.

The Effect of a Delay Error. Suppose there is an error in our caculation of τ_g such that the geometric delay used for fringe tracking is $\tau_g + \Delta\tau$. The variable delay and LO phase are then in error by $-\Delta\tau$ and $\omega_{lo}\Delta\tau$ respectively. The signal at the correlator becomes

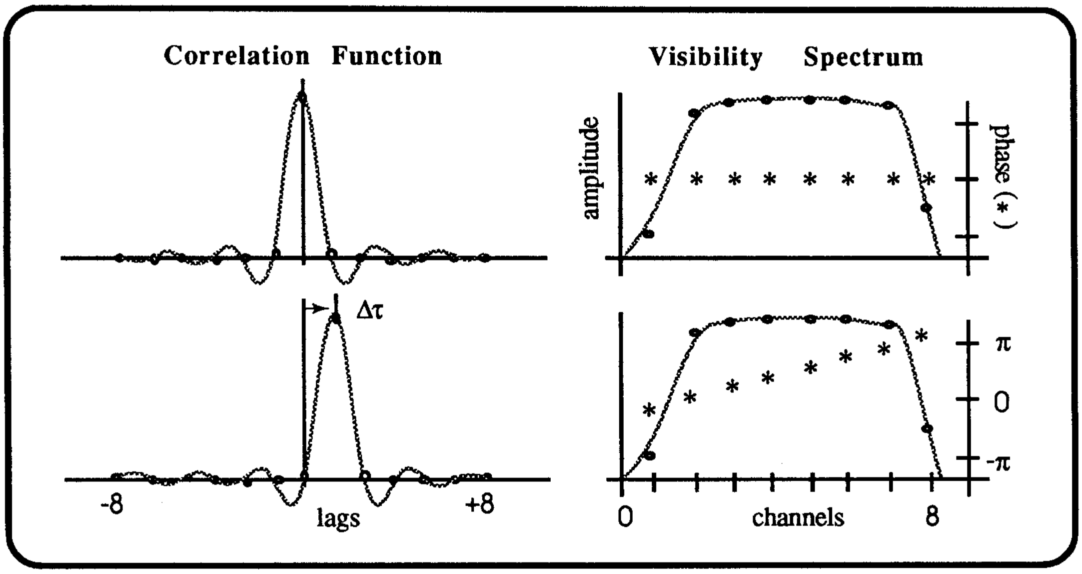
$$\cos [\omega_{if} (t - \Delta\tau) - \omega_{lo}\Delta\tau] = \cos [\omega_{if} (t) - \omega_0\Delta\tau]$$

which differs from the desired signal by the phase term $\omega_0\Delta\tau$ equivalent to an rf delay $\Delta\tau$. Note that this phase is proportional to the observing frequency and so produces a linear phase slope across the observed band. What happens at the correlator is shown schematically below using a 64 MHz bandwidth observation with 8 frequency channel resolution as an example.

Although the sampling rates (nominally 1 / 2b where b is the bandwidth) at the two antennas are necessarily different in order to track the delay, the sample intervals at the correlator are equal after passing through the FIFO. If delay tracking were not done at the sampler then the sampled wavetrains from two antennas can be misaligned by up to half a sample period. This is currently the situation for VLBI observations, and results in a few percent loss of sensitivity compared to observations with fine delay tracking.



At the correlator the sampled wavetrains from two antennas are multiplied together and added for different lags (the lags are in units of one sample period). The * marks equivalent samples - when these are aligned the relative delay between the two pulse trains is zero. In order to develop 8 frequency channels a total of 8 positive and 8 negative lags must be generated. The correlation function and its Fourier transform (the visibility spectrum) are sketched below for zero delay error and for a delay error $\Delta\tau$. The delay error transforms into a phase error proportional to frequency.



