AT 39.3/015

also { AT 31.6.2/020, 20.1.1/054, 21.3.1/030 }

# AT Polarisation Calibration

R.J. Sault N.E.B. Killeen M.J. Kesteven

September 19, 1991

This memo builds on the analysis of Komesaroff (AT/21.3.1/010 and /011) and Kesteven (AT/20.1.1/005 and other memos) on the calibration of AT data, in particular polarisation data. It concludes with a description of the implementation of a calibration scheme in Miriad. The Appendix gives some derivation outlines for most of the equations used in this memo.

## The Response of Ideal Linear Feeds

The AT antennae are equipped with orthogonal, linear feeds (the $X$ and $Y$ feeds). As the antennae are on alt-az mounts, the feeds rotate with respect to the source during the course of an observation. Forming the cross-correlations between the two feeds on two antennae gives the four products, $V_{XX}, V_{YY}, V_{XY}$ and $V_{YX}$.

For perfect feeds, these correlations are related to the Stokes parameters by:

$$
\begin{aligned}
V_{XX} &= I + Q\cos 2\chi + U\sin 2\chi \\
V_{YY} &= I - Q\cos 2\chi - U\sin 2\chi \\
V_{XY} &= -Q\sin 2\chi + U\cos 2\chi + iV \\
V_{YX} &= -Q\sin 2\chi + U\cos 2\chi - iV.
\end{aligned}
$$

The sign conventions are those use in Thompson, Moran and Swenson (1976). Here $\chi$ is the angle between the $X$ feed and the meridian to the North celestial pole, and is measured in the direction celestial North towards East. The $Y$ feed is assumed to be offset another $+90°$ (i.e. towards the East) from the $X$ feed.

The angle $\chi$ differs from the parallactic angle, $\psi$, only in the angle that the $X$ feed is offset from Celestial North at transit. In the AT's case, the $X$ feed is offset by 45° towards the East[1] (i.e. $+45°$). So for the AT, $\chi$ is the parallactic angle plus 45°. We have assumed that the parallactic angle is the same for each antenna. This assumption is usually adequate (but see later).

## Gain Errors and Polarisation Leakage

The measured correlations are clearly affected by feed-dependent complex gains, $g_X$ and $g_Y$. These gains include both the effects of the atmosphere and the instrument as a whole. Additionally the feeds are not perfect. They are not perfectly aligned, nor do they transmit a purely linear wave (the feeds would transmit an elliptically polarised wave). Misalignment of, say, the $X$ feed results in some of the $Y$ signal being present in the $X$ channel (i.e., the $Y$ signal leaks into the $X$ channel). Similarly the ellipticity of the $X$ feed means that it has a finite response to the $Y$ signal, but with a phase lag of 90°. The net

[1] Because the two AT feeds are offset by plus and minus 45°, the assignment of the names "$X$" and "$Y$" to the two feeds is fairly arbitrary. The RPFITS writer names the feed at $+45°$ as the $X$ feed, and the feed at $-45°$ as the Y feed. This naming convention has been preserved in both AIPS and Miriad. However, the convention used here (the normal convention?) is for the Y feed to be $+90°$ relative to the $X$ feed (rather than $-90°$). To follow this convention, the $V_{XY}$ and $V_{YX}$ correlations have to be negated after being read from the RPFITS file. This step is currently performed by the ATLOD tasks in both AIPS and Miriad.

result is that each channel is a sum of the desired polarisation signal, plus some complex factor, $D$ (the leakage parameter), times the orthogonal polarisation signal. That is

$$\begin{aligned} E'_X &= E_X + D_X E_Y \\ E'_Y &= E_Y + D_Y E_X, \end{aligned}$$

where $E'$ is the actual signal, and $E$ is the error free or ideal signal. If the leakage is only a result of feed misalignment, $\phi$, and feed ellipticity, $\theta$, it will be given by

$$\begin{aligned} D_X &= \frac{-\cos\theta_X \sin\phi_X + i\sin\theta_X \cos\phi_X}{\cos\theta_X \cos\phi_X + i\sin\theta_X \sin\phi_X} \\ &\approx -\phi_X + i\theta_X \\ D_Y &= \frac{\cos\theta_Y \sin\phi_Y - i\sin\theta_Y \cos\phi_Y}{\cos\theta_Y \cos\phi_Y + i\sin\theta_Y \sin\phi_Y} \\ &\approx \phi_Y - i\theta_Y. \end{aligned}$$

The approximations are for small $\phi$ and $\theta$.

Apart from the the sign convention for $D_X$, the leakage parameters are the same as Kesteven's $\zeta$ parameters.

If misalignment and ellipticity are small and are the only causes of polarisation leakage, the leakage parameters are simply related to misalignment and ellipticity errors. For the sake of physical intuition, as well as brevity, it is convenient to speak of the real and imaginary parts of the leakage as being a result of misalignment and ellipticity. However, the results in this memo do not dependent on small angle approximations of misalignment and ellipticity.

