

## Calibration Notes

### I - Antenna Errors

mjk,mrc 17 April 1986

This note is the first of a series examining AT calibration questions. We address here the problem of the antenna errors, as they affect the LO control (phase and delay errors), and as they affect the antenna pointing. We also discuss the implications for the database.

#### 1. Some preliminary considerations.

1.1 The reflector surface: There are several axes of interest:

a. Symmetry axis. This is defined by the reflecting panels. It is expected that this axis will be close to orthogonal to the Elevation axis, and will come close to intersecting the Elevation axis (to within several mm).

b. Optical axis, defined by the actual position of the feed - ie, this is the axis that is normal to the principal wavefront. This axis should, in principle, be the symmetry axis; positioning errors of the feed will produce an offset. 1 mm feed displacement shifts the axis by 3.5 arcsecs. The turret should be able to reposition a feed to within 0.5 mm, ensuring reproducibility of the pointing.

The structure will sag under gravity - a displacement of 5 mm or so between the zenith and the horizon.

#### 1.2 The geometry

During the course of an observation we compute a delay appropriate to the antenna in question and the source we are tracking. The expression is:

$$\tau = \underline{r} \cdot \hat{e} / c$$

Where  $\underline{r}$  is the end-point of  $\underline{r}$ ? We argue that a suitable reference is the point on the elevation axis nearest to the azimuth axis. (Their intersection in an ideal antenna).

Assume, for the moment that the dish does not deform with Elevation. The structure above our reference is then invariant as the telescope points in different directions. Consider the plane defined by the wavefront at the instant when it first encounters the reflecting surface. This is at a fixed distance from our reference: call this distance  $p$ . The travel time of the wavefront can then be considered as:

$$T = (\text{time to our reference point on the elevation axis}) \\ - (p) \\ + (\text{fixed transit time to the focus}) \\ + (\text{fixed transit time through receiver, etc})$$

## 2. Antenna defects (& glossary of terms)

we list here the problems to be treated (and determined by calibration)

We will use a right-hand reference frame, z-axis in the direction of the azimuth axis, x-axis along the elevation axis.

$\Delta X$  .. an offset between the station reference point and the base of the azimuth axis.

$a_x, a_y$  .. tilt of the azimuth axis, described in terms of two infinitesimal rotations.

$h$  .. distance along the Az. axis to the point nearest the Elevation axis.

$e_y$  .. tilt of the Elevation axis: again, an infinitesimal

rotation. Two rotations provide the full description. but one of these, about the z-axis is absorbed into the azimuth encoder offset.

$d$  .. distance between the Azimuth and Elevation axes.  $r$  is measured from the El. axis to the Az. axis.

$f_x, f_z$  .. tilt of the optical axis, due to feed positioning.

$\epsilon_a, \epsilon_e$  .. encoder offsets, azimuth and elevation.

$c_a, s_a, c_e, s_e$  .. periodic terms in the encoders.

$d_z$  .. a parameter to describe the sag of the structure.

$u$  .. a fixed delay.

3. Delay calculation

$$\begin{aligned} \tau = u - d \cdot \cos(E) &+ (\Delta X - h \cdot a_y) \sin(A) \cos(E) \\ &+ (\Delta Y - h \cdot a_x) \cos(A) \cos(E) \\ &+ (\Delta Z + h) \sin(E) \end{aligned}$$

$[\Delta X + h]$  is invariant over the course of an observation, and will be treated as a fine adjustment to the antenna location.  $u$  and  $d \cdot \cos(E)$  will be treated as adjustments to the delay and phase, and applied after the ephemeris computations.

in detail:

$$\underline{r} = \Delta X + \underline{h} + \underline{d}$$

$$\underline{h} = \begin{bmatrix} 1 & 0 & -a_y \\ 0 & 1 & -a_x \\ a_y & a_x & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix} = \begin{bmatrix} -ha_y \\ -ha_x \\ h \end{bmatrix}$$

$$\underline{d} = \begin{bmatrix} d \cdot \sin(A) \\ d \cdot \cos(A) \\ 0 \end{bmatrix}$$

(Since  $|d|$  is  $< 2.5$  mm we can neglect second order terms such as  $a_x d$ .)