In the continuum case the visibility integrated over the bandpass is desired. The effect of a delay error in this case is a reduction in visibility amplitude by $\sin(\pi b \Delta\tau) / (\pi b \Delta\tau)$ caused by the varying phase across the band. In order to avoid more than a 2% amplitude reduction the delay error $\Delta\tau$ must satisfy

$$\sin(\pi b \Delta\tau) / (\pi b \Delta\tau) > 0.98 \quad \text{or} \quad \Delta\tau < 0.11 / b.$$

For the largest AT bandwidth ($b = 256$ MHz) less than 2% decorrelation requires that $\Delta\tau < 0.43$ nsec. In the example above with $b = 64$ MHz the delay error is 1 sample period or about 8 ns. The phase variation across the band is 180 degrees and the continuum visibility amplitude is reduced to 0.64 of its maximum value. The individual channel amplitudes are reduced by $\sin(\pi b'\Delta\tau) / (\pi b'\Delta\tau)$ where b' is the bandwidth of a frequency channel (twice the channel separation for hanning smoothing).

If the delay error is very large the correlation function may be shifted to the edge of the measured lag range. The phase slope across an individual frequency channel then becomes large. In order to have any correlated signal left in a frequency channel the delay error must be less than the lag range measured, *ie*. $\Delta\tau < n / 2b$. In the 64 MHz example this is 62.5 ns. A nice discussion of the effects of a finite lag window and tapering the measured correlation function can be found in Appendix 2 of Chapter 2 of the *"Westerbork Users Manual"* (Willis & Kahlmann, 1980). Rather serious bandpass calibration problems can arise under certain circumstances.

The effect of a delay error on the interferometer phase depends on the source of the delay error. Consider a delay error $\Delta\tau_1$ of say, 30 ps due to an unaccounted delay somewhere (equivalent to 1 cm of pathlength) at an intermediate frequency ω_1, and an rf delay error $\Delta\tau_o$ of say, 1 ps at ω_o. The total delay error results in a phase variation across the observing bandwidth with a 31 ps slope. The total interferometer *phase* error is given by $\phi = \phi_1 + \phi_o = \Delta\tau_1\omega_1 + \Delta\tau_o\omega_o$. For an rf frequency of 116 GHz and an IF delay error at *eg*. 600 MHz, the phase contributions are 6° at the IF frequency and 42° at rf. Thus an rf delay error (*eg*. the atmosphere) can produce a large instrumental phase without appreciably changing the total delay.

The Effect of a Phase Error. Even in the absence of delay errors a constant phase offset is expected due to random oscillator startup phases. Long-term drifts are also generally present due to diurnal temperature variations and other slowly varying sources of phase instability. These phase errors are removed by periodically monitoring the interferometer phase using calibration sources. Provided there is no delay error the phase error is constant over the passband and there is no decorrelation. Time dependent phase errors are particularly serious because they can shift the positions of sources and produce false structure in maps. Phase offsets due to the atmosphere and ionosphere usually limit the phase stability of a connected interferometer like the compact array. Integrations longer than the phase stability time will cause decorrelation. The phase stability of the local oscillators used at the various AT sites will limit the coherent integration time of the LBA unless oscillators of hydrogen-maser quality are used (Rogers and Moran 1981, IEEE Trans. vol. IM-30, No.4, p.283).

Baseline and Position Errors. It is useful to derive the time dependent delay error caused by an offset in the source position and baseline vector. Various sources of delay error can then be described in terms of these offsets by comparing coefficients of the time dependent terms. The error in the geometrical delay when phase tracking is based on slightly erroneous values of **s** and **B** is:

$$\Delta\tau_g = (\partial\tau_g/\partial\alpha)\Delta\alpha + (\partial\tau_g/\partial\delta)\Delta\delta + (\partial\tau_g/\partial B_x)\Delta B_x + (\partial\tau_g/\partial B_y)\Delta B_y + (\partial\tau_g/\partial B_z)\Delta B_z$$

$$\begin{aligned}\Delta\tau_g = \quad & \Delta\alpha\,(\;B_x \cos\delta \sin h \;-\; B_y \cos\delta \cos h \;)\,/\,c \\ & + \Delta\delta\,(\;-B_y \sin\delta \sin h \;-\; B_x \sin\delta \cos h \;+\; B_z \cos\delta\;)\,/\,c \\ & + \;(\;\Delta B_y \cos\delta \sin h \;+ \Delta B_x \cos\delta \cos h \;+ \Delta B_z \sin\delta\;)\,/\,c.\end{aligned}$$

Timing Errors. There are two kinds of timing errors: *absolute* and *relative*. An error in *absolute* time means that the AT station clock is wrong in its prediction of the transit time of a calibration source (*ie.* a point source with accurately known position). *Relative* time refers to differences in time between the central site and individual AT antennas. *Stability* determines the amount of drift expected over a certain time interval.

