Beam Geometry in ASKAP

John Reynolds, initial draft 24/6/2014.

Coordinate Systems in the Focal Plane

Figure 1 shows the numbering of the 188 ports of the BETA (Mk-I) PAF feed. Active ports with the E-vector vertical in the diagram are numbered from 1 to 94 and are designated, left-to-right, top-to-bottom. Conceptually, the PAF is then rotated anticlockwise by 90 degrees and ports of the orthogonal polarization are then numbered consecutively (from 95 to 188), again left-to-right, top-to-bottom.

Rectilinear coordinates (x,y) in the feed plane are defined with the origin at the centre of the feed and with orientation shown in the following Figure. Polar coordinates (r,FA) are defined with Feed Angle FA measured anticlockwise from the red-leg direction.

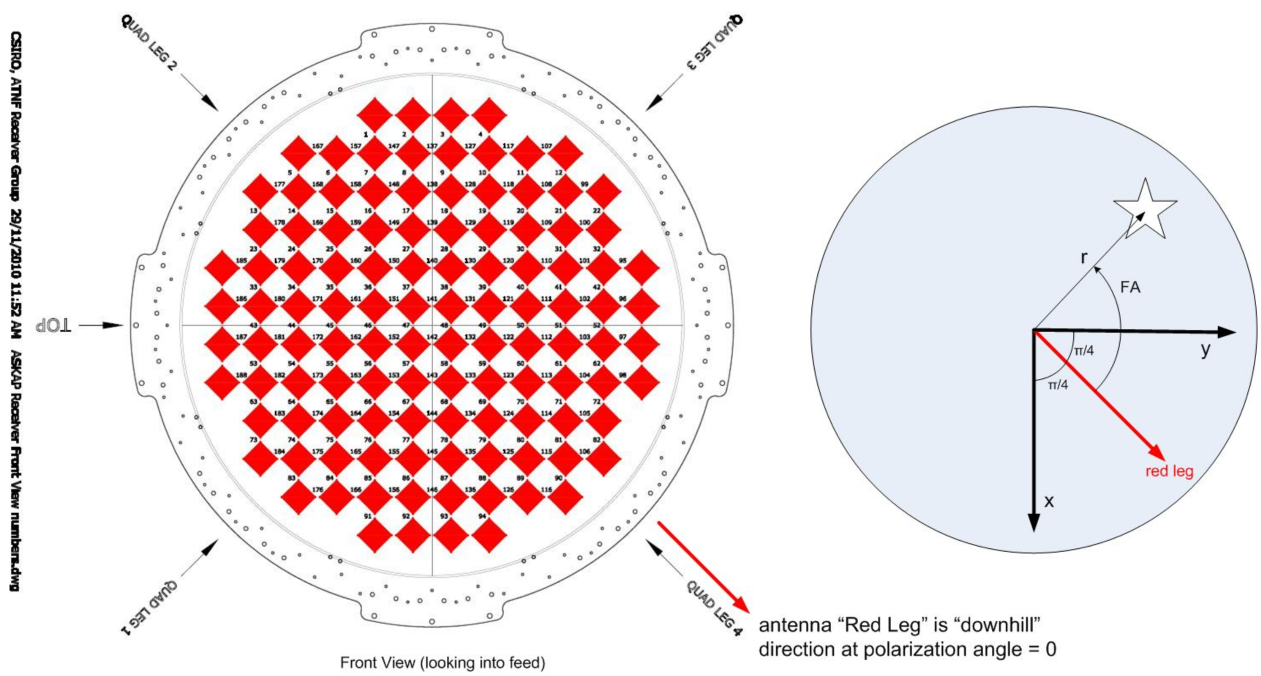


Figure 1. Front view of PAF (feed side) showing location and numbering of the 188 PAF ports and the orientation of the (x,y) axes in the feed plane. The foot of the antenna “red leg” is closest to the ground when the polarization axis angle is zero (neutral position). Polar coordinates (r, FA) are defined as shown with Feed Angle FA measured anticlockwise from the red-leg. The pitch of the regular tesselation in the feed design is 90mm.

