Steering the AT - the ephemeris routines

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Introduction

In this note we describe the procedures that we propose to follow to "steer" the AT - to ensure that a point source located at the specified field centre is seen to have zero phase, on all baselines.

Let $\hat{\varepsilon}$ be the unit vector in the direction of the field centre, and let $\underline{r}_i$ be the position vector of antenna (i) in the same reference frame.

$$l = \underline{r}_i \cdot \hat{\varepsilon}$$

This quantity prescribes the delay and phase which must be added to the signal from antenna (i).

This note is devoted to the questions involved in calculating $\hat{\varepsilon}$ and $\underline{r}_i$.

1. Let (α_0, δ_0) be the source position at some reference coordinate frame (J2000 mean equator and equinox, of preference). We need to convert this to a direction in the reference frame of date - we need to allow for: precession, nutation, annual aberration, and the gravitational deflection of light by the sun. Proper motion, annual parallax and radial motion are included. The end product is a direction specified in a geocentric frame.

2. There are then a few corrections which are antenna specific: diurnal aberration, atmospheric and ionospheric refraction, diurnal parallax (comets).

   Put $\hat{\varepsilon} = (\alpha, \delta)$

3. Each antenna's position on the earth is known. But we

need to calculate the earth's location within the geocentric frame (of date). We need to calculate the earth's rotation (local apparent sidereal time), the location of the pole, and the effect of the earth tides; we also allow for ocean loading displacement and continental drift.

When all this is done we can specify the antenna's position in the same frame as the source.

Put $\underline{r}_i$ for antenna i.

4. We then form the scalar product:

$$p_i = \underline{r}_i \cdot \hat{s}$$

(p_i / c) is the delay which should be added to i's IF.

$2\pi f_1 (p_i / c)$ is the phase to be added to i's LO.

$2\pi f_s (p_i / c)$ is the phase to be added to i's sampler.

5. We will also calculate χ, the parallactic angle, needed at the stage when the cross-correlation products are combined to form the Stokes parameters.

6. We will then determine the differential precession, needed to align the resulting map in the output reference frame of epoch. (The CA requires a minor fudge to the baselines to place them in the J2000.0 frame).

In essence, steps 1 and 3 follow the precepts of the Astronomical Almanac pp B36 to B41 (step 1), and B58 to B61 (step 3), using the latest (1980) IAU recommendations. The topocentric corrections (step 2) follow a new scheme - they are applied to the antenna position vectors.

Step 1 is concerned with establishing the celestial reference frame of date. The task is largely satisfied by the subroutines written by P.Wallace, for COCO. These subroutines claim an accuracy better than 0.3 milliarcsec, which should be adequate for the Compact Array, and will probably suffice for the LBA.

Step 3 relates the geodetic reference frame to the celestial frame. This operation is somewhat messier: the necessary algorithms are detailed below. The basic reference data and procedures will be drawn from the Project MERIT standards. (USNO circ. # 167).

Steps 5 and 6 are intrinsically of lower precision, and pose no problem.

In section 2 we discuss the factors which influence the design, and in 3 we outline the implementation.

2 Factors which influence the design

2.1 Reference coordinate frame

J2000 mean equator and equinox

Our basic plan is to operate under the IAU (1976/1980) recommendations. This is a consistent package: the relation sidereal time/UT1: the precession constants: the nutation theory; the equinox. The ephemeris package will therefore be expecting positions referred to J2000, and it will output data in this frame.

We will maintain a file of calibrator source positions: these positions will be in J2000.

We will provide assistance to observers who arrive with positions specified in other reference frames. Conversion of source positions from one catalogue epoch to another is a messy business, and we should eschew all requests to provide that service on-line. (The interested reader is referred to : Clark, VLA mem. #s 145, 167; Kaplan, USNO circ. 163; Aoki et al. A. & A. 128, 263.)

In spite of the above, we note that cometary ephemerides are presently computed with respect to B1950., so we may nonetheless have to provide for other epochs. The STARLINK subroutines do allow for precession according to the pre-1976 scheme (ie. FK4), so provided we are prepared to recompute sidereal time on the old scheme we should be able to cope. Non-J2000 input will produce non-J2000 output.

2.2 Comets and planets

These sources will accomodated by computing a "proper motion" correction to their positions. That is, the observing program (CAOBS) will pass the the tracking program a position and a proper motion.

2.3 Phase centres and LO control

It would clearly be simplest to use the earth's centre as reference, as we could then use the same algorithms for CA and LBA, as well as be sufficiently general for international collaboration. This avenue poses severe problems. The "Day one" LO will provide a continuously variable, (calibrated) range of +/- 1 KHz, with large coarse steps beyond. (That is, the phase of the array will need recalibrating whenever an oscillator is switched to a new frequency). This is adequate for the CA alone, where fringe rates of up to ~40 Hz/λ(cm) at 6 km are required. The problem of positioning a spectrum within the output band - ie. the problem of correcting for doppler shifts can be accomodated with the "80 MHz" oscillator. If we wish to track at the rate specified by the geocentric phase centre, then we encounter much higher rates (~400 KHz) which will mean more frequent updates, and calibration difficulties when the 1 KHz range limit is reached.

The proposed solution is to adopt Culgoora as reference point for the CA antennas, then account for the motion of the reference point in a separate operation. This is discussed in greater detail in the appendix.

2.4 Accountability

Our philosophy in the processing of the data, between the antennas and the archival medium is that the archived data should be as close to finally calibrated data as possible, but that it should always be possible to reverse all corrections applied to the data, and to recover the raw data. This means that we need to archive the algorithms employed in the entire LO/delay tracking operations. This will be achieved through our use of the Digital CMS routines to oversee the integrity of the software - it will always be possible to recover to code used for any observation. Complementing the accountability of the algorithms, the observing process ("CAOBS") will archive all the parameters which control the software.

2.5 Frequency of computation : How often should we compute?

* Conversion from mean to apparent:

precession

$$d(\alpha\cos\delta)/dt = m*\cos(\delta) + n*\sin(\alpha)\sin(\delta)$$

$$= 0.126\cos(\delta) + 0.055\sin(\alpha)\sin(\delta) \quad \text{arcsecs/day}$$

$$d\delta/dt = n*\cos(\alpha)$$

$$= 0.055\cos(\alpha) \quad \text{arcsecs/day}$$

aberration

$$d(\alpha\cos(\delta)/dt = -0.352\cos(\lambda)\sin(\alpha) + 0.324\sin(\lambda)\cos(\alpha)$$

$$\begin{aligned} d\delta/dt = & -0.352\cos(\lambda)\cos(\alpha)\sin(\delta) \\ & + 0.324\cos(\lambda)\sin(\alpha)\sin(\delta) \\ & + 0.137\cos(\delta) \qquad \text{arcsecs/day} \end{aligned}$$

nutation

Similar expressions, with terms of order 0.1 arcsecs/day.

It seems therefore that we should re-evaluate the source vector at intervals of about one hour, and use linear interpolation at each integration interval. We could, for example, simply evaluate the source vector at each hour, and use the difference for the interpolation - this would ensure a smooth, continuous position/time function.

The topocentric corrections are applied in full for every integration period.

* Conversion from apparent to topocentric.

Diurnal aberration.

Although the basic diurnal aberration effect is smaller than the preceding effects, it varies much more rapidly - in effect, we would find that linear interpolation would be unsatisfactory at the 1 milliarcsec level over 1/2 hour spans.

$$\cos(\delta) d\alpha/dt = 1.74 \sin(t_s - \alpha) \quad \text{arcsec/day}$$

$$d\delta/dt = -1.74 \cos(t_s - \alpha)$$

(t_s = sidereal time)

2.6 Computation precision requirements;

1 arcsec ~ 1 part in 10^6

Thus, if we wish to attain the mas region, double precision will be required.