Absolute Time. First a few words on the definition of absolute time. The time interval in current use is the International Second, defined as 9192631779 periods of a transition of the cesium atom. The second is therefore not directly related to the rotation of the earth. International Atomic Time (IAT) is reckoned by counting standard seconds from some starting point. The AT clock can be set to a resolution of 1 μs (AT/25.1/001 - GWC *"Preliminary Clock Proposal"*). Agreement with international time depends on the method used to synchronize the clock with IAT.

In order to calculate the source position vector **s** accurately in equatorial coordinates we need to know exactly when a source transits the local meridian and also the orientation of the earth's equator (or pole). Coordinated Universal Time (UTC) is kept in synchronism with the earth's rotation by adding offsets of 1 standard second to IAT. Further corrections are still needed to bring UTC into more precise agreement with the earth's rotation. These corrections include small irregularities in the earth's rotation and are supplied by the US Naval

Observatory to subscribers. Corrected UTC is referred to as UT1 or simply UT. This is the best approximation to the mean solar time (at longitude zero) available without astronomical observations. Because there are a number of unpredictable effects which affect the earth's rotation the accuracy to which these corrections can be predicted is about 5 ms. A 5 ms timing error is equivalent to an error in right ascension of 0.075 arcsec. The AT clock is designed to allow UTC timing corrections in 1 ms increments. Other things which affect the apparent position of an astronomical source (*eg*. precession, nutation, etc.) are discussed in AT/25.1.1/025 - MJK & MRC *"Steering the AT - The Ephemeris Routines"*. The accuracy of these corrections is of order 0.001 arcsec.

Local sidereal Time (LST) is used to calculate the local hour angle of an astronomical source. LST is generated from UT1 by adding the east longitude of the observatory and the difference in rate between universal time and sidereal time (1 sidereal day = 0.99726956634 solar days, or about 4 minutes/day slower). Hour angle (h) is related to LST (t) and the source right ascension (α) by: $h = t - \alpha$. At transit the hour angle is zero. An error in the sidereal time t is exactly equivalent to an error in h or in $-\alpha$. The effect of an absolute timing error on the interferometer phase and delay is considered below.

Delay Error: For a timing error Δt (radians) the error in the geometric delay $\Delta\tau_g$ is equivalent to an error $-\Delta t$ in right ascension:

$$\Delta\tau_g = \Delta t \; (-B_x \cos\delta \sin h + B_y \cos\delta \cos h) / c.$$

For an east-west interferometer this error is

$$\Delta\tau_g = 0.243 \, \Delta t \; B_y \cos\delta \cos h \quad \text{(nsec)}$$

for B_y in km and Δt in sec. A 10 ms absolute timing error produces a maximum delay error of about 0.015 ns on the 6 km baseline. The corresponding decorrelation over a 256 MHz bandwidth is negligible (0.002%).

Phase Error. The phase of a calibration source measured by an interferometer with a timing error Δt is:

$$\phi = \omega_o \Delta\tau_g = \omega_o \Delta t \, \tau'_g.$$

At the highest compact array fringe rate (170 Hz) a 10 ms timing error gives a phase error of 1.7 lobes or about 600 degrees. Fortunately, other sources observed with the same timing error have a similar phase error so the effect calibratres out to first order. However, a residual phase error remains whose size depends on the separation in the sky between the source and its calibrator. The phase difference between a source and calibrator separated by (Δx, Δy) radians

in RA and Dec is:
$$\Delta\phi = \omega_o \Delta t \; [(\partial\tau'_g/\partial\alpha)\, \Delta x + (\partial\tau'_g/\partial\delta)\, \Delta y]$$
$$\Delta\phi = \omega_o \Delta t \; [\Delta x\, (B_y \cos\delta \sin h + B_x \cos\delta \cos h) + \Delta y\, (B_x \sin\delta \sin h - B_y \sin\delta \cos h)] / c.$$

