Wide-band Full-polarization Imaging

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S. Bhatnagar
NRAO

The Scientific Problem

- The scientific problem addressed here requires precise measurement of the sky brightness distribution.
  - Continuum science
  - Wide-field polarimetry
  - Spectral behaviour
  - Temporal behaviour

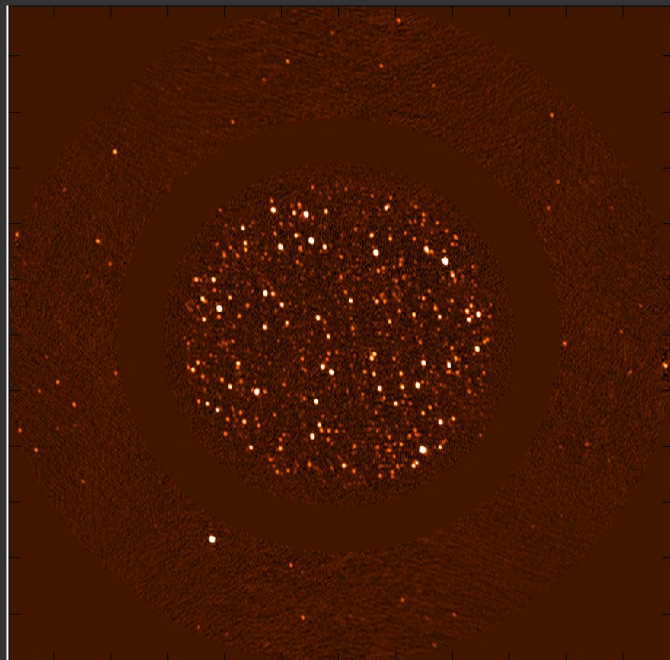
- The goal of calibration, imaging and deconvolution therefore is to derive a precise model for the sky brightness
  - Imaging is **not** inter-changeable with *somehow* removing the foreground emission (the scientific result is not in the background emission)

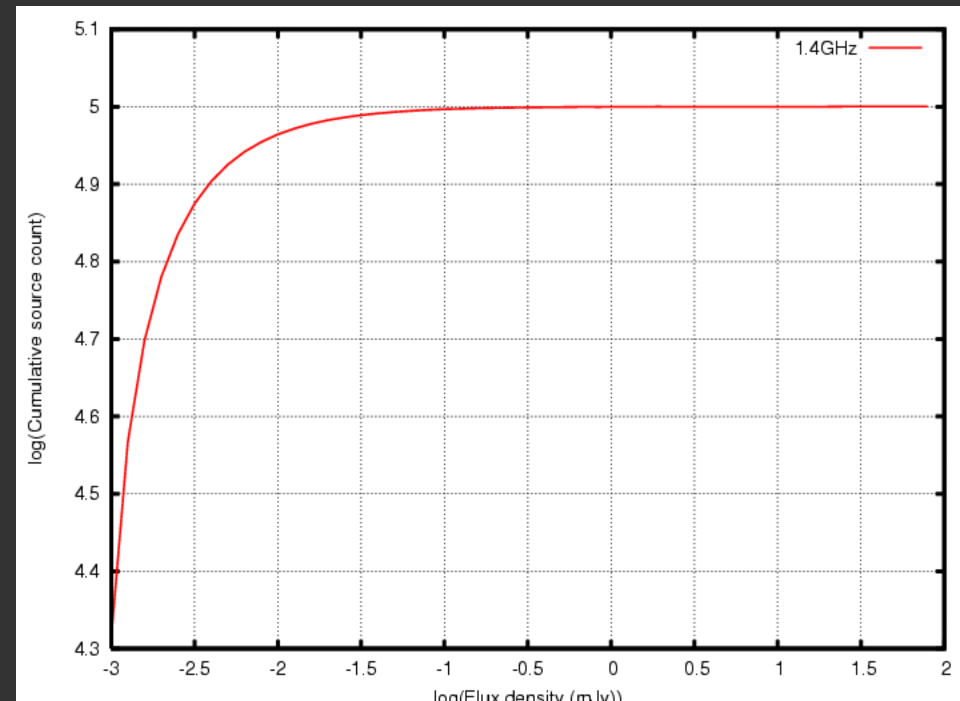
  - Even where the scientific result is in the background, requirement will be precision measurement of the *real* background emission

The Summary

- Sky brightness at low radio frequencies
- Reminder of some fundamental principles behind interferometric imaging based on the physics of the measurement process
  - Fundamental separation of noise, signal and instrumental/atmospheric terms based on the physics of the measurement process
- Equivalence between DI calibration and DD corrections
  - Show that Projection algorithms are in-fact a true DD generalization of DI calibration. E.g., WB A-Projection == DD Band-pass calibration
- Single pointing wide-band wide-field imaging
  - Projecting-out the dominant effects (PB effects)
  - Results: Simulations (for understanding)
  - Results: Application to real data and comparison with known facts (for verification)

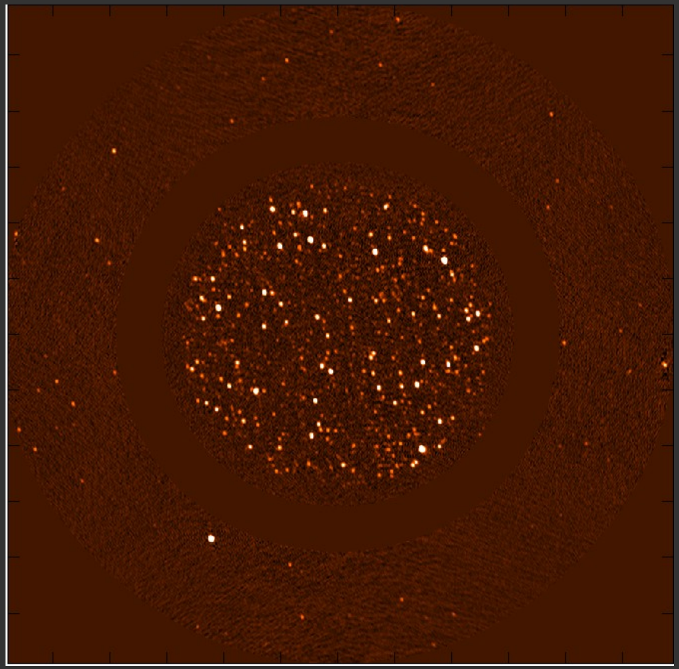
Sky at low frequencies: No. of sources

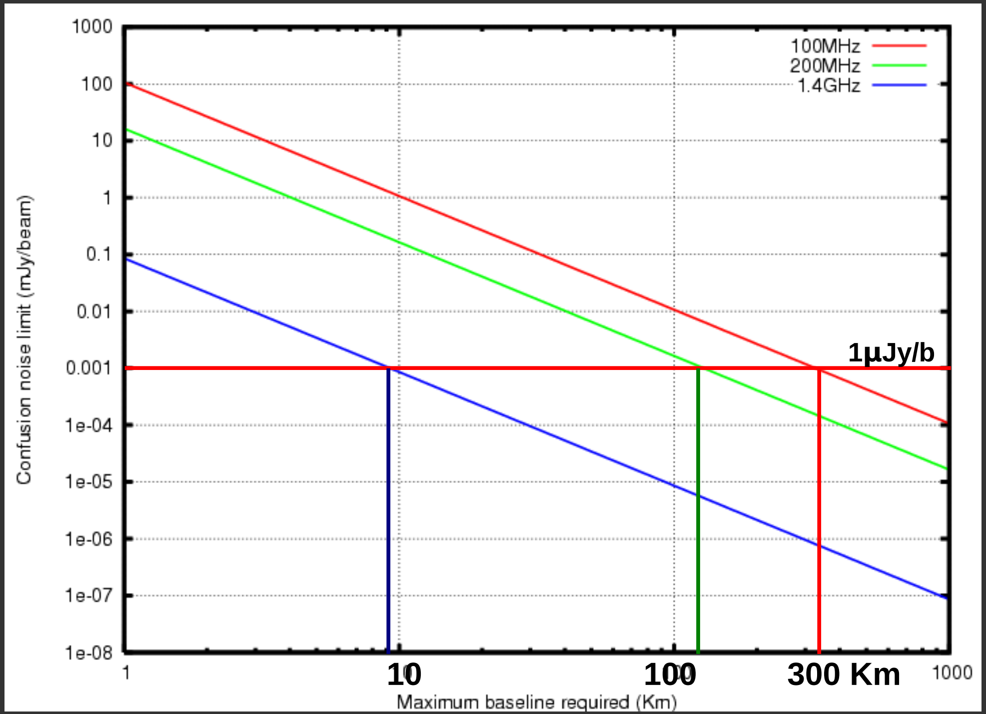




- PSF side-lobe at 1% level → deconvolve sources >100µJy for 1µJy/beam RMS
- $10^{3\text{-}4}$ sources per deg^2 >10µJy @ >=600 MHz
  - Source size distribution important at resolution < ~2”
- Implications for imaging
  1. Wide-field imaging
  2. HDR imaging: few X 100 mJy – 1 Jy source ~few sq. deg.
  3. Deconvolution of crowded fields (same problem as deconvolution of extended emission)

