Allsky, direction-dependent polarization calibration
&
Lessons for future fix mount telescopes (SKA-lo, SKA-mid)

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Can you use background sky to pol-calibrate?

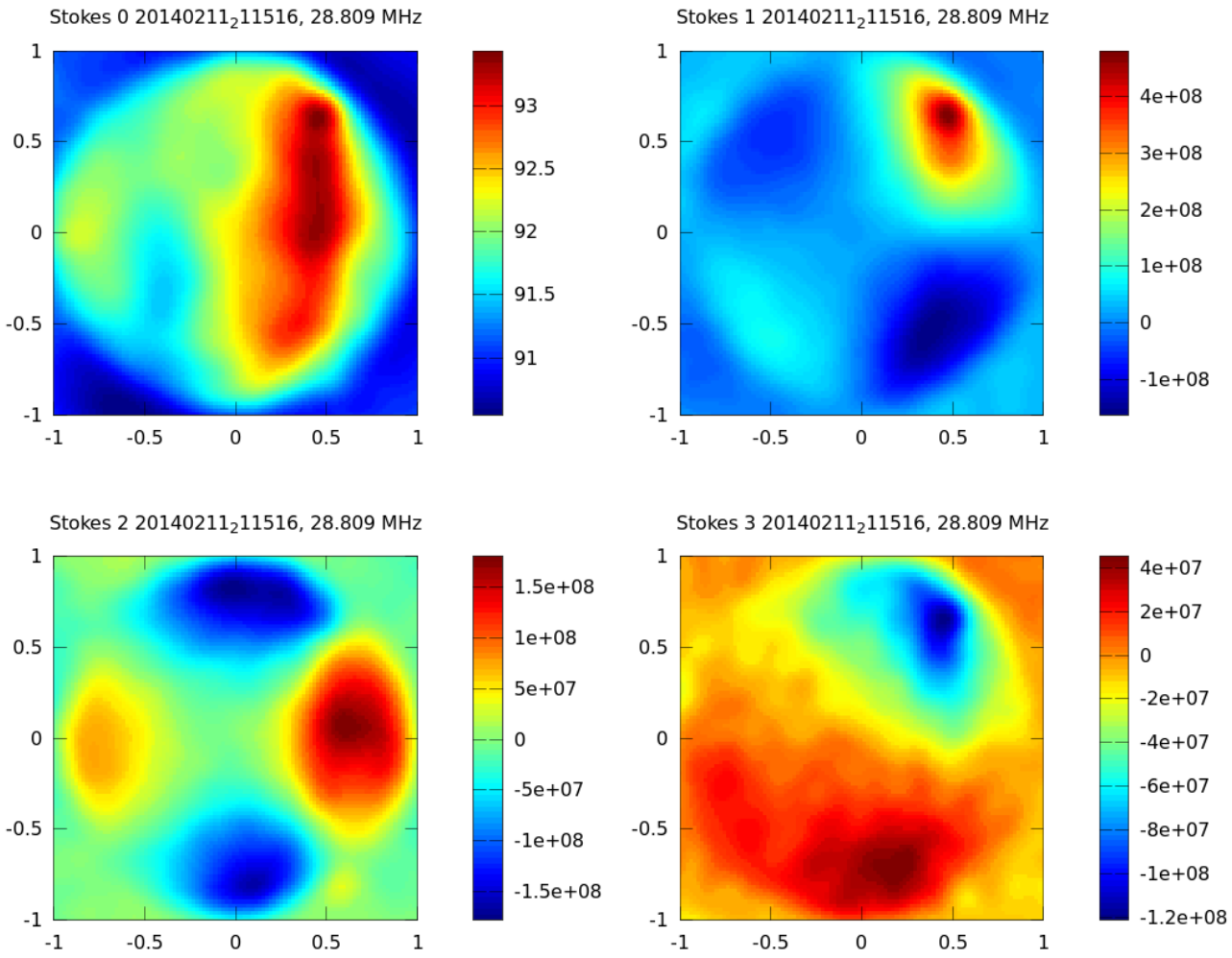
- Calibration ideally uses ”Calibrators”
  - Isolated
  - Known position
  - High flux
  - Point sources
- But these conditions can be difficult to meet
- Background sky, on the hand, is ubiquitous
- Can we use it for quick & dirty, cheap polarization calibration?

