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# Initial results obtained with the 30 GHz Radiometer

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During the week commencing August 26, 1991, a 30 GHz water vapour radiometer, on loan from TELECOM AUSTRALIA, was installed at the Paul Wild Observatory. On the evening of August 29, 1991, a sky dip measurement was made by Dan Cerchi and Joe Gravina. A copy of the chart record obtained is shown in Fig. 1. The results were analysed as outlined in a note written by Dick Flavin[1].

The equations are summarized in the Appendix, and the results are given in Table 1. $T_{ar}$ is the antenna temperature at the reference point, the switching circulator, calculated using eqn. (1), and $T_a$ is the antenna temperature calculated using eqn. (5).

| Elevation (Degrees) | $d_S - d_a$ (V) | $d_C - d_a$ (V) | $T_{AMB}$ (K) | $T_{ar}$ (K) | $T_a$ (K) |
|---|---|---|---|---|---|
| 10 | 2.87 | 3.09 | 18.0 | 133.6 | 74.7 |
| 15 | 3.08 | 3.09 | 17.9 | 122.0 | 58.7 |
| 20 | 3.21 | 3.09 | 17.8 | 114.7 | 48.8 |
| 30 | 3.42 | 3.10 | 17.6 | 103.5 | 33.5 |
| 45 | 3.57 | 3.11 | 17.5 | 95.8 | 22.9 |
| 60 | 3.62 | 3.13 | 17.4 | 94.2 | 20.8 |
| 90 | 3.69 | 3.17 | 17.1 | 92.6 | 18.6 |

Table 1.

$T_a$ is plotted as a function of 1/sin(Elevation) in Fig. 2. In an ideal case these measurements would lie on a straight whose slope would be the equivalent noise

[1] Flavin, R.K., "30 GHz Radiometer Data Analysis", July 8, 1991.

temperature of the loss in the atmosphere, and whose zero intercept would be 2.7 Kelvin, the cosmic microwave background noise. Fig. 3 shows $T_a$ is plotted as a function of elevation.

The straight line shown in Fig. 2 is the one defined by the antenna temperatures measured at elevations of 45° and 10°. This line has a slope of 11.9 K, a zero intercept of 6 K, and is within ±1 K of the antenna temperatures measured at elevations of 90° and 60°. It is clear, however, that the antenna temperatures measured at elevations of 30°, 20° and 15° do not lie on this line. This could be due to an elevation dependent contribution to $T_a$ from ground radiation picked up in the sidelobes for elevations below 45°, or some other effect.

At an elevation of 45°, we can estimate that the measured $T_a$ , 22.9 K, is the sum of 16.9 K due to atmospheric loss, 2.7 K due to cosmic microwave background, and 3.3 K from spillover and ground radiation scattered off the antenna structure[2]. The equivalent noise temperature of the atmospheric loss at the zenith is 70% of that at an elevation of 45°, that is 11.9 K.

When the sky dip was made, the air temperature was 11.4°C and the relative humidity was 59.7%. We can estimate the surface water vapour density, $p_0$, from the zenith atmospheric loss at 30 GHz using eqns. (6) - (8) in the Appendix. This gives an estimate for the surface water vapour density of 5.9 g/m³.

Fig. 4 shows the equivalent noise temperature, $T_{atmosphere}$ , of the loss in the atmosphere as a function of the surface water vapour density. Atmospheric transmission is the parameter of most interest to astronomers working at millimetre wavelengths. The atmospheric transmission at 30 GHz can be calculated from $T_{atmosphere}$ using eqn. (9) in the Appendix. In Fig. 5, the atmospheric transmission at 30 GHz has been plotted as a function of $T_{atmosphere}$ .

To verify the plausibility of our estimate for the surface water vapour density, 5.9 g/m³, I have calculated the expected water vapour density under the prevailing surface meteorological conditions. Using eqns. (10) - (11) in the Appendix, I estimate the surface water vapour density, $p_0$ , under these conditions, to be 6.3 g/m³. The atmospheric loss at the zenith due to this water vapour density would be 0.21 dB, which would increase the radiometer system temperature by 12.5 K. This agrees well with the estimate of the surface water vapour density of 5.9 g/m³ and the estimate of $T_{atmosphere}$ , at the zenith, of 11.9 K, derived from the radiometer measurement. Tables 2 and 3, in the Appendix, give the surface water vapour density and $T_{atmosphere}$ for a range of surface temperatures and relative humidities.

The estimates of surface water vapour density deduced from the radiometer measurements depend on the accuracy of the measurement of the radiometer outputs and the validity of the assumptions made. If we take the measurement made at an elevation of 45°, we can calculate the sensitivity of the value of $T_a$ , calculated using eqn. (5), to uncertainties in the radiometer outputs. A 1 degree

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[2] The 6 K zero intercept is the sum of the 2.7 K ( cosmic microwave background ) and 3.3 K.

increase in $T_{AMB}$ will increase $T_a$ by 1.4 K; and a 1% increase in $d_s$ - $d_a$ (or a 1% decrease in $d_c$ - $d_a$) will decrease $T_a$ by 2.7 K.

