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FILE AT

On line AT corrections

mjk, 29 June 1984
revised, 26 oct.84

The main revisions to this note relate to shadowing (A.9), and polarization (A.11 and .12).

The purpose of this note is to spell out in some detail the various tasks that will be required of the on-line vax, as far as the correction of data is concerned. We list the tasks, then provide the algorithms.

A. AP tasks.

We assume as input the raw, uncorrected correlations.

A.1 Correct the correlations for DC offsets.

A.2 Van Vleck correction.

A.3 Hanning - if requested by the observer.

A.4 Fourier invert.

A.5 Antenna gain-elevation.

Real scalar; this will be reset by the vax during the course of a scan.

A.6 Atmospheric attenuation

Real scalar; will vary during a scan - requires a model atmosphere, plus atmospheric conditions monitor data.

A.7 IF gain

Real scalar; ? One might use the noise tube associated with the complex gain monitor (see A.11 and D.1) to provide a measure of the temporal gain variations. In that case, we would update the IF gains at relatively frequent intervals.

A.8 Tsys

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A.8 Tsys

Real scalar; updated every 10 secs.

A.9 Shadowing

Real scalar: in effect, there is a gain change for the shadowed antenna. This correction will be needed infrequently (ie., only occasionally will there be shadowing; but when it does set in the correction factor must be reevaluated every integration interval).

A.10 Bandpass correction

Complex vector; probably invariant, although enthusiasts may update after Cal source obs.

A.11 Complex Relative Gain Factors.

Complex scalar; reset frequently (eg, several minute time constant).

A.12 Stokes parameters

Every integration period the 4 correlator products from each baseline must be combined to form the 4 Stokes parameters.

A.13 Output.

The precessed values of (L,M) or (U,V,W) must be evaluated, and attached to the spectra; a weight (is the data believable?) based on the vax's monitoring of the array must also be found.The data is then sent to the archive medium. (L,M) for the Compact Array, (U,V,W) for the LBA and tied array).

Note: we envisage Real*4 format - the data storage now seems tractable; see RPN's file note on the subject.

B. <u>Vax LO control.</u>

The vax will need to perform a variety of computations to steer the array; a number of phase correction factors can be inserted at this stage, via the LO.

B.0 Precession: provide the rotation matrix to give apparent coordinates of the field centre.

B.1 Phase tracking of the field centre: requires the solution of the projected baseline geometry; to this we add:

station errors (antenna positioning)
antenna phase (antenna geometry).

B.2 Receiver/IF phase offsets.

B.3 Refraction. See appendix A for a discussion of the phase shift algorithm required.

B.4 Ionosphere
B.5 Troposphere. (B.4 and B.5 will likely be slowly varying)

Amplitude correction: Some gain control could be transferred from software (eg. item A.7) if a switchable attenuator is provided on each IF. This would also be desirable for the tied array.

C. <u>On-Line Real-Time diagnostics.</u>

C.1 Monitor the array performance - check whether parameters are within range. Provide a weight (data reliability) for each IF.

C.2 Closure phase and amplitude - form sufficient closure triangles (_quadrilaterals) to monitor all IFs. Average over the observer specified channels.

C.3 Provide an averaged spectrum at one (several ?) map points. (Vector averaging of the phase centre is easy; the others are possible, given sufficient enthusiasm).

C.4 Provide a scalar average spectrum from all visibilities - whereas C.3 gives an accurate spectrum at a point, this should give an indication of the global spectrum.

D.<u>Nasties assumed, but not spelt out.</u>

D.1 The Complex Relative Gain Monitor (the "CRGM"). This needs to monitor the IF pairs to measure the relative gain and phase. Straightforward in principle, but at present (oct.84) it is no more than a principle. Since some number fiddling may well be needed, vax intervention may be required.

D.2 Tsys. Again, simple in principle.