For an east-west interferometer the phase difference is:

$$\Delta\phi = \omega_o \Delta t \; [\Delta x\, (B_y \cos\delta \sin h - \Delta y\, (B_y \sin\delta \cos h)] / c$$

which is equivalent to a position offset:

$$\Delta\alpha = \Delta t\, \Delta y \sin\delta / \cos\delta$$

$$\Delta\delta = -\Delta t\, \Delta x \cos\delta / \sin\delta,$$

or approximately the sky offsets (swapped RA for Dec) scaled by the timing error in radians. For a 10 degree sky offset between source and calibrator a 10 ms timing error shifts the source position by 0.026 arcsec from its true position relative to the calibrator.

Baseline Calibration. In order to calibrate the baseline vector a set of sources with accurately known position ($\Delta\alpha = \Delta\delta = 0$) are observed over a range of hour angle and declination. The delay offset due to a baseline error is:

$$\Delta\tau_g = (\Delta B_x \cos\delta \cos h + \Delta B_y \cos\delta \sin h + \Delta B_z \sin\delta) / c.$$

If an error in timing is present the delay error is:

$$\Delta\tau_g = (\Delta t\, B_y \cos\delta \cos h - \Delta t\, B_x \cos\delta \sin h) / c.$$

A timing error is equivalent to a rotation of the baseline by Δt radians about the polar axis. Since this is a pure rotation, so long as the timing error is the same for all observations including the baseline calibration the error would never be noticed; the positions of the calibrator sources will be rotated by the same angle as the baseline vector. If a jump in the clock occurs between baseline calibration and source observations a corresponding jump in the RA of all sources results. This mostly calibrates out as shown above, unless the error is large enough to produce a substantial error in fringe rate.

Basically our absolute timing accuracy determines the accuracy to which we know the baseline direction. This determines the accuracy of geodetic experiments and our ability to measure apparent time astronomically. A reasonable goal would be the ability to track a calibration source to within some fraction (say 1/2) of a lobe. This is equivalent to a maximum phase error of 180 degrees at the highest fringe rate of 170 Hz, or an absolute timing accuracy better than 3 ms (0.045 arcsec). For the (ultimate) LBA fringe rates 10^3 times greater may be present. A similar specification for the LBA would require an absolute timing accuracy of 3 μs (0.045 milliarcsec). In view of irregularities in the earth's rotation of order 5 ms this may seem excessive. However, absolute timing accuracy to better than 1 μs is not difficult to obtain. Timing accuracy to 1 μs would avoid any fringe ambiguities in either the compact array or the LBA and enable the AT to join in international VLBI astrometric and geodetic experiments.

Relative Timing Error. Insofar as tracking the geometrical delay is concerned the total time error for a particular baseline is the sum of the reference clock error and the relative errors at the two antennas. The geometrical delay and phase errors described above actually apply to the total time error. The practical difference between absolute and relative time errors is stability. The stability requirement for the station master clock is considered in the next section. The distribution system used to dispense time to the antennas is the primary source of relative time error. If the error is constant the normal calibration proceedure will remove most of its effect. A much more important aspect of relative timing errors for the AT compact array involves changes in the instrumental delay. This problem is discussed below.

Time Distribution. In order to synchronize the compact array antennas the station clock time is sent to the antennas along optical fibre in 1 ms frames containing 1000 bits each. The clock bit stream therefore has 1μs timing built into it, plus the absolute time of transmission encoded on the bit stream. Absolute time is needed at the antennas in order to start the fringe rotator and sampler at the correct instant. These devices will have been preset with the proper start phase and rate for the beginning of the next integration period, but they need to know exactly when that event occurs. To get the time right at the antenna the propogation delay must be taken into account. This delay (about 5 μs/km) and the slow rise time of the 1 μs clock pulses (about 30 ns) limits the relative timing accuracy to about 50 ns. Since a 3 ms total timing error is tolerable, a 50 ns relative timing error is of little consequence in tracking the geometric delay provided it remains constant.