The figure of 1.38 dB used for the loss, $L$, in the input waveguide is the sum of the loss in the portion of input line used in the calibration ( with the cryogenic termination ) and the loss in the antenna horn feed. A 0.01 dB over-estimation of the 0.44 dB loss in the antenna horn feed will cause a 0.6 K under-estimation of $T_a$ ; a 0.01 dB over-estimation of the 0.94 dB loss in portion of input line used in the calibration with the cryogenic termination will cause an under-estimation of $T_a$ of less than 0.01 K. The effect of the extra 0.01 dB loss in the input line would be almost completely cancelled by the effect of a 0.4 K reduction in the estimated noise temperature of the reference noise diode.

If we were to use eqn. (4) to calculate $T_a$ , using the measured air temperature of 284.6 K for $T_{LOSS}$ ( rather than $T_{LOSS} = T_r = 290.7$ ), $T_a$ would increase by about 2.3 K at all elevations, but there would be a negligible change in the estimate of the equivalent noise of the loss in the atmosphere.

There are, in fact, a number of straight lines we could draw through the data of Fig. 2, depending on which points we choose to believe. The three lines are shown in Fig. 6 are:

(a) a line of best fit through the antenna noise temperatures measured at elevations of 90°, 60° and 45°. This line has a slope of 10.6 K and a zero intercept of 7.8 K, that is 2.7 K cosmic microwave background plus 5.1 K from spillover and ground radiation scattered off the antenna structure. The atmospheric loss at the zenith deduced from the slope of this line is not very accurate as a ±1 K error in $T_a$ leads to a ±5 K error in the slope.

(b) the line ( shown in Fig. 2 ) which is defined by the antenna temperatures measured at elevations of 45° and 10°. This line has a slope of 11.9 K and a zero intercept of 6 K, that is 2.7 K cosmic microwave background plus 3.3 K from spillover and ground radiation scattered off the antenna structure. The atmospheric loss at the zenith deduced from the slope of this line is more accurate than (a): a ±1 K error in $T_a$ leads to a ±0.4 K error in the slope.

(c) a line of best fit through the antenna noise temperatures measured at elevations of 90°, 60° 45°, 30°, 20° and 15°. This line has a slope of 14.8 K and a zero intercept of 3.4 K, that is 2.7 K cosmic microwave background plus 0.7 K from spillover and ground radiation scattered off the antenna structure.

The measurements at high elevations are probably more accurate, but the slope of line (a) is very sensitive to measurement errors. Line (c) leads to an implausibly low spillover and ground radiation component. This leads us to favour line (b).

There is probably a larger contribution from spillover at an elevation 10° than at 45°, leading to an over-estimation of the slope in (b), but it is not clear what, if any, correction should be made.

Most importantly, however, the range of slopes described above constrains our estimate of the equivalent noise temperature of the loss in the atmosphere ( at the zenith ) to 12.7 ±2 K.

# APPENDIX

## A1 Calculation of $T_{ar}$

The antenna temperature, $T_{ar}$, at the reference point, the switching circulator, is calculated using eqns. (1) - (3)[3].

$$T_{ar} = T_r - (T_n - T_r)\frac{d_s - d_a}{d_c - d_a} \tag{1}$$

where

$d_a$ is the chart reading of zero output ( switching between ambient terminations, or input switch off,

$d_c$ is the chart reading of noise diode input during periodic calibration,

$d_s$ is the chart reading during normal operation,

$$T_r = T_{AMB} + 273.2 \tag{2}$$

$$T_n = 464.05\left(10^{0.0007(T_{AMB}-23.3)}\right) \tag{3}$$

and $T_{AMB}$ is the temperature of the ambient reference load in degrees Celsius.

## A2 Calculation of $T_a$

We can calculate the antenna temperature, $T_a$, by correcting $T_{ar}$ for the 1.38 dB loss in the input waveguide using

$$T_a = T_{ar}L - (L-1)T_{LOSS} \tag{4}$$

where $L$ equals 1.374 ( a loss of 1.38 dB ) and $T_{LOSS}$ is the temperature, in Kelvin, of the loss in the input waveguide. If we assume that $T_{LOSS}$ equals $T_r$, eqns. (1) and (4) can be combined to give

$$T_a = T_r - L(T_n - T_r)\frac{d_s - d_a}{d_c - d_a} \tag{5}$$

## A3 Calculation of the surface water vapour density from $T_{atmosphere}$.

We can estimate the surface water vapour density, $p_0$ ( in g/m$^3$ ), from the zenith atmospheric loss, $L_{atmosphere}$ at 30 GHz, expressed in dB, as[4]

$$p_0 = 7.5 + 44.25\left(L_{atmosphere} - 0.22 - 0.0014(21 - T_{AMB})\right) \tag{6}$$

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[3] Flavin, R.K., "30 GHz Radiometer Data Analysis", July 8, 1991.