In more detail:

A-5 Antenna Gain-Elevation

For antennae A and B, there will be gain terms $G_A(El)$ and $G_B(El)$; we assume that the same gain term applies to both IF channels of an antenna. G are assumed REAL, as the phase variations with antenna position will have been allowed for in the LO (cf B.1)

All correlation products involving A and B will need to be scaled:

$$Sc = Sm.\sqrt{(G_A.G_B)}$$

(Sc . . corrected; Sm . . measured)

A-6 Atmospheric attenuation

This will require essentially the same treatment as A-5, but with the simplification that the same factor applies to all antennas.

A-7 IF gain

We will determine the relative gain of the various IF pairs from each antenna (see D-1); this leaves us with one factor for each antenna/frequency band:

G_A and G_B

These are in effect complex, but if we compensate the phase in the LO then we are left with REALs, and these can also be absorbed with A-5.

A-8 T(sys)

Again, REALs, to be absorbed with A-5.

To summarize: A-5 to A-8 provide 1 factor for each baseline, for each frequency band.

$$f = \underbrace{[G_A(E).G_B(E)]^{1/2}}_{5}.\underbrace{[\mu]}_{6}.\underbrace{[G_A.G_B]}_{7}.\underbrace{[T_A(sys).T_B(sys)]^{1/2}}_{8}$$

A-9 Shadowing

This effect will be equivalent to a gain change, and so can be incorporated into f. See appendix B for the algorithm.

A-10 Bandpass

For each correlator product, and for each channel, there will be a complex correction ε. (i is the channel number)

$$Sc_i = Sm_i(\varepsilon_{Ai}.\varepsilon^*_{Bi})$$

A-11 Relative gains

and A-12 Stokes parameters

The question of Stokes parameters, polarization corrections and relative gains is dealt with in detail in AT/25.1.1/008. Briefly, we associate with each antenna 3 matrices, each 2 by 2: a gain matrix, a cross-polarization matrix and a parallactic angle rotation matrix. The 4 correlator products from a given baseline are placed in a 2 by 2 matrix. We then perform a series of matrix operations to correct, in sequence, for gain, polarization and parallactic angle, using the matrices of each antenna involved. The end result is a matrix which contains the 4 Stokes parameters.

$$[M] = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix}$$

It is important to recognize that the end result of these operations will all be the stokes parameters corrupted by the phase shifts introduced by the atmosphere/ionosphere - the phase shifts that need self-calibration or redundancy. In effect all 4 stokes parameters have been multiplied by the term:

$$\exp[i(\psi_A - \psi_B)]$$

A-13 <u>Precession</u>

The CA and the LBA need different treatment. Appendix C contains a fuller discussion.

<u>Compact array</u>:

1. Precess the field centre to J2000

2. Let $\underline{B}$ be the baseline at date, in the equatorial system.

   Precess $\underline{B}$ to J2000: $\underline{B}_J = [P].\underline{B}$

3. Rotate $\underline{B}_J$ to align the y-axis with the pole/source plane:

$$L = -B_x.SIN(\alpha) + B_y.COS(\alpha)$$

$$M'' = -B_x.COS(\alpha) + B_y.SIN(\alpha)$$

$$N' = B_z$$

4. Transform the coordinates to provide a small stretch in the y-direction:

$$M' = M'' + N'.COT(\delta)$$

5. Provide a further y-direction stretch to generate an approximately equi-angular map:

$$M = M'.SIN(\delta)$$

   L and M are the "(U,V)" coordinates needed.

Long Baseline Array

Since we don't have a coplanar array, and since we aren't interested in wide-field maps, we can go directly to (U,V,W):

Express $\underline{B}_J$ in the (U,V,W) frame:

$$\underline{U} = [R].\underline{B}_j$$

where [R] is a rotation matrix:

$$[R] = \begin{pmatrix} -SIN(\alpha) & -COS(\alpha) & 0 \\ -SIN(\delta).COS(\alpha) & SIN(\delta).SIN(\alpha) & COS(\delta) \\ -COS(\delta).COS(\alpha) & COS(\delta).SIN(\alpha) & SIN(\delta) \end{pmatrix}$$

APPENDIX A

Refraction

mjk - 27 June 1984

References: Brouw, thesis, p.76
VLA comp. mem. 105

We need to provide a phase correction to each baseline:

$$\phi = 2\pi.L.(\mu-1).SIN(\theta).SIN(A)/COS(z-\theta)$$

where :

L = baseline length in λ

A = azimuth

μ = refractive index

z = zenith angle

θ = the angle between the radius vector from the earth's centre to the observatory, and the radius vector from the earth's centre to the point of intersection between the incoming ray from the source and the outer boundary of the atmosphere of reduced height H.