For a thermal expansion coefficient of $2\ 10^{-5}$/°C the largest expected timing variation due to changes in fibre length is about 0.4 ns for a one degree temperature change. However, the jitter due to the 30 ns risetime may be many nanoseconds. This drift is above the limit of tolerable delay error at the widest AT bandwidth. The LO distribution system is stabilized against pathlength variations and its wider bandwidth allows much shorter risetimes. It may be necessary to use a combination of clock and LO reference signals to achieve the level of relative timing stability required at the antennas.

The Instrumental Delay. In order to bring the sampled bit streams into alignment at the correlator the instrumental delays τ_a and τ_c must be calibrated and removed along with the geometric delay. If there is a relative time error at the antenna, that error is equivalent to an instrumental delay in the sampler. This delay is distinguishable from the IF cable delay only in that it is not quantized in units of a sample period - the FIFO removes any fractional delay in τ_c. The antenna delay τ_a may be distinguished from τ_c by its phase dependence which is proportional to the observing frequency.

Provided the relative time error is constant between instrumental delay calibrations the sampler delay is removed by a constant delay correction. In essence, the sampler timing error is discovered in the delay calibration and removed by re-adjusting the phase of the sampler. If the relative time error changes, the sampler delay changes by the same amount. Since the delay accuracy required to maintain less than 2% decorrelation in continuum observations is 0.11/b, relative timing must be stable to this level; *ie.* 0.43 ns for 256 MHz bandwidth. It is hoped that instrumental delay calibration will be required only once after each antenna move. Relative timing variations should therefore be less than 0.4 ns over a few weeks. The integration time required to measure the instrumental delay is not large, however. In a 64 MHz bandwidth containing 8 frequency channels the rms noise in an individual channel after one minute integration is about 20 mJy for AT bands between 20cm and 3cm (AT/01.17/003 - JRF *"Compact Array Resolution and Sensitivity"*). It should therefore be possible to measure the phase slope across the band to about 1° in one minute using a 1 Jy continuum source.

An upper limit on the relative time error which can be corrected in the compact array is set by the range of the bulk delay. The maximum delay which needs to be accommodated is about 50 µs; the total length of the bulk delay is 64 µs. Thus an upper limit on the relative time error which can be safely removed by the bulk delay is 14 µs. For the LBA the amount of relative delay uncertainty determines the delay space over which the astronomer must search for fringes on calibration sources. The European VLBI network specifies a maximum relative time error of 10 µs in order to avoid lengthy calibration periods during setup.

Clock and LO Stability. The *stability* of a clock depends on the *frequency standard* used to run it. Different types of frequency standard (hydrogen-masers, cesium, rubidium, quartz, *etc.*) have varying degrees of stability. Stability is usually specified in terms of the variation in an oscillator's frequency or period over a certain time interval. For example, the HP5065A rubidium oscillator has a long-term stability of 10^{-11} (one part in 10^{11}) over a month and a short term stability of 10^{-10} in 10 ms, 5 10^{-12} in 10 seconds, and 5 10^{-13} in 100 seconds.

A long-term stability of 10^{-11} can give a time error of about 1μs/day. At an observing frequency of 116 GHz a 1 μs diurnal time error amounts to a total phase drift of over 10^5 lobes in 24 hours! Fortunately for the CA this phase drift is common to all antennas in the array and therefore cancels, except for an error in the calculated fringe rate. A 1 μs time error results in a maximum phase change over a 24 hour period of 0.05 degrees (170 Hz fringe rate) for the compact array. Slow clock drifts are therefore not a problem for the compact array.

