AT/20.1/006

Systems and Performance

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WIDE FIELD MAPPING: EFFECTS INCREASING WITH DISTANCE FROM THE FIELD CENTRE

Introduction

The AT's compact array is being designed to produce high quality maps over a wide field of view. Some effects which may limit the accuracy of wide field mapping are considered here. In this note I am concerned with effects which originate in the (u,v,w) plane. Another class of problems which are not considered here are related to the antenna elements, principally the primary beam. These include effects due to pointing, beam asymmetry, sidelobes and polarization characteristics.

Summary

Expressions for the maximum sampling interval in the uv plane (ΔB), integration time (Δt) and percentage bandwidth ($\Delta\nu/\nu$) are given in terms of the parameters

- D = Antenna Diameter (metres)
- B = Baseline Length (km)
- F = Field-of-View/Primary Beam Diameter to 1st Null
- f = Intensity reduction factor for a point source at the edge of the field-of-view

The sampling theorem requires that

$\Delta B \leq D/2F$ metres

$\Delta t \leq 150/FB$ seconds

Circumferential smearing gives

$\Delta t \leq 570\sqrt{f}/FB$ seconds

and Bandwidth smearing gives

$$\frac{\Delta\nu}{\nu} \leq \sqrt{5f} \times 10^{-3}\ D/FB$$

The constraint on the size of the ω (non-coplanar) baseline term for a maximum phase error $\Delta\phi$ radians is

$$\omega \leq \frac{\Delta\phi}{\pi\lambda} \left(\frac{D}{F}\right)^2 \text{ metres.}$$

For a maximum error of ~ 1% at the 3 dB point of the primary beam we require that

$$\Delta t \leq 20 \text{ seconds}$$

$$\Delta\nu/\nu \leq 1.7 \times 10^{-3}$$

$$\omega \leq 40 \text{ metres.}$$

Discussion

A fundamental requirement for mapping over a field of view of radius r is that the u,v plane be sampled at intervals

$$\frac{\Delta B}{\lambda} \leq \frac{1}{2r}$$

where ΔB is the sampling interval on the ground. This is the sampling theorem due to Bracewell, and it guarantees that complete information about the sky brightness distribution within that field is contained in the measurements. We shall define our field of view in terms of the angle r measured from the beam axis to the first null of the primary beam pattern

$$r = F.\frac{\lambda}{D}$$

where D is the diameter of the antenna elements making up the interferometer. In order to faithfully map the sky out to the first null of the primary beam (F = 1) the sampling theorem requires

$$\Delta B \leq D/2 \text{ metres.}$$

For the compact array with D = 22 m, this implies sampling at intervals no greater than 11 m on the ground.

It should be pointed out that mapping over an area corresponding to $F = 1$ using two grid points per synthesized beam requires the generation of maps with

$$N = 4 \frac{B}{D}$$

points on a side. With $B = 6$ km and $D = 22$ m the maps will be ~ 1024 x 1024 points.

For an east-west array the u,v plane is sampled along concentric ellipses with major axes equal to twice the baseline length in wavelengths.

For a uniformly filled array the spacing increment along the u axis is constant. For the compact array Δu is currently specified at 15 m. The sampling theorem is satisfied in this direction for a field corresponding to $F = 0.73$. The sampling increment along the v axis is $\Delta v = \Delta u \sin \delta$ where δ is the declination of the field centre. For $\Delta v = 15$ m the sampling theorem for $F = 1$ is satisfied along the v axis for sources south of $\delta = -47°$, but not for more northern declinations. The increment along an elliptical track is non-uniform except at $\delta = -90°$. The maximum increment occurs near the u axis and has a value

$$\Delta B \approx 4.36 \, B \Delta t \text{ metres}$$

for a baseline of length B km and an integration time of Δt minutes. In order to satisfy the sampling theorem along an ellipse the maximum integration time is

$$\Delta t \approx 150/B \, F \text{ seconds.}$$

For the 6 km baseline with $F = 1$ the maximum integration time is $\Delta t = 25$ sec. It should be noted that this integration time scales inversely with baseline length.

There is another related limit on the integration time caused by the rapid fringe rate of points far from the phase centre. In the map, a point source at a distance corresponding to F (primary beams) from the field centre will have its intensity reduced by a factor

$$I_{time} \simeq 1-\left(\frac{FB\Delta t}{570}\right)^2$$

due to integration of the time varying visibility. Here B is the maximum baseline length in km and Δt is the integration time in seconds. For a 25 s integration time and B = 6 km, the intensity of a point source at F = 1 is reduced by ~ 7%. In order to keep the intensity reduction to less than 1% an integration time less than 9.5 seconds is required. It might be worthwhile remembering here that at F = 1 the primary beam response is down by many dB. To recover the true, intensity of a source located near F = 1 requires accurate knowledge of the primary beam response. Such a measurement would be severely affected by noise.

It is clear that the u,v plane is not uniformly sampled along most directions and that the sampling theorem is not always satisfied for F = 1. Especially serious is the hole in the u,v plane near the origin caused by the absence of spacings shorter than the minimum. The minimum spacing currently specified for the compact array is 30 m. This means that the telescope acts as a (spatial frequency) high pass filter and is insensitive to structure on scales of order $\lambda/2B_{min}$. As an example, a unit Gaussian source of FWHM = F (primary beams) has a visibility amplitude on baseline B(metres) of

$$|V| \simeq \exp(-3.56\ FB/D)$$

For B = 30 m, D = 22 m and F = 1, the measured amplitude is ~ 0.008, i.e. less than 1% of the total flux is detected. For a minimum spacing of 11 m, about 17% of the flux is detected. Only if the total flux is measured and included at the origin of the u,v plane is the sampling theorem satisfied and the time brightness distribution recoverable.