[4] Derived from a private communication from M.W. Sinclair.

where

$$L_{atmosphere} = -10\log_{10}(1 - \frac{T_{atmosphere}}{T_m})\ (\text{dB}) \tag{7}$$

$$T_m = 1.12(T_{AMB} + 273.2) - 50 \tag{8}$$

and where $T_{atmosphere}$ is the equivalent noise, in Kelvin, of the loss in the atmosphere at the zenith and $T_{AMB}$ is the temperature of the ambient reference load in degrees Celsius. The atmospheric transmission at 30 GHz can be calculated from $T_{atmosphere}$ using

$$Transmission_{atmosphere} = 1 - \frac{T_{atmosphere}}{T_m} \tag{9}$$

## A4 Calculation of the surface water vapour density from the surface temperature and relative humidity.

We can estimate the surface water vapour density, $p_0$ ( in g/m$^3$ ), from $T_{AIR}$ , the surface temperature in degrees Celsius, and relative humidity, $RH$, using[5]

$$p_0 = \frac{(RH)e_s}{0.461Jg^{-1}K^{-1}(T_{AIR} + 273)} \tag{10}$$

where the saturated partial pressure of water vapour, $e_s$ ( in Newtons/m$^2$ ), which corresponds to $T_{AIR}$ , can be estimated as

$$e_s \approx 690 \cdot 10^{(T_{AIR}/37.7)} \tag{11}$$

The estimate of $e_s$ given by eqn. (11) is accurate at 20° C, but about 10% high at 0° C and 40° C.

Surface water vapour density (g/m/m/m)

| RH | Temperature (C) 10 | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|---|
| 0.40 | 3.9 | 5.2 | 6.9 | 9.2 | 12.3 | 16.5 |
| 0.50 | 4.9 | 6.5 | 8.7 | 11.6 | 15.4 | 20.6 |
| 0.60 | 5.8 | 7.8 | 10.4 | 13.9 | 18.5 | 24.7 |
| 0.70 | 6.8 | 9.1 | 12.1 | 16.2 | 21.6 | 28.8 |
| 0.80 | 7.8 | 10.4 | 13.9 | 18.5 | 24.7 | 33.0 |
| 0.90 | 8.8 | 11.7 | 15.6 | 20.8 | 27.8 | 37.1 |

Table 2

[5] Sinclair, M.W., private communication.

Frequency =30.00 T(atmosphere) (K)

| RH | Temperature (C) | | | | | |
|---|---|---|---|---|---|---|
| | 10 | 15 | 20 | 25 | 30 | 35 |
| 0.40 | 9.298 | 10.845 | 13.042 | 16.116 | 20.363 | 26.165 |
| 0.50 | 10.601 | 12.608 | 15.422 | 19.318 | 24.649 | 31.868 |
| 0.60 | 11.897 | 14.360 | 17.781 | 22.481 | 28.867 | 37.449 |
| 0.70 | 13.187 | 16.099 | 20.119 | 25.606 | 33.018 | 42.912 |
| 0.80 | 14.470 | 17.827 | 22.435 | 28.694 | 37.102 | 48.260 |
| 0.90 | 15.747 | 19.543 | 24.731 | 31.745 | 41.121 | 53.494 |