The ASKAP antennas comprise a 12-metre parabolic reflector with the PAF receiver mounted at the prime focus supported by a tetrapod or "quad" [sic]. The design of the antenna allows the entire reflector with tetrapod and receiver to be rotated bodily about the boresight direction. This third axis is called 'polarization angle' with a travel of a full turn (360 degrees).

The mounting points of the four tetrapod legs are shown in Figure 1 with the matching leg number. Leg 4, the "red leg" is aligned with the downhill direction on the reflector surface with the antenna tipped over and rotated to polarization angle 0°. An alternate naming scheme (little used in practice) is to label each leg with the nominal polarization angle that aligns it in the downhill direction. Thus the red leg is the 0º leg in this scheme. This leg carries water and air to the PAF while the other three legs carry the 188 PAF outputs, on coaxial cable.

Leg name	Alternate name	Cabling	Notes
leg 4 or "red" leg	0°	Services	origin for feed angle
leg 3	-90°	RF coax	
leg 2	180°	RF coax	
leg 1	+90°	RF coax	

With this mounting arrangement it proves convenient to define the origin of Feed Angle FA in the red leg direction, rather than aligned with the x or y-axis. Hence we have;

$$x = r\cos(FA + \pi/4)$$

$$y = r\sin(FA + \pi/4)$$

To convert between (x,y) coordinates and polar coordinates.

Note that the annotation "TOP" in the various drawings of the receiver defines an arbitrary reference orientation for receiver design and assembly purposes, and should not be imbued with any deeper significance such as defining the orientation of the mounted receiver.

Figures 2 and 3 are included for completeness, and to demonstrate the rationale for the particular choice of orientation for the (x,y) system. The x-axis is consistently defined in receiver drawings such as Figure2. Ports 1-94 of the PAF are fed from feed dipoles that are aligned in this direction (E-vector parallel to x-axis). Ports 95-188 are orthogonal and inspection of Figures 2 and 3 reveals that the natural orientation of the y-axis is as shown in Figure 3, in the sense that a plane polarized wave with E-vector aligned at 45degrees to both the (positive) x-axis and y-axis will produce in-phase voltages in the x and y ports (c.f. polarization parameter FD_SANG, introduced later). Note the (x,y) system appears left-handed when viewed from the rear of the receiver and right-handed when viewed from the front, looking into the feed.