Single LOFAR LBA station

- Data from Swedish LBA
- Freq. 10-80 MHz
- Sky noise is expected to dominate system temperature
- 96 dual polarized dipole antennas
- Data product e.g. visibilities/covariance matrix of antenna voltages

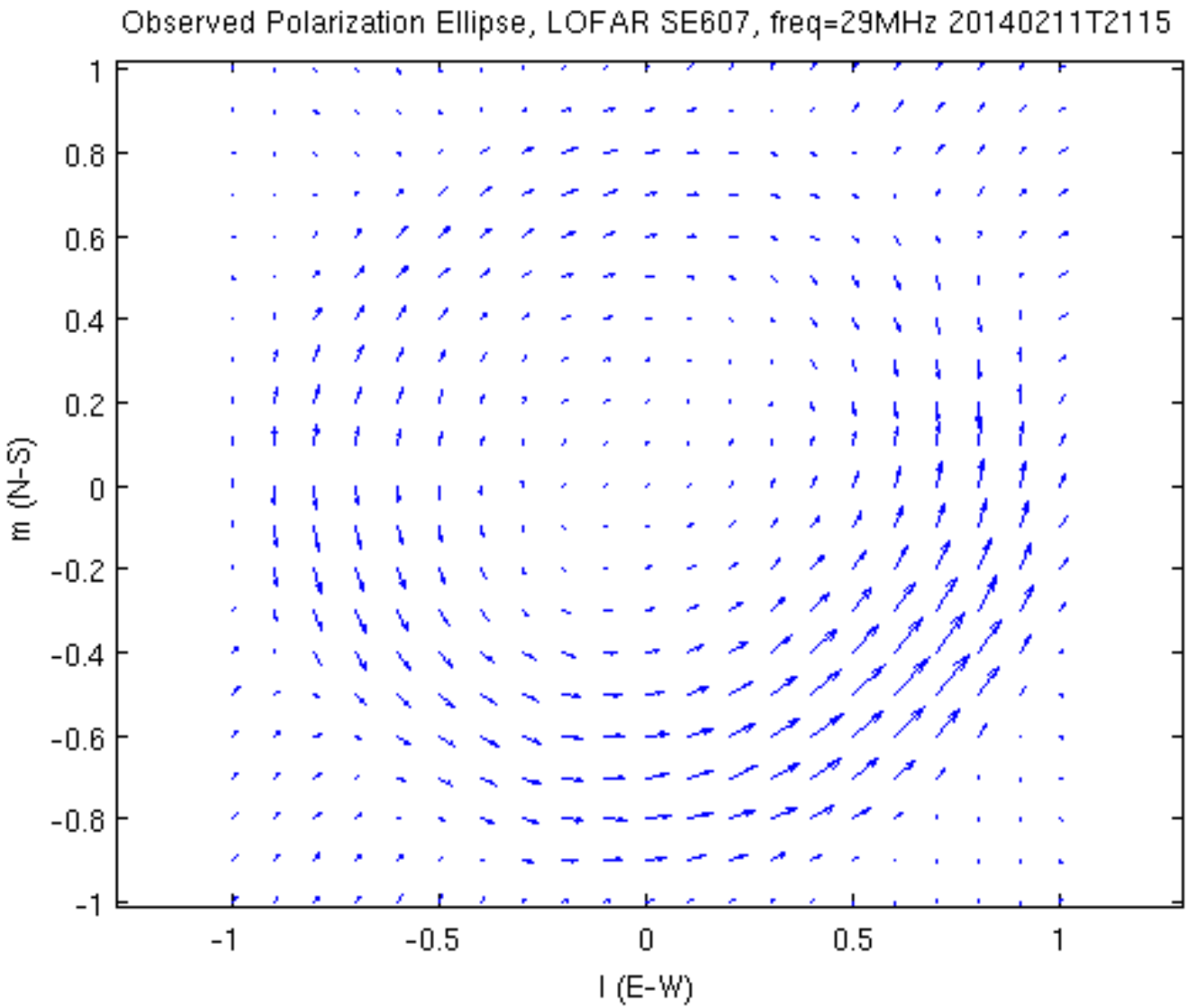
Dirty allsky polarization images

- Based on 1 sec raw single station covariances
- Allsky Stokes I,Q,U,V



Polarization bias

- Same data shown as polarization ellipses



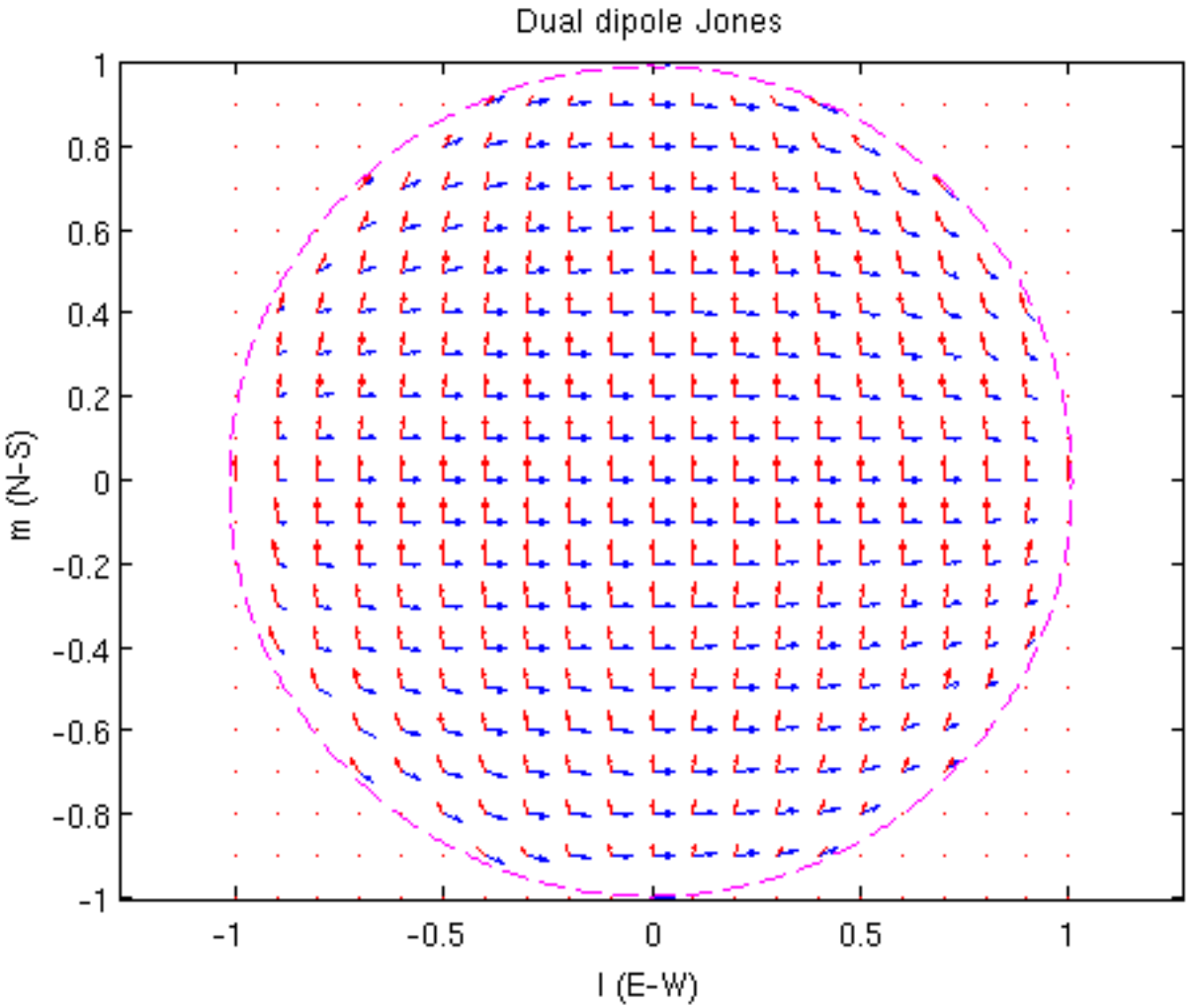
Model explanation

- At low frequencies, sky-dominated noise
- Most of this noise is unpolarized on sky
  - But after passing through antennas becomes polarized
  - Thus image is instrumentally polarized via ”filtering” of unpolarized background sources

$$\mathbf{B}_{sky} = B_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{B}_{obs} = \mathbf{J}\mathbf{B}_{sky}\mathbf{J}^{\mathbf{H}} = B_0 \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}^{H}$$