Feed gain and polarisation leakage results in the measured correlations being related to the desired ones by

$$\begin{aligned} V'_{XX}/(g_{X1}g^*_{X2}) &= V_{XX} + D_{X1}V_{YX} + D^*_{X2}V_{XY} + D_{X1}D^*_{X2}V_{YY} \\ V'_{YY}/(g_{Y1}g^*_{Y2}) &= V_{YY} + D_{Y1}V_{XY} + D^*_{Y2}V_{YX} + D_{Y1}D^*_{Y2}V_{XX} \\ V'_{XY}/(g_{X1}g^*_{Y2}) &= V_{XY} + D_{X1}V_{YY} + D^*_{Y2}V_{XX} + D_{X1}D^*_{Y2}V_{XY} \\ V'_{YX}/(g_{Y1}g^*_{X2}) &= V_{YX} + D_{Y1}V_{XY} + D^*_{Y2}V_{YX} + D_{Y1}D^*_{Y2}V_{XX}, \end{aligned}$$

where the subscripts 1 and 2 refer to the antenna numbers of the baseline.

As the leakages are typically small ($10^{-2}$), we can neglect the products of leakages ($D \times D$ terms). We can now write the measured correlations in terms of the source Stokes parameters

$$\begin{aligned} V'_{XX}/(g_{X1}g^*_{X2}) &= I - iV(D_{X1} - D^*_{X2}) \\ &\quad + Q(\cos 2\chi - (D_{X1} + D^*_{X2})\sin 2\chi) + U(\sin 2\chi + (D_{X1} + D^*_{X2})\cos 2\chi) \\ V'_{YY}/(g_{Y1}g^*_{Y2}) &= I + iV(D_{Y1} - D^*_{Y2}) \\ &\quad - Q(\cos 2\chi + (D_{Y1} + D^*_{Y2})\sin 2\chi) - U(\sin 2\chi - (D_{Y1} + D^*_{Y2})\cos 2\chi) \\ V'_{XY}/(g_{X1}g^*_{Y2}) &= I(D_{X1} + D^*_{Y2}) + iV \\ &\quad - Q(\sin 2\chi + (D_{X1} - D^*_{Y2})\cos 2\chi) + U(\cos 2\chi - (D_{X1} + D^*_{Y2})\sin 2\chi) \\ V'_{YX}/(g_{Y1}g^*_{X2}) &= I(D_{Y1} + D^*_{X2}) - iV \end{aligned}$$

$$-Q(\sin 2\chi - (D_{Y1} - D^*_{X2})\cos 2\chi) + U(\cos 2\chi + (D_{Y1} + D^*_{X2})\sin 2\chi).$$

The above equations describe the general, or "strongly polarised" case.

If we assume that the source is weakly polarised (or unpolarised) and neglect terms in $Q \times D$, $U \times D$ and $V \times D$, we get

$$\begin{aligned}
V'_{XX}/(g_{X1}g^*_{X2}) &= I + Q\cos 2\chi + U\sin 2\chi \\
V'_{YY}/(g_{Y1}g^*_{Y2}) &= I - Q\cos 2\chi - U\sin 2\chi \\
V'_{XY}/(g_{X1}g^*_{Y2}) &= I(D_{X1} + D^*_{Y2}) - Q\sin 2\chi + U\cos 2\chi + iV \\
V'_{YX}/(g_{Y1}g^*_{X2}) &= I(D_{Y1} + D^*_{X2}) - Q\sin 2\chi + U\cos 2\chi - iV.
\end{aligned}$$

We call these the "weakly polarised" equations. Though these equations are never used in the calibration procedure that we describe, it is instructive to examine these to see what can (and cannot) be determined from an observation of a weakly polarised source.

## Time Scales for the Gain and Leakage Terms

The leakage parameters are most likely determined by the mechanics of the feeds, and so should be stable with time. It would seem a good assumption that they do not vary over the length of an observation.

However the phases of the feed gains, $g$, may change comparatively rapidly (the order of minutes to hours). These phases will be determined by the atmosphere and, on a longer time scale, instrumental variations.

The atmospheric contribution for the $X$ and $Y$ gain should be identical, as they are looking through the same atmosphere. Indeed this has been implicitly assumed in the derivation of the above equations. On the other hand, the instrumental phase contribution will differ between the two channels, and this difference may vary with time. For the AT, this difference is measured by coupling a common noise signal (of the order of 10% of $T_{sys}$) into both feeds of each antenna. The phase of the correlation between the two feeds of the same antenna is a measure of the instrumental phase difference between the two channels. There is reason to believe that any such instrumental X-Y phase measurement will be offset from the correct value by a small constant value. For example, this may be because of asymmetries in coupling the noise signal into the two feeds. In the current AT hardware, this measurement has some shortcomings. At best, it contains a certain amount of noise (up to several degrees), while at worst it can produce erratic measurements (especially at 20 cm). Although plans are well advanced to improve this situation, there are some suggestions that the $X$-$Y$ phase differences for the AT may be constant over the course of an observation (provided setup parameters such as the delays are not changed).

## Current Calibration Practice

Although the equations for helical feeds (measuring circular polarisations) are mildly simpler than the those for the linear feeds, there is a more important practical difference. Here $V_{RR}$ and $V_{LL}$, for ideal feeds, are purely a sum and difference of $I$ and $V$ (i.e., independent of $Q$ and $U$). For many sources, particularly calibrators, it is a good assumption that Stokes $V = 0$. This means that the $V_{RR}$ and $V_{LL}$ correlations are good measures of Stokes $I$ (this also ignores the leakage of $Q$ and $U$ into the estimate of $I$, which is presumed negligible here). This allows the AIPS task CALIB to determine feed gains of $V_{RR}$ and $V_{LL}$ independently, and without regard to source polarisation (apart from the assumption that $V$ is zero).

