The SYN Projection

ATNF document series

Mark Calabretta and Michael Kesteven

Australia Telescope National Facility
1995/03/03

1 Introduction

The SYN (synthesis) projection provides an exact coordinate description for any co-planar synthesis array. This includes east-west arrays and VLA snapshots. It includes SIN and NCP as special cases. SYN has been included in the FITS WCS proposal as a generalization of the SIN projection.

2 Derivation of the SYN projection

From the basic synthesis equation, the phase term in the Fourier exponent is

\[ \text{phase} = (e - e_0) \cdot B \]  

(1)

where \(e\) and \(e_0\) are the unit vectors pointing towards a point in the field and the field centre, \(B\) is a baseline vector, and we measure phase in rotations so that we don't need to carry factors of 2\(\pi\). We can write

\[ \text{phase} = p_u u + p_v v + p_w w \]  

(2)

where \((u, v, w)\) are components of the baseline vector in a coordinate system with the \(w\)-axis pointing from the geocentre towards the source and the \(u\)-axis lying in the J2000.0 equatorial plane, and

\[ p_u = -\cos \theta \sin \phi \]  
\[ p_v = -\cos \theta \cos \phi \]  
\[ p_w = \sin \theta - 1 \]  

(3)

are the coordinates of \((e - e_0)\), where \((\phi, \theta)\) are the longitude and latitude of \(e\) in the (left-handed) native coordinate system of the projection with the pole towards \(e_0\). Now, for a planar array we may write
3 SYN Projection equations in equatorial coordinates

\[ n_u u + n_v v + n_w w = 0 \]  \hspace{1cm} (4)

where \((n_u, n_v, n_w)\) are the direction cosines of the normal to the plane. Then

\[ w = -\frac{n_u u + n_v v}{n_w} \]  \hspace{1cm} (5)

Combining (2) and (5) we have

\[ \text{phase} = [p_u - \frac{n_u}{n_w} p_w]u + [p_v - \frac{n_v}{n_w} p_w]v \]  \hspace{1cm} (6)

From equations (3) and (6) the equations for the "SYN" projection for a planar synthesis array are thus

\[ x = -[\cos \theta \sin \phi + p_1 (\sin \theta - 1)] \]

\[ y = -[\cos \theta \cos \phi + p_2 (\sin \theta - 1)] \]  \hspace{1cm} (7)

where

\[ p_1 = \frac{n_u}{n_w} \]

\[ p_2 = \frac{n_v}{n_w} \]  \hspace{1cm} (8)

3 SYN projection equations in equatorial coordinates

If \((\alpha, \delta)\) and \((\alpha_0, \delta_0)\) are the J2000.0 right ascension and declination of \(\mathbf{e}\) and \(\mathbf{e}_0\) then

\[ \cos \theta \sin \phi = \cos \delta \sin(\alpha - \alpha_0) \]  \hspace{1cm} (9)

\[ \cos \theta \cos \phi = -\sin \delta \cos \delta_0 + \cos \delta \sin \delta_0 \cos(\alpha - \alpha_0) \]

\[ \sin \theta = \sin \delta \sin \delta_0 + \cos \delta \cos \delta_0 \cos(\alpha - \alpha_0) \]

These may be substituted into equations (7) to obtain the SYN projection equations in J2000.0 equatorial coordinates.

4 Special cases of the SYN projection: SIN and NCP

Note in equations (7) that since theta is approximately 90° the terms involving \(p_1\) and \(p_2\) are small; neglecting them, as is usually done, gives us the equations for the "SIN" projection:
\[ x = - \cos \theta \sin \phi \]
\[ y = - \cos \theta \cos \phi \]  
(10)

From equations (7), for an array which lies in the J2000.0 equatorial plane, we have

\[ n_u = 0 \]
\[ n_v = \cos \delta_0 \]
\[ n_w = \sin \delta_0 \]  
(11)

where \( \delta_0 \) is the declination of the field centre, whence

\[ p_1 = 0 \]
\[ p_2 = \cot \delta_0 \]  
(12)

and

\[ x = -[\cos \theta \sin \phi] \]
\[ y = -[\cos \theta \cos \phi + \cot \delta_0 (\sin \theta - 1)] \]  
(13)

These are the equations for the "NCP" projection. To first order the difference between equations (10b) and (13b) is

\[ \frac{r^2}{2} \cot \delta_0 \]  
(14)

where \( r \) is the distance from the field centre in radians. This amounts to nearly 1' for a position 1° from the field centre at \( \delta_0 = 30° \).

5 Correction for precession

The plane of an east-west array coincides with the apparent equatorial plane at the date of the observation and this is tilted slightly with respect to the J2000.0 equatorial plane. If \( (\alpha_p, \delta_0) \) are the J2000.0 right ascension and declination of the apparent pole then

\[ n_u = - \cos \delta_p \sin (\alpha_p - \alpha_0) \]
\[ n_v = \sin \delta_p \cos \delta_0 - \cos \delta_p \sin \delta_0 \cos (\alpha_p - \alpha_0) \]
\[ n_w = \sin \delta_p \sin \delta_0 + \cos \delta_p \cos \delta_0 \cos (\alpha_p - \alpha_0) \]  
(15)
These may be substituted directly into equations (7) and (8). Precession from 1990 to 2000 amounts to about 3'. For $\alpha_p = \alpha_0$ and $\delta_0 = 30^\circ$ we get

\begin{align*}
    p_1 &= 0 \\
    p_2 &= 1.72857
\end{align*}

Equations 12 with no precession correction give

\begin{align*}
    p_1 &= 0 \\
    p_2 &= 1.73205
\end{align*}

For a position 1° from the field centre at this declination the difference between these amounts to about 0'.'1.

6 Field shifts

A phase shift may be applied to the visibility data at the time a map is synthesized in order to translate the field centre. If the phase shift applied to the visibilities is

\begin{equation}
    \text{phase shift} = q_u u + q_v v + q_w w
\end{equation}

where $(q_u, q_v, q_w)$ is constant then equation (2) becomes

\begin{equation}
    \text{phase} = (p_u - q_u) u + (p_v - q_v) v + (p_w - q_w) w
\end{equation}

whence equation (6) becomes

\begin{equation}
    \text{phase} = [(p_u - q_u) - p_1 (p_w - q_w)] u + [(p_v - q_v) - p_2 (p_w - q_w)] v
\end{equation}

Equations (7) become

\begin{align*}
    x &= -[\cos \theta \sin \phi + p_1 (\sin \theta - 1)] - [q_u - p_1 q_w] \\
    y &= -[\cos \theta \cos \phi + p_2 (\sin \theta - 1)] - [q_v - p_2 q_w]
\end{align*}

From which we see that the field centre is shifted by
\[ \Delta x = q_u - p_1 q_w \]
\[ \Delta y = q_v - p_2 q_w \]

(20)

The shift is applied to the coordinate reference pixel. For the SIN projection \((p_1, p_2) = (0, 0)\) and the shift is just

\[ \Delta x = q_u \]
\[ \Delta y = q_v \]

(21)

For the NCP projection the shift is

\[ \Delta x = q_u \]
\[ \Delta y = q_v - q_w \cot \delta_0 \]

(22)

In the general case the correction for precession, although small, applies systematically to the whole field. For a shift of 1° at \(\alpha_0 = \alpha_p\), and \(\delta_0 = 30°\) the whole map is shifted by about 0°.1 in declination.