Sky at low frequencies: Confusion limit

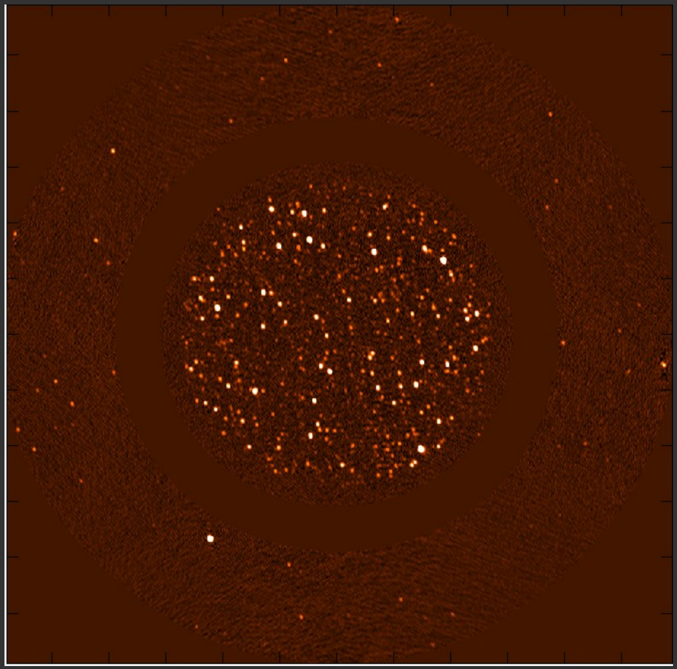


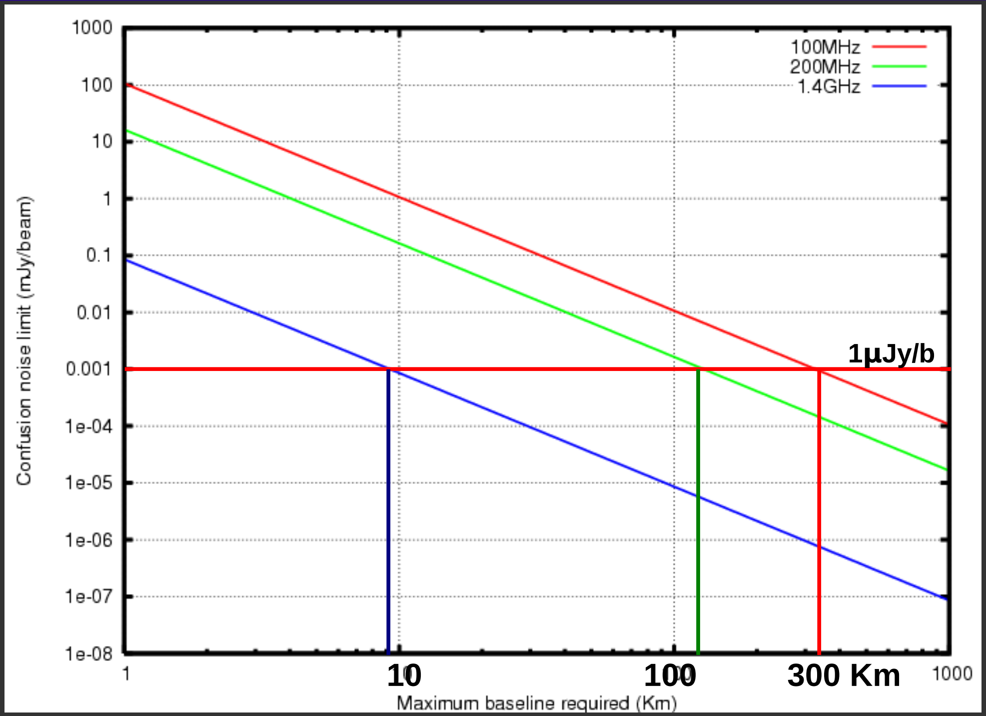


- $\sigma_{confusion} \propto (\nu^{-2.7}/B^2_{max})$: B_{max} ~100 Km at 200MHz for $\sigma_{confusion}$ ~ 1μJy/beam
- Implications for imaging
  1. Long baselines: B_{max} > 10 Km & DR > 10^4
  2. Wide-field effects: W-term, PB effects, ionospheric effects
  3. Larger data volume

Wide-field, wide-band, high resolution, HDR imaging using large data volumes is a natural consequence of low frequency and high sensitivity

Sky at low frequencies: Confusion limit





- $\sigma_{confusion} \propto (\nu^{-2.7}/B^2_{max})$: B_{max} ~100 Km at 200MHz for $\sigma_{confusion}$ ~ 1µJy/beam
- Implications fo
  1. Long
  2. Wide- effects
  3. Large

Point source sensitivity 1-sigma 12hr. Synthesis:		
VLA	EVLA	Factor
10uJy	1uJy	10
Data volume:		
~1GB	TB	10^4

Wide-field, wide-band, high resolution, HDR imaging using large data volumes is a natural consequence of low frequency and high sensitivity

Principles of interferometric CAL

$$V_{ij}(\nu) = G_{ij} W_{ij} \int P_{ij}(s, \nu, t)\; I(s, \nu)\; e^{\iota s.b_{ij}}\; ds$$

Data

DI Terms

DD Term Instrumental Ionospheric

Sky Also DD

Geometry Also DD

- Calibration model

$$V_{ij}^{Obs} = G_{ij} V_{ij}^{M} + \epsilon_{ij}$$

  - G_{ij} is separable into antenna-based quantities $G_{ij} = G_i \otimes G_j^T$
  - ϵ_{ij} is **not** separable into antenna-based quantities

Principles of interferometric CAL

- Calibration terms are necessarily separable into antenna-based quantities

- Closure quantities encodes this physics of the measurement in the calibration (and imaging!) process.

  - Triple Product is a Good Measurable of the phase due to *only* the sky emission

$$\arg\left(G_{ij} G_{jk} G_{ki}\right) = \arg\left(V^o\right) \textit{ for identical antennas}$$
$$= \textit{Pancharatanam/Geometric Phase of Phyiscal Optics}$$

(A&A, 375, 355-350, 2001)

- The *final* calibration products (one used to construct a calibrated image) must remain expressible as antenna-based quantities

Principles of interferometric CAL

- Measurement noise is a Gaussian random process (Central Limit Theorem). Therefore, χ^2 is the optimal estimator

- These fundamental principles, based on physics alone, lead to the antsol algorithm (solver engine behind DI SelfCal)

$$min \sum_{ij} |V_{ij}^{Obs} - G_{ij} V_{ij}^{M}|^2 \quad w.r.t.\ G_i$$

- Simple Steepest Descent iterative solver (not LM!)

$$\boxed{G_i^N = \frac{\sum_{j,i\neq j} X_{ij} G_j^{N-1} W_{ij}}{\sum_{j,i\neq j} G_i^{N-1} G_j^{N-1^T} W_{ij}} \quad where \quad X_{ij} = V_{ij}^{Obs} V_{ij}^{M^{-1}}}$$

- In use for the past many decades
  - See Cornwell MNRAS '81, Thompson&D'Addario Radio Sc. '82 for the first papers
  - See Bhatngar & Nityananda A&A 2001 for a modification to include “pseudo closure terms”
  - See Bhatnagar (PhD thesis) for a pedagogical derivation