For a shortest spacing of S metres, baseline forshortening brings the projected baseline length to its minimum value of B_{min} = S sin δ at ± 6 hours of hour angle. The minimum projected spacing is B_{min} = D, at which point one antenna becomes shadowed by the other. With S = 30 m, shadowing occurs only for declinations north of -47°. In order to reach a projected spacing of 22 m at -60° declination (the southernmost extension of the galactic plane) a minimum spacing S = 25.4 m is required.

Holes elsewhere in the u,v plane have a similar effect, although the visibility amplitudes there are generally smaller. One way of looking at

it is the following: the sampling theorem gives a measure of how quickly the visibility function changes with distance on the ground. As long as samples are taken often enough, the visibility between sample points is well estimated by interpolation. This interpolation is the prime function of image restoration techniques like CLEAN or MEM. When holes larger than $0.5/r$ occur, the interpolation is less certain to give correct values. The difficulty of interpolating across the hole near the origin is compounded by a number of additional problems. Among them is the generally large fringe amplitude which can produce large flux errors for the same rate of change as elsewhere. Another problem is that without the zero spacing datum the restoration is forced to extrapolate to the origin. In the presence of large-scale structure in the field, the region near the origin of the u,v plane is perhaps the most difficult to predict based on the value of surrounding points. Of course if large-scale structure is not present, a hole near the origin is no worse than a hole anywhere else.

Another important effect for wide field mapping is bandwidth smearing. The effect is to smear a point source in the radial direction by an amount

$$\frac{\Delta\nu}{\nu} \cdot \frac{r}{\theta}$$

where $\Delta\nu/\nu$ is the fractional bandwidth, r is the distance of the source from the phase centre and θ is the FWHM of the synthesized beam. Letting $r = F\lambda/D$ and $\theta = \lambda/B_{max}$ we have

$$\psi_{bandwidth} = \frac{\Delta\nu}{\nu} \; F \; \frac{B_{max}}{D} . \qquad \text{(size factor)}$$

Since the total flux of a smeared source is conserved, the peak intensity of a point source is reduced, for moderate smearing, by approximately

$$I_{bandwidth} \simeq 0.2 \left(\frac{\Delta\nu}{\nu} \; F \; \frac{B_{max}}{D}\right)^2 \qquad \text{(intensity factor)}$$

This approximation is valid for $I_B \leqq 20\%$ or for $\psi_B \leqq 1$ (smeared by a factor 2).

For B_{max} = 6 km and D = 22 m, allowing a maximum of 0.5 smearing at F = 1 gives

$$\frac{\Delta\nu}{\nu} = 1.8 \times 10^{-3}.$$

The corresponding intensity reduction is 5%. If this criterion is used to determine the number of frequency channels required to synthesize the maximum instantaneous bandwidth available (160 MHz), the worst case for the AT is at L-band where ~ 60 channels are required (AT/05.4/001).

The final effect to be considered here is the effect of non-coplanar baselines. If a 2-dimensional FFT is used to transform visibilities measured on a 3-dimensional surface, the maximum phase error introduced by the required approximation is

$$\Delta\phi = \pi r^2 \omega \text{ radians}$$

where r is the distance from the phase centre and ω is the non-coplanar component of spatial frequency. In terms of F,D and $h = \omega\lambda$ we have

$$\Delta\phi = \pi F^2 \frac{h\lambda}{D^2} \text{ rad.}$$

This is the only error discussed here which is wavelength dependent for a given value of F. The largest error occurs at the lowest frequency, so letting $\lambda = 1$ m (300 MHz) and D = 22 m, the maximum phase error at F = 1 is

$$\Delta\phi = 0.007 \text{ h radians}$$

for h in metres. If we allow a maximum phase error of 0.1 radian (~ 6°) for a source at the first null in the primary beam, the maximum allowable non-coplanar baseline component is ~ 16 metres. Note that the phase error is directly proportional to λ, and is proportional to distance squared from the phase centre.

Conclusions

1) For a minimum u,v increment of 15 m, sampling is adequate over the entire uv plane for fields of view corresponding to F = 0.73 with a 22 m dish. For full-field mapping (F = 1) parts of the uv plane are under-sampled unless the increment is reduced to 11 m.

2) For a uv increment of 15 m, the sampling is adequate over the entire uv plane for fields of view less than or equal to F = 0.33 with a 10 m dish.

This will probably be the case for frequencies above 50 GHz with the AT. This is likely to cause problems only for CO mapping at 115 GHz.

3) For accurate wide field mapping, good measurements of the visibility function near the origin of the uv plane is essential. In order to reach the minimum (non-shadowed) projected spacing of 22 m at - 60° declination, a shortest spacing of ~ 25 m is required.

4) For most observations with the compact array an integration time of 20 seconds on the 6 km baseline is appropriate from the standpoint of the sampling theorem and circumferential smearing. The integration time required for shorter baselines is correspondingly longer. However, in order to keep the circumferential smearing amplitude error for a point source at F = 1 below 0.3% (25 dB), a 5 second integration time is necessary.

5) In order to have similar errors due to bandwidth and circumferential smearing, the percentage bandwidth for mapping should be

$$\frac{\Delta\nu}{\nu} = 3.92 \times 10^{-6}\, \Delta t\, D$$

With D = 22 m and Δt = 20 seconds, the appropriate value is $\Delta\nu/\nu = 1.73 \times 10^{-3}$.

6) Distortion due to non-coplanar components in the compact array visibility coverage will be negligible.

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