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A noise budget for SKA imaging applications

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Introduction

Dynamic range is not a suitable figure of merit

Effective noise

- thermal noise
- classical source confusion noise
- calibration noise (estimation noise + penalty for corrections)
- calibration artefacts
- far sidelobe confusion noise (FSCN)
- psf sidelobe confusion noise (PSCN)

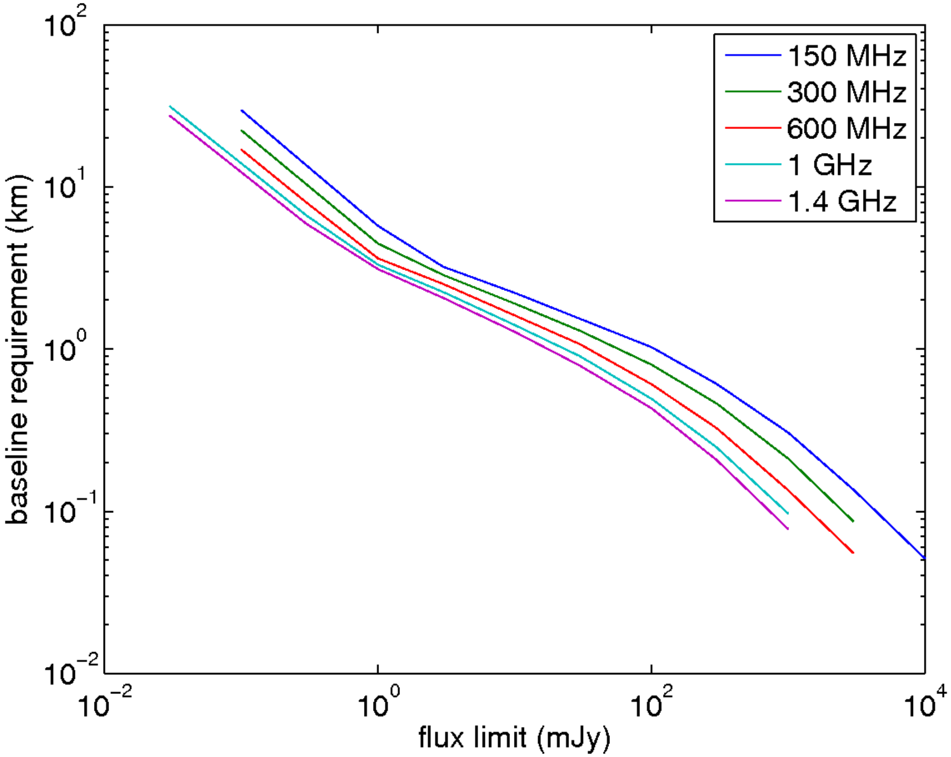
Last 5 factors can be mitigated by **design-for-calibratability**

Classical source confusion

Definition used: 10 psfs / source to avoid cluttering of sources

LFAA can do a relatively shallow all-sky continuum survey

EoR/CD needs frequency independent psf



f (MHz)	A/T (m^2/K)	τ (h)
50	144	15.9
100	760	1.0
160z	1070	2.1
220	1060	4.7

Calibration noise – part 1

Wijnholds & Van der Veen, IEEE JSTSP, Oct 2008
Wijnholds, Ph.D. thesis, Mar 2010

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calibration extracts information that can't be used for imaging

- simple view
  - each data point is an equation
  - each calibration parameter is an unknown
  - each image parameters is an unknown
- rigorous approach: Cramer-Rao bound analysis
  - partitioned FIM: $\mathbf{F} = \begin{bmatrix} \mathbf{F}_{\text{im}} & \mathbf{F}_{\text{im,cal}} \\ \mathbf{F}_{\text{cal,im}} & \mathbf{F}_{\text{cal}} \end{bmatrix}$
  - invert to get CRB: $\text{CRB} = \mathbf{F}^{-1} = \begin{bmatrix} [\mathbf{F}^{-1}]_{\text{im}} & [\mathbf{F}^{-1}]_{\text{im,cal}} \\ [\mathbf{F}^{-1}]_{\text{cal,im}} & [\mathbf{F}^{-1}]_{\text{cal}} \end{bmatrix}$
  - use Schur complement: $[\mathbf{F}^{-1}]_{\text{im}} = \left(\mathbf{F}_{\text{im}} - \mathbf{F}_{\text{cal,im}} \mathbf{F}_{\text{cal}}^{-1} \mathbf{F}_{\text{cal,im}}\right)^{-1}$