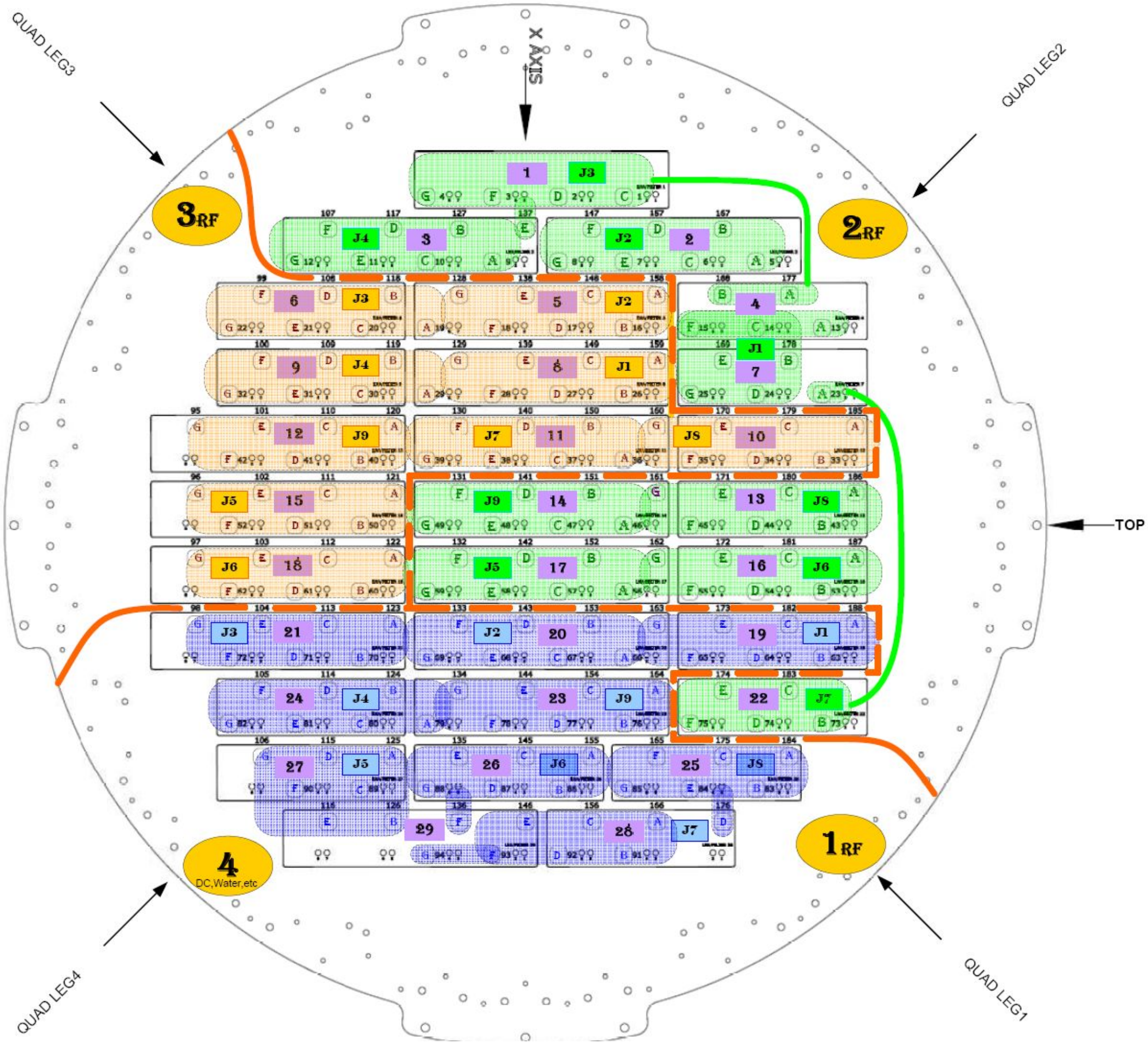


Figure 2. View of the PAF from the rear (weather-shield side) showing layout of individual LNA cards (black rectangles) and gain cards (coloured rectangles with rounded corners). The small rounded squares enclosing a letter A-G show the location of the feed ports, numbered as in Figure 1. The smaller numbered circles show the LNA outputs to the gain cards.