In the reference frame of the station, the required source direction is given by :

$$\hat{\epsilon} = (\cos(E) \sin(A), \cos(E) \cos(A), \sin(E))$$

4. Antenna pointing

Since the error angles are all small (arcsecs), we can compute the pointing corrections ( $\Delta A$  and  $\Delta E$ ) from each effect separately.

$$E' = E + \Delta E$$

$$A' = A + \Delta A$$

$$\begin{aligned} \Delta E = & a_y \sin(A) \\ & + a_x \cos(A) \\ & - f_z \\ & + \epsilon_e \\ & + (c_e + d_z) \cos(E) \\ & + s_e \sin(E) \end{aligned}$$

$$\begin{aligned} \cos(E) \Delta A = & - a_y \sin(E) \cos(A) \\ & + a_x \sin(E) \cos(A) \\ & - e_y \sin(E) \\ & - f_z \\ & + \epsilon_a / \cos(E) \\ & + c_a \cos(A) / \cos(E) \\ & + s_a \sin(A) / \cos(E) \end{aligned}$$

In detail:

#### 4.1 Tilt of the azimuth axis.

$$\begin{array}{l} \cos(E')\sin(A') \\ \cos(E')\cos(A') \\ \sin(E') \end{array} = \begin{bmatrix} 1 & 0 & -a_y \\ 0 & 1 & -a_x \\ a_y & a_x & 1 \end{bmatrix} \begin{bmatrix} \cos(E)\sin(A) \\ \cos(E)\cos(A) \\ \sin(E) \end{bmatrix}$$

$$\Delta E = a_y \sin(A) + a_x \cos(A)$$

$$\cos(E)\Delta A = -a_y \sin(E)\cos(A) + a_x \sin(E)\sin(A)$$

#### 4.2 Tilt of the elevation axis

(the y-axis is at the correct azimuth)

$$\begin{array}{l} \cos(E')\sin(A') \\ \cos(E')\cos(A') \\ \sin(E') \end{array} = \begin{bmatrix} 1 & 0 & -e_y \\ 0 & 1 & 0 \\ e_y & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \cos(E) \\ \sin(E) \end{bmatrix}$$

$$\Delta E = 0$$

$$\cos(E)\Delta A = -e_y \sin(E)$$

#### 4.3 Tilt of the optical axis

(y-axis now pointing at the source)

$$\begin{array}{l} \cos(E')\sin(A') \\ \cos(E')\cos(A') \\ \sin(E') \end{array} = \begin{bmatrix} 1 & -f_z & 0 \\ f_z & 1 & -f_x \\ 0 & f_x & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Delta E = -f_x$$
$$\cos(E)\Delta A = -f_z$$

#### 4.4 Encoder offsets

$$\Delta E = \varepsilon_e$$
$$\Delta A = \varepsilon_a$$

#### 4.5 Periodic terms

$$\Delta E = c_e \cos(E) + s_e \sin(E)$$
$$\Delta A = c_a \cos(A) + s_a \sin(A)$$

#### 4.6 Gravitational deflection (one possibility)

$$\Delta E = d_z \cos(E)$$
$$\Delta A = 0$$

## 5. Database considerations

As part of the calibration phase (before each scan) the OBS task will prepare a table of parameters for the ephemeris routine (POINTER); the Antenna Control Computer will also need a number of these parameters for its antenna pointing program. These parameters will be compiled from four files:

1. a station file, which describes the station characteristics: location and tilt of the plane defined by the four concrete pedestals; azimuth of the line defined by the two survey plates embedded in the diagonal pedestals.

2. an antenna file, which describes the antenna: location of the azimuth axis relative to the plane defined by the feet; tilt of the azimuth axis; offset between the azimuth and elevation axes; distance to the focus.

In addition, this file will contain the information needed by the Antenna Control Computer to point the telescope in the right direction: azimuth and elevation encoder offsets; gravitational deflection parameters; offset between the optical axis and the dish axis.

3. a location file, which describes the positioning imperfections: offset between the station and antenna reference points (defined by the optical positioners - see appendix).

4. the calibration file. This will contain the residual errors that will have been determined by observation of calibration sources. For example, rotation in azimuth of the antenna (mainly needed for pointing); additional tilt (dirty feet?). It will also contain delay and phase offsets. The intention here is to accumulate the corrections needed to ensure proper tracking; analysis of these corrections as to station error, for example, will be postponed until sufficient data has been accumulated.

These files will reside in the database, and therefore will be archived. Thus it will always be possible to correct data retrospectively, should the observer so wish.