LBA allsky Jones



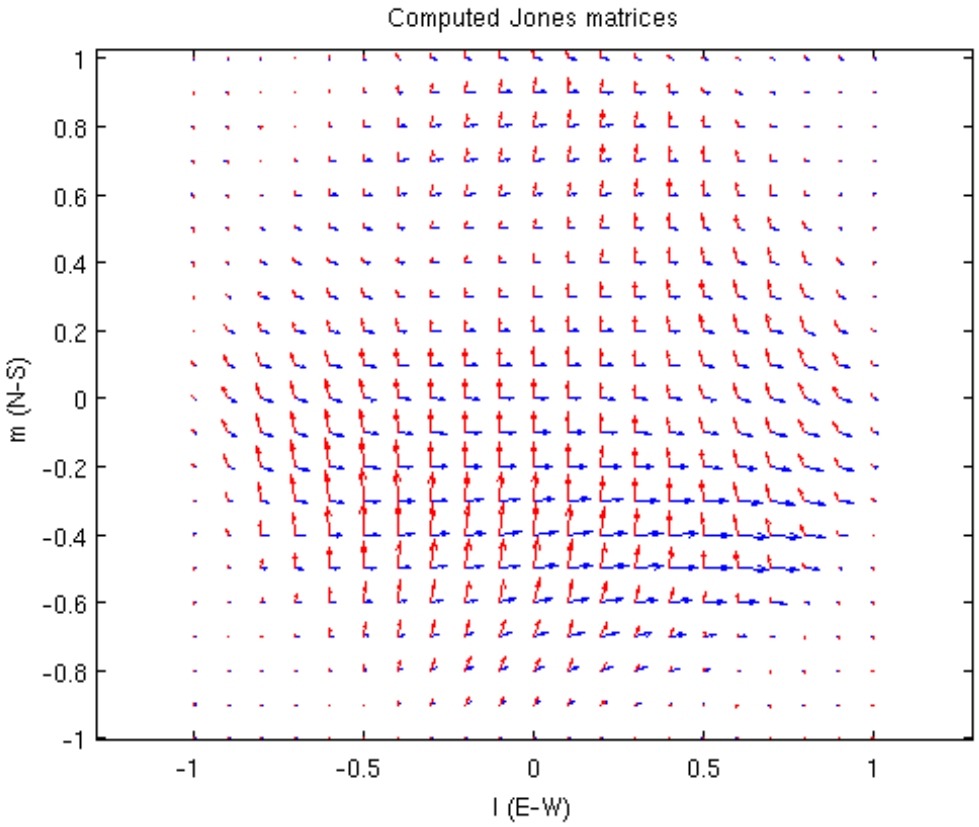
- Fixed horizontal dipoles do NOT see the vertical component

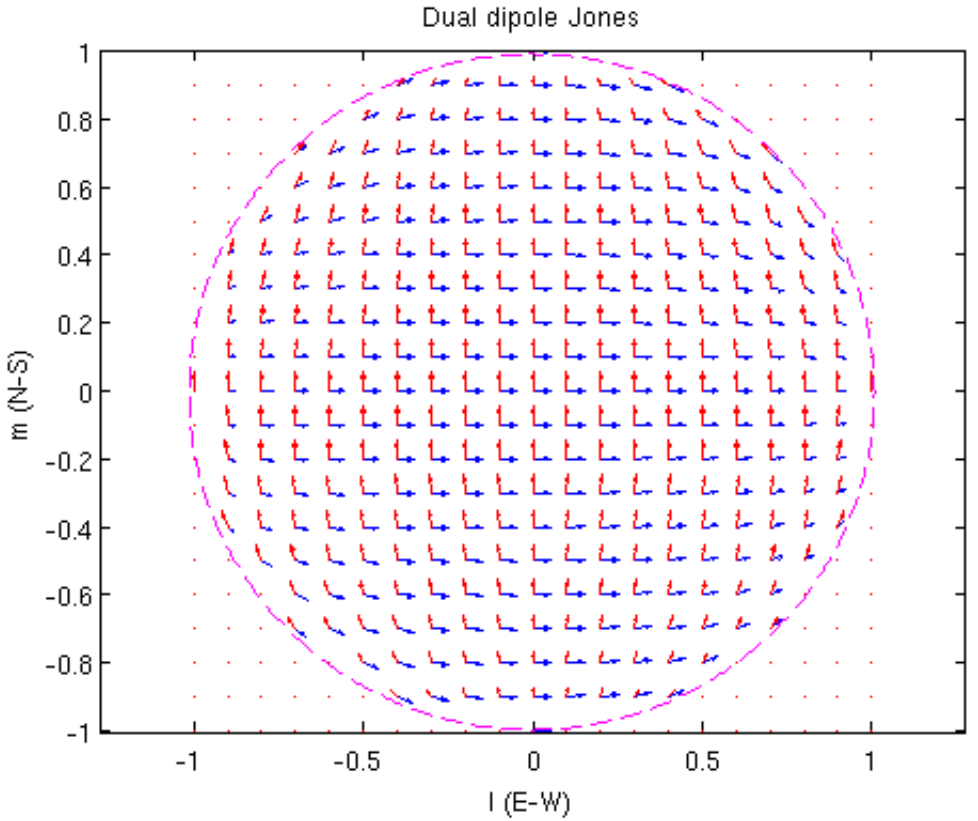
How to get a direction dependent Jones estimate