The Westerbork defaults are:

μ = 1.00031
H = 8000 m.

<u>Note</u>: It should be recognized that although the refraction produces no gross pointing offset for the array as a whole, nonetheless a pointing correction is required for each antenna - given by:

$$\Delta z = (21''\!.3B/T + 19''\!.8f/T + 1''\!.031\mathrm{x}10^5 f/T^2).\mathrm{TAN}(z)$$

where:

z = true zenith distance.
Δz = true - apparent
B = atmospheric pressure in mm Hg
f = partial pressure of water, in mm Hg
T = temperature, in K.

General formulation, and CA specifics.

Let: $\hat{s}$ be the unit vector to the source, and $\underline{r}_t$ be the vector from the earth centre to the telescope.

Consider a plane normal to $\hat{s}$ and tangential to the earth. The path length from the telescope to this plane, P, is the appropriate quantity to calculate, as far as the array is concerned. Now, with no atmosphere, we have:

$$R = (\underline{r}_t + \underline{P}).\hat{s}$$

(Ie., R is the distance from the centre to the tangential plane). So, since $\underline{P}$ is parallel to $\hat{s}$:

$P = R - \underline{r}_t . \hat{s}$ ...(1)

If we assume the earth to have an atmosphere of reduced height H, and refractive index μ, then eqn. (1) is modified to include the path in the atmosphere:

$R+H = (\underline{r}_t + \underline{P}_a + \underline{P}_v).\hat{s}$ ...(2)

where P_a is the path in the atmosphere, in a direction slightly different from $\hat{s}$.

$P_v = (R+H) - (\underline{r}_t - \underline{P}_a).\hat{s}$

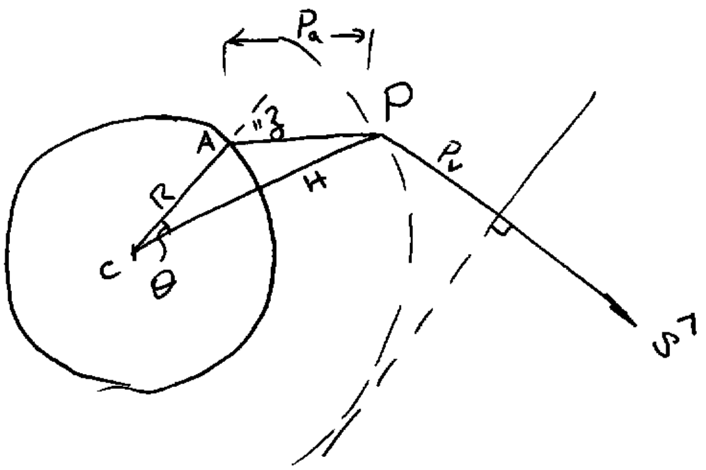
The phase lengths are thus:

$\phi_v = 2\pi P_v / \lambda$

$\phi_a = 2\pi \mu P_a / \lambda$

With an array we are interested in the difference in the quantities $(\phi_v + \phi_a)$ evaluated for each telescope of the array.

The term $(\underline{r}_t .\hat{s})$ is the standard interfermøeter term, and will be fully allowed for in the phase and delay tracking, and will not be further discusseed here. We are left with a differential refrction term - no refraction correction would be required for a plane atmosphere.