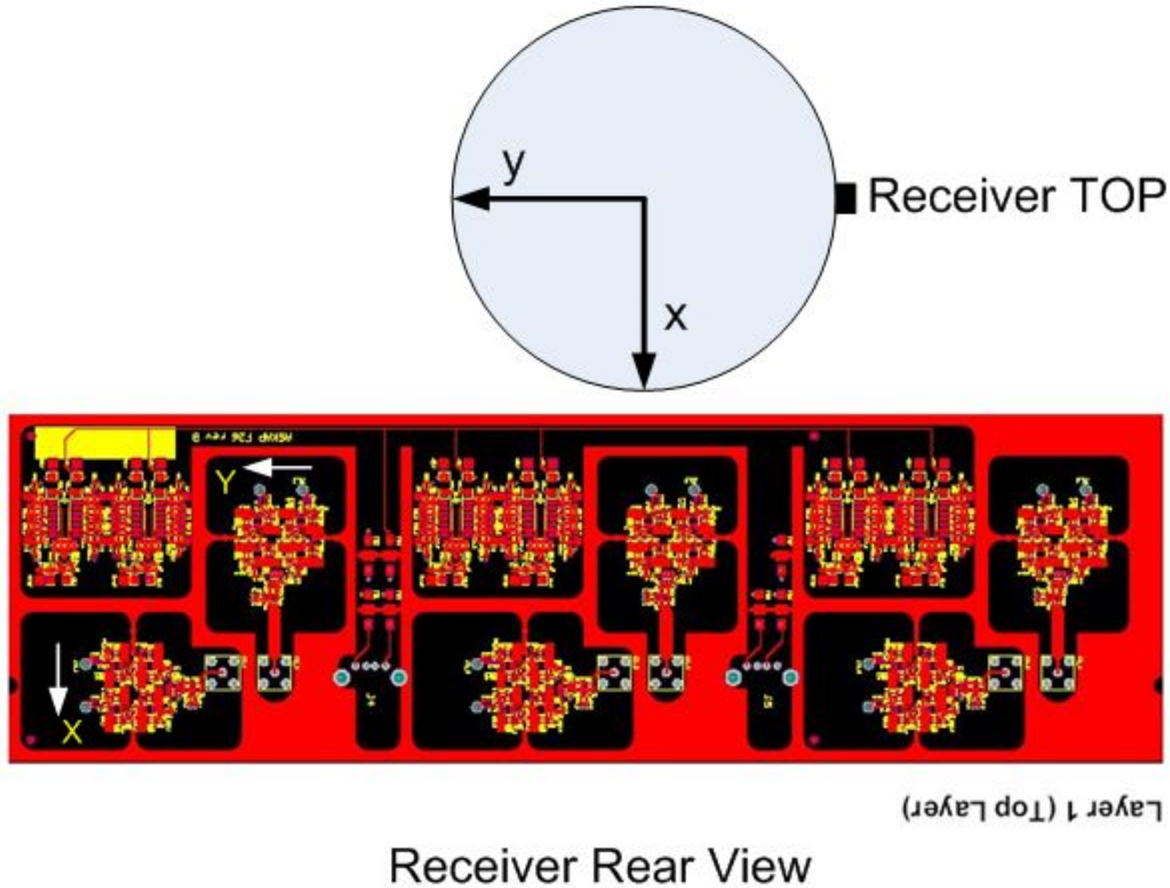


Figure 3. schematic of an LNA card (F26) with same orientation and viewing side (rear) as in Figure 2. The PAF uses a combination of this 6-port card and a similar 8-port version to populate the 188 active PAF ports. The 94 LNA circuits for ports aligned in the x-direction all have the same orientation, while their y-direction counterparts are functionally identical but rotated clockwise by 90 degrees. This determines the sense of the y-axis needed to define a consistent basis for polarization.

Coordinate Systems in the Focal Plane

In Figure 4, at left, a radio source is shown displaced from the boresight direction by angle θ at orientation, or position angle, PA (by the IAU definition of this latter quantity). The image of this source falls in the focal plane as shown at right, as seen looking into the feed along the boresight direction. The image is inverted owing to the single reflection involved.