In the initial operation of the AT the data in the first two files will largely be nominal: with time we expect to improve our knowledge of the array, and to upgrade the files. The location file will contain data obtained when the antenna is positioned at the station. The calibration file will contain the results of the calibration observations run immediately after an array reconfiguration. It may also be upgraded at a later date after many calibration sources have been observed.

Contents of the files.

The reference frame for these files is the local tangent plane, with its origin at the central site.

### 1. Station file

- Reference point - this is defined as the point mid-way between the two survey plates on the diagonal pedestals. (ST\_X(3))
- Tilt of the normal to the best fitting plane (ST\_az\_tilt(2); infinitesimal rotations about the x and y axes)
- Azimuth of the diagonal (ST\_diagonal)

### 2. Antenna file

- Azimuth axis base - offset in the horizontal plane between the azimuth axis and the reference point, defined as the mid-point between the two optical locators. (Azimuth\_X(3))
- Tilt of the azimuth axis (Ant\_az\_tilt(2))
- Height of the elevation axis above the foot (h)
- Distance between the axes (d)
- Non-orthogonality of the elevation axis to the azimuth axis (An\_el\_tilt .. we need rotation about y-axis

only)

- Distance between the elevation axis and the focus (p)
- Azimuth encoder offset (Az\_off)
- Elevation encoder offset (El\_off)
- Tilt of the optical axis (Ant\_optical\_tilt(3); we use rotation about x and z axis only)
- RF delay (RF\_delay)

There are some additional parameters that will be needed by the ACC in the antenna pointing computation:- some algorithm for the gravitational distortion; some model for non-linearities in the encoders.

### 3. Location file

The optical locator will determine the items.

- Location offset (LOC\_X(3))
- Rotation (LOC\_rot)

### 4. Calibration file

- Offset (CAL\_X(3))
- Tilt (CAL\_az\_tilt(2))
- Delay (CAL\_delay)
- Phase (CAL\_phase)

How are these entries combined?

## 1. Offsets.

The optical locators have their x-axis parallel to the diagonal, pointing away from the azimuth axis: (see appendix). Thus we get:

$$Tx = (x_n - x_s) / 2$$

$$Ty = (y_n - y_s) / 2$$

$$Loc\_X(1) = Tx * \sin(ST\_diagonal) - Ty * \cos(ST\_diagonal)$$

$$Loc\_X(2) = Tx * \cos(ST\_diagonal) + Ty * \sin(ST\_diagonal)$$

$$Loc\_rot = \tan^{-1}((y_n + y_s) / (2L + x_n + x_s))$$

2L is the spacing between the optical locators (~ 13.5 m)

$$\Delta X = ST\_X + Az\_X + Loc\_X + Cal\_X$$

$$+ (s * \text{tilt}(2), s * \text{tilt}(1), s)$$

## 2. Tilts.

We assume that all the tilt angles (departures from vertical) are small. Using subscripts 1,2 and 3 for the station, location and antenna tilt angles:

$$\text{tilt}(1) = \sum \text{tilt}(1)_i$$

$$\text{tilt}(2) = \sum \text{tilt}(2)_i$$

## Appendix I

## Antenna Position Locators - requirements

mjk, 3 April 1986

In order to measure the location of the antenna at each station, it is proposed to install two telescopes on diagonally opposed feet, to sight the targets embedded in the concrete pad. An x-y translation table at the eyepiece will allow us to measure the offset between the telescope axis and the targets.

We would like to be able to determine the azimuth of the antenna platform to within 1 arc sec. and the location of the platform's reference point to within 100 microns.

I recommend that the x-y table have its axis aligned along the diagonal, as shown in figure 1.

The azimuth error is related to the measurements by:

$$\zeta = \tan^{-1}((y_n + y_s)/(2L + x_n + x_s))$$

where 2L is the separation of the telescopes (~13.5 m); the antenna location error is:

$$\Delta x = (x_n - x_s)/2.$$

$$\Delta y = (y_n - y_s)/2.$$

The accuracy requirements:

The azimuth limit sets a tolerance on  $y$  of ~ 30 microns;

The location limit sets a tolerance on x of ~ 50 microns;  
The x-y table axis needs to be aligned to an accuracy of ~ 3  
degrees, allowing for +/- 5 mm. travel.  
There appears to be no serious restriction on the  
orientation of the telescope axis; the fact that we have to  
be able to see the target is probably sufficient.

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file [mjk,calibration]antenna.rno