Principles of Interferometric CAL-IM

- ME: $V_{ij}^{Cal} = A_{ij} I^M + A_{ij} \epsilon_{ij}$
- Imaging: Solve for the coefficients of $I^M = \sum_k c_k I_k$
- Fundamental principle(s) required to enforce separation of I from A_{ij}
  - For the Calibration-Imaging process to converge, and
  - The result to be provably consistent with the truth

1. Sky brightness is <u>not</u> expressible as antenna based quantities
2. Correlation-lengths in the image domain fundamentally also separates signal (sky brightness) from noise

Principles of Interferometric CAL-IM

- CAL-IM algorithms obeying these principles can solve ME of type

$$V_{ij} = G_{ij} F \left[c_0 I_0 + c_1 I_1 + \ldots \right] + \epsilon_{ij}$$

- Another model to describe the measurements

$$V_{ij} = F \left[G_{ij,0}(c_0 I_0) + G_{ij,1}(c_1 I_1) + \ldots \right] + \epsilon_{ij}$$

  - No constraints on solvers mixing terms with incompatible physics
  - Formulation inconsistent with the basic physics of the measurement process
    - Measurements are corrupted by strictly antenna-based quantities
  - i.e., hard to imagine a solver which *also* obeys the physics of observation

Principles of Interferometric DD CAL-IM

- Therefore, we include *all* calibration terms, ***even if DD***, in the ME as
  - Antenna-based term(s)
  - Fundamentally separate from parameters that model the sky brightness

$$V_{ij} = G_{ij} F \; M_{ij} \left[c_0 I_0 + c_1 I_1 + \ldots \right] + \epsilon_{ij}$$

1. Solvers generalized for DD calibration ***without*** violating Closure Principle (e.g. Pointing SelfCal)
2. Deconvolution algorithms designed to ***not*** mix instrumental/atmospheric terms and sky brightness model (e.g. RMSynth, MS-MFS)
3. Combined DD-correcting image deconvolution algorithms obey the fundamental principles: designed which converge to results that are provably consistent with the truth (e.g. Projection+MS-MFS)

Equivalence between DI and DD terms

- ME with DI terms $\quad V_{ij}^{Obs} = \left[J_i \otimes J_j^*\right].\left[V_{ij}^{o}\right] = \left[M_{ij}^{DI}\right].\left[V_{ij}^{o}\right]$
- ME with DD terms $\quad V_{ij}^{Obs} = \left[E_i \circledast E_j^*\right].\left[V_{ij}^{o}\right] = \left[M_{ij}^{DD}\right] * \left[V_{ij}^{o}\right]$

- DI Mueller: Outer product of two antenna-based terms
- DD Mueller: Outer convolution of two antenna-based terms (ApJ, 2013)

- DI operator: Matrix multiplication
- DD operator: Matrix multiplication algebra with convolutions

$$\begin{bmatrix} V_{pp}^{Obs} \\ V_{pq}^{Obs} \\ V_{qp}^{Obs} \\ V_{qq}^{Obs} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} . \begin{bmatrix} V_{pp}^{o} \\ V_{pq}^{o} \\ V_{qp}^{o} \\ V_{qq}^{0} \end{bmatrix}$$

Diagonal: “pure” poln. products
Off-diagonal: Include poln. leakage

Equivalence between DI/DD corrections

- Full-pol DI correction

$$V_{ij}^{Corr} = \left[M_{ij}^{DI^{-1}} \right] . \left[V_{ij}^{Obs} \right] = \frac{adj\left(M_{ij}^{DI} \right)}{det\left(M_{ij}^{DI} \right)} . \left[V_{ij}^{Obs} \right]$$

$$\text{Equivalent Complex math.: } G_i^{-1} = \frac{G^*}{|G|^2}$$

- A-Projection (and related) algorithms: Full-pol DD correction
  - Generalization of DI correction

$$V_{ij}^{Corr} = \left[M_{ij}^{DD^{-1}} \right] * \left[V_{ij}^{Obs} \right] = \frac{adj\left(M_{ij}^{DD} \right)}{det\left(M_{ij}^{DD} \right)} * \left[V_{ij}^{Obs} \right]$$

- Projection algorithms are a DD generalization of the DI calibration

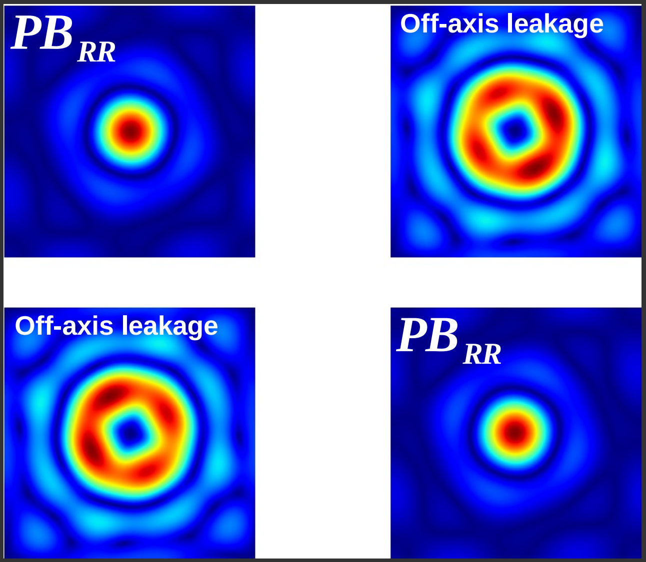
Equivalence between DI/DD corrections

- DI Jones Matrix: Each term is a complex gain (a number)
  - Receiver gains and polarization leakages

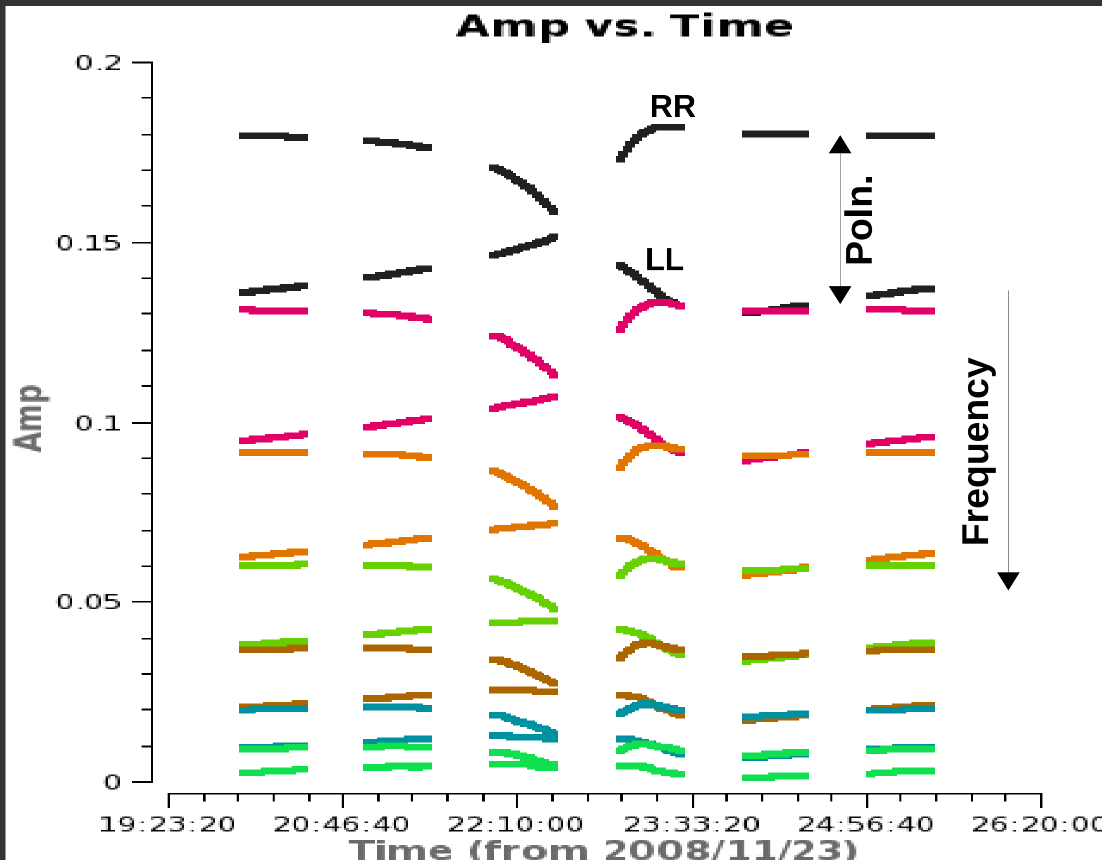
$$J_i = \begin{bmatrix} G_p & -D_{p\to q} \\ D_{q\to p} & G_q \end{bmatrix}$$