The essential difference between the compact array and the LBA (if independent oscillators are used) is that no single reference signal is distributed to the LBA stations for phase-locking the LOs. The LBA therefore relies on the relative stability of the oscillators used at the different sites. The equivalent problem for the CA is the stability of its LO distribution system. While phase variations in the Culgoora oscillator do not affect compact array observations they do appear in LBA correlations. The best stability using a rubidium oscillator is about 5 10^{-13} with an integration time of 100 s. At an observing frequency of 10 GHz the phase change on this timescale is 0.5 cycles or 180 degrees. Although a frequency standard of rubidium quality might be usable at 10 GHz, integration times less than 100 s would be necessary in order to maintain coherence.

The phase error in a single integration period t for an LBA observation at frequency f is $\phi = 2ftv$ lobes, where v is the stability of the oscillator (the factor 2 assumes similar stability at both LBA sites). In order to keep the phase error below 10° the required stability on the timescale of an integration period is $v = (72ft)^{-1}$. For 100 s integrations at 116 GHz the stability requirement is $\approx 10^{-15}$. Currently only the best hydrogen-maser oscillators are able to achieve this level of stability. The current AT specification is 5 10^{-15}.

Effect of a Frequency Error. An error $\Delta\omega$ in the assumed frequency of the compact array master oscillator produces a residual fringe rate equal to $\Delta\omega\tau'_g$ which looks like a scaling error of $(1+\Delta\omega/\omega_0)$ in the baseline vector **B**. Position offsets (and therefore the synthesized maps) scale in the same way, and so does the velocity axis in molecular line spectra. If the frequency error changes between the baseline calibration and subsequent observations a phase error equivalent to a baseline error appears in the interferometer phase. In order to keep the baseline error to a small fraction of a wavelength (say $\lambda/360$ or 1° of phase) then $\Delta f B = c/360$, or Δf = 140 Hz for a baseline length (B) of 6 km. This sets a maximum allowable frequency error $\Delta f/f_0 = 1.2\ 10^{-9}$ at 116 GHz. The map and velocity scaling error is then $(1 + 1.2\ 10^{-9})$. Only quartz oscillators do not have this level of long-term stability.

Another requirement on the AT LO frequency stability is that it maintain the position of a spectral line feature to within 0.1 of a channel width in all configurations of the correlator throughout an observation. For spectral line observations in a 0.5 MHz band using all available channels on a single correlation product the narrowest channel width is 500 / 8192 KHz = 61 Hz. Use of this spectral resolution at an observing frequency of 116 GHz requires a frequency stability of at least $5\ 10^{-11}$. Since some large-field spectral line observations may require months to complete, this is a long-term stability requirement.

For the LBA a *relative* error in LO frequency produces a residual fringe rate which can wash out the signals if fringe rotation is done at the antennas. An error of 10^{-11} at 116 GHz gives a residual fringe rate of 1.16 Hz which will completely average out the correlated signal in a 1 second integration. A long-term frequency stability better than 10^{-13} is required to avoid this problem with integration times of order 100 s. By measuring the accumulated time drift at the LBA sites (using GPS for example) the corresponding frequency drift can be calculated and corrected for. Alternatively, real-time fringe detection can provide the frequency calibration as part of the LBA observing setup procedure (AT/17.3.1/006 - RPN *"A Lower Limit to LBA Clock Accuracy"*).

Phase Stability of the LO Distribution System. Reference LO signals are provided at 5 MHz and 160 MHz to all CA antennas on a stabilized coaxial cable. Both frequencies are derived from the master oscillator and these are in turn used to derive LO frequencies at the antennas. Phase changes in the maser oscillator are of no consequence for the compact array since a phase offset common to all antennas does not appear in the correlation product. Phase changes which are not identical at all antennas do produce a phase error. A common source of this kind of phase error is thermal and other effects in LO generating components which are outside the phase stabilization system. Another is differences in atmospheric delay at the antennas.

The atmosphere produces unpredictable rf delay changes (and therefore phase changes which are proportional to the observing frequency) on a timescale of tens of minutes (AT/10.3/004 - JRF *"LO Phase Stability"*). The LO stability need only be somewhat better than the atmospheric phase stability in order to not make matters worse. Therefore phase drifts of about 1 degree/GHz on timescales of 10's of minutes are tolerable. On times short compared to the atmospheric timescale the LO phase variation should be only a few degrees in order to avoid decorrelation in an integration period. For a 10 degree phase error at 116 GHz the required LO stability is about 10^{-14} on a timescale of 10s of seconds. In order to use longer integration times the LO stability requirement increases proportionally. The relative delay caused by the atmosphere is typically 10 ps on a 6 km baseline. This is equivalent to a phase error of 1.16 lobes or about 420 degrees at 116 GHz. High frequency observations on long baselines will therefore require frequent phase referencing or self-calibration to remove the atmospheric effects.