3. The Implementation

3.1 The Celestial frame

At the start of every scan (or every hour, if the scan is of more than one hour duration), we precess, nutate aberrate and gravitationally deflect (α_0, δ_0), to produce, in the reference frame of date, $\hat{\varepsilon}$ and $d(\hat{\varepsilon})/dt$. The derivatives may most easily be evaluated by calculating $\hat{\varepsilon}$ at the scan ends, and differencing.

To do this: we need time (TDB), and position.

Time

the observatory will maintain TAI

$$TDT = TAI + 32^s.184$$

(Terrestrial dynamic time)

$$TDB = TDT + 0^s.001658\sin(\;..\;)$$

(Barycentric dynamic time)

(cf. Astronomical Almanac, p.B6)

TDB is the argument required by the starlink subroutines.

position

this is the tracking centre specified by the observer. (When dealing with comets, these positions will need to be appropriately updated).

It will be convenient, at this time, to compute some other quantities that will be needed later on - the earth's velocity, to be used in correcting spectral line observations.

The results of these operations will be stored in common, available to the next stage.

3.2 The terrestrial frame

These computations are antenna-specific.

On the same time scale as the previous operation (every hour), we will tweak up the antenna positions - correct for earth tides.

At the start of every integration, locate the terrestrial frame within the celestial frame - calculate sidereal time, and obtain the rotation matrix. We will also precess the baselines to the reference frame of the output maps.

Terrestrial reference frame

The terrestrial frame used for the antenna locations is the geodetic frame, defined by the average north pole (Conventional International Origin), and the Greenwich mean astronomic meridian.

Sidereal time

$$UTC = TAI + \Delta AT$$

$$UT1 = UTC + \Delta UT1 + \text{seasonal variations}$$

ΔAT and $\Delta UT1$ are distributed by the BIH. (We receive a version from National Mapping). The USNO provides an expression for UT1-UTC:

$$UT1-UTC = C11 + C12(MJD - \text{ref.date}) + \text{seasonal variations.}$$

The coefficients C11 and C12 are available on a weekly basis. (Bulletin A of National Mapping, in our case.)

The prescription for the seasonal variations is given in two parts: the annual terms (cf. Mueller: also BIH circular-D), and the earth zones as given in the Annual report of the BIH. The full treatment involves a 41-term summation, but the first 8 terms will probably suffice.

$$GMST = 67310^{s}.54841$$

$$+ (876600^{hrs} + 8640184^{s}.812866)T_U$$

$$+ 0^{s}.09310T_U^2 - 6^{s}.2E\text{-}6T_U^3$$

$$GAST = GMST + \text{eqn. of equinoxes } (= \Delta\psi\cos(\varepsilon))$$

T_U is the number of Julian centuries of 36525 days that have elapsed since 2000 January 1, 12 hrs, UT1 (JD 2451545.0)

$$T_U = ((Y-2000)*365 + \mathrm{int}((Y-2000)/4) + D + T - 0.5)/36525$$

(year Y, day D, time T). It will be unecessary to recompute this entire expression every 10 secs - a full computation every hour, followed by an interpolation every integration interval will be adequate.

Earth tides

The arguments are sidereal time, latitude and longitude. The prescription comes from Appendix A5, USNO 167.

The antenna locations : $\underline{A}_i$

To these we add $\Delta\underline{A}_i$ (earth tide correction):

$\underline{A}_i^f = \underline{A}_i + \Delta\underline{A}_i$

Polar motion

We compute the rotation matrix, **R**, to take us from the geodetic (terrestrial) frame to the celestial frame.

We need the polar motion (x_p, y_p). As with the predictions for UT1, we should use the USNO predictions here:

$$x_p = C1 + C2\cos(A) + C3\sin(A) + C4\cos(B) + C5\sin(B)$$

$$y_p = C6 + C7\cos(A) + C8\sin(A) + C9\cos(B) + C10\sin(B)$$

$A = 2\pi(MJD - ref.date)/365.25$

$B = 2\pi(MJD - ref.date)/435.0$

the rotation matrix is thus:
(put G for Greenwich apparent ST)

$$\mathbf{R} = \mathbf{R}_1\mathbf{R}_2$$

$$\mathbf{R}_1 = \begin{matrix} \cos(G) & -\sin(G) & 0 \\ \sin(G) & \cos(G) & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$\mathbf{R}_2 = \begin{matrix} 1 & 0 & -x_p \\ 0 & 1 & y_p \\ x_p & -y_p & 1 \end{matrix}$$

and finally: $\underline{r}_i = (\mathbf{R})\underline{A}_f$

Ocean tide loading

The MERIT standards describe a further set of corrections that we will implement: local deformation of the crust due to the ocean tides. The effect is a change in height (of order 1/10 cm), and a tilt of the local vertical (of order milliarcsecs). The change in height presents no problem - it can be incorporated directly into the station coordinates. The tilt would seem to be of no importance for the LBA. The East-West component of the tilt will appear as a timing error for the CA: the simplest procedure would be to compute the small phase correction to add to each antenna, taking the local phase centre (control building) as reference point.

Continental drift

Since this is of order cm/year, and is thus larger than some of the effects discussed above, it would seem that a case could be made for including this as a correction. An annual computation, plus interpolation will probably suffice.

Diurnal aberration and refraction. These are corrections to the source vector that are based on the antenna's location. As such, they fall between celestial and terrestrial frames.

diurnal aberration

We calculate the explicit delay at the antenna, rather than calculating the apparent displacement in position of the source.

$$\tau[\underline{V}.\hat{\varepsilon} + c] = \underline{r}.\hat{\varepsilon}, \quad \text{for antenna position } \underline{r}$$

$$\underline{V} = \Omega\hat{\ }\underline{r}$$

Ω is the earth's rotational velocity,

Atmospheric refraction

Refraction can also be dealt with as a delay - see appendix. (We need only correct for the spherical nature of the atmosphere).

For each antenna we will compute the additional path length through the atmosphere, due to the atmospheric refraction.

$$\Delta s = 0.002277\sec(z)[p + (1255/T + 0.05)e - B\tan^2] + \delta$$

where:

Δs is in metres:
p is the barometric pressure in millibars:
e is the partial pressure of water vapor, in mb:
T is the surface temperature, in Kelvin
B is a height correction, ~ 1.1 mb;
δ is a further correction related to height, and for culgoora, (at 216 m above sea level), is :

$$\delta \sim 6.7e{-}4\tan^3(z)$$

e can be determined from the relative humidity:

$$e = 6.108RH\exp[(17.15T-4684)/(T-38.45)]$$

Ionospheric refraction

A small correction for a spherical ionosphere may be applied at the lower AT frequencies.

Ionospheric faraday rotation ?

Perhaps.

Diurnal parallax

The basic idea is to compute a time delay to be added each antenna to correct for the parallax. The algorithm is outlined in the appendix. The result is:

$$\tau = (\underline{r}.\hat{s}/c)[1 - \sin(\pi_0)(r^2-(\underline{r}.\hat{s})^2)/(2a_e\underline{r}.\hat{s})]$$

where π_0 is defined in terms of the distance to the source:

R, the distance from the geocentre to the source,

$$R = a_e/\sin(\pi_0)$$

and π_0 is the horizontal parallax, and a_e is the radius of the earth.