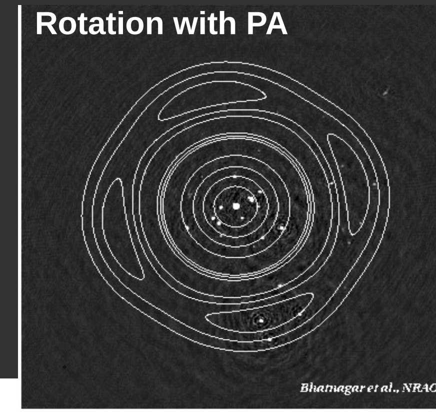
- DD Jones Matrix: Each term is a complex gain pattern (a 2D function)
  - Antenna off-axis gains and polarization leakages

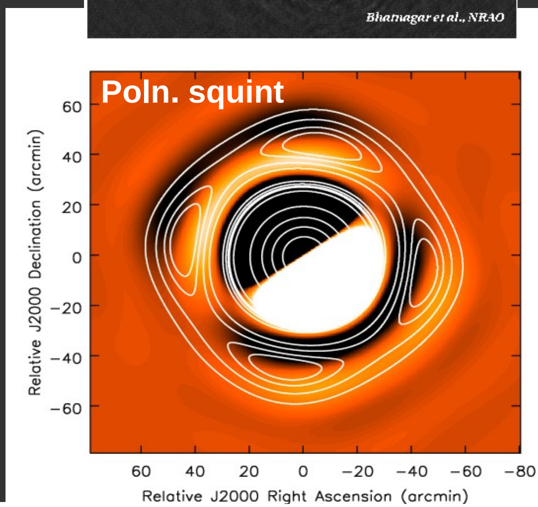
$E_i =$

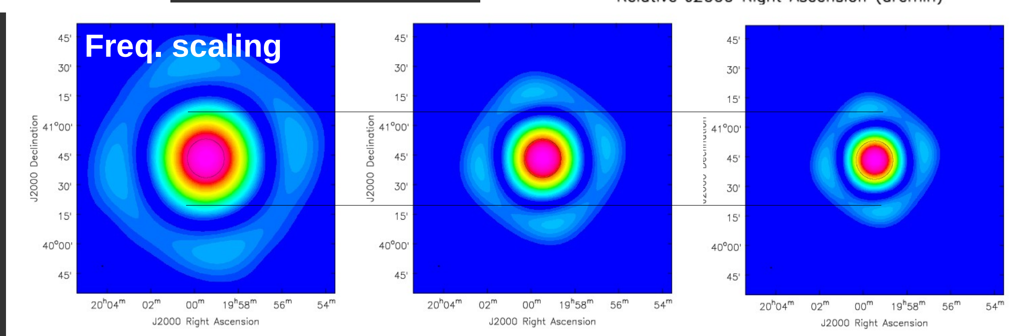


All PB effects together









Projection algorithms=DD corrections

- DD corrections cannot be done independent of imaging
  - But the imaging algorithms must fundamentally (explicitly) separate them from sky-brightness parameters

- Projection algorithms project-out DD effects as part of the transform to image domain

$$V_{ij}^{Corr} = \frac{adj(M_{ij}^{DD})}{det(M_{ij}^{DD})} * [V_{ij}^{Obs}]$$

During Imaging

$$I^{Corr} = \frac{F \sum_{ij} [adj(M_{ij}^{T})] * [V_{ij}^{Obs}]}{F \sum_{ij} det(M_{ij})}$$

Image plane normalization

DD Corrections: Projection Algorithms

- Construct an operator X which projects-out the undesirable effects of A?

$$X_{ij}\ V_{ij}^{DI-Cal} = X_{ij}\ A_{ij}\ V^{True}$$

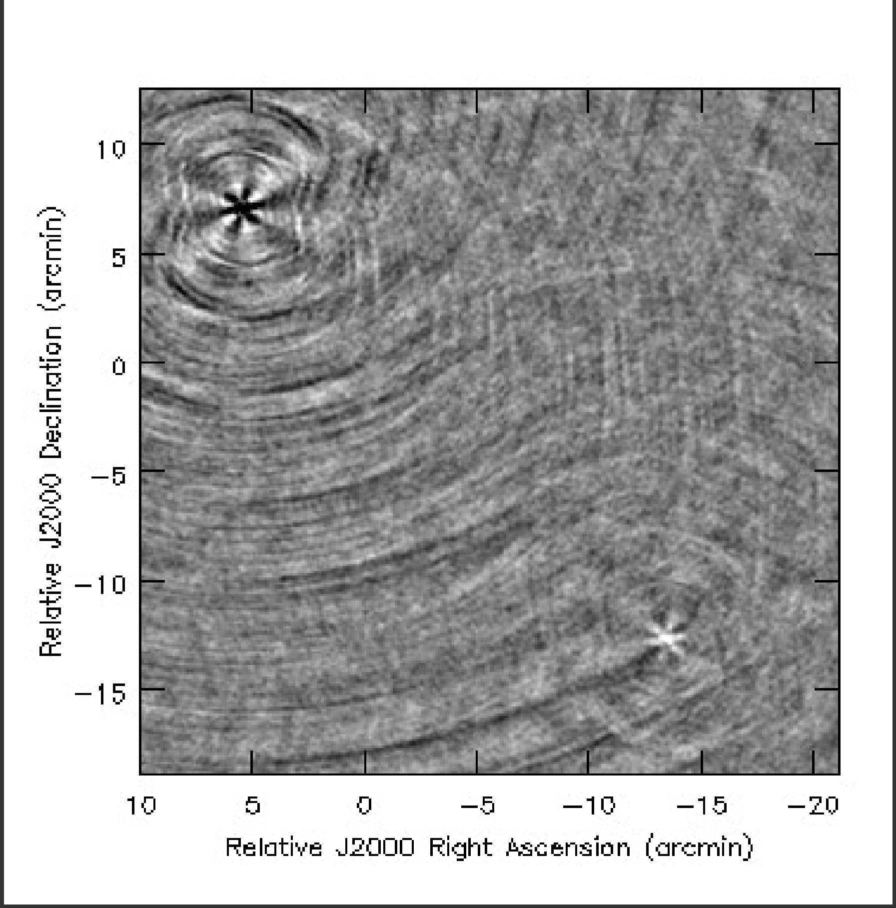
$$such\ that\ X_{ij}\ A_{ij} = \mathbf{1}$$