- Assume that observed image brightness matrix is produced through instrumental filtering of unpolarized radiation
- Then square root image brightness matrix is proportional (scalar & unitary factor) to instrumental/total Jones matrix
  - Not unique of course but doesn't matter

$$\mathbf{B}_{obs} = \mathbf{J}\mathbf{B}_{sky}\mathbf{J}^{\mathbf{H}} = B_0 \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}^{H}$$

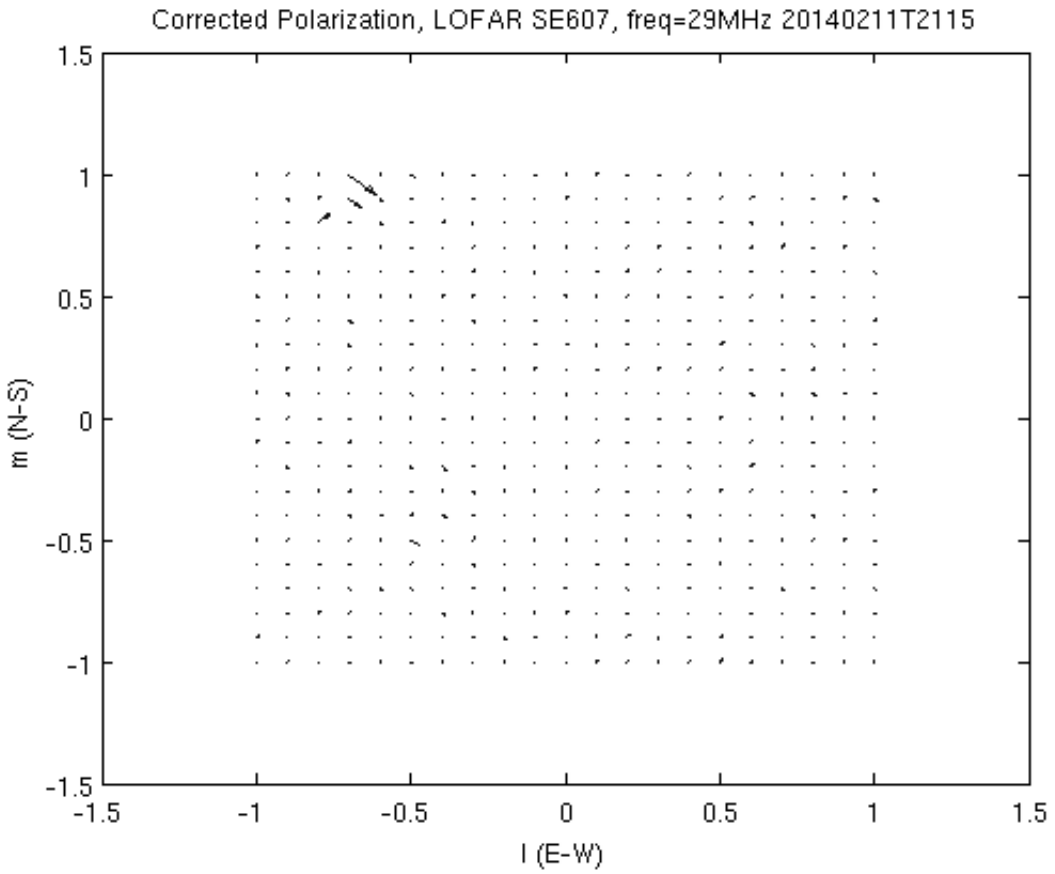
Get to the root of Jones: results vs theory