Table 3

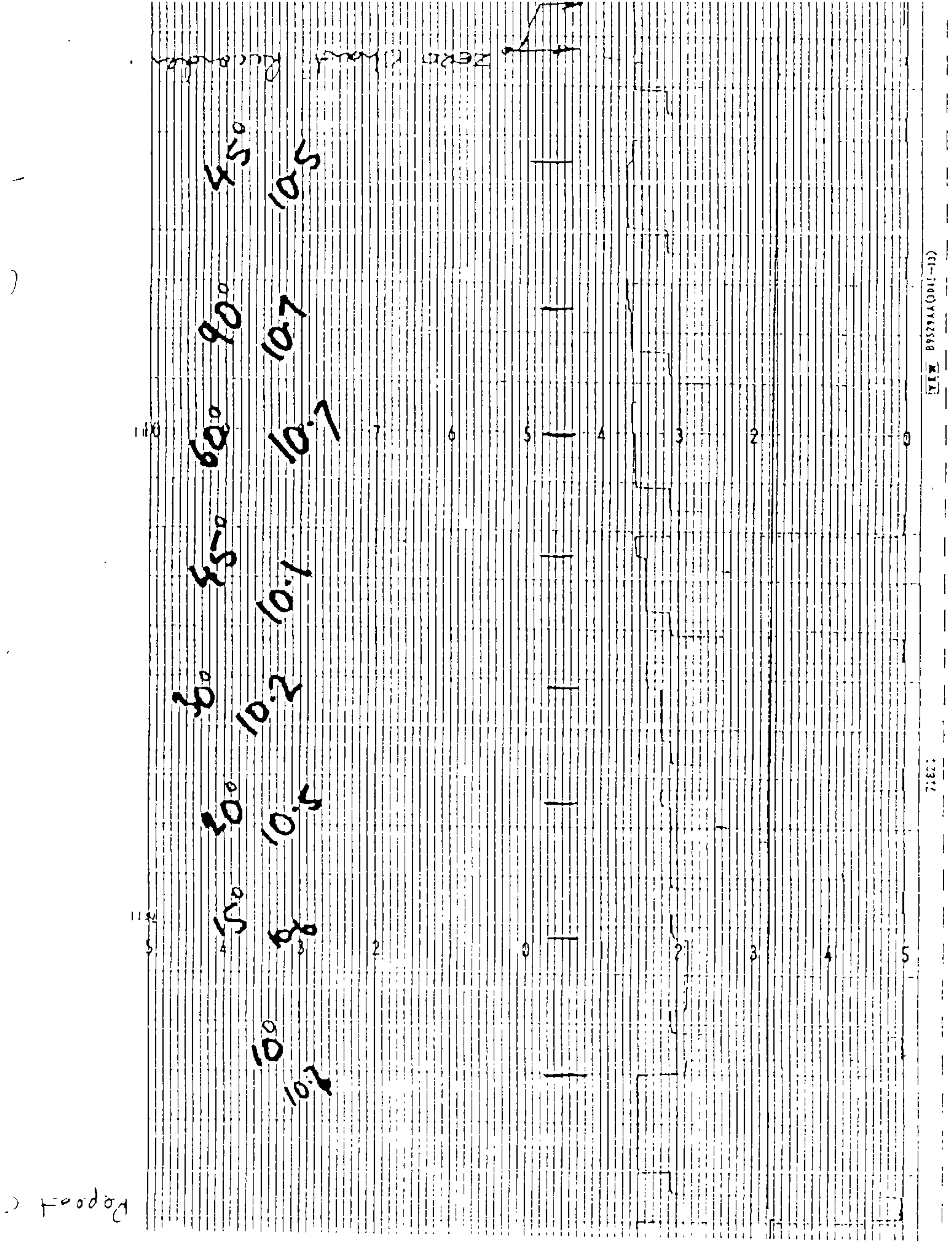
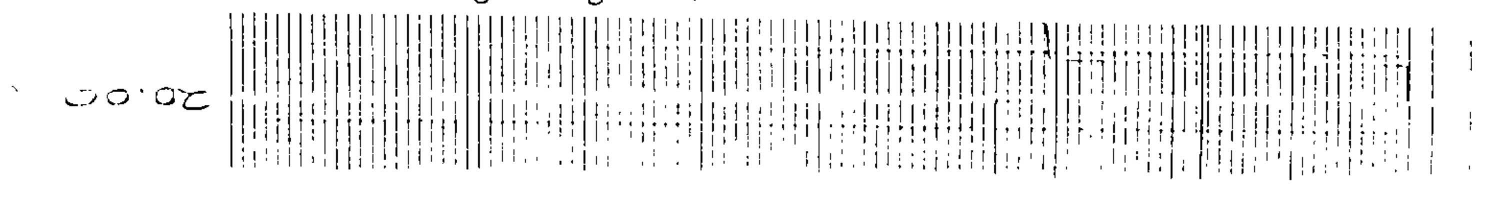


Fig 1. Chart record obtained during the sky dip measurement made on the evening of August 29, 1991



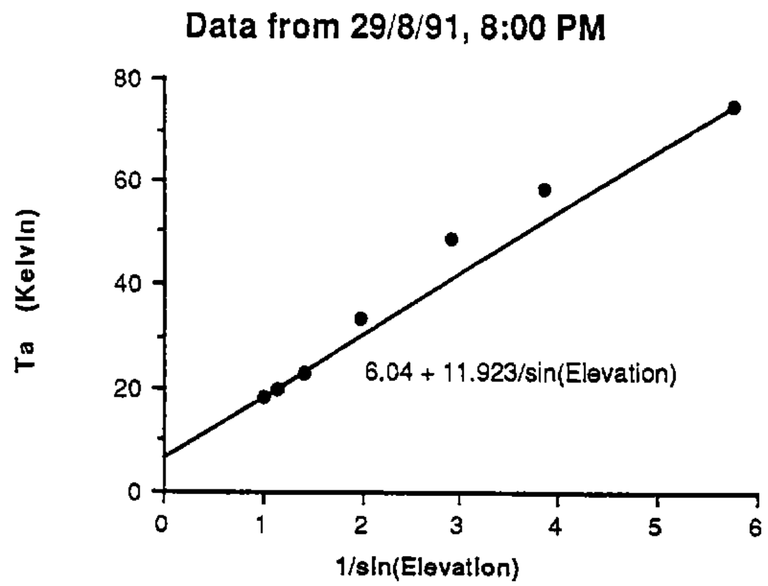



Fig 2. $T_a$ plotted as a function of 1/sin(Elevation).

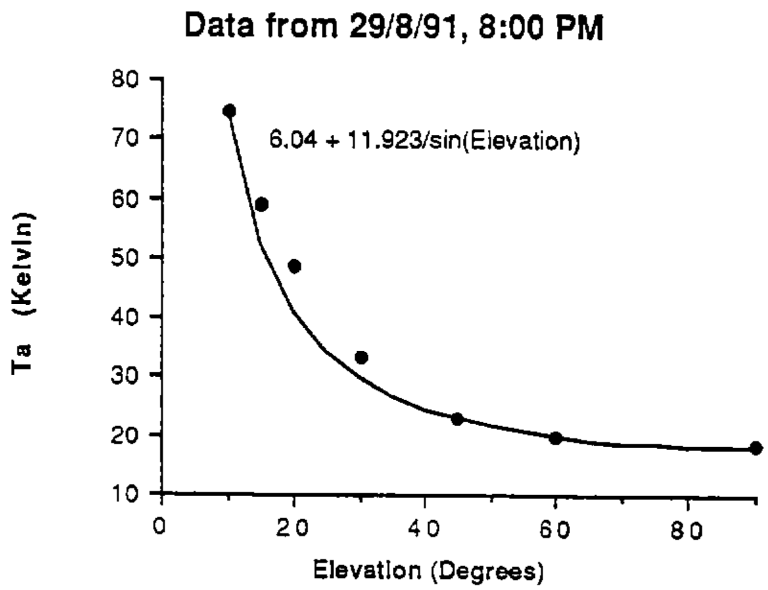



Fig. 3 $T_a$ plotted as a function of elevation.