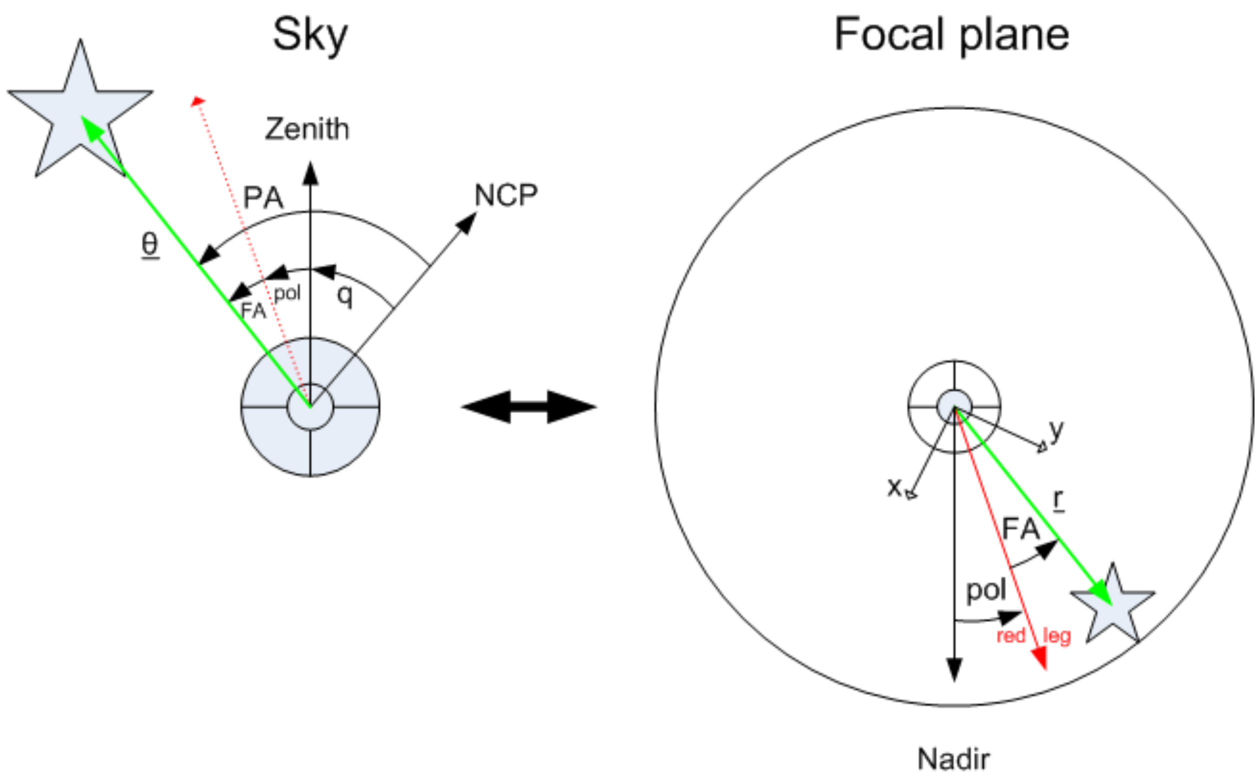


Figure 4. Relationship between image plane and sky coordinates. The vector (green) from the boresight position on the sky to an offset position (top left) has angular length θ and position angle PA. In the image plane (right) this vector is inverted owing to the single reflection in this prime-focus system. The image plane is seen looking into the feed along the boresight axis. The angles *pol* and *q* are the polarization angle of the antenna, and the parallactic angle respectively.

The two basic equations relating coordinates on the sky to those in the image plane are;

$$PA = FA + q + pol$$

$$tan|\underline{\theta}| \cong s * |\underline{r}|,$$

where q is the parallactic angle of the boresight position, and pol is the antenna polarization angle from its neutral position. The angular separation $|\underline{\theta}|$on the sky is to first order proportional to the linear distance in the image plane $|\underline{r}|$ where s is the so-called "plate-scale". In the limit of long focal length f/D ≫ 1 the plate scale approaches 1/f but in practice is smaller. For a uniformly-illuminated aperture with focal ratio f/D=0.5 the plate scale is approximately;

$$s = 0.86/f$$

This linear relationship is approximate only as the diffraction pattern (Airy disc) of a point source in the image plane becomes increasingly asymmetric with increasing angular separation (exhibiting "coma lobe").

In a typical imaging observation with ASKAP the polarization axis is adjusted continuously to keep the image of the sky fixed in the focal plane. That is, the polarization angle *pol* is tracked to keep the quantity *pol* + *q* constant. This practice is sometimes referred to by the cumbersome term "parallactification" (not to be found in any dictionary, even the MacQuarie).The ASKAP scheduler or operator specifies an angle PA_fixed which defines the position angle to be maintained for the red leg direction when projected onto the sky, as shown in Figure 5.