Raybans for Lofar station

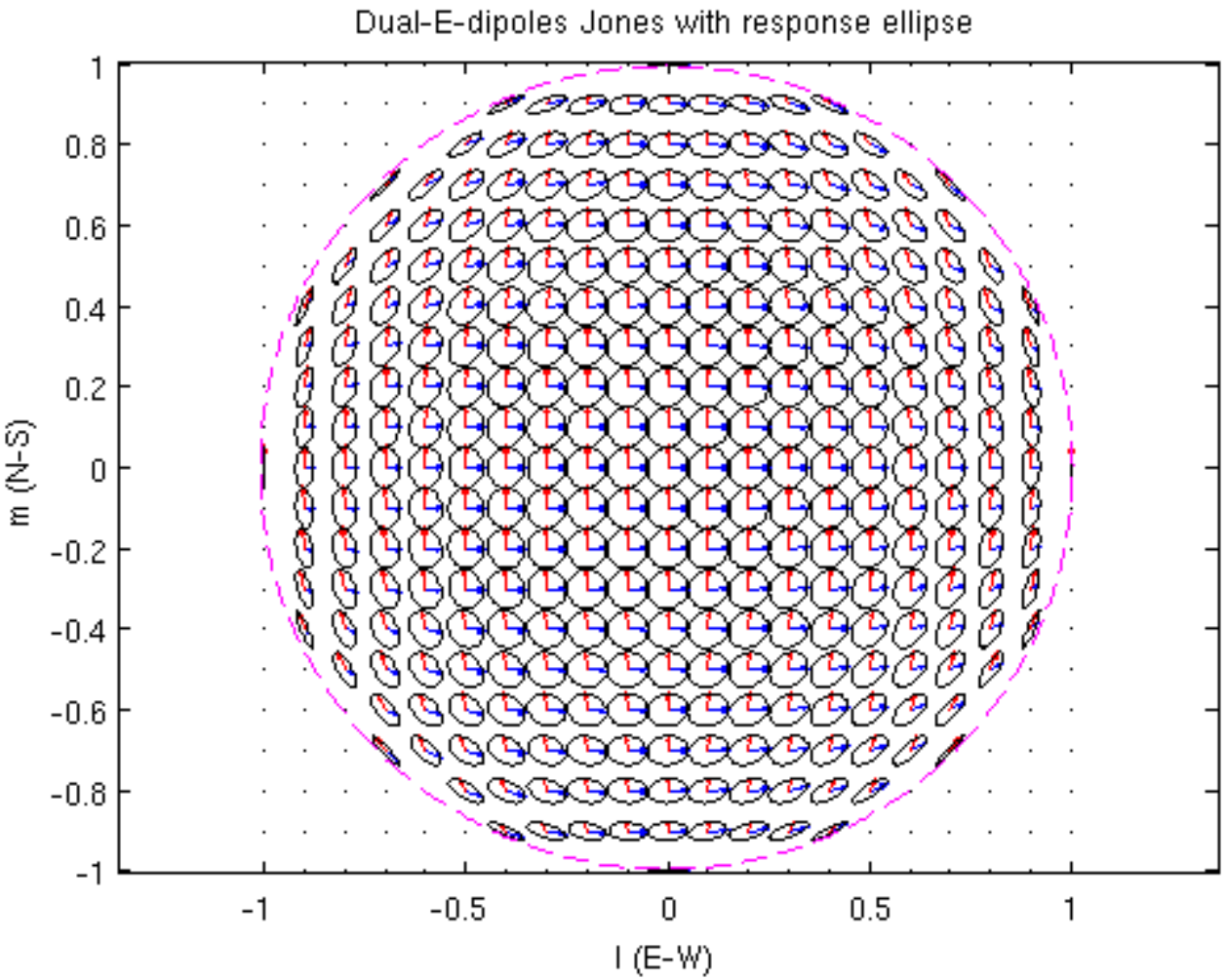
- After applying the estimated Jones to the data we get



Limits of direction dependent calibration

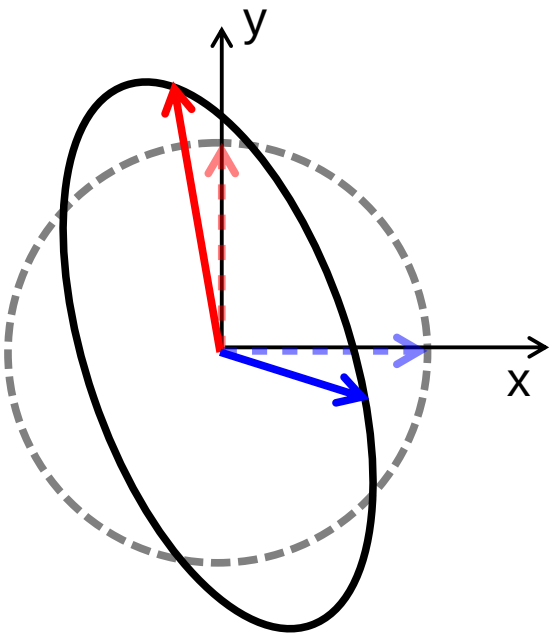
- Shown one method of polarimetrically calibrating images
  - What are the fundamental limits? i.e. what are limits even if
    - all model assumptions are true
    - estimates of Jones are correct
- Depends on conditioning of Jones matrix
  - Degenerate/singular Jones implies not even total intensity can be properly calibrated
- How can one determine Jones matrix condition?