Parallactic angle

$$\sin(\chi) = \sin(H)\cos(\phi)/\cos(E)$$

where

H is the hour angle (at the antenna) of the tracking centre.

E is the elevation of the tracking centre

ϕ is the antenna latitude

Precession of the baselines

The task here is to ensure that the baselines are calculated in a reference frame consistent with the required epoch of the output maps. For the LBA, we define a frame which has one axis in the direction of the source (tracking centre), and one axis in the equatorial plane of epoch. This frame can be obtained from the precession/nutation rotation matrix. Thus we perform a matrix operation on each baseline:

$$\underline{b}_{ij} = (\mathbf{PN})(\underline{r}_j - \underline{r}_i)$$

The CA will use the Celestial frame of epoch (NCP projection), in order to be able to exploit the exact fourier transform capability (wide field of view). This poses a minor problem (cf. mjk, AT/25.1.1/002B) in that the equator of epoch (J2000) is not the same as the equator of date.

The algorithm is discussed in detail in the appendix. The sequence will be:

Compact array

1. Place each antenna in the frame defined by the polar axis, the equatorial plane and the source.

$$\underline{r}'_i = \mathbf{R}\underline{r}_i$$

2. Locate each antenna in the NCP frame of epoch. This is a differential precession, since we want the field centre to remain at the centre of the map. By this operation a vector which originally lay in the equatorial plane (of date) will acquire a small polar component (r'_z).

3. Since r'_z is small, we can incorporate it into the "y"term :

$$r''_x = r'_x$$

$$r''_y = r'_y + \cot(\delta) r'_z$$

$$r''_z = 0$$

4. For each baseline, compute the baseline vector:

$$\underline{b}_{ij} = \underline{r}''_j - \underline{r}''_i$$

The visibility coordinates (l,m,n) - the components of $\underline{b}$ - are passed to the correlator array processor, to be incorporated into the data set.

LBA

1. Express each antenna vector in the (u,v,w) frame, defined by the triad: source vector as "z-axis", "x-axis" in the equatorial plane, and "y-axis" in the plane defined by the source vector and the polar axis.

2. Apply the differential precession, a rotation of $Nt\sin(\alpha)/\cos\delta$, about the w axis. (N is ~ 20"/year, expressed in radians), t is the time interval between the date of observation and the epoch of the output map.

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refraction corrections

mjk, 27 Feb. 1986

1. Atmospheric

Saastamoinen (1973) has examined the question of range correction to a satellite. This is applicable to our case, and provides an estimate of the refraction correction to the path length.

$$\Delta s = 0.002277\sec(z)[p + (1255/T + 0.05)e - B\tan^2] + \delta$$

where :

Δs is in metres;
z is the zenith angle of the source;
p is the barometric pressure in millibars;
e is the partial water vapor pressure, in mb;
T is the surface temperature, in degrees K;
B is a height correction, equal to about 1.1 mb;
δ is a height dependant factor, equal, for Culgoora, to about:

$$\delta \sim 6.7e\text{-}4\tan^3(z)$$

The partial pressure of water vapor is derived from a measurement of the relative humidity:

$$e = 6.108RH\exp[(17.15T - 4684)/(T - 38.45)]$$

(This is equivalent to the Goff-Gratch equation . Smithsonian meteorology tables, p350).

Moran and Rosen (1981) have shown that with these expressions one can predict the atmospheric refraction contribution to the vertical path length with an RMS $\sigma \sim 5$ cm (summer), and ~ 2 cm (winter), (in the north-east of the

USA). The computed path length is ~ 2.3m, so σ ~ 2

The difference between antennas will therefore be substantially less: the differential path length over the compact array,

$$\delta(\Delta s) = (\Delta s/\cos(z))\sin(z)\delta z$$

$$= (\Delta s/\cos(z))(L/R)\sin(\phi)\cos(\delta)\cos(H)$$

for an East-West baseline of length L. R is the earth's radius, ϕ is the CA latitude, and H the local hour angle of the field centre.

Since z is limited to angles > 80 $^{\circ}$ (hard limit set by the antenna) we have a maximum differential path length of ~ 7 cm for the CA. The 2% uncertainty in the vertical path thus translates to an acceptably low phase error (< 0.14mm). At this level the error will be incorporated in the phase calibration (in the general run of observations): it is less than the uncertainty set by the random phase fluctuations due to weather cells.

This optimistic view does not apply to the LBA operations. The results of Moran and Rosen indicate that the errors are uncorrelated over antenna separations greater than 200 km. LBA operations will thus be rather more heavily dependent on phase referencing, as differential path errors of order 30 cm are likely to be the norm.

Ionospheric refraction

Extensive work by Spoelstra (1983) at WSRT indicates that some ionospheric refraction can be expected at the lower frequencies of the AT. A simple spherical model of the ionosphere will be employed to remove some portion of the predictable component.

(typical phase errors at 610 MHz , 3 km baseline, ~ 50 $^{\circ}$, which reduces to 9 $^{\circ}$ at 1420 MHz).

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Bean,B and Dutton,E (1966) Radio Meteorology
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The LO and sampler operations

mjk, 2 March 1986

This note attempts to offer a definitive description of the LO and sampler operations, from the computational requirements point of view.

Fig. 1 shows a stylized interferometer.

$\underline{r}$ is the vector from the phase centre to an antenna

$\hat{s}$ is the unit vector towards the source.

We follow the signal as it progresses down the chain, from the antenna to the correlator.

Let $t = 0$ at the start of an integration interval. This instant is defined by the clock at the central site. We will place the phase centre at the central site.

If the RF signal at the central site is : $\cos(2\pi ft)$

then at an antenna it is : $\cos[2\pi f(t + \tau)]$

$$\text{where } \tau = \underline{r}.\hat{s}$$

$$\sim \tau_0 + \tau' t$$

$$(\tau' = d\tau/dt)$$

The algorithms involved in computing $\underline{r}$ and $\hat{s}$ are given in AT/25.1.1/025

Local oscillator

The "local oscillator" consists of a number of units (two for most frequency bands). Only the last of these is continuously tunable (over a range of +/- 1 KHz); the others operate in steps (0.3 GHz, 10 MHz. 1 MHz). f_l is the sum of all oscillators.

The local oscillator signal going into the mixer is :

$$S = \cos(2\pi f_L + \phi)$$

the signal from the mixer (and into the sampler) is thus:

$$S = \cos[2\pi(f-f_L)t + 2\pi f\tau - \phi]$$

$$\text{where } \phi = 2\pi f_L \tau$$
$$= 2\pi f_l (\tau_0 + \tau' t)$$

That is, at the start of each integration interval we will specify the required starting phase of the tunable oscillator, and the rate at which its phase should increase.

In this formulation we are ensuring that the antennas of the CA are all correct relative to a phase centre (at the central site). The question of the motion of the phase centre is dealt with separately:

let $\underline{r}_c$ be the vector from the geocentre to the site;
let $\underline{V}$ be the total velocity that needs to be accounted for:

$$\underline{V}_1 = d(\underline{r}_c)/dt \quad : \quad \underline{V}_2 = \underline{V}(\text{orbital}) + \underline{V}(\text{radial})$$

$$\underline{V} = \underline{V}_1 + \underline{V}_2$$

CA -

The problem here is the removal of the bulk doppler term. so that a given spectral feature remains in the same spectral channel throughout the course of the observation. For "narrow" spectral work (Bandwidths < 16 Mhz) this is accomplished with an additional oscillator, at the central site. This is mixed with the signal from the digital-analogue unit. For higher bandwidths a different scheme is required, described elsewhere (Norris, AT/23.4/011)

We need to provide a frequency offset of:

$\sim f(\underline{V}.\hat{\varepsilon}/c)(1+\underline{V}.\hat{\varepsilon}/2c)$

At the higher observing bands some portion of the frequency offset could be provided at the antenna local oscillators.