One AT calibration scheme would be to use this approach, that is to determine gains from $V_{XX}$ and $V_{YY}$. Unfortunately $V_{XX}$ and $V_{YY}$ differ from Stokes $I$ by $Q\cos 2\chi + U\sin 2\chi$. This term may well be a few percent, and will vary with parallactic angle. This will result in time variable errors of a few percent in the amplitude solutions. This is complicted further by the linear polarisation characteristics of many calibrators being unknown and probably variable with time (over months) and frequency.

An alternate calibration scheme is to form Stokes $I = (V_{XX} + V_{YY})$ and use this in determining the gains. Although this eliminates the terms in $Q$ and $U$, it introduces another problem – if there are mismatches between the gains of the $X$ and $Y$ channels, then these do not manifest themselves as a closure error in $I$. They are closing in $V_{XX}$ and $V_{YY}$, not $V_{XX} + V_{YY}$. Thus these instrumental gain mismatches will not be correctly eliminated by CALIB. The AT's on-line system attempts to reduce these mismatches, firstly by producing data which has been nominally amplitude calibrated (with the primary calibrator say, at the beginning of the observation), and secondly by measuring the $X$-$Y$ phase. However at some level instrumental gain mismatch errors will exist. Some measurements suggest that the time varying amplitude mismatches can be several percent under bad conditions. Errors in the current $X$-$Y$ phase measurement system probably introduces much worse mismatches, particularly at 20 cm.

The problems with these schemes affect CALIB whether it is used in its primary calibration or self calibration mode. It is, of course, possible to do primary calibration in one of the schemes, and self calibration in the other. This choice would depend on whether instrumental gain mismatches or source linear polarisation would result in a smaller error in the gain solutions. Additionally, if the program source is unpolarised and $V_{XX}$ and $V_{YY}$ are self calibrated (rather than self calibrating $V_{XX} + V_{YY}$), then any errors made in the primary calibration can be self calibrated out.

These problems also affect observations regardless of whether the observer is interested in polarisation information or not. However gain errors are more significant in polarisation observations, as they result in portions of $I$ corrupting the other (usually much weaker) Stokes parameters. In these observations, the polarisation leakage parameters, $D$, also become significant.

## Determinable Quantities – Weakly Polarised Case

A natural question is what we can determine about the gain and leakage parameters from an observation of a calibrator. Firstly we consider the weakly polarised case (i.e., we consider the "weakly polarised" equations). Generally the gains, leakages and $X$-$Y$ phases can be determined (down to some offsets – see later). Roughly speaking, the gains are determined from the $V_{XX}$ and $V_{YY}$ correlations, whereas the leakages are derived from the $V_{XY}$ and $V_{YX}$ correlations. Note that, like gains, leakage parameters are antenna based – not baseline based, and so whereas the number of unknown leakages goes as the number of antenna, $N$, the number of measurements goes as the number of baselines, $N(N-1)/2$.

In addition to gains and leakages, if there is good parallactic angle coverage, the values of $Q$ and $U$ for the calibrator can also be determined. The fact that $Q$ and $U$ can be disentangled from the leakages, even if their effect is smaller than leakage of $I$, is a consequence of the parallactic angle coverage. Leakage of $I$ into $V_{XY}$ and $V_{YX}$ lead to a constant signature, whereas $Q$ and $U$ lead to a signature that varies as sinusoids of the parallactic angle. Note that in considering the $V_{XX}$ and $V_{YY}$ correlations alone, it is not possible to distinguish variations caused by polarised flux density from amplitude gain variations in $X$ and $Y$.

Given good parallactic angle coverage, this ability to disentangle $Q$ and $U$ from leakage of $I$ is an advantage of an alt-az mount over an equatorial mount (or any telescopes with fixed $\chi$). That is, from the observation of a calibrator, an alt-az telescope can simultaneously determine instrumental leakage parameters and linear polarisation, whereas an equatorial telescope cannot.

Unfortunately, for a weakly polarised calibrator, there is no way to distinguish between $V$ and leakage of $I$. Note that we can incorporate an unknown $V$ within the leakage terms by

$$D_X - D_X + i\frac{V}{I}.$$

Note that observing a weakly polarised calibrator gives "relative" solutions only. The misalignment and ellipticity of one feed, and the X-Y phase of one antenna have to be assumed in the solution process. This is somewhat similar to solving for relative antenna phases when calibrating data (the phase of one antenna can be arbitrarily set to 0). However the "relative" solutions of misalignment, ellipticity and X-Y phase differ fundamentally from "relative" antenna phase, in that "absolute" quantities are needed for the calibration of some sources.

To see that only relative solutions are possible, consider the "weakly polarised" equations. Here we can add an arbitrary offset to all the $D_X$ leakages, then subtract the conjugate of this offset from all the $D_Y$ leakages, and the $D_X + D_Y^*$ term remains unchanged. Physically, at least for the real part, this tells us that we cannot solve for source polarisation position angle and absolute alignment of the feeds simultaneously. This certainly agrees with intuition. We can determine the alignment of the feeds relative to each other, but we cannot solve for the absolute alignment of the feeds as we have not given some absolute reference frame. We have to assume one feed (the reference feed, e.g., the $X$ feed of the reference antenna) is perfectly aligned.