- W-Projection: X is the conjugate of the w-term

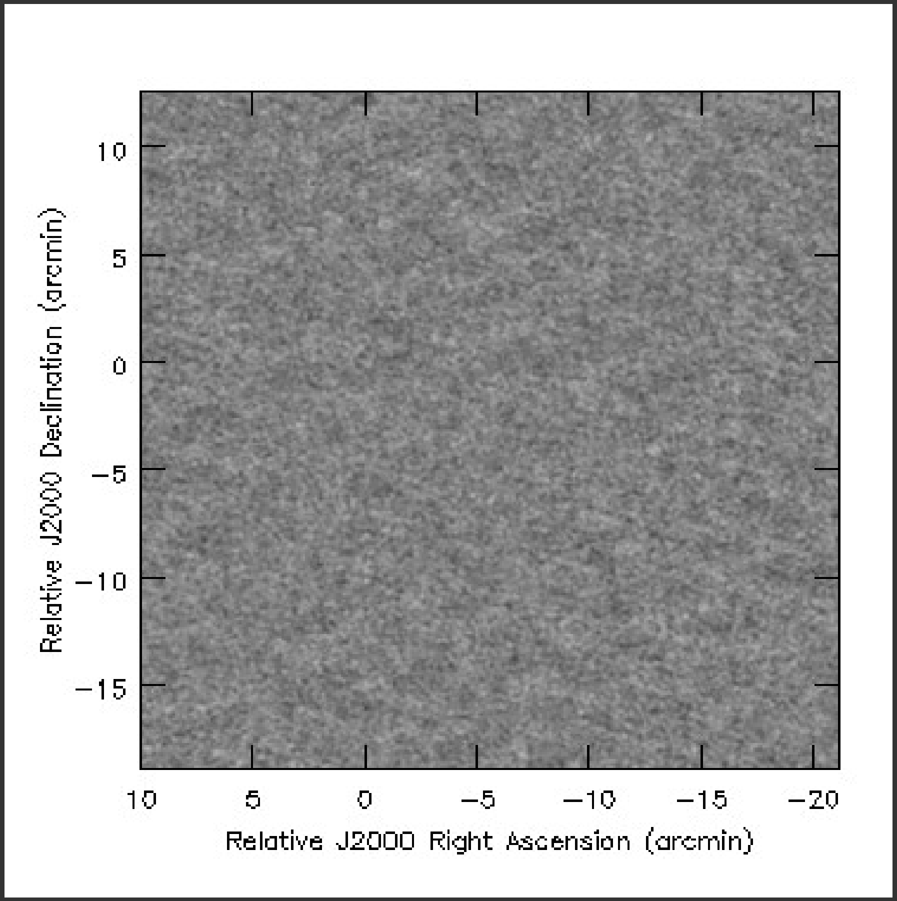
- A-Projection: X is the polarization conjugate of the PB term
  - Does not project-out WB effects of the PB (A&A, 2008)

PB Polarization Effects

Stokes-V Images ("Narrow band")



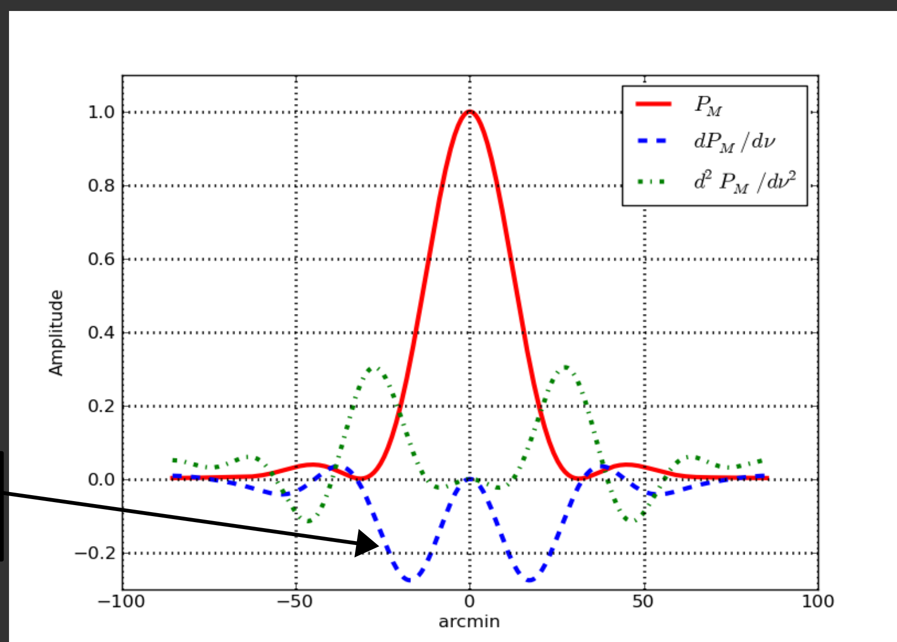
A-Projection



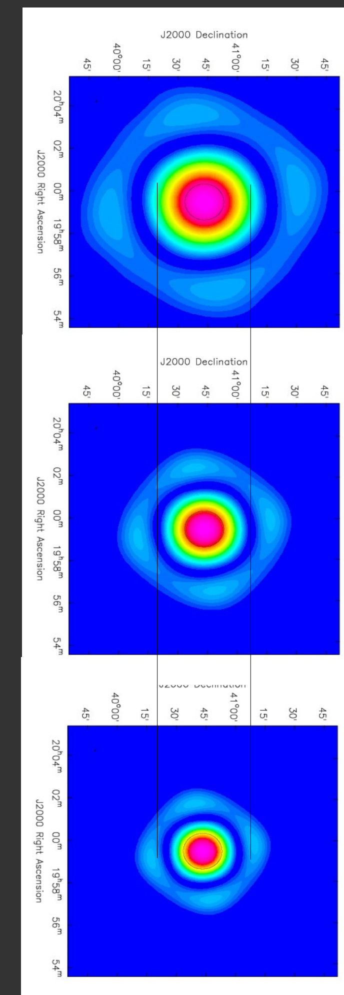
Instrumental frequency dependence

- Continuum imaging $I^{continuum} = \int P_{ij}(\mathbf{s},\nu,t)\;\; I(\mathbf{s},\nu)\;\; d\nu$

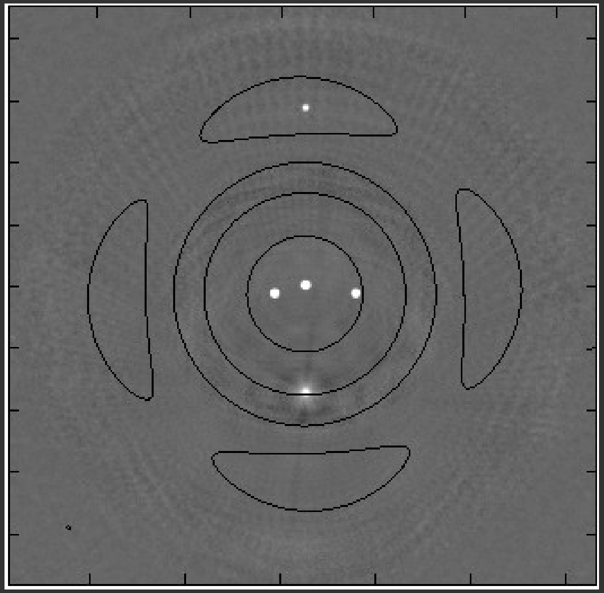
- Antenna PB ($The\;\; P_{ij}(\mathbf{s},\nu,t)$)
  - Frequency dependence

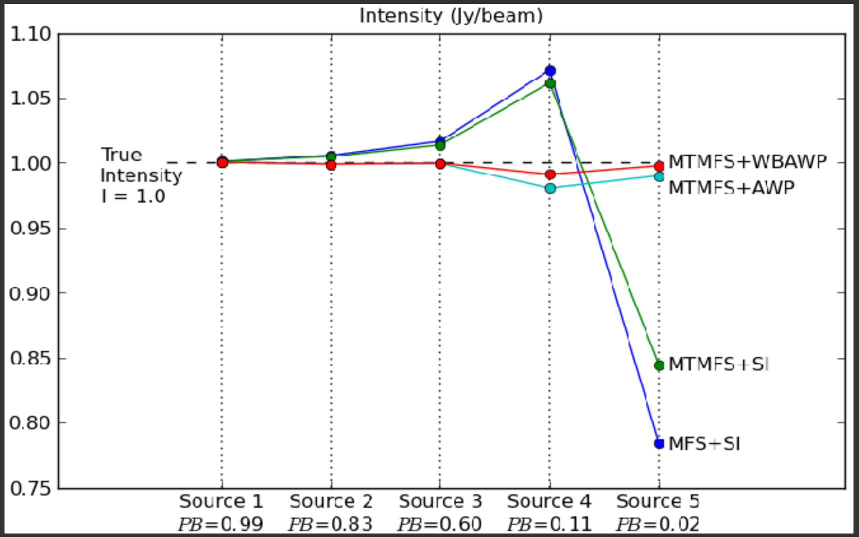


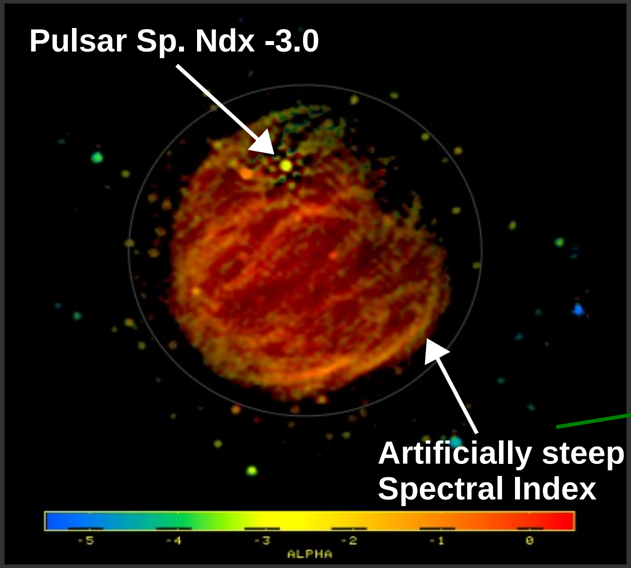
PB Freq. dependence (blue curve)