LBA -

The signals from the antennas of the CA are summed at the central site. At this stage a further phase shift may be required, depending on where the phase centre is set. The natural choice, and the one which seems increasingly necessary for international cooperation, is to place the phase centre at the geocentre.

The geocentre reference requires us to provide a phase of

$$\phi = \underline{r}_c . \varepsilon$$

and a frequency offset of $f \sim (\underline{V}_2.\hat{\varepsilon}/c)(1+\underline{V}_2.\hat{\varepsilon}/2c)$

Sampler

A synchronizing signal is sent from each antenna at the start of each integration period: the sampler will start at that instant: the first sample will be taken an interval $\Delta\tau$ after the start of the integration interval, where $\Delta\tau$ is the fine delay - the difference between τ_0 and the delay of an integral number of samples (INT($\tau_0 f_s$)). This fine delay is set by the initial phase of the sampling frequency.

The signal at the sampler can be written:

$$S = \cos[2\pi(f-f_L)t + 2\pi f\tau - \phi]$$

$$= \cos[2\pi(f-f_L)(1+\tau')t + 2\pi(f-f_L)\tau_0)]$$

If we put $t_1 = t(1+\tau')$

Then $S = \cos[2\pi(f-f_L)t_1 + 2\pi(f-f_L)\tau_0)]$

Thus the sampler should operate at a rate:

$$f_s(t) = f_s \cdot (1+\tau')$$

FIFO

Data is fed into top of a FIFO after the receipt of the synchronizing signal. The delay required (τ_0) can be of either sign, depending on the source position, and on the location of the phase reference. Define the mid-point of the FIFOs to be zero delay. The data is extracted from the FIFO at a point $N_s = (\tau/f_s)$ below the mid-point. (see fig.3). Correlating cannot start until valid data has arrived from the most distant antenna - we need to wait at least $MAX\{-\tau_i\}$

Fine detail

The phase specified for the LO is an approximation - we should specify the initial phase and phase rate to minimize the error over the entire interval.

We will be approximating $\tau(t)$ with the function of the form:

$$\alpha + \beta t + \gamma t^2$$

We can determine the coefficients on the assumption that the main contribution to an error arises from the fourth term of the Taylor expansion of τ. We have examined several possibilities: we could ask for a least squares error over the entire interval; a simpler scheme is to compute τ at three points equispaced over the interval, then obtain the coefficients of the parabola passing through these points. This second approach (which we will adopt) differs only slightly (and only in the third order) from the first.

A least squares criterion yields:

$$\alpha = \tau(0)$$

$$\beta = \tau^{1}(0) + T^2\tau^{3}(0)/40$$

$$\gamma = \tau^{3}(0)$$

The parabolic solution differs only in β :

$$\beta = \tau^{1}(0) + T^{2}\tau^{3}(0)/24$$

where $\tau^{i}(0)$ are the coefficients of the Taylor expansion, evaluated for t = 0 at the mid-point of the interval.

Since a worst case analysis (source at the equator, at 90 deg. hour angle), has $\tau^{3}(0) = 0$, our proposed solution is adequate. (The phase error is less than 0.01 deg over a ten second interval at a wavelength of 1 cm.)

We derive α, β and γ from the values of τ evaluated at 3 equispaced points in the interval. In general the "interval" will be an integration interval (10 secs).

$$\alpha = \tau(0)$$

$$\beta = (\tau(T/2) - \tau(-T/2))/T$$

$$\gamma = (\tau(T/2) + \tau(-T/2) - 2\tau(0))(2/T^{2})$$

With t = 0 at the start of an interval the coefficients become:

$$\alpha' = \alpha - \beta T/2 + \gamma T^{2}/4$$

$$\beta' = \beta - \gamma T$$

$$\gamma' = \gamma$$

Phase offsets.

Provision has been made for the introduction of a phase offset between the two polarization IFs. This is separate from the fringe rotation phasing which is common to both IFs. Thus the antenna computer must provide, for each frequency band:

τ_0 and τ'

phase offset, H pol.
phase offset, V pol.

A phase input to the LBA sampler will be required, to track the motion of the site relative to the geocentre.

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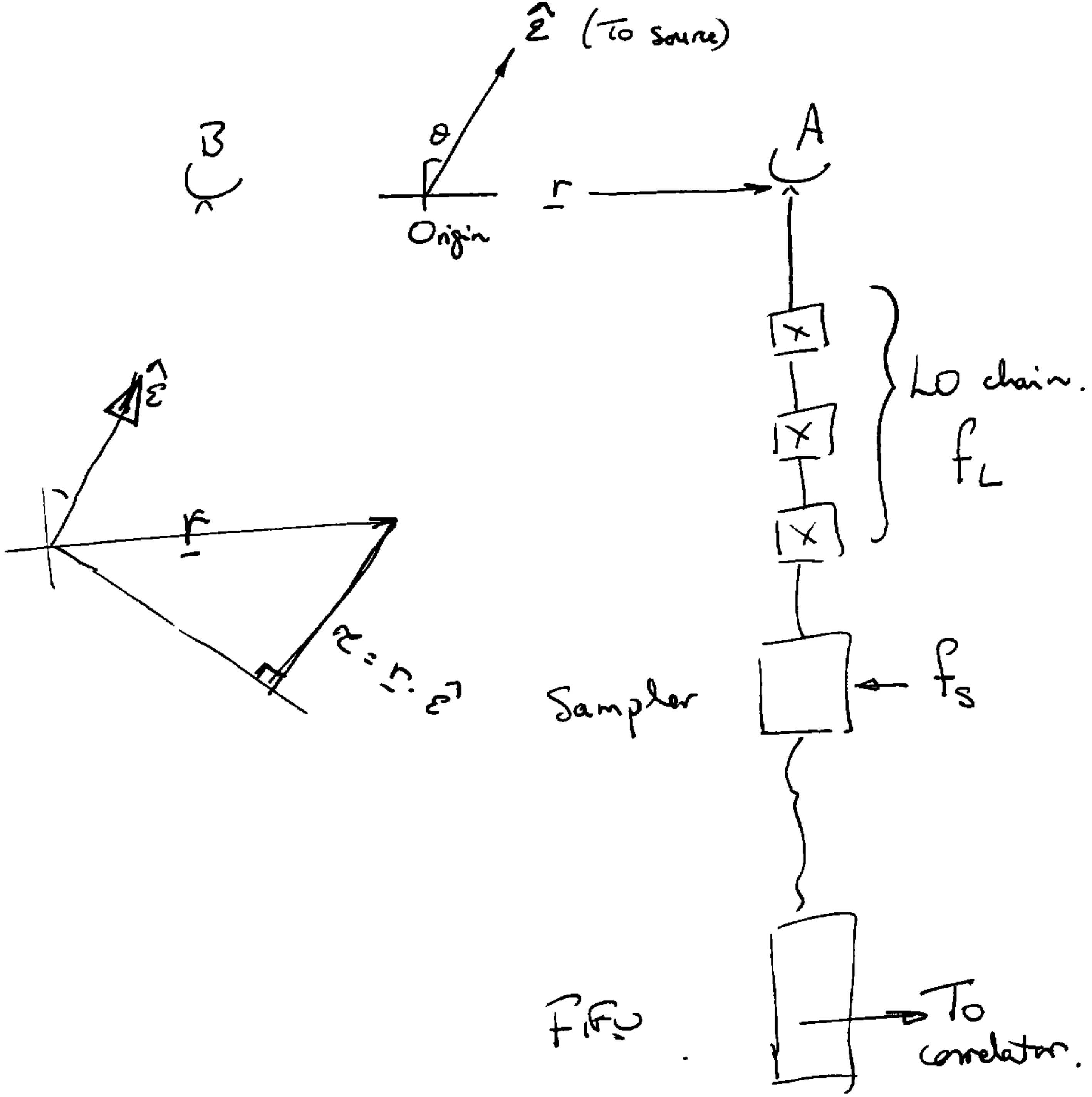