Pictorial assessment of Polarimetry Quality of Image/Beam



Polarimetry Quality of Beam: *"Jones ellipses"* and *"IXR"*

- Note that this ellipse is NOT the polarization ellipse
- It is a visual representation of the Jones matrix ellipse (for real valued matrix)
- Normally this ellipse is assessed in terms of Cross-polarization ratio (XPR) in the reference frame
- Condition of Jones is the axial ratio of ellipse
- For compatibility with mathematicians one would use this number to assess limits of calibratability
  - But most common parameter is XPR
- But equivalently one can use a XPR in an invariant reference frame
- This is instrinsic cross-polarization ratio (IXR) Carozzi, Woan IEEE-t.a.p. (2011)



Polarimetry: Traditional Figures of Merit

- Cross-polarization ratio (XPR)
  - Given in some nominal coordinate system
  - 4 unique possibilities
  - is essentially equivalent to D-terms in VLBI
- Axial ratio
  - Only considers one antenna at a time
- Polarization efficiency
  - Restricted to Aperture antennas
  - Does not give directional information
- Instrumental polarization

$$\begin{pmatrix} f_p \\ f_q \end{pmatrix} = \begin{pmatrix} J_{px} & J_{py} \\ J_{qx} & J_{qy} \end{pmatrix} \begin{pmatrix} e_x \\ e_y \end{pmatrix}$$

$$\mathbf{f} = \mathbf{Je}$$

$$XPI_p = | J_{px}/J_{py} |^2$$

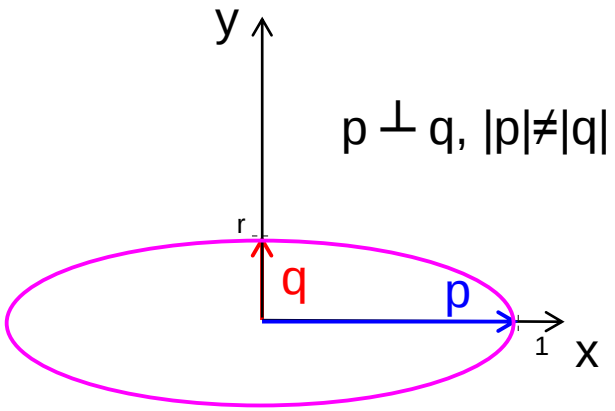
$$XPI_q = | J_{qy}/J_{qx} |^2$$

$$XPD_x = | J_{px}/J_{qx} |^2$$

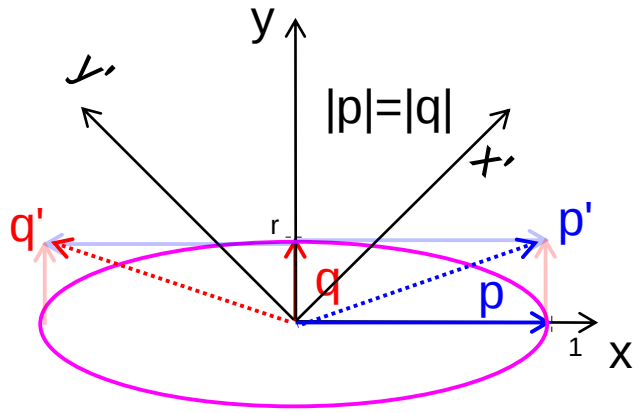
$$XPD_y = | J_{qy}/J_{py} |^2$$

IXR: a rehashed conditioning number or alt. an invariant XPR

$$J = \begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix}$$



$$J' = \begin{pmatrix} 1+r & r-1 \\ r-1 & 1+r \end{pmatrix} = \begin{pmatrix} r & r \\ r & r \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$



- IXR incorporates differential channel gains, which fixed coord. sys. XPR does NOT despite diff gains will induce polarimetric errors
- Thus you could say intrinsic cross-polarization is a misnomer but this is because the concept of cross-polarization is an arbitrariness of coordinate system
- Anyways: IXR conveys polarimeter error better than fixed coordinate system cross-polarization ratio

Uncalibrated (raw) Polarimetric Error

- Error in uncalibrated polarimetry is given by IXR (noise-free case)

$$\frac{\Delta|\,|\,\mathbf{f}\,|\,|}{|\,|\,\mathbf{f}\,|\,|} \leq \frac{1}{\sqrt{IXR}}$$

- Interpretation is simple
  - IXR=10 dB gives at most 10% error in power estimate (uncalibrated)

$$\frac{1}{\sqrt{IXR}} = \frac{g_{max} - g_{min}}{g_{max} + g_{min}} = \frac{g_{max}/g_{min} - 1}{g_{max}/g_{min} + 1} = \frac{cond(\mathbf{J}) - 1}{cond(\mathbf{J}) + 1}$$