Similarly we cannot solve for absolute ellipticity, though the physical reasons behind this are more arcane.

While the relative $X$ and the relative $Y$ phases are well measured, the phase offset which ties together the $X$ and $Y$ phases is poorly measured by an observation of a weakly polarised source. To see this, note that the $V_{XX}$ and $V_{YY}$ correlations do not depend on this X-Y phase offset – only the $V_{XY}$ and $V_{YX}$ correlations do (i.e., the $g_{X1}g_{Y2}^*$ like terms are only present in the cross polarisations). For an unpolarised source, and ideal feeds, the $V_{XY}$ and $V_{YX}$ correlations will be pure noise, and we cannot hope to use this to infer the $X$-$Y$ phase! If, however, there was appreciable leakage, then the $X$-$Y$ phase could be determined from the leaked signal in the $V_{XY}$ and $V_{YX}$ correlations. However any such estimate of the offset would be fairly noisy – it is probably better to measure the $X$-$Y$ phase directly, and to observe a strongly polarised source to determine errors in this measurement.

## Determinable Quantities – Strongly Polarised Case

The equations dealing with the strongly polarised case are somewhat more involved. We can again solve for all the quantities in the weakly polarised case. In addition we can solve for absolute ellipticity and the $X$-$Y$ phase offset (i.e., we can tie the $Y$ phases to the $X$ phases). Obviously, though, we still cannot solve for both the source polarisation position angle as well as absolute alignment. We can, however, solve for either one, given the other.

## Needed Quantities

It is useful to determine what errors are introduced if the program source is calibrated with the relative, rather than the absolute parameters. To examine this, we give linearised equations expressing the nominal Stokes parameters (i.e., the Stokes parameters formed by using the nominal gains, and ignoring the leakage terms) in terms of the true Stokes parameters.

Assuming that the nominal gains, $g'$, have a relative error $\epsilon$, that is:

$$\frac{g_X}{g'_X} = 1 + \epsilon_X$$
$$\frac{g_Y}{g'_Y} = 1 + \epsilon_Y,$$

then the nominal Stokes parameters $(I', Q', U', V')$ are related to the true Stokes parameters $(I, Q, U, V)$ by:

$$
\begin{aligned}
I' &= I + \frac{1}{2}(I\epsilon^{++} - Q(\epsilon^{-+}\sin 2\chi - \zeta^{++}\cos 2\chi) + U(\epsilon^{-+}\cos 2\chi + \zeta^{-+}\sin 2\chi) + iV\zeta^{--}) \\
Q' &= Q + \frac{1}{2}(-I(\epsilon^{-+}\sin 2\chi - \zeta^{++}\cos 2\chi) + Q\epsilon^{++} - U\zeta^{-+} - iV(\epsilon^{--}\cos 2\chi + \zeta^{+-}\sin 2\chi)) \\
U' &= U + \frac{1}{2}(+I(\epsilon^{-+}\cos 2\chi + \zeta^{++}\sin 2\chi) + Q\zeta^{-+} + U\epsilon^{++} - iV(\epsilon^{--}\sin 2\chi - \zeta^{+-}\cos 2\chi)) \\
iV' &= iV + \frac{1}{2}(-I\zeta^{--} - Q(\epsilon^{--}\cos 2\chi + \zeta^{+-}\sin 2\chi) - U(\epsilon^{--}\sin 2\chi - \zeta^{+-}\cos 2\chi) + iV\epsilon^{++})
\end{aligned}
$$

where

$$
\begin{aligned}
\epsilon^{++} &= (\epsilon_{X1} + \epsilon_{Y1}) + (\epsilon^*_{X2} + \epsilon^*_{Y2}) \\
\epsilon^{--} &= (\epsilon_{X1} - \epsilon_{Y1}) - (\epsilon^*_{X2} - \epsilon^*_{Y2}) \\
\epsilon^{-+} &= (\epsilon_{X1} - \epsilon_{Y1}) + (\epsilon^*_{X2} - \epsilon^*_{Y2}) \\
\zeta^{-+} &= (D_{X1} - D_{Y1}) + (D^*_{X2} - D^*_{Y2}) \\
\zeta^{+-} &= (D_{X1} + D_{Y1}) - (D^*_{X2} + D^*_{Y2}) \\
\zeta^{--} &= (D_{X1} - D_{Y1}) - (D^*_{X2} - D^*_{Y2}) \\
\zeta^{++} &= (D_{X1} + D_{Y1}) + (D^*_{X2} + D^*_{Y2}).
\end{aligned}
$$

Of the seven parameters, four of them are not dependent on knowing absolute alignment, ellipticity and $X$-$Y$ phase. That is, these four can be determined from the relative values alone, as the offsets cancel out. So these four can be determined purely from observations of a weakly polarised (or unpolarised) source. Of the remaining three:

- $\epsilon^{-+}$ depends on knowing absolute $X$-$Y$ phase. Not knowing the absolute $X$-$Y$ phase causes $Q$ and $U$ to corrupt $I$, and $I$ to corrupt $Q$ and $U$. If the program source is strongly linearly polarised, and/or if $Q$ and $U$ are of interest, it is important to have a good estimate of the absolute $X$-$Y$ phase.