In triangle ACP:

$$R.\cos(\theta) + P_a.\cos(z-\theta) = R+H \quad ..(1)$$

$$R.\sin(\theta) - P_a.\sin(z-\theta) = 0 \quad ..(2)$$

rearranging,

$$P_a = (R.(1-\cos(\theta)) + H) / \cos(z-\theta) \quad ..(1')$$

$$\sin(\theta) = P_a.\sin(z-\theta)/R \quad ..(2')$$

We will assume that θ will remain constant over the extent of the interferometer, but that z will not. To calculate the change in zenith angle over the interferometer of length B:

$$\sin(z) = \cos(l).\cos(\delta).\cos(HA) + \sin(l).\sin(\delta)$$
$$\sin(A).\cos(z) = \cos(\delta).\sin(HA)$$

for latitude l, hour angle HA, and azimuth A.

$$\Delta z.\cos(z) = -\cos(l).\cos(\delta).\sin(HA).\Delta HA$$

but: $\Delta HA = B/(R.\cos(l))$ where we take the west telescope as
reference.

so:

$$\Delta z = -B.\sin(A)/R$$

$$\Delta P_a = -B.\sin(\theta).\sin(A)/\cos(z-\theta)$$

Next we need to calculate P_v. Since μ is close to 1, we find it a fair approximation that

$\underline{P}_a.\mathbf{s} = P_a$ (The deflection is $<0.5^\circ$ for all usable z).

Thus:

$$P_v = (R+H) - \underline{r}_\perp.\mathbf{s} - P_a$$

and

$$\Delta P_v = -\Delta(\underline{r}_\perp.\mathbf{s}) - \Delta P_a$$

thus the refractive phase difference is:

$$\Delta\phi_{va} = 2\pi.\Delta P_a.(1-\mu)/\lambda$$

$$\Delta\phi_{va} = (1-\mu).2\pi.(B/\lambda).\sin(\theta).\sin(A)/\cos(z-\theta)$$

APPENDIX B

shadowing

With an EW array there is the occasional difficulty, for sources near the equator, at hour angles near +/- 6 hours, that one telescope will shadow its neighbor.

The (U,V) plane is normal to the line from the telescopes. In this plane the spacing between the optical axes is related to the (u,v) coordinate. In this plane, also, the cross-section of the telescope's beam is circular, so the question of shadowing reduces to the geometrical one of measuring the overlap of 2 circles, each 22 m. in diameter, separated by

$$S = \sqrt{(u^2 + v^2)}.$$

In the observer's coordinate frame, the perpendicular separation between the optical axes is given by:

$$S = B.\sqrt{(\cos^2(Az) + \sin^2(El).\sin^2(Az))}$$

Shadowing will occur if $S < d$, where d is the telescope diameter (22 m. in our case).

If $S < d$, then the shadowing factor (the amount by which the voltage signal is reduced) is:

$$f = 2.(\phi - \sin(2\phi)/2) / \pi$$

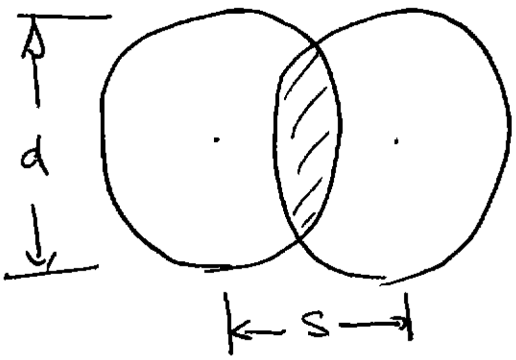
where $\cos(\phi) = S/d$

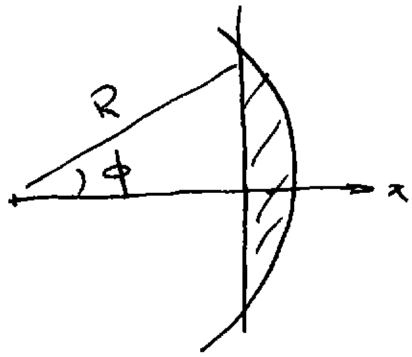
For a given baseline, (B), there is a limiting declination, (δ_1): sources at declinations closer to the equator than δ_1 will require some shadowing corrections at some time - near rising and setting. The relation is:

$$B.\sin(\delta_1) = d$$

this function is shown in fig. 1.