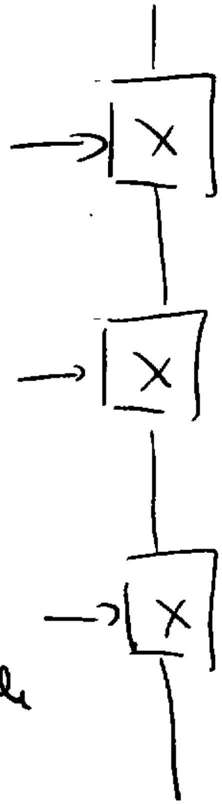
ẑ (To source)
θ
B
A
Origin
r
LO chain.
f_L
Sampler
f_S
FIFO
To correlator.
ŝ
r
τ = r · ŝ

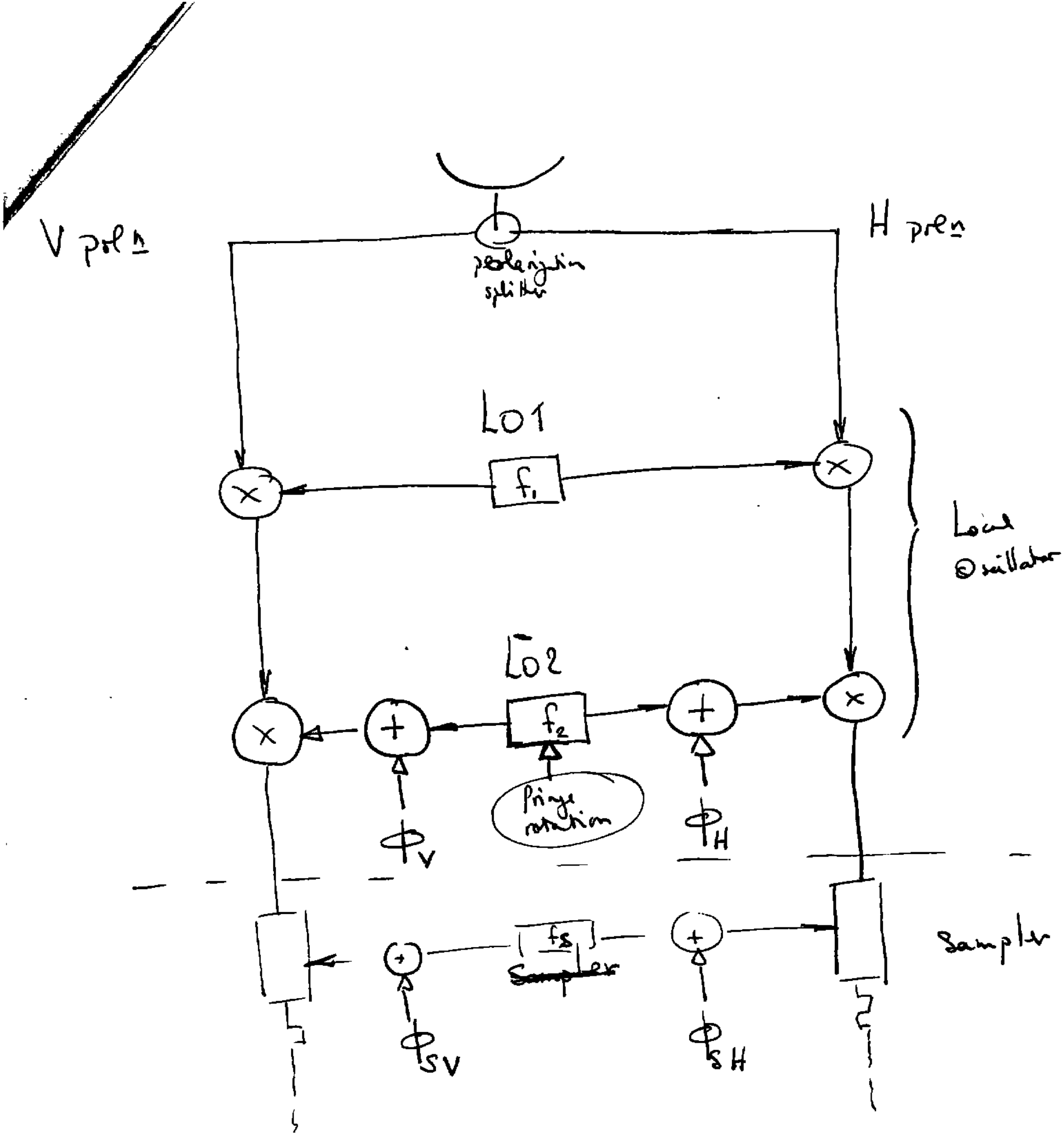
LO chain

fixed ~7 GHz; .3 GHz steps

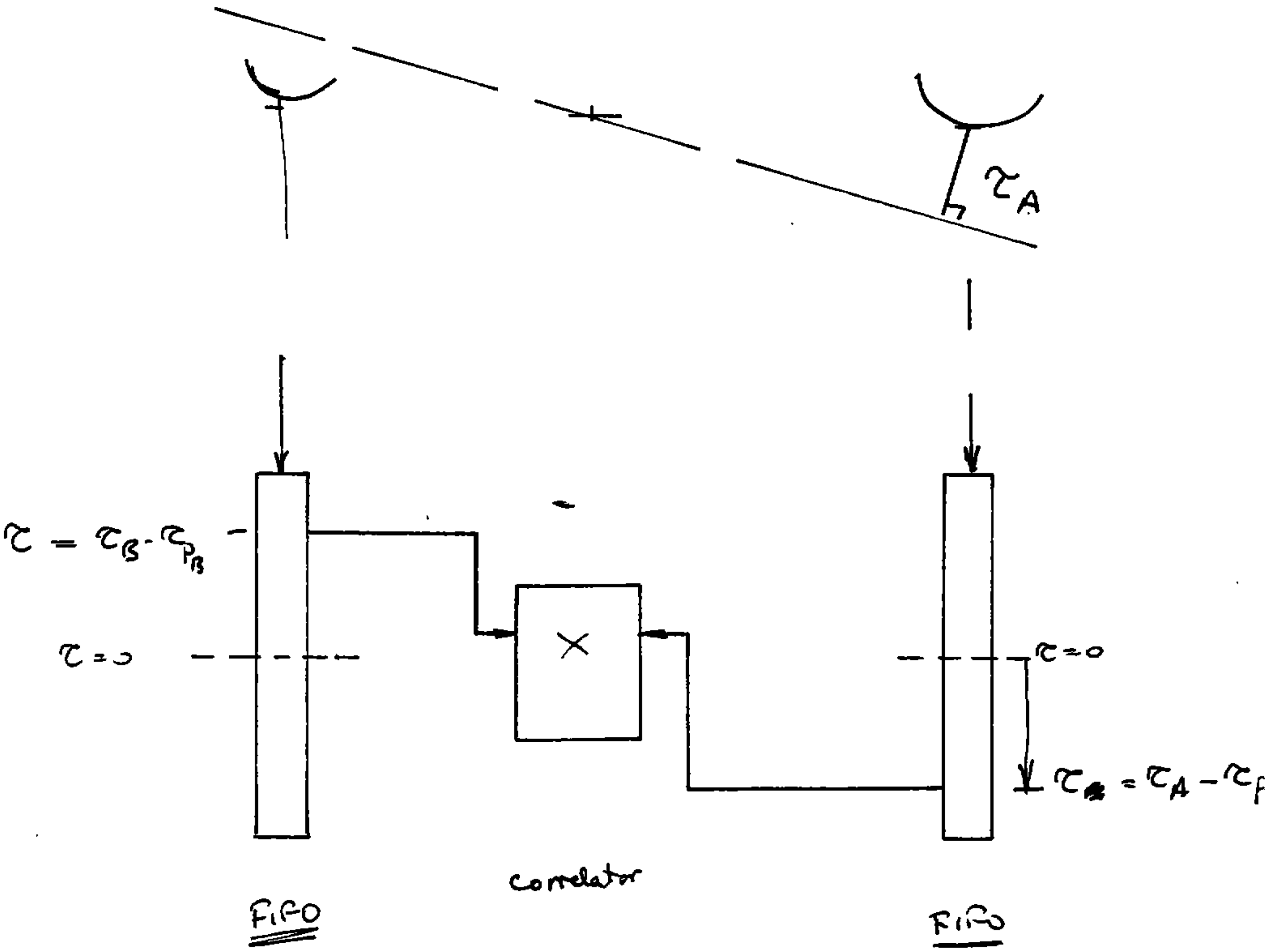
fixed ~2 GHz; .01 GHz steps

tunable ~.7 GHz; 1 MHz steps
±1 kHz tunable





V polⁿ
H polⁿ
polarization splitter
LO1
f_1
Local Oscillator
LO2
f_2
Fringe rotation
ϕ_V
ϕ_H
f_S
~~Sampler~~
Sampler
ϕ_{SV}
ϕ_{SH}



τ_p = propagation time antenna → FIFO

Precession of the baselines

mjk. 26 Feb. 1986

This note addresses the question of the calculation of the (u,v,w) coordinate of each baseline, correctly expressed in the desired output coordinate frame of reference.

In essence, we need to express each baseline in a coordinate frame suitable for the fourier inversion operation at the mapping stage. There are two options:

a. A source-aligned equatorial (NCP) frame is suitable for the CA. The point here is to exploit the wide-field mapping capability which arises because the array is accurately placed in the equatorial plane. The frame we use (l,m,n) has the $\hat{n}$ axis aligned with the polar axis, and the $\hat{m}$-$\hat{n}$ plane contains the source.

b. The conventional (u,v,w) will be used for the LBA. This frame has the $\hat{w}$ axis directed towards the source, and the $\hat{u}$ axis in the equatorial plane.

The interferometer output for a baseline $\underline{b}$, and a tracking field centre $\hat{\epsilon}_0$ is:

$$v(\underline{b},\hat{\epsilon}_0) = \int B(\hat{\epsilon}) e^{j2\pi(\hat{\epsilon}-\hat{\epsilon}_0)\cdot\underline{b}} d\hat{\epsilon}$$

Since the scalar product is invariant over precession, we can produce maps in the reference frame of epoch by precessing the baseline. The computational burden is not severe, as we are basically interested in the differential precession - this amounts to a simple rotation in the case of the (u,v,w) frame:the accuracy required in the rotation angle is of order 6′ for a 512 by 512 map.

The differential precession is small - a map rotation of ~ 5.7 arcmin over 12 years (1988 to 2000). In effect, we do well enough with :

$$\phi_{rot} = \tan^{-1}[Nt\sin(\alpha)/\cos(\delta)]$$

where N is the usual precession term, (~ 20"/year) expressed in radians, and t is the time interval between the observation and the epoch of the output map.

For the CA we rotate the baselines by $Nt\sin(\alpha)\tan\delta)$, then add a component to the baseline parallel to the polar axis, $NtL\sin(T)$, where N is the precession term, as above, and T is the local sidereal time; t is the interval between date and epoch; L is the baseline length.

<u>The problem</u>: The equatorial plane of date is not the equatorial plane at epoch. Over the period 1988 to 2012 the angle between the 2 planes will be <= 4 arc min. This misalignment has some implications for the manner in which precession is treated at the AT.

Some possibilities:

a. Make the maps in (l,m) of date, and subsequently deal with the precession - eg., superimpose on the maps a precessed coordinate grid.