- $\zeta^{-+}$ depends on knowing absolute alignment. This misalignment causes some error when resolving the linearly polarised component into $Q$ and $U$ (i.e., $Q$ and $U$ corrupt each other). The percentage linear polarised flux density will be correctly determined, but the deduced polarisation position angle will be misaligned by the unknown feed misalignment offset. As this will typically be less than a degree, knowing absolute alignment is only necessary if polarisation position angles are needed to better than a degree.

  Uncertainty of the absolute alignment is similar to uncertainty in absolute position. It is still possible to produce an artifact-free, self-consistent image, which is still adequate for many purposes.

- $\zeta^{+-}$ depends on knowing absolute ellipticity. Not knowing absolute ellipticity causes $V$ to corrupt $Q$ and $U$, and $Q$ and $U$ to corrupt $V$. Absolute ellipticity is consequently needed only for those sources which are both linearly and circularly polarised.

The conclusion is that the relative parameters determined from a weakly polarised calibrator are adequate in calibrating the program source if either:

- Polarisation of the program source is not of interest, and either it is weakly polarised, or the on-line $X$-$Y$ phase measurements are good, or

- Linear polarisation of the program source is of interest, and the on-line X-Y phases are good, the source has little circular polarisation, and polarisation position angles to better than a degree are not needed, or

- Circular polarisation of the source is of interest, and there is little or no linear source polarisation.

## Observing Strategies

An observing strategy is:

- Intermix the observations of the program source with the secondary phase calibrator, as usual. This should give good parallactic angle coverage of the secondary. Assuming the secondary is weakly polarised, feed gains (including relative *X*-*Y* phase offsets), relative leakage parameters and calibrator *Q* and *U* can be determined.
- Observe a "position angle" calibrator (normally 3C286 or 3C138) for a short period (10 minutes). As this calibrator is strongly polarised, with known position angle, this observation can be used to determine the X-Y phase, alignment and ellipticity offsets to convert the relative parameters determined above to absolute parameters.
- Observe a flux calibrator (normally 1934-638) for a short period (10 minutes). This calibrator is used to determine an absolute flux density scale.

Depending on the astronomical requirements and the quality of the on-line *X*-*Y* phase measurements, observing the position angle calibrator may not be necessary.

## Implementation of a Scheme in Miriad

A scheme to calibrate AT data has been implemented in Miriad. The main task, GPCAL, can be used to solve for feed gains, source *Q* and *U* and polarisation leakage parameters. There are a set of switches that the user can set to turn on or off the various solver options. Some pertinent details include:

- The feed gains are assumed to vary on a short time scale (specified by the user), and independent gain solutions are found in each interval. However the *X*-*Y* phase difference is assumed to be constant over the observation, so the phase of the Y feed gain is constrained to be a time-invariant offset from the *X* feed (there is no constraint on the amplitudes). This *X*-*Y* phase can also solved for, using all the data present. By default, the *X*-*Y* phase of the reference antenna is not changed from its initial value (i.e., GPCAL solves for relative X-Y phase only), but a switch can enable solving for the reference antenna X-Y phase as well (i.e., for a strongly-polarised source).
- The values of *Q* and *U* for the calibrator can be solved for. This requires good parallactic angle coverage, so that *Q* and *U* can be disentangled from the leakage terms.
- The polarisation leakage terms can be solved for, giving either relative or absolute solutions. For the relative case, the leakage parameter for the *X* feed of the reference antenna is not changed from its initial value (normally zero).
- In addition to determining relative solutions for *X*-*Y* phase and leakage parameters, it is possible to take previously determined relative solutions, and solve for just their absolute offset. This would be used when converting the solution determined from the secondary to absolute parameters using a "position angle" calibrator.

GPCAL has knowledge of the Stokes parameters of 3C286 and 3C138. These are based on values given by Perley (private communication, 1991). Figure 1 plots these polarisation characteristics against frequency together with spectral fits.

Algorithmically, GPCAL finds a least squares solution. It consists of two basic steps performed iteratively. The first step assumes the source *Q* and *U*, leakages and X-Y phases are known, and so solves for the unknown feed gains (enforcing the X-Y phase constraint). This gain solving step uses an algorithm similar to that advocated by Schwab ("Adaptive calibration of radio interferometer data", *SPIE International Optical Computing Conference*, vol. 231, 1980). The second step assumes the gains are known. For whatever other parameters are being solved for (e.g. *Q*, *U*, leakage and X-Y phase), the polarisation equations are linearised and then solved. The iteration of the two steps continues until the scheme converges to a self-consistent solution. This typically take 4 to 10 iterations.

A number of other Miriad tasks are generally needed to perform the various steps in polarisation calibration. See Killeen's manual "Analysis of Australia Telescope Data" (Appendix D) and the Miriad manuals and help files.