<u>The shadowing fraction</u>





$$A = \int_0^{\phi} 2R\sin(\phi')dx$$

put $x = R\cos(\phi)$; $dx = -R\sin(\phi)d\phi$

$$A = 2\int_0^{\phi} R^2 \sin^2(\phi')d\phi'$$

$$= 2\int_0^{\phi} R^2 (1-\cos 2\phi')d\phi'$$

$$A = R^2 \{\phi - \sin(2\phi)/2\}$$

The full shadowing effect is twice this. Since the unobstructed area is πR^2, we have the shadowing factor:

$$f = 2\{\phi - \sin(2\phi)/2\} / \pi$$

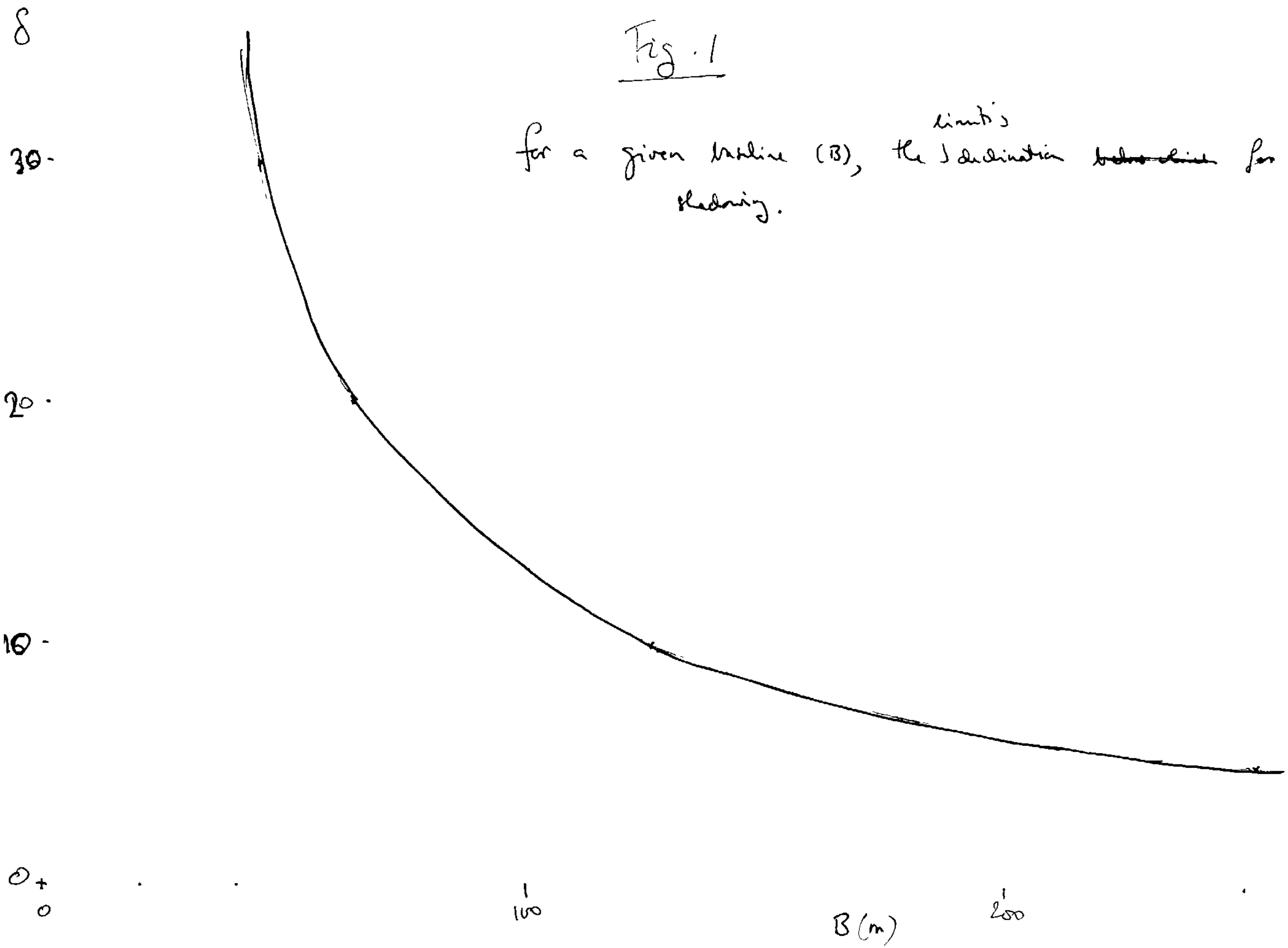


Fig. 1

for a given baseline (B), the limiting declination ~~below which~~ for shadowing.

APPENDIX C

Precession

mjk, 20/5/84

<u>The problem</u>: The equatorial plane of date is not the equatorial plane at epoch. Over the period 1988 to 2012 the angle between the 2 planes will be <= 4 arc min. This misalignement has some implications for the manner in which precession is treated at the AT.

Some possibilities:

a. Make the maps in (L,M) of date, and subsequently deal with the precession - eg., superimpose on the maps a precessed coordinate grid.

b. Make maps in (L,M) of epoch.

c. Make maps in (U,V,W) of epoch.

Option a. will generate maps which, (to the extent that the construction errors are as small as expected), will have a coordinate grid that is correct. The subsequent precession is straight forward, but messy; AIPS has a number of tasks which would need modifying. Option b. has some slight errors at the edge of wide field maps; however, these are small enough that they can be neglected. Option c. would likely have significant errors in wide field maps. These options are discussed in detail in the following pages.

Recommendation: Choose option b. - the procedure is simple, and the concomitant errors bearable.

references:

VLA comp. memo. #105 (Clark)

Brouw, thesis, p. 76.

Some definitions:

By (L,M,N) we mean a coordinate frame which has:

L,M in the equatorial plane, and N aligned with the polar axis, such that the plane (M,N) contains the source, M pointing away from the source. The unit along the M axis is compressed by SIN(δ) in order that the final map be more or less equi-angular.

The (U,V,W) frame is the conventional one, normal to the line of sight to the source.

<u>Option</u> <u>a.</u> - make maps in (L,M) of date.

It is intended that the AT be a wide field instrument. The compact array will therefore be accurately aligned E-W. Thus all the observed baselines (of a single day's synthesis) will lie in a plane parallel to the equatorial plane. The 2-D FT of the (U,V) data is then exact. The same would be true for a map based on several days' data, provided that the separation in time between the various dates is not large.

There are a number of difficulties associated with this approach.

1. Which is the reference date of the map? The first day of data? If not, how do you decide? If the data extends over a number of months, the (U,V) coordinates may need changing - at which point the procedure is becoming messy.

2. All the AIPS tasks which return a coordinate would need attention. It is possible that the subroutine which makes the projection correction could be modified, and that this is the only modification needed; but life is never that simple.

<u>Option</u> <u>b.</u> - Make maps in (L,M) of epoch.

Three steps are involved:

Let the field centre be (α_c, δ_c), at epoch.

1. Form $\underline{B}_e = \underline{B}.\underline{R}$

where $\underline{B}_e$ is the baseline in the coordinate frame of epoch,

$\underline{B}$ is the baseline at date.
$\underline{R}$ is the precession rotation matrix.

for an ideal E-W baseline, of length L, (in units of λ):

$$\underline{B} = [L.COS(T),\ L.SIN(T),\ 0],\ \text{and}$$

$$\underline{B}_e = [B_{ex},\ B_{ey},\ B_{ez}]$$

$$B_{ex} \sim L.COS(T+\nu t)$$
$$B_{ey} \sim L.SIN(T+\nu t)$$
$$B_{ez} \sim N.L.SIN(T)$$

where: T = local sidereal time,
$N = (\nu t).SIN(23.5)$
ν = 50"/yr
t = time interval (epoch - date)