b. Make maps in (l,m) of epoch.

c. Make maps in (u,v,w) of epoch.

Option a. will generate maps which, (to the extent that the construction errors are as small as expected), will have a coordinate grid that is correct. The subsequent precession is straight forward, but messy; AIPS has a number of tasks which would need modifying. Option b. has some slight errors at the edge of wide field maps; however, these are small enough that they can be neglected. Option c. would likely have significant errors in wide field maps. These options are discussed in detail in the following pages.

<u>Recommendation</u>: Choose option b. - the procedure is simple, and the concomitant errors bearable.

references:

VLA comp. memo. #105 (Clark)

Brouw, thesis, p. 76.

Some definitions:

By (l,m,n) we mean a coordinate frame which has:

l,m in the equatorial plane, and n aligned with the polar axis, such that the plane (m,n) contains the source, m pointing away from the source. The unit along the m axis is compressed by SIN(δ) in order that the final map be more or less equi-angular.

The (u,v,w) frame is the conventional one; the w-axis points towards the source, u is in the equatorial plane, normal to the plane containing the w-axis and the polar axis.

Option a. - make maps in (l,m) of date.

It is intended that the AT be a wide field instrument. The compact array will therefore be accurately aligned E-W. Thus all the observed baselines (of a single day's synthesis) will lie in a plane parallel to the equatorial plane. The 2-D FT of the (l,m) data is then exact. The same would be true for a map based on several days' data, provided that the separation in time between the various dates is not large.

There are a number of difficulties associated with this approach.

1. Which is the reference date of the map? The first day of data? If not, how do you decide? If the data extends over a number of months, the (u,v) coordinates may need changing - at which point the procedure is becoming messy.

2. All the AIPS tasks which return a coordinate would need attention. It is possible that the subroutine which makes the projection correction could be modified, and that this is the only modification needed; but life is never that simple.

Option b. - Make maps in (l,m) of epoch.

1. Precess the field centre position from epoch to date. This places the vector in the reference frame of date. Call this vector. ε_0, = (α_0,δ_0). (Let $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ be the unit vectors of the reference frame, $\hat{k}$ parallel to the polar axis).

2. Locate the antenna in the same reference frame: $\underline{r}_i$

(Steps 1 and 2 are needed in the LO and delay tracking calculations).

3. Compute the components of $\underline{r}$ in the NCP frame:

$$r_x = \underline{r}.\hat{\imath}$$

$$r_y = \underline{r}.(\hat{k}\times\hat{\imath})$$

$$r_z = \underline{r}.\hat{k}$$

(In practice, this amounts to a simple rotation in the equatorial plane, by an angle (α_0). r_z is very small - it is expected to be of order mm).