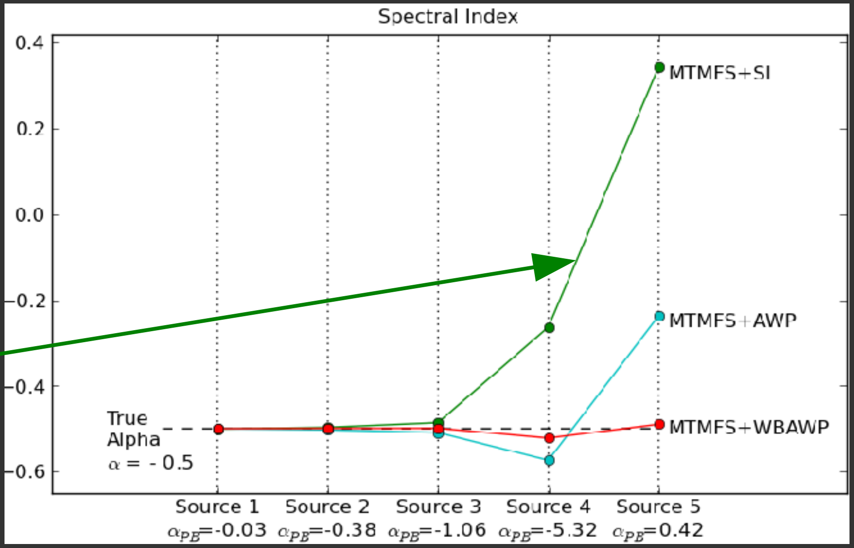


Instrumental frequency dependence



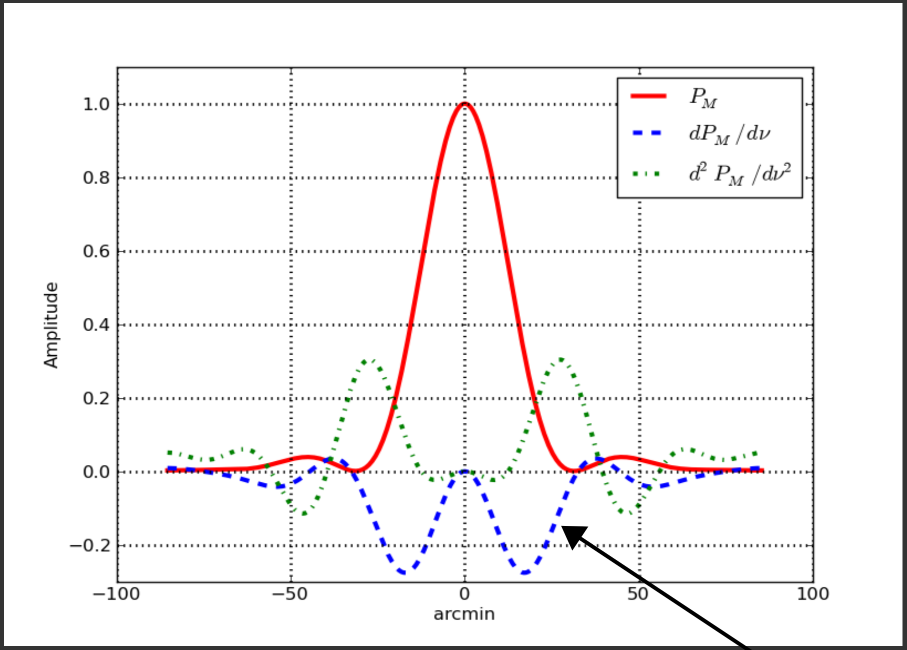


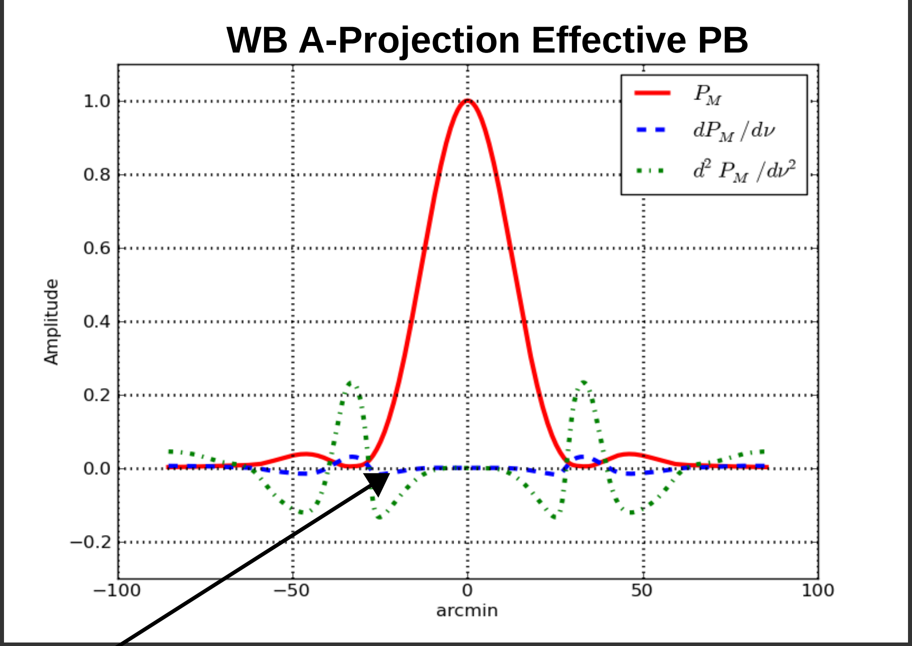




Wide-Band AW-Projection

- Construct X such that it is *also* a frequency-conjugate for PB
- Correct for PB effects + W-term
  - Polarization: Squint + in-beam polarization
  - Time variability: Rotation with Parallactic Angle



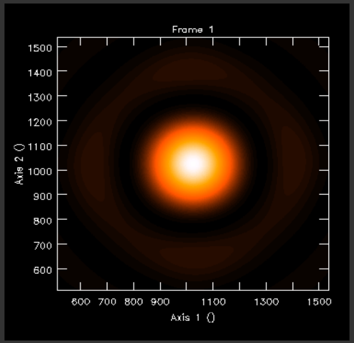


PB Frequency dependence (blue curve)