## Calculation of Parallactic Angle

For a given antenna, the parallactic angle $\psi$ is given by

$$\tan\psi = \frac{\cos\mathcal{L}\sin H}{\sin\mathcal{L}\cos\delta - \cos\mathcal{L}\sin\delta\cos H},$$

where $\mathcal{L}$ is the angle between the antenna's azimuth plane and the equatorial plane, $H$ is the source hour angle, and $\delta$ is the source's apparent declination. If the azimuth plane is parallel to the local horizontal, $\mathcal{L}$ will be the geodetic latitude of the antenna. For an east-west array, $\mathcal{L}$ is nominally constant. The compact array was constructed on a tangent plane to the Earth (at station 35) so that the source hour angle and declination are independent of telescope. The result is $\psi$ will be independent of telescope for the compact array.

It is instructive to consider the error introduced in the parallactic angle calculation ($\Delta\psi$) by errors in $\mathcal{L}$, $\delta$ and $H$ ($\Delta\mathcal{L}$, $\Delta\delta$ and $\Delta H$). Using various small angle approximations, near transit (i.e. small $H$)

$$\begin{aligned}
\Delta\psi &= \cos\mathcal{L}\csc(\mathcal{L}-\delta)\Delta H.\\
\Delta\psi &= -H\cos\delta\csc^2(\mathcal{L}-\delta)\Delta\mathcal{L}\\
\Delta\psi &= -H\cos\mathcal{L}\cos(\mathcal{L}-\delta)\csc^2(\mathcal{L}-\delta)\Delta\delta
\end{aligned}$$

Not surprisingly, errors may become significant when the source declination approachs the observatory latitude. For an error in $H$, the largest error will occur at transit. On the other hand errors in $\mathcal{L}$ and $\delta$ will cause no error at transit, but a maximum at some other time. The small angle approximations used in deriving these equations break down at an hour angle in proportion to $\csc(\mathcal{L}-\delta)$. Because the error equations for $\delta$ and $\mathcal{L}$ increase in proportion to $\csc(\mathcal{L}-\delta)$, the maximum error must increase at least at a rate proportional to $\csc(\mathcal{L}-\delta)$. Figure 2 plots the error in the calculated parallactic angle, for a 1 arcminute error in $\mathcal{L}$, for $\mathcal{L} = -30°$ and $\delta$ ranging from $-31°$ to $-40°$. This shows a relatively small error of 1 arcminute in $\mathcal{L}$ can get amplified significantly for sources near the declination of the observatory's latitude. For the AT, the value of $\mathcal{L}$ may well vary between telescopes, and vary after moves at the level of 1 arcminute. Consequently, for sources within a few degrees of $\delta = -30°$, it may be necessary to use a value of $\mathcal{L}$ determined from the pointing model (derived after each move) rather than just the nominal value. Of course, such a value of $\mathcal{L}$ will then vary between telescopes.

## Appendix: Some Derivations

In this section, we give a sketch of the derivation of some of the basic equations.

For a single antenna, and assuming ideal feeds, the electric field vectors, $E_X$ and $E_Y$, can be resolved into components, $E_\delta$ and $E_\alpha$. If $\chi$ were zero, $E_X$ would equal $E_\delta$ and $E_Y$ would equal $E_\alpha$. For arbitrary $\chi$,

$$\begin{aligned}
\begin{pmatrix} E_X \\ E_Y \end{pmatrix} &= \begin{pmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{pmatrix}\begin{pmatrix} E_\delta \\ E_\alpha \end{pmatrix}\\
&= R\begin{pmatrix} E_\delta \\ E_\alpha \end{pmatrix}.
\end{aligned}$$

That is, we can relate the electric field for arbitrary $\chi$ to that for $\chi = 0$ using a rotation matrix, $R$.

Similarly we can relate the correlations for a particular value of $\chi$ ($V_{XX}$, etc) to the electric field vectors, and the correlations that would be measured if $\chi$ were zero:

$$\begin{aligned}\begin{pmatrix} V_{XX} & V_{XY} \\ V_{YX} & V_{YY} \end{pmatrix} &= \begin{pmatrix} E_{X1} \\ E_{Y1} \end{pmatrix} \begin{pmatrix} E^*_{X2} & E^*_{Y2} \end{pmatrix} \\ &= R \begin{pmatrix} E_{\delta 1} \\ E_{\alpha 1} \end{pmatrix} \begin{pmatrix} E^*_{\delta 2} & E^*_{\alpha 2} \end{pmatrix} R^T. \\ &= R \begin{pmatrix} V_{\delta\delta} & V_{\delta\alpha} \\ V_{\alpha\delta} & V_{\alpha\alpha} \end{pmatrix} R^T. \end{aligned}$$

Thompson, Moran and Swenson give the relationship between linear correlations and the Stokes parameters, when $\chi$ is zero:

$$\begin{pmatrix} V_{\delta\delta} & V_{\delta\alpha} \\ V_{\alpha\delta} & V_{\alpha\alpha} \end{pmatrix} = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix}.$$

So the correlations for a particular value of $\chi$ can be related to the Stokes parameters using the rotation matrix.

$$\begin{pmatrix} V_{XX} & V_{XY} \\ V_{YX} & V_{YY} \end{pmatrix} = R \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} R^T.$$

Expanding this gives the relationship between the linear correlations and the Stokes parameters for ideal feeds.