Compute the baseline vectors : $\underline{b}_{ij} = \underline{r}_j - \underline{r}_i$

4. Precess to the epoch of the output map. (Which need not necessarily be that of the input coordinates, although different epochs is not recommended). The precession can be viewed as two operations - locate the field centre in the output frame of reference, and rotate the map to account for differential precession over the map. At this point we are concerned solely with the differential precession. The output (NCP) frame is defined exactly as is the frame of date, with the polar axis, the source vector, and the equatorial plane. Our problem is to locate $\underline{r}$ in this new frame. We will find that the baseline vectors acquire a component parallel to the polar axis. In simple terms: between date and epoch, the main precession is given by the usual expression -

$$\Delta\alpha = M + N\sin(\alpha)\tan(\delta)$$

$$\Delta\delta = N\cos(\alpha)$$

Aplying these expressions to the unit vectors, we find that in the new (NCP) frame the baseline has acquired a "z" component :

$$b_z' = b_z - b_x N\sin(\alpha_0) - b_y N\cos(\alpha_0)$$

(b_z' is still a small quantity: N is ~ 20"/year, so from 1988 to 2000, adds up to 0.1% of a baseline length to b_z).

We will use the full accuracy of COCO to compute the transformation matrix.

5. We combine b_y and b_z in order to retain the wide-field mapping ability of the CA within the mapping routines:

$$b_y' = b_y + b_z'\cot(\delta_0)$$

The point here is that we want to retain as best we can the exact two-dimensional fourier relation between visibility and map brightness. In the NCP frame a source's position is given by the vector:

$$\hat{\varepsilon} = (\cos(\delta)\sin(\Delta\alpha), -\cos(\delta)\cos(\Delta\alpha), \sin(\delta))$$

and the phase of the tracking interferometer is:

$$\phi = 2\pi(\varepsilon-\varepsilon_0).\underline{r}$$

If $b_z = 0$, then we have an exact two-dimensional transform. For modest angular offsets from the field centre, we have:

$$\hat{\varepsilon}-\hat{\varepsilon}_0 \sim (\cos(\delta)\Delta\alpha,\ \sin(\delta)\Delta\delta,\ \cos(\delta)\Delta\delta)$$

so that:

$$b'_y \sin(\delta)\Delta\delta = b_y \sin(\delta)\Delta\delta + b_z \cos(\delta)\Delta\delta, \text{ as required.}$$

The accuracy of this approximation is examined below.

6. We scale b'_y by $\sin(\delta_0)$ in order to produce equi-angular maps. (Furthermore, since this is the WSRT standard, AIPS expects this scaling).

$$b''_y = b'_y.\sin(\delta_0)$$

7. We then pass to the correlator the baseline, to be attached to each visibility:

$$(l,m,n) = (b_x,\ b''_y, 0)$$

We now examine the errors which result from this procedure. The basic conclusion is that the errors are small. We examine the phase error which will arise in the FFT for a point source at $(x_s,\ y_s,\ z_s)$.

$$\underline{B} = L.COS(T).\hat{\imath} + L.SIN(T).\hat{\jmath} \qquad \text{at date.}$$

after precession, we have (approximately):

$$\underline{B}_e = L.COS(T).\hat{\imath} + L.SIN(T).\hat{\jmath} + N.L.SIN(T).\hat{k}$$

at epoch,

At the map making stage we have:

$$U = L.SIN(T-\alpha_c)$$

$$V' = L.COS(T-\alpha_c)$$

$$W' = N.L.SIN(T)$$

We then form:

$$V = V' + COT(\delta_c).N.L.SIN(T)$$

Compare the phase observed with the phase of a point source at (x_s, y_s) ; the source position derived from the FFT will correspond to the smallest phase difference: our estimate should therefore be an upper limit.

$$\Delta\phi = 2\pi\{y_s.V - y_s.V' - z_s.W'\}$$

$$= 2\pi.N.L.(y_s.COT(\delta_c) - z_s)$$

For a point source in the corner of a 1° field, $\delta_c = 60^\circ$, we have , after 12 years (1988 to 2000), a phase gradient of about 2E-5 degrees/(λ), which corresponds to a pointing offset of about 0.01 arcsec.

<u>Option</u> <u>c.</u> Make maps in (u,v,w) of epoch.

1. and 2. compute ε_0 and $\underline{r}$ in the frame of reference of date.

3. Locate each antenna in the (u,v,w) frame:

$$r_u = \underline{r}.(RxQ)/\cos(\delta)$$

$$r_v = \underline{r}.(QxO)$$

$$r_w = \underline{r}.Q$$

(A rotation of α about the polar axis, then by δ about the u-axis).

calculate the baselines. $\underline{b} = \underline{r}_i - \underline{r}_j$

4. Rotate each $\underline{b}$ about the w-axis to provide the differential precession. The magnitude of the rotation is:

$$\phi_{rot} = \tan^{-1}[Nt\sin(\alpha)/\cos(\delta)]$$

where N, the precession term, is expressed in radians.

5. Pass the visibiity coordinates to the correlator:

$$u = b_u\cos(\phi_{rot}) + b_v\sin(\phi_{rot})$$

$$v = -b_u\sin(\phi_{rot}) + b_v\cos(\phi_{rot})$$

$$w = b_w$$

Estimate of the error associated with a wide-field map:

We calculate the derived position for a source at (x_s, y_s, z_s) :

$$x = \mathrm{COS}(\delta).\mathrm{SIN}(\Delta\alpha)$$

$$y = \mathrm{COS}(\delta_c) - \mathrm{SIN}(\delta_c).\mathrm{COS}(\delta).\mathrm{COS}(\Delta\alpha)$$

$$z = -1 + \mathrm{SIN}(\delta_c).\mathrm{SIN}(\delta) + \mathrm{COS}(\delta_c).\mathrm{COS}(\delta).\mathrm{COS}(\Delta\alpha)$$

$$(z \sim x^2 + y^2)$$

The observed phase is:

$$\phi = x_s.L.COS(H) + y_s.L.SIN(H).SIN(\delta_c) - z_s.L.SIN(H).COS(\delta_c)$$

(The approximate expressions suffice here to outline the problem)

If we set up a 2-D FFT, we will find a point source in the map, but its position will be in error -

put: $u = L.COS(H)$

$v = L.SIN(H).SIN(\delta_c)$

$$f(x,y) = \int F(u,v).e^{j2\pi(xu+yv)}.dudv$$

But $F(u,v) = e^{-j2\pi(x_s u+y_s v-z_s v.COT(\delta_c))}$

so that $f(x,y) \quad \delta(x-x_s,\ y-y_s+z_s.COT(\delta_c))$

ie., an error in y of $z_s.COT(\delta_c)$

The magnitude of the error:

at $\delta = 60^{\circ}$, at the corner of a 1 degree map, we have a 5" error.

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Diurnal parallax

mjk/mrc 26 feb 1986

The approach here is to compute the delay at an antenna of a spherical wavefront from a nearby source.

let $\underline{R}$ be the vector from the geocentre to the source:

and $\underline{r}$ the vector from the geocentre to the antenna.

$\underline{R}' = \underline{R} - \underline{r}$ is the vector from the antenna to the source.