2. Rotate in α to align the y-axis with the (polar axis/source) plane, and form:

$$U = -B_{ex}.SIN(\alpha_c) + B_{ey}.COS(\alpha_c)$$

$$V' = -B_{ex}.COS(\alpha_c) + B_{ey}.SIN(\alpha_c)$$

$$W' = B_{ez}$$

3. Transform the coordinates to provide the small stretch in the y-direction:

$$V = V' + W'.COT(\delta_c)$$

This takes advantage of the fact that :

$$y.COS(\delta_c) \sim z.SIN(\delta_c)$$

How good is this approximation?

We examine the phase error which will arise in the FFT for a point source at (x_s, y_s, z_s).

$$\underline{B} = L.COS(T).\hat{\imath} + L.SIN(T).\hat{\jmath} \qquad \text{at date.}$$

after precession, we have (approximately):

$$\underline{B}_e = L.COS(T-\zeta).\hat{i} + L.SIN(T-\zeta).\hat{j} + N.L.SIN(T).\hat{k}$$

at epoch, where

$$\zeta \sim \nu t$$

At the map making stage we have:

$$U = L.SIN(T-\zeta-\alpha_c)$$

$$V' = L.COS(T-\zeta-\alpha_c)$$

$$W' = N.L.SIN(T)$$

We then form:

$$V = V' + COT(\delta_c).N.L.SIN(T)$$

Compare the phase observed with the phase of a point source at (x_s, y_s) ; the source position derived from the FFT will correspond to the smallest phase difference; our estimate should therefore be an upper limit.

Thus : $\Delta\phi = 2\pi\{y_s.V - y_s.V' - z_s.W'\}$

$$= 2\pi.N.L.(y_s.COT(\delta_c) - z_s)$$

At $\delta = 60^\circ$, and assuming a point source in the corner of a 1 degree field, we have , at the map corners, a phase gradient of 2.E-5 degrees/(λ), which corresponds to a pointing offset of about 0.01 arcsec.

$$y_s = COS(\delta_c) - COS(\delta_s).COS(\Delta\alpha_s)$$

$$z_s = SIN(\delta_s) - SIN(\delta_c)$$

<u>Option c.</u> Make maps in (U,V,W) of epoch.

(This is the domain of the VLA wide field mapping problem)

We calculate the derived position for a source at (x_s, y_s, z_s). The observed phase is:

$$\phi = x_s L\cos(H) + y_s L\sin(H)\sin(\delta_c) - z_s L\sin(H)\cos(\delta_c)$$

(The approximate expressions suffice here to outline the problem)

If we set up a 2-D FFT, we will find a point source in the map, but its position will be in error -

put: $U = L.\mathrm{COS}(H)$

$V = L.\mathrm{SIN}(H).\mathrm{SIN}(\delta)$

$f(x,y) = \int F(U,V).\exp j2\pi(xU+yV).dUdV$

But $F(U,V) = \exp -i2\pi(x_s U+y_s V- z_s V.\mathrm{COT}(\delta_c))$

so that $f(x,y) = \delta(x-x_s, y-y_s+z_s.\mathrm{COT}(\delta_c))$

ie., an error in y of $z_s.\mathrm{COT}(\delta_c)$

The magnitude of the error:

at $\delta = 60^\circ$, at the corner of a 1 degree map, we have a 5" error.

$x = \mathrm{COS}(\delta).\mathrm{SIN}(\Delta\alpha)$

$y = \mathrm{COS}(\delta_c) - \mathrm{SIN}(\delta_c).\mathrm{COS}(\delta).\mathrm{COS}(\Delta\alpha)$

$z = -1 + \mathrm{SIN}(\delta_c).\mathrm{SIN}(\delta) + \mathrm{COS}(\delta_c).\mathrm{COS}(\delta).\mathrm{COS}(\Delta\alpha)$

$(z \sim x^2 + y^2)$