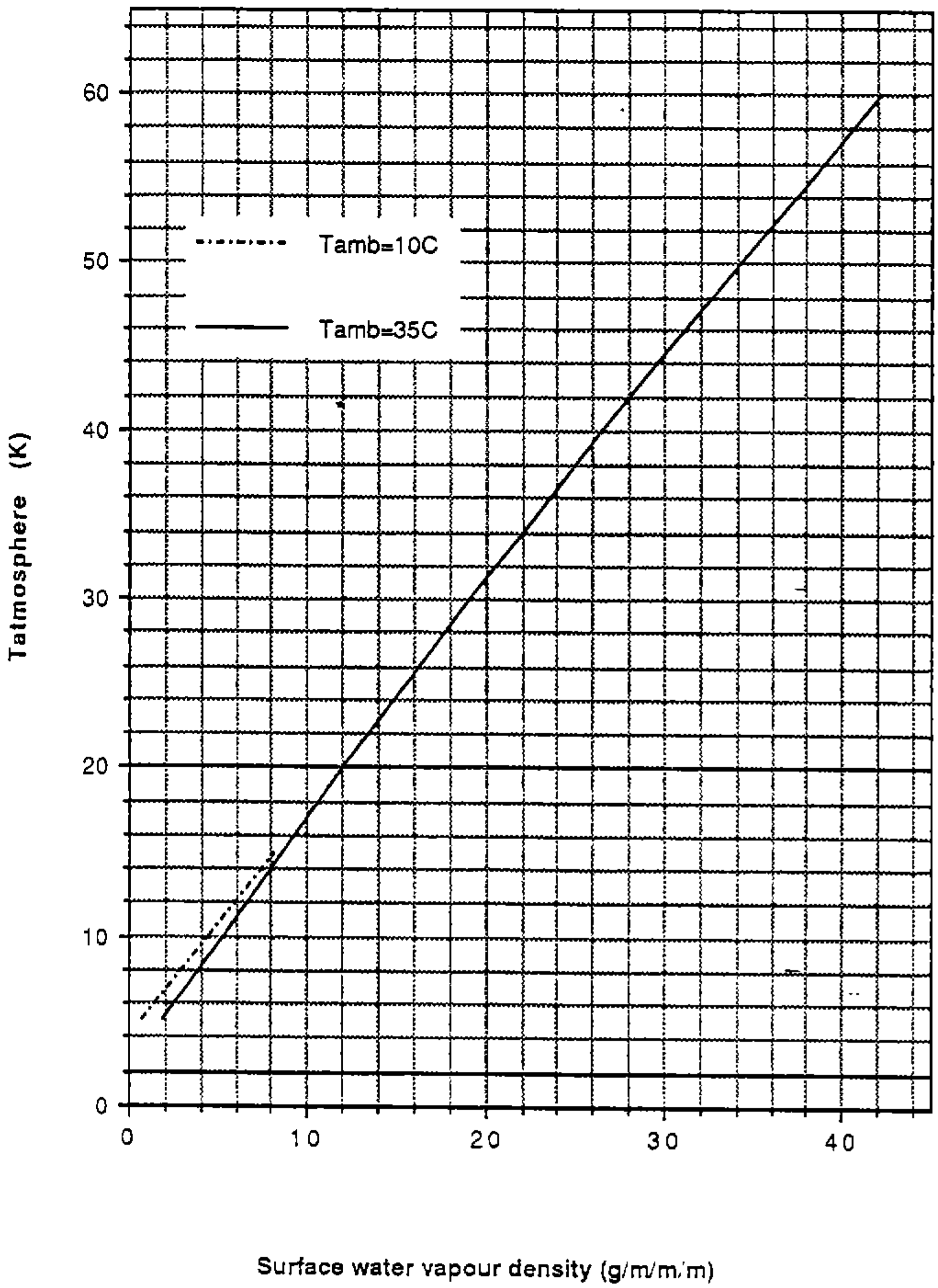



Fig. 4 Equivalent noise of the loss in the atmosphere, *Tatmosphere* , as a function of surface water vapour density.