Calibrated Polarimetric Error

- Polarimetric calibration is inverting Jones matrix to raw voltages

$$\mathbf{e} = \mathbf{J}^{-1}\mathbf{f}$$

- Error in inversion is given by condition number of Jones matrix

$$\mathbf{f} + \mathbf{\Delta f} = (\mathbf{J} + \mathbf{\Delta J})(\mathbf{e} + \mathbf{\Delta e})$$

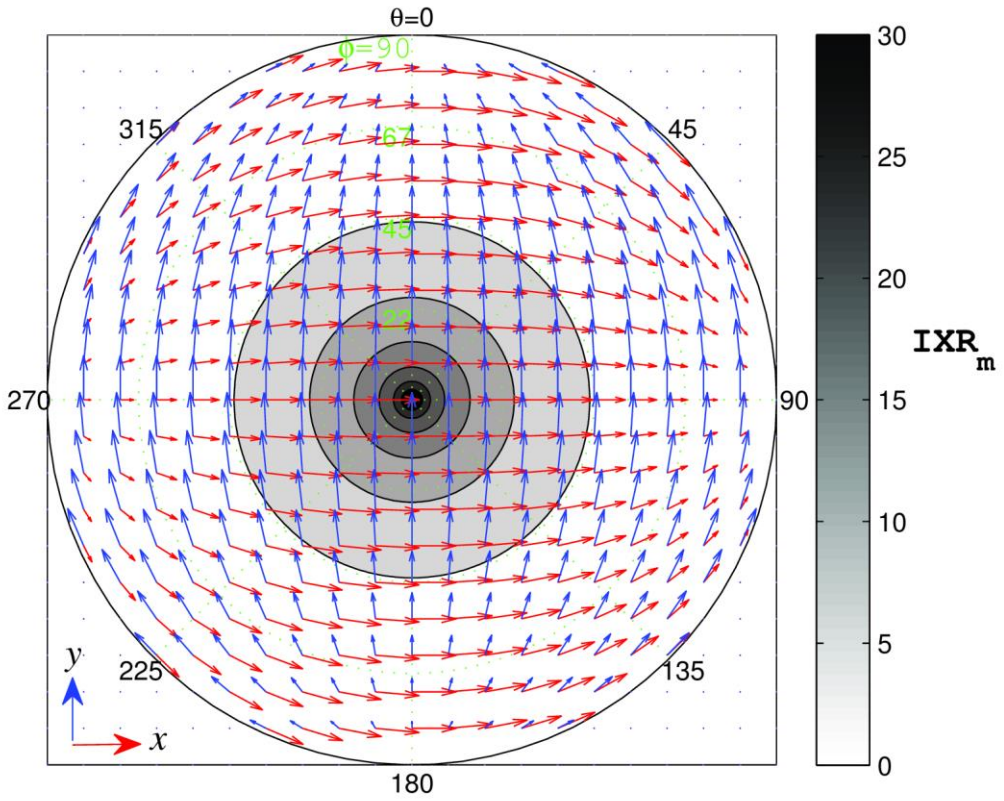
- ”Intrinsic” is meant to denote the fact that it the XPR that cannot be calibrated out

$$||\frac{\mathbf{\Delta e}}{\mathbf{e}}|| \leq \left(1 + \frac{2}{\sqrt{IXR}}\right)\left(||\frac{\mathbf{\Delta f}}{\mathbf{f}}|| + ||\frac{\mathbf{\Delta J}}{\mathbf{J}}||\right)$$

  - Error in calibrated polarimetry is much smaller than IXR prescribes for uncalibrated data

IXR for Lofar LBA

- Going back to previous data with estimates of Jones matrices for Lofar LBA
- We can produce estimated IXR values
  - Ambiguities do not affect IXR due to it's invariance
- Main conclusion: IXR is readily observed in telescope data
- Main consequences: precision polarimetry best done around zenith
- Note that no amount of beamforming can change this
- For SKA-lo & SKA-mid (fixed zenith pointing mounted) this needs consideration



Conclusions

- Direction dependent Jones can be determined and corrected for ”within limits” using only background sky at low frequencies
  - Limits are same as fundamental limits of polarimetry set by Intrinsic Cross-polarization ratio (IXR)
- As a by-product IXR can be estimated uniquely
- Fixed zenith pointing mount telescopes need to be designed carefully for polarimetry science cases so as to not be limited to fields close to zenith