To account for leakage, the measured electric fields ($E'_X$ and $E'_Y$) are related to the ideal ones ($E_X$ and $E_Y$) by the leakage matrix:

$$\begin{aligned}\begin{pmatrix} E'_X \\ E'_Y \end{pmatrix} &= \begin{pmatrix} 1 & D_X \\ D_Y & 1 \end{pmatrix} \begin{pmatrix} E_X \\ E_Y \end{pmatrix} \\ &= L \begin{pmatrix} E_X \\ E_Y \end{pmatrix}\end{aligned}$$

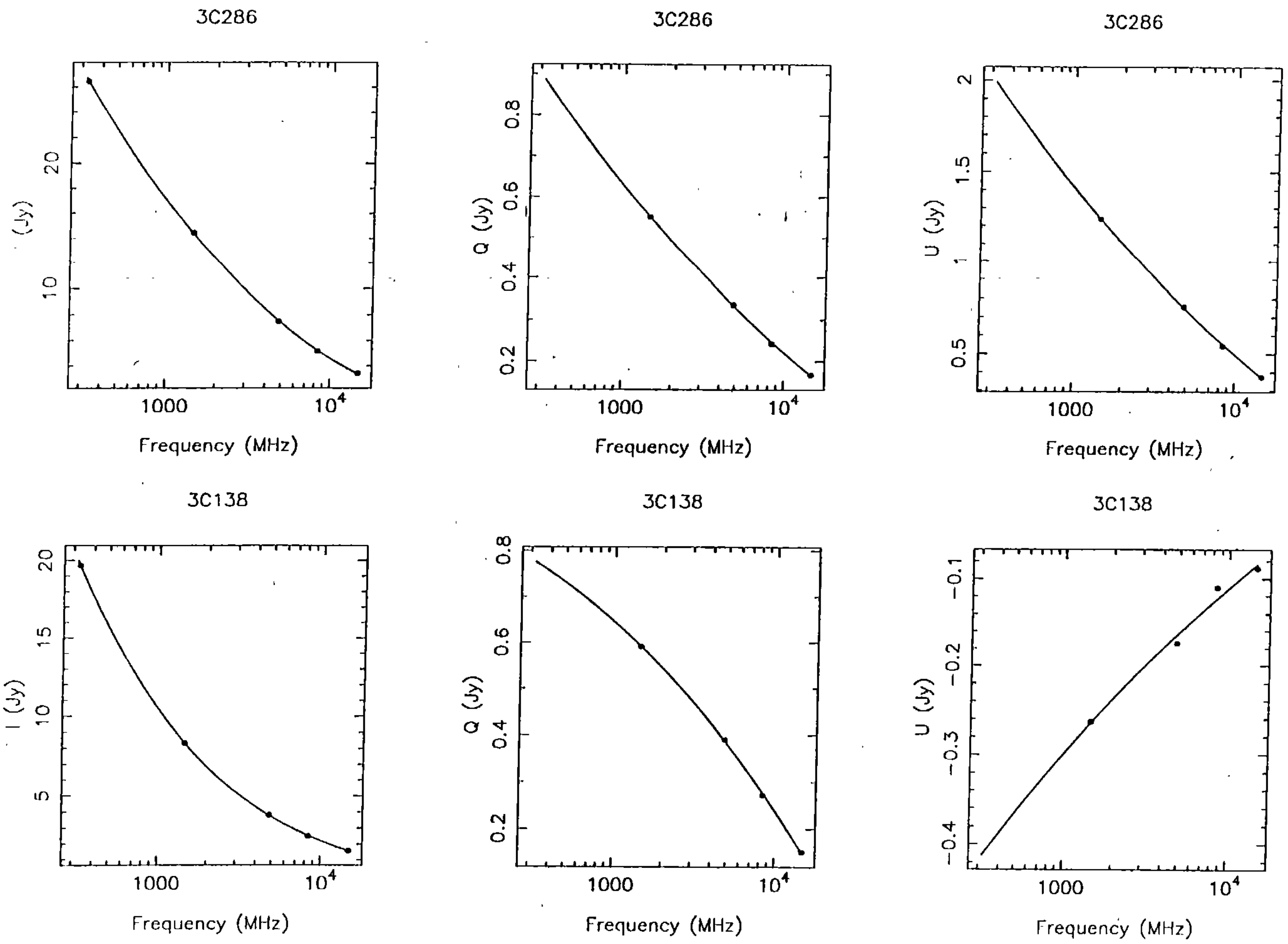
The measured correlations ($V'_{XX}$, $V'_{XY}$, $V'_{YX}$ and $V'_{YY}$) are related to the ideal correlations ($V_{XX}$, $V_{XY}$, $V_{YX}$ and $V_{YY}$), and the Stokes parameters, by:

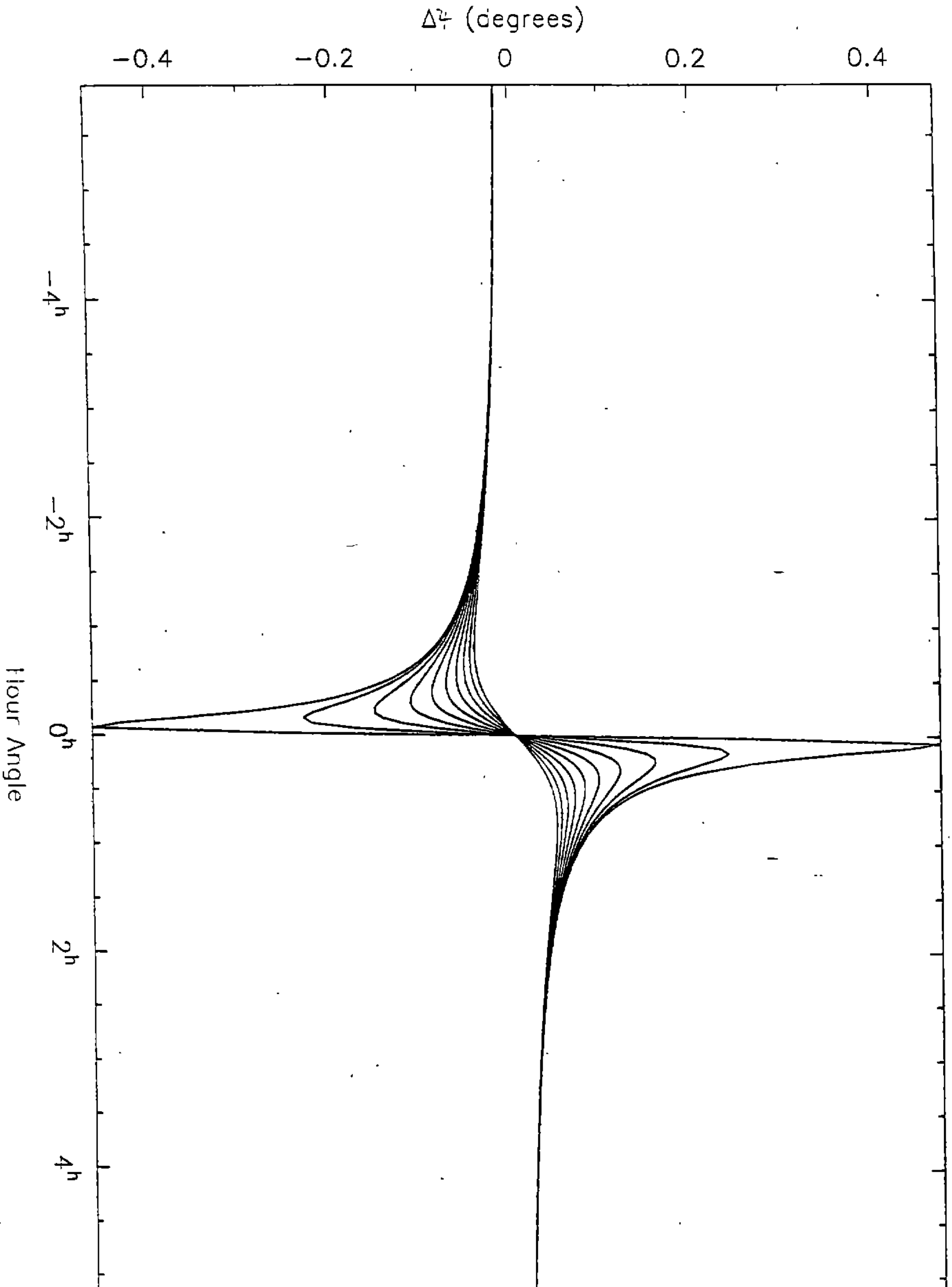
$$\begin{aligned}\begin{pmatrix} V'_{XX} & V'_{XY} \\ V'_{YX} & V'_{YY} \end{pmatrix} &= \begin{pmatrix} E'_{X1} \\ E'_{Y1} \end{pmatrix} \begin{pmatrix} E'^*_{X2} & E'^*_{Y2} \end{pmatrix} \\ &= L_1 \begin{pmatrix} E_{X1} \\ E_{Y1} \end{pmatrix} \begin{pmatrix} E^*_{X2} & E^*_{Y2} \end{pmatrix} L_2^{*T} \\ &= L_1 \begin{pmatrix} V_{XX} & V_{XY} \\ V_{YX} & V_{YY} \end{pmatrix} L_2^{*T} \\ &= L_1 R \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} R^T L_2^{*T}.\end{aligned}$$

# Figure Captions

1. Polarised flux density of 3C286 and 3C138: The lines give a least squares fit through the measured points. The measurements were at 327, 1465, 4866, 8434 and 14934 MHz. Note Q and U have not been measured at 327 MHz.

2. Error in Parallactic angle calculation: This gives the error in the calculated parallactic angle resulting from a 1 arcminute error in the assumed antenna azimuth plane. The observatory latitude was $-30°$, and the source declinations varyed from $-31°$ to $-40°$.


3C286
(Jy)
10
20
1000
10^4
Frequency (MHz)
3C286
Q (Jy)
0.2
0.4
0.6
0.8
1000
10^4
Frequency (MHz)
3C286
U (Jy)
0.5
1
1.5
2
1000
10^4
Frequency (MHz)
3C138
I (Jy)
5
10
15
20
1000
10^4
Frequency (MHz)
3C138
Q (Jy)
0.2
0.4
0.6
0.8
1000
10^4
Frequency (MHz)
3C138
U (Jy)
−0.4
−0.3
−0.2
−0.1
1000
10^4
Frequency (MHz)


Δψ (degrees)
−0.4
−0.2
0
0.2
0.4
Hour Angle
−4h
−2h
0h
2h
4h