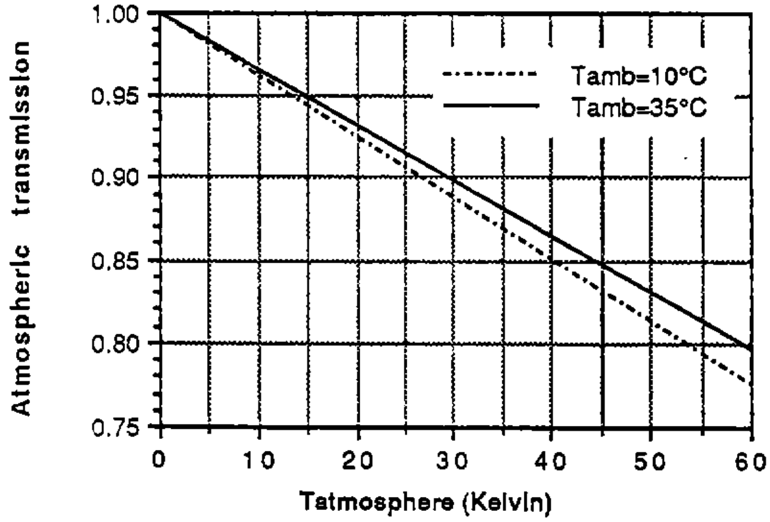



Fig. 5 Atmospheric transmission at 30 GHz as a function of $T_{atmosphere}$ for ambient temperatures of 10°C and 35°C.

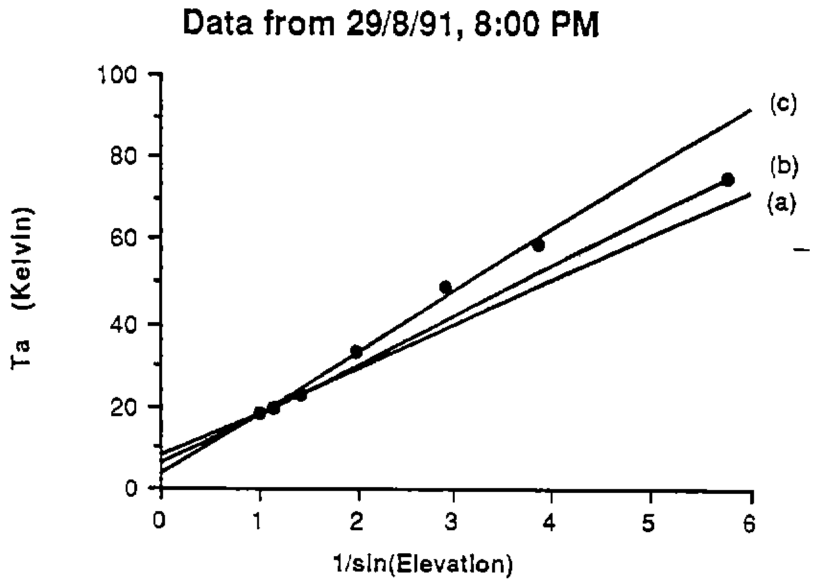



Fig 6. $T_a$ plotted as a function of 1/sin(Elevation) and approximations
(a) $T_a = 7.82 + 10.62/\sin(\text{Elevation})$
(b) $T_a = 6.04 + 11.92/\sin(\text{Elevation})$
(c) $T_a = 3.37 + 14.76/\sin(\text{Elevation})$.

# 30 GHz Radiometer Data Analysis

**by R. K. Flavin**
(8 July 1991)

## Introduction

A radiometer monitors (measures) incoherent thermal radiation (noise) at the design frequency. The associated antenna temperature, when one is looking at the sky, is essentially made up of two components - (1) thermal radiation from the sky, and (2) thermal radiation from the ground due to the antenna pattern. The antenna temperature $T_a$ (in degrees Kelvin) can be represented by the relationship,

$$T_a = hT_s + (1-h)T_g \quad (K) \qquad (1)$$

where h is the part of the antenna pattern directed towards the sky (h<1), $T_s$ is the sky radiation noise temperature, and $T_g$ is the ground radiation noise temperature. In general, h is approximately 0.945 for typical parabolic antenna designs, and $T_g$ can be assumed to be around 270K.

The "sky" temperature ($T_s$) includes both incoming cosmic radiation and the radiation from the atmosphere as a "lossy" medium. For gaseous absorption, with no scattering contribution, the sky temperature is given by the equation,

$$T_s = T_m(1 - 1/L) + T_c/L \quad (K) \qquad (2)$$

where $T_c$ = the noise temperature due to cosmic background radiation, $T_m$ = an "effective" ambient temperature of the atmosphere which validates equation (2), and L = the total atmospheric losses (due to molecules, water vapour, rain, etc) taken as a positive power ratio.

At 30 GHz, where scatterers are not small compared to a wavelength, scattering effects should be taken into account when determining the "true" attenuation L. Because of the complexity of simulating the real world model, the scattering problem will not be addressed in this note. Suffice it to say that the "real" attenuation is always greater than the "apparent" attenuation measured by a radiometer, and the difference increases with attenuation level. At 30 GHz the "real" attenuation is approximately 20% higher than the "apparent" attenuation of 10 dB, with the error being negligible for clear sky conditions (no rain for scattering).

At 30 GHz, $T_c$ is approximately 2.7K. The effective ambient sky temperature ($T_m$), or the equivalent temperature of the atmosphere as a lossy medium, is not easily determined. Until more exact data is available the temperature $T_m$ (K) can be approximated by the simple relationship,

$$T_m = 1.12*T_{ag} - 50 \quad (K) \qquad (3)$$

where $T_{ag}$ is the ambient temperature on the ground in degrees Kelvin.