These specifications apply to the absolute stability of the AT LBA oscillators (since there is no reference other than the absolute for the LBA), or to the relative stability of the compact array LO disteribution system. Since the (tied) compact array is part of the LBA the absolute stability requirement applies to this oscillator as well.

Time and Frequency Calibration. Time is measured (conceptually) by counting cycles of a clock's oscillator, multiplying the number counted by the period in seconds, and adding this to the time present when the count began. A rubidium oscillator is not a primary frequency standard like a cesium oscillator; the rubidium's frequency must be calibrated against another (primary) frequency standard. Once this has been done the period of an oscillation can be used to count time, and the frequency can be used to generate reference tones with accurately known frequencies.

Drifts in a clock/oscillator can be monitored either by comparing the oscillator's current period with another standard, or by checking the clock's time against another time standard. Our ability to monitor time or frequency drifts in the AT clocks and oscillators depends on how often we are able to calibrate them against another standard and the accuracy of that standard. The GPS (Global Positioning System) receivers currently on the market claim absolute time (UTC) measurement to 50 ns accuracy, and 1-5 ns relative accuracy, at a continually decreasing cost (currently about US$ 17K). Installation of these systems at the AT sites would allow accurate and continuous calibration of the AT clocks and LOs. If 5 ns relative timing accuracy is achieved daily the long-term stability implied is better than 10^{-13}.

Although long-term drifts in the AT LOs may be monitored by GPS timing calibrations the variability of the oscillator on short timescales determines the maximum coherent integration time posssible at a given frequency on the LBA. This integration time determines the weakest sources which can be observed with the LBA because, unlike the phase stable CA, interference fringes must be found in each integration period in order to provide a phase useful in the closure relations. A similar thing happens to the CA at the highest frequencies for integration times approaching the timescale for large atmospheric phase changes. This situation can be alleviated only if the drift in interferometer phase between integration periods is predictable.

Conclusions. Absolute timing accuracy to at least 1 μs is recommended for the AT in order to assure astrometric and geodetic capability, and accurate synchronization of the AT stations.

Relative timing stability of the compact array antennas must be good to better than 0.4 ns on a timescale of weeks for accurate delay compensation without repeated calibration. The maximum relative timing error admissible is set by the range of the bulk delay to 14 μs.

The relative phase stability of the AT local oscillator signals (both CA and LBA) should be better than 10° in 10 seconds at all frequencies. For integration times longer than 10 seconds the stability requirement increases proportionally. On timescales of 10s of minutes the stability requirement is about 1°/GHz.

If a secondary frequency standard is used for the AT local oscillators the frequency should be calibrated to a percentage accuracy of 10^{-9} to avoid scaling errors in the measurement of baseline length, radial velocity and source position.

A long-term LO frequency stability better than 5 10^{-11} in about 6 months is required for the AT in order to avoid velocity drifts in spectral line observations. A long-term stability better than 10^{-13} per day is required to avoid lengthy fringe searches and large residual fringe rates on the LBA.

The short-term stability of the AT oscillators determines the maximum coherent integration time possible and therefore the weakest sources observable with the LBA. A stability of 10^{-15} is required for coherent integrations of 100 seconds at 116 GHz. These specifications are based entirely on astronomical requirements and do not take into account constraints imposed by the budget. It is unlikely that frequency standards with the short-term stability required for high frequency LBA observations will be available initially. Hopefully such oscillators will be available when the AT attains its full high frequency capability in the future.

A GPS receiver capable of providing absolute time to 50 ns and relative time to 5 ns would easily satisfy the long-term time and frequency calibration requirements for both the CA and the LBA.