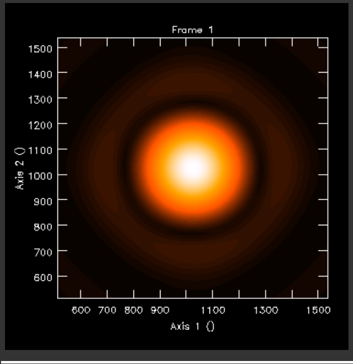
Wide-Band AW-Projection

$$A_{ij}(\nu_*) \quad where\ \nu_* = \sqrt{2\nu_{ref}^2 - \nu^2}$$

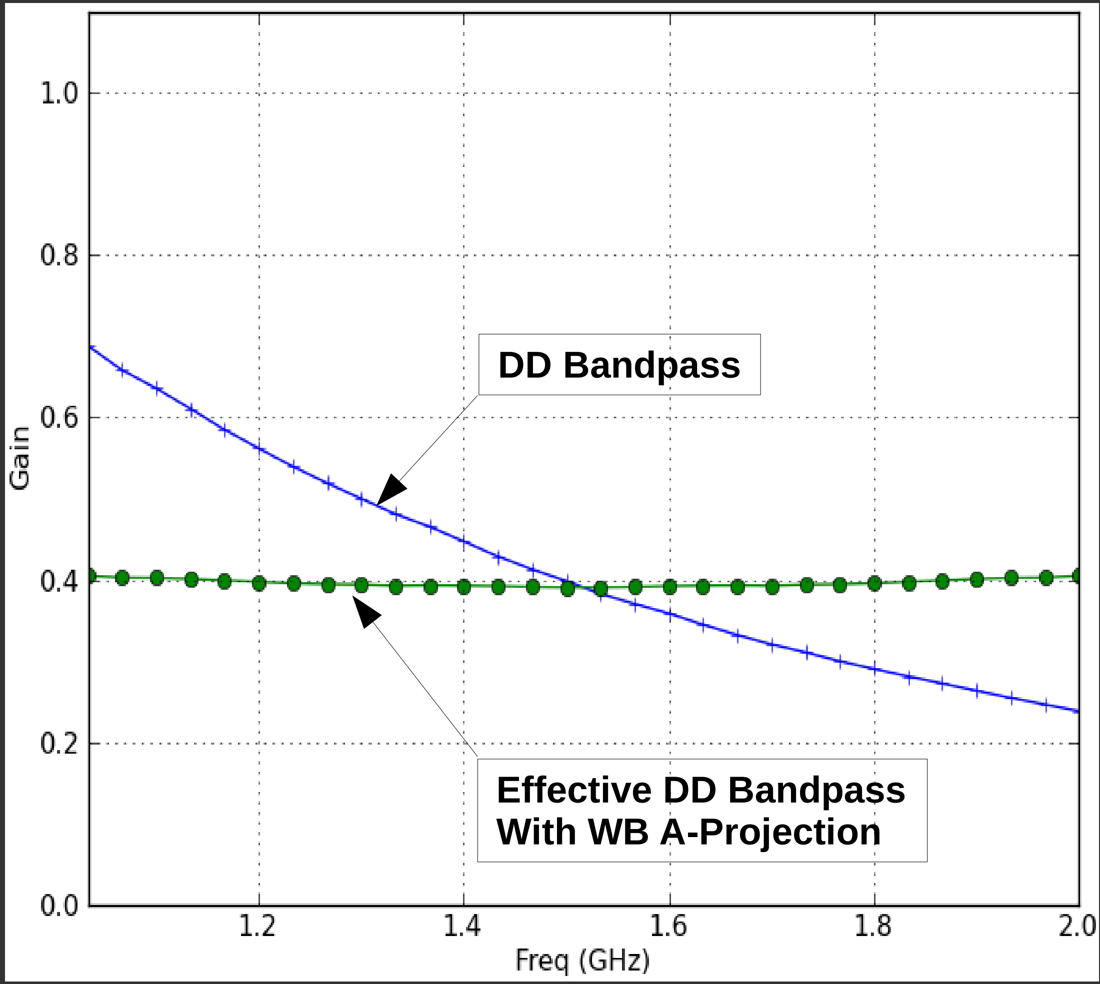
Frequency along the animation axis



$PB(\nu)$



$$\sqrt{PB(\nu)\, PB(\nu_*)}$$



WB AW-Projection + MT-MFS

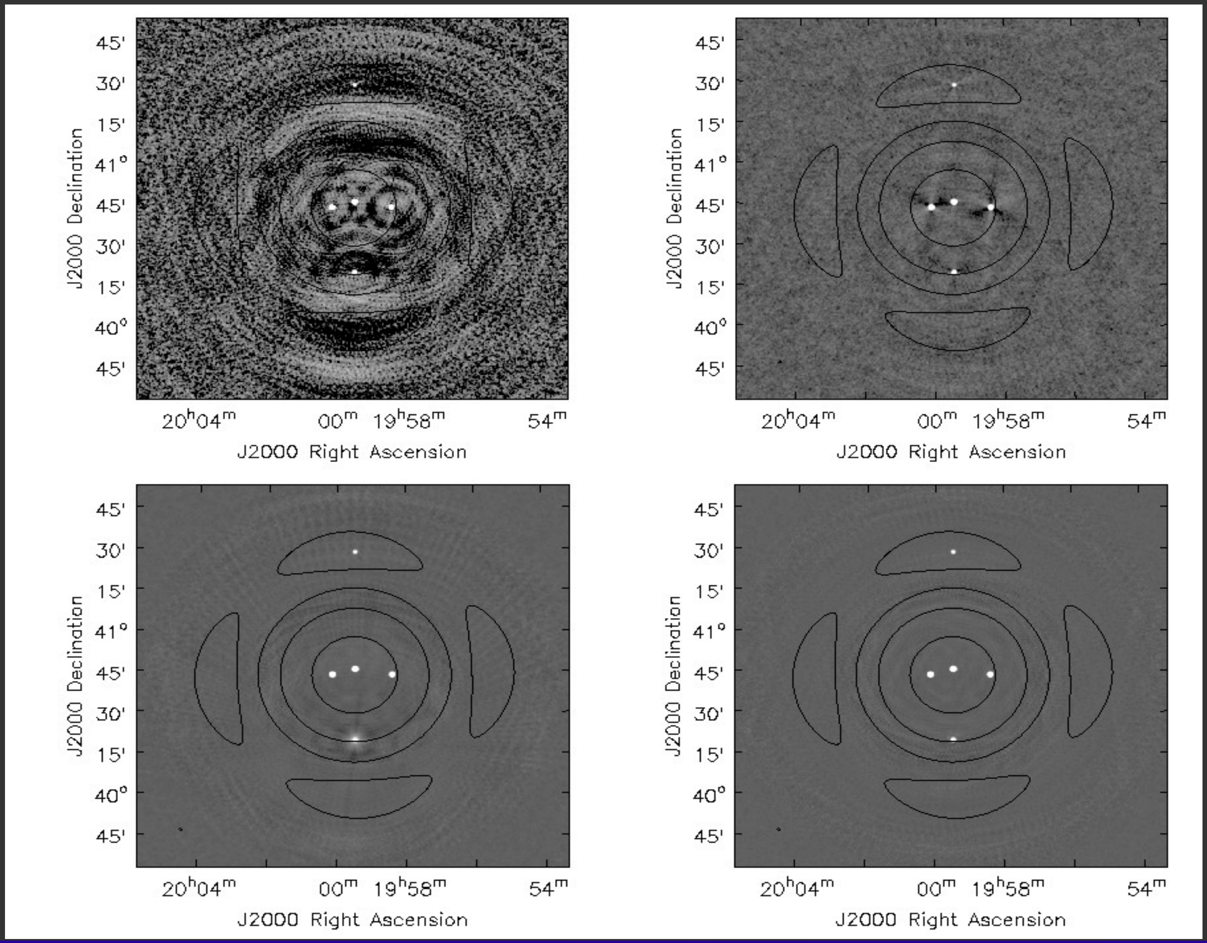
- Simultaneously account for the PB effects and frequency dependence of the sky
  Separation of instrumental calibration and sky brightness terms:
  - PB effects corrected by WB A-Projection
  - PB-corrected image used in MT-MFS for model the frequency dependence of the sky brightness