Combining equations (1) and (2), the atmospheric loss is given by the relationship,

$$L = 10*\log[(T_m - T_c)/(T_m + (1/h - 1)T_g - T_a/h)] \quad (dB) \qquad (4)$$

where L is expressed in decibels. Equation (4) suggests a way of estimating the effective atmospheric temperature ($T_m$) during very high attenuation events, such as with heavy rain. As the atmospheric losses approach infinity, or more practically, when the losses are greater than roughly 30 dB, the antenna temperature will approach $[hT_m + (1 - h)T_g]$ - or, the sky noise temperature ($T_s$) approaches the "effective" atmospheric ambient temperature ($T_m$). Measurements of such events can provide insight into the validity of equation (3).

## 30 GHz Radiometer

The 30 GHz radiometer designed and built at the Telecom Research Laboratories is a switched radiometer, wherein the receiver input is switched between the antenna and a reference ambient temperature termination, and then synchronously detected at the switching rate. Since the radiometer output is proportional to the difference between the two signal inputs, the impact of short term gain variations is reduced, but long term gain variations must still be accounted for. Normally, long term gain changes are reduced by operating the radiometer in a null balancing mode, wherein a compensating noise is injected into the "cold" antenna arm to make the antenna temperature equal to the fixed reference termination temperature, and then using the amount of injected noise as the radiometer output. This mode of operation is not available in the Telecom 30 GHz radiometer. The effects of long term gain variations are offset by calibrating the radiometer every half-hour with two levels of noise temperature input - a 30 second period with the reference termination being the only input (no switching - zero radiometer output), followed by a 30 second period wherein a calibrated noise signal is injected into the system. The calibrated noise input is provided by an avalanche diode noise source with a temperature coefficient of better than +-0.01 dB/K. The half-hourly calibration has proven in the past to be a good way of minimising the effects of long term receiver gain variations. Practise has shown that the long term gain variations can be approximated as linear between calibrations, and the data analysis can be accommodated accordingly.

## Antenna Temperature Measurement

The accuracy of the radiometer measurement of antenna noise temperature depends mainly on a knowledge of - (a) certain waveguide line losses, (b) the "effective" input cryogenic temperature during calibration of the avalanche diode noise source, (c) the stability of the diode noise source during the operation of the radiometer, (d) the ambient temperature of the reference termination during radiometer calibration and operation, and (e) the stability of the receiver gain between hourly calibrations.

For any transmission line with loss $L_t$, the output noise temperature ($T_{out}$) for a given input noise temperature ($T_{in}$), is given by the expression,

$$T_{out} = T_{in}/L_t + (1 - 1/L_t)T_{amb} \quad (5)$$

where $T_{amb}$ is the ambient temperature of the transmission line, and the line loss ($L_t$) is expressed as a positive power ratio.

The radiometer line losses pertinent to the measurement of antenna noise temperature are the antenna horn feed and the total transmission line between the antenna feed and the receiver mixer-preamp input. The receiver input is taken as the reference point for comparing noise temperatures since that is the point of injection for the noise diode as an hourly calibration source.

When the radiometer is calibrated before operation in the field, a cryogenic termination is connected to the radiometer at the point where the antenna feed is normally connected. The total line losses from this point to the reference point (mixer-preamp input), which we shall call $L_c$, has been measured at 30 GHz to be 0.94 dB ($L_c$ = 1.24), and the antenna horn feed, which is approximately 710 mm long, has been calculated to have a line loss ($L_f$) at 30 GHz of 0.44 dB ($L_f$ = 1.11). The cryogenic termination has an output noise temperature ($T_{cry}$) of 86.1K under normal atmospheric conditions. Using equation (5) with a typical ambient temperature for the line loss of $T_{amb}$ = 295K, and line losses $L_c$ = 1.24, the cryogenic temperature at the reference point becomes 126.5K. This temperature will be referred to as $T_{cal}$.

The calibration avalanche diode noise source is fed into the radiometer at the reference point (mixer-preamp) through a variable attenuator and a directional coupler. With the cryogenic termination in place, and the radiometer operating in the normal mode with the output proportional to the <u>difference</u> between $T_{cal}$ and the reference termination temperature $T_{rcal}$, the noise diode is turned on, modulated in anti-phase to the normal input, and adjusted (attenuated) so that the radiometer output is zero. At this point the absolute difference $|T_{cal} - T_{rcal}|$ is equal to the absolute difference $|T_{nd} - T_{rcal}|$, where $T_{nd}$ is the noise temperature of the avalanche diode noise source injected at the radiometer reference point to make the cold input look like the reference termination. For a typical reference ambient temperature of $T_{rcal}$ = 295K, the calibration diode noise temperature becomes $T_{nd}$ = 463.5K.

At the radiometer reference point (mixer-preamp input) the temperature difference $T_r - T_{ar}$ is measured, where $T_r$ is the operating reference termination temperature (ambient), and $T_{ar}$ is the antenna temperature at the reference point. Assume the following parameters for the radiometer output :-

- $d_a$ = chart reading of zero output (switching between ambient terminations, or input switch off)
- $d_c$ = chart reading of noise diode input during periodic calibration
- $d_s$ = chart reading during normal operation

The antenna temperature at the reference point is then given by the formula,

$$T_{ar} = T_r - [(T_n - T_r)/(d_c - d_a)](d_s - d_a) \qquad (6)$$

where, $T_n$ is the calibration noise diode temperature compensated for ambient temperature changes (see next section - equation 8), and $T_r > T_{ar}$, $T_n > T_r$, $d_c > d_a$, and $d_s > d_a$.