The delay, referenced to the geocentre, is :

$$\tau = (\underline{R} - \underline{R}') / c$$

now, $R'^2 = r^2 + R^2 - 2rR\cos(z)$

where, $\cos(z) = (\underline{R}.\underline{r}) / (rR)$

so that,

$$\tau = [R - (r^2 + R^2 - 2rR\cos(z))^{1/2}] / c$$

$$= [1 - (1 + (r/R)^2 - 2(r/R)\cos(z))^{1/2}](R/c)$$

Taking terms to second order in (r/R) in the binomial expansion we have:

$$\tau = [\cos(z) - (r/2R)\sin^2(z)](r/c)$$

or,

$$\tau = (\underline{r}.\hat{\varepsilon}/c)[1 - (\sin(\pi_0)/2a_e)(r^2 - (\underline{r}.\hat{\varepsilon})^2)/(\underline{r}.\hat{\varepsilon})]$$

where we have used : $R = a_e/\sin(\pi_0)$

π_0 is the horizontal parallax, a_e is the equatorial radius of the earth. and $\hat{\varepsilon} = \underline{R}/R$

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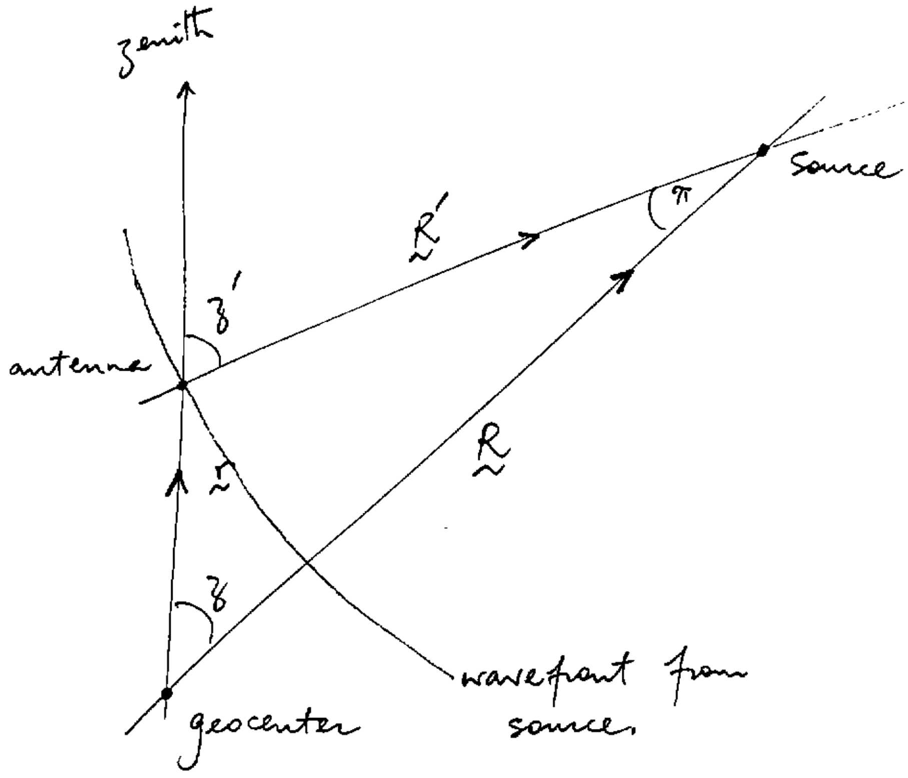


zenith
source
π
$\underset{\sim}{R}'$
z'
antenna
$\underset{\sim}{r}$
$\underset{\sim}{R}$
z
geocenter
wavefront from source.

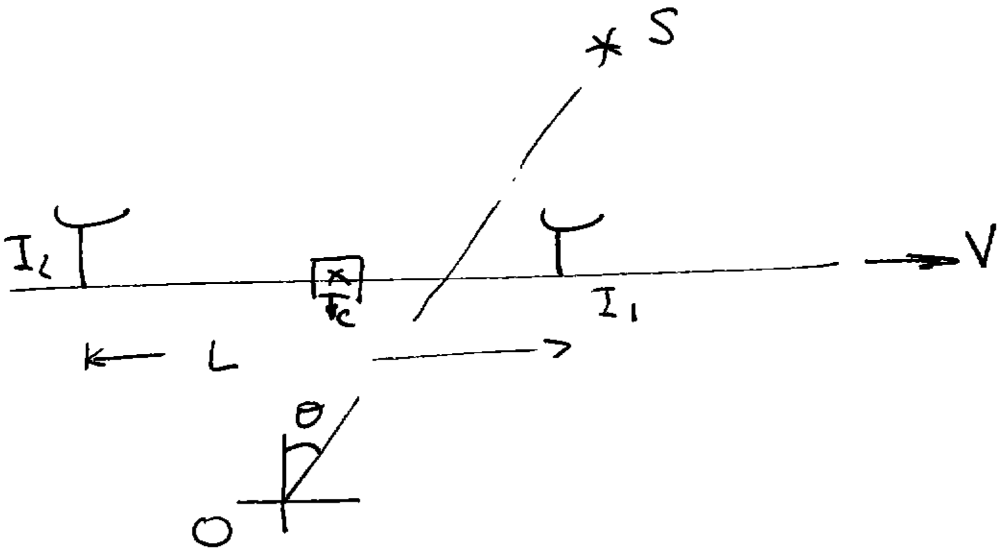
Aberration and the interferometer

mjk. 20 Jan 1986

Conclusion The AT should use all the aberration terms, except for the constant part of the diurnal aberration ($L\Omega R/c^2$).

Simple minded derivation

Consider an all-seeing observer stationary with respect to a distant source (S) and an interferometer moving along the observer's x-axis with velocity V.



The observer follows a wavefront from S to the 2 antennas, and then to the correlator. (For the moment we will propagate the signals from the antennas to the correlator along vacuum pipes).

From I_1 to the correlator: $\tau_1(c + V) = L/2$

From I_2: $\tau_2(c - V) = L/2$

Time between wavefront at I_1 and I_2 :

$$\tau_3 (V\sin(\theta) + c) = L\sin(\theta)$$

The interferometer operator will interpret the delay in terms of pointing:

$$(L/c)\sin(\theta') = \tau(\text{total})$$

$$(L/c)[\sin(\theta) + \Delta\theta\cos(\theta)] \sim$$

$$(L/c)\sin(\theta)/(1 + (V/c)\sin(\theta)) + LV/(c^2 - V^2)$$

$$\Delta\theta \sim (V/c)\cos(\theta)$$

In the AT (both CA and LBA) we in effect interrupt the transmission (antenna - correlator). but we maintain clocks at the antenna synchronized with the clock at the correlator. This means that the constant term is lost, and :

$$\tau' = (L/c)\sin\theta/(1+(V/c)\sin(\theta))$$

This argument applies to the diurnal aberration only. The annual aberration term will enter in full force.

A relativistic discussion would invoke the Sagnac effect, as the earth is a rotating frame, but the conclusion will be the same.

J2000. (FK5) and B1950. (FK4)
Converting the catalogues

mjk, 20 Jan 1986

We would like the AT to adopt the J2000. mean equator and equinox as its basic reference coordinate frame. Catalogues currently referenced to B1950. will therefore need to be upgraded. This is, in principle, a messy business, as detailed in the references. I suspect that for most of the RP-specific data there will be no problem as the inherent accuracy of the catalogues is insufficient to detect the fine details of the transformation.

magnitude of the effect:

difference in precession rates: 1."3 /century

Equinox error (FK4 in FK5) : 0.035 + 0.085T (secs), where T is in centuries. (This correction may not be relevant - it depends on just how the right ascension of the catalogue was established).

E-terms of aberration (if relevant) : 0."34 (max)

A command procedure involving COCO will be set up to provide a general conversion. Users requiring the full accuracy should consult the references to ensure that the adopted procedure accomodates their requirements.

references:
Aoki et al (1983) Ast. and Ast <u>128</u>, p.263
Clark (1982) VLA Sci. Mem #145
Clark (1983) VLA Comp. Mem #167
Kaplan (1981) USNO #163
Wallace (1985) COCO prologue
Williams and Melbourne (1981) IAU coll. #63, p.293