MFS+SI

MT-MFS+SI

MT-MFS+ A-Projection

MT-MFS+ WB A-Projection

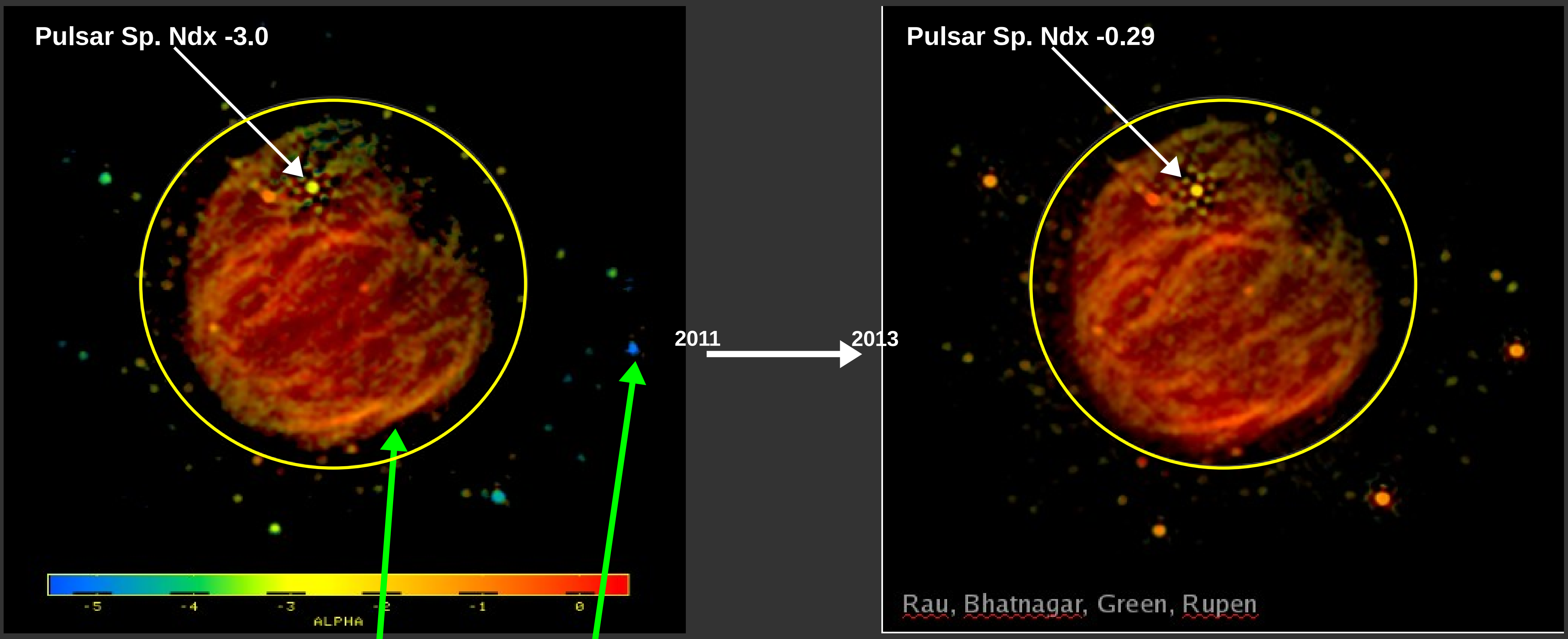


Ap.J., 2013

Wide-Band AW-Projection + MT-MFS

- **Results verifiable consistent with the truth**

Intensity weight Spectral Index Map
Wide-field Spectral Index maps comes out in the wash correctly



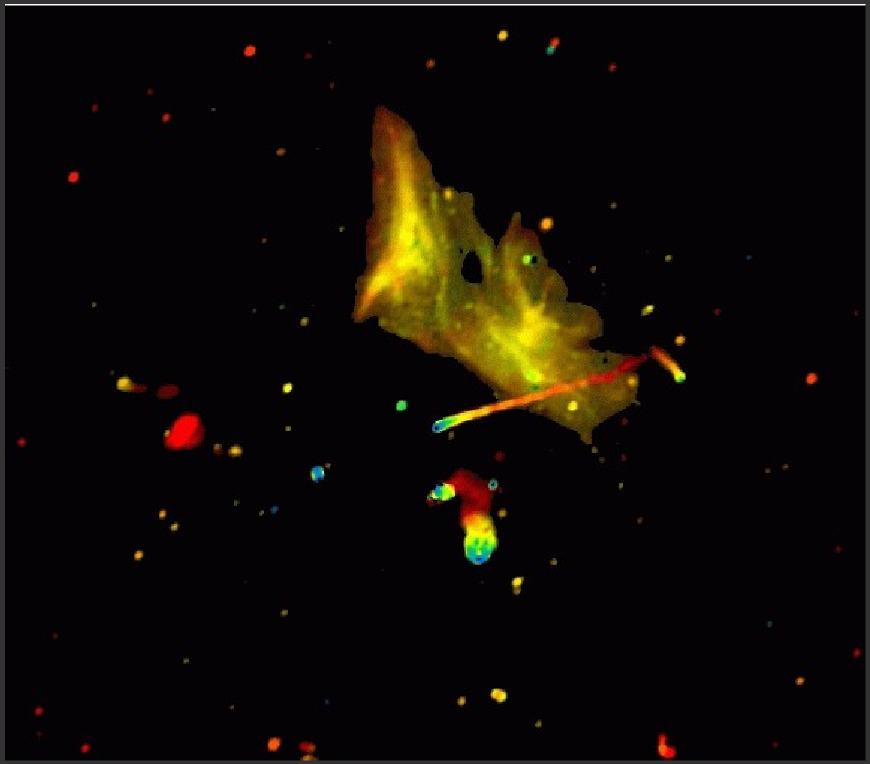
Artificially steep (due to PB)

ApJ, 2011, 2013

The Conclusion

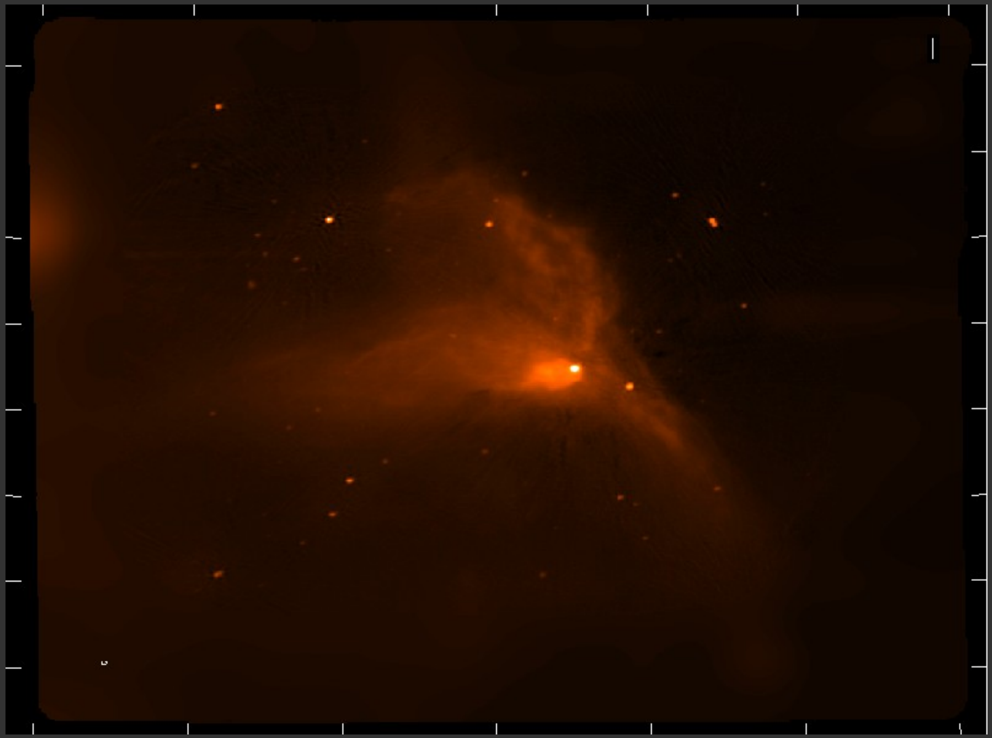
- Wide-band, wide-field, high dynamic range imaging is a natural consequence of high sensitivity imaging at low radio frequencies
  - Large data and large images is also a natural consequence
- Fundamental principles behind interferometric imaging based on the physics of the measurement process
  - Algorithms must show that they obey closure relations
  - Algorithms must obey fundamental separation of signal, noise and calibration parameters
- Projection algorithms are true DD generalization of DI corrections
  - NB A-Projection is the DD equivalent of DI gain correction
  - WB A-Projection is the DD equivalent of DI bandpass correction
  - Possible to invent DD solvers which can be shown to obey physics principles
- Single pointing wide-band wide-field imaging
  - WB A-Projection + MT-MFS: Verified with simulations and real data
  - Further work in progress to test other terms

Imaging with the EVLA @ L-Band



Intensity-weighted Sp. Ndx. Map

Single pointing, narrow field, wide-band image
(Owen, Rau)



Wide-band mosaic+Single Dish
Working on Stokes-I + Sp.Ndx. mapping