Calibration noise – part 2

Propagation of calibration errors

- Errors in calibration cause beam perturbations
- Assumed imaging process $\boldsymbol{\sigma} = \mathbf{M}(\boldsymbol{\theta})\,\mathrm{vec}(\mathbf{R})$
- Rigorous error propagation to image

$$\mathrm{cov}(\boldsymbol{\sigma}) = \left(\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\theta}^{\mathrm{T}}}\right) \mathrm{cov}(\boldsymbol{\theta}) \left(\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\theta}^{\mathrm{T}}}\right)^{\mathrm{T}}$$

- Apply this to single point source test image to see impact on beam
- Can be done for AA and PAF beams

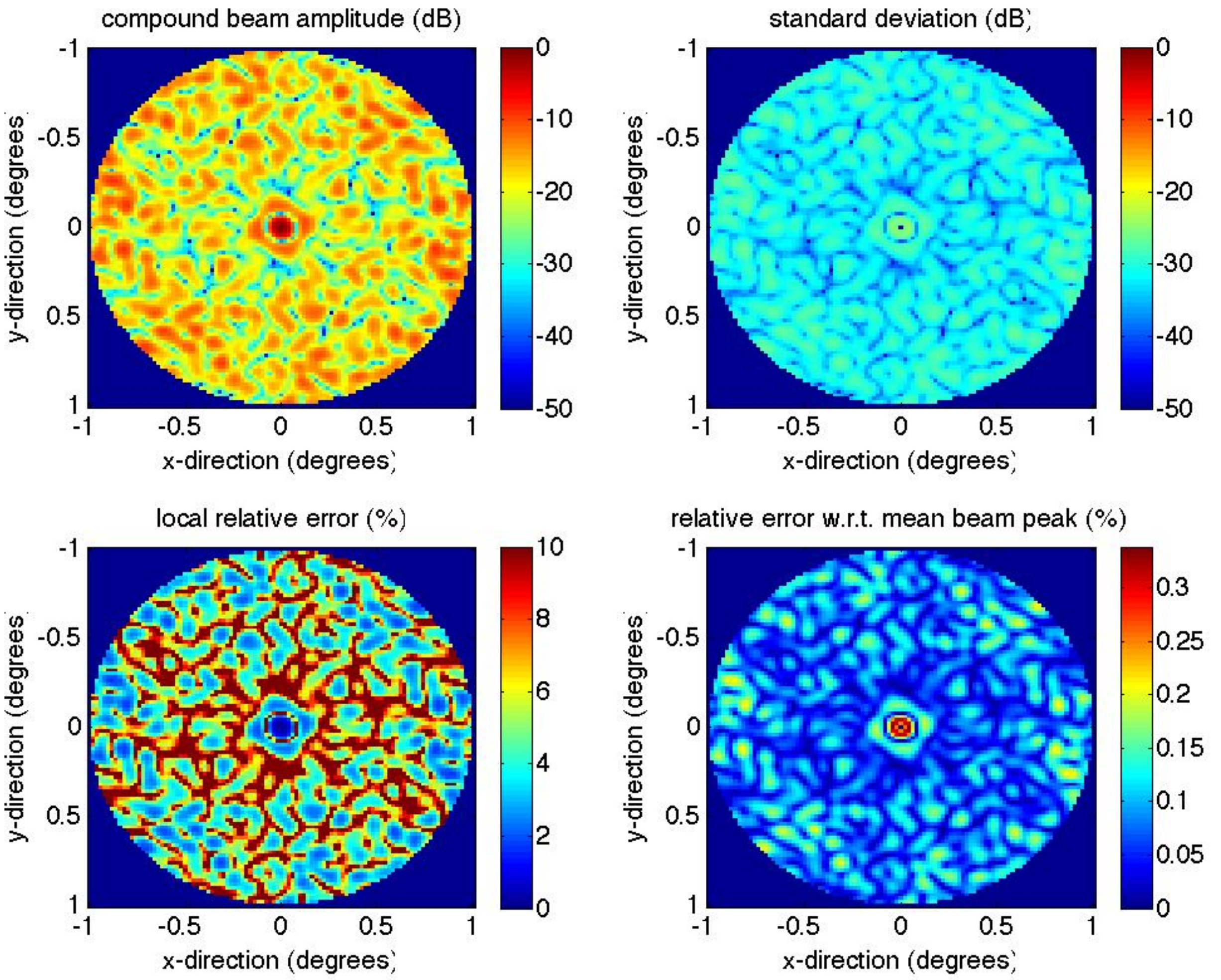
Example: cal. errors in AA beam

Wijnholds, Grainge & Nijboer, SKA-low, Sep. 2011

Impact station calibration errors on LOFAR LBA station beam

Assumptions:

- LBA_OUTER, CS302
- calibration on 4-9-'11, 15:00 UTC at 50 MHz
- 1 s, 195 kHz
- calibration errors derived from CRB
- $SNR_{Cas} = 0.01$
- constraint on peak



Calibration artefacts

- **Calibration processes may produce artefacts**
  - biased solutions (self-cal bias)
  - ghost sources (talk by Trienko Grobler on Tuesday)
- **Many different and often subtle causes**
  - weights based on data (Wijnholds & v.d. Veen, TSP, 2009)
  - incomplete sky model
- **Calibration processes need very careful checking**
  - algorithms should be unbiased
  - algorithms should be statistically efficient

PSCN

Source statistics: density of sources increases as we image deeper

- sky is sparse in (relatively) bright sources
- sky is filled with ~10^{11} weak sources (# of galaxies in universe)
- example:
  - LFAA @ 110 MHz after 1000 hours / 1 MHz integration
  - to reach 100σ level, 2.10^{5} need subtraction
  - after that, still 10^{8} sources remain in FoV.
  - to image one source, 10^{8} sources need to be suppressed

This requires a RMS psf sidelobe level of -49 dB

FSCN

- Strong sources outside FoV are treated individually
- Side lobes become more sensitive in longer integrations
  $\rightarrow$ weaker sources will start to affect our data as well
- We cannot treat ***all*** theses sources individually
  $\rightarrow$ we need to suppress sources outside FoV sufficiently
- Sources outside FoV are suppressed by
  - primary beam sidelobes
  - psf sidelobes
  - time and frequency smearing (if applicable)

FSCN analysis

Squared addition of all untreated flux in sidelobes

$$\Delta S^2 = \int_0^{S_{max}} \int_0^{2\pi} \int_{\theta_0}^{\pi/2} \rho(S) S^2 E_{stat}^2(\theta,\phi) E_{psf}^2(\theta,\phi) \, dS \, d\theta \, d\phi$$

Assume random distribution of sources, balance with thermal noise

$$E_{stat} = \frac{1}{\sqrt{2\pi}} \left(\frac{\Delta S}{E_{psf} \Delta S_0} \right)$$

where ΔS_0 is the RMS flux of all sources weaker than S_{max}

ΔS is the thermal noise level

With E_{psf} = -49 dB this gives $\mathbf{E_{stat}}$ **= -45 dB**

Achievable with differently randomized / rotated LFAA stations

Overall noise budget

	Assumption	value
thermal noise		1σ
calibration noise	10% penalty for extraction of information in selfcal process	0.1σ
	20% penalty for calibraiton corrections	0.2σ
	thermal noise level after selfcal	***1.3σ***
source confusion	negligible	0
cal. artefacts	absent	0
PSCN	balanced with thermal noise	1σ
FSCN	balanced with thermal noise	1σ
	effective noise	***2.05σ***

Discussion

- Balancing FSCN and PSCN against thermal noise implies
  - Requirement of -49 dB RMS psf sidelobe level
  - Requirement of -45 dB RMS average station sidelobe level
  - effective noise of at least $\sqrt{3}\,\sigma$
- This assumes (for 1000 h / 1 MHz LFAA observation)
  - no confusion $\rightarrow$ maximum baseline ~500 km
  - no calibration artefacts

Getting within a factor 1.5 – 2 from thermal noise is a challenge

EoR/CD needs frequency independent psf to remove confusion

Summary

Deep osbervations

- EoR/CD: 1000 hours / 1 MHz
- Continuum surveys: 10 hours / 100 MHz

Getting within a factor 1.5 – 2 of thermal noise is a challenge

- PSCN $\rightarrow$ -49 dB RMS psf sidelobe level
- FSCN $\rightarrow$ -45 dB RMS average station sidelobe level

Theoretically, getting within 50% of thermal noise is possible

- PSCN and FSCN need to be (almost) absent
- even more stringent requirements