The line losses back to the antenna horn feed are given by $L_g$(dB) = $L_f$ - $L_c$ (dB). Using equation (5), the antenna temperature $T_a$ can be shown to be given by the formula,

$$T_a = T_r - [L_g(T_n - T_r)/(d_c - d_a)](d_s - d_a) \qquad (7)$$

where the total line loss $L_g$ is expressed as a power ratio. From the loss measurements discussed previously, $L_g$ = 1.38 (dB), or $L_g$ = 1.374.

The antenna temperature can be monitored as a function of time using equation (7) for the data reduction, or the equivalent atmospheric loss can be calculated using equations (7), (3), and (4). The major uncertainty is the assumption of equation (3), which should be experimentally verified. The error in antenna temperature measurement using equation (7) is mainly due to the stability of $T_n$ which is specified as ~0.01 dB/K. If the reference termination temperature $T_r$ varies over the range 273-323K (0-50 C), the error in $T_{nd}$ is roughly ~ 6%. This is seen as the greatest possible error in the antenna temperature measurement (see below for a reduction of this error).

## Noise Diode Temperature Variation

The avalanche noise diode output, which is used as an hourly calibration noise source, has been measured in the laboratory as a function of the ambient temperature. Let the calibration noise diode temperature, determined in the field (before operation) at an ambient temperature $T_{rcal}$, be given by $T_{nd}$. The best linear fit to data taken over an ambient temperature range of 295-326K is given by the equation, Y = 0.007*X, where Y is the change in the diode noise power output (expressed in dB), and X is the ambient temperature differential (referenced to the calibration temperature $T_{rcal}$). Using the experimental data the following equation should be used to determine $T_n$ in equations (6) and (7):-

$$T_n = T_{nd}\{10^\wedge[(T_r - T_{rcal})*0.007/10]\} \quad (8)$$

where $T_r$ is the reference termination temperature (K), $T_{rcal}$ is the same parameter at time of laboratory (or field) calibration, and $T_{nd}$ is the diode noise source output at the time of calibration.

## Radiometer Sensitivity

For a switched radiometer with square-law detection, the output voltage V is given by the expression,

$$V = G*(T_{ar} + T_{rc}) - G*(T_r - T_{rc}) \quad (9)$$

where, $T_{rc}$ is the receiver system noise temperature at the reference input (mixer-preamp), and G is the system gain (assumed constant during the switching cycle). Let $(T_{ar} - T_{rc}) = T_x$, and $(T_r - T_{rc}) = T_o$. Then,

$$V = G*(T_x - T_o) \quad (10)$$

and,

$$d(V) = G*[d(T_x) - d(T_o)] + d(G)*(T_x - T_o) \quad (11)$$

where, _d_ is the differential.

For an ideal system, a total power radiometer has a sensitivity given by the equation,

$$d(T_{sys}) = T_{sys}/SQR(B*t) \quad (K) \quad (12)$$

where _d_ is the differential, $T_{sys}$ is the system noise temperature input (K), SQR is the square root, B is the receive noise signal bandwidth (Hz), and t is the integration time (sec) of the measurement (which for a square-wave switched radiometer is the time for one-half of the complete switching cycle).

If we refer the output fluctuation to the input, equation (11) becomes,

$$d(V)/G = d(T_x) - d(T_o) + [d(G)/G]*T_{diff} \qquad (13)$$

where $T_{diff}$ is the temperature difference between the "sky" (antenna) temperature and the reference termination temperature, and d is the differential. The rms value of equation (13) is given by the following expression,

$$[d(V)/G]_{rms} = SQR\{d(T_x)^2 + d(T_o)^2 + [T_{diff}*d(G)/G)]^2\} \qquad (14)$$

where again, SQR is the square-root. Using equation (12) for an ideal power radiometer, we can substitute first $T_x$ and then $T_o$ for the system temperature $T_{sys}$ to obtain the differentials in equation (14). In general, for most modern day transistor amplifiers, the differential gain term [d(G)/G] is approximately 0.01.

The following parameters apply to the Telecom 30 GHz radiometer: B = 80 MHz (double-sideband); switching rate = 493 Hz, therefore integration time t = 1.014 msec; mixer-preamp double-sideband noise figure = 3.5 dB, therefore $T_{sys}$ = 359.2K; $T_o$ = 295K; with a "clear sky" antenna temperature of 50K and line losses of 1.38 dB, $T_{ar}$ = 116.7K; and $T_{diff}$ = 178.3K. Substituting these values into equations (12) and (14), the radiometer sensitivity is approximated as,

$$[d(V)/G]_{rms} = SQR\{ 2.792 + 5.275 + 3.179 \} = 3.35K$$

where SQR is the square-root. This temperature differential can be looked at as the minimum change in temperature that is definitely discernible. In actual practise, the minimum discernible temperature difference for the 30 GHz radiometer is better than that value.