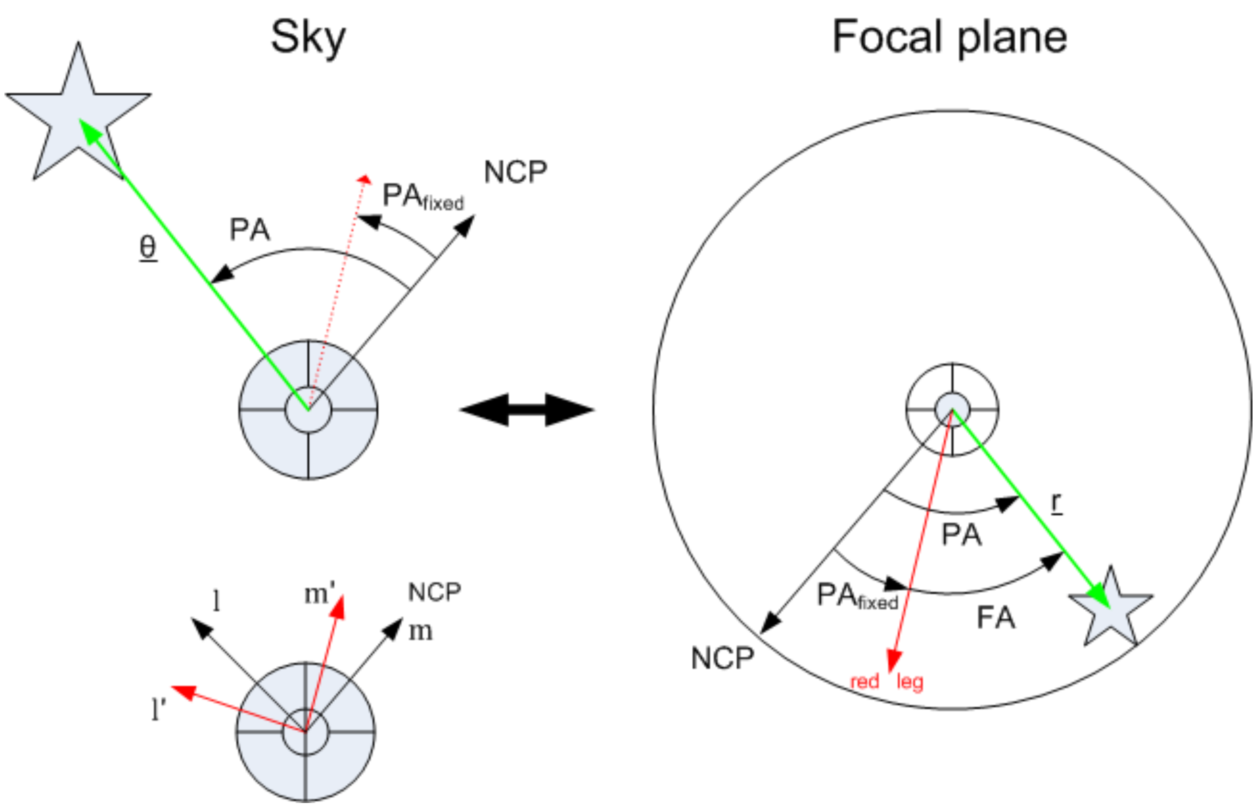


Figure 5. Illustration of relationship between Position Angle PA, Feed Angle FA with polarization axis tracking at angle PA_{fixed} . The orientation of the axes for the direction-cosine coordinate systems (l,m) and (l', m') centred on the boresight are shown at bottom left.

This leads to the following relationship between the position angle PA of a source in the field-of-view, and its Feed Angle FA in the image plane;

$$PA = PA_{fixed} + FA$$

$$FA = PA - PA_{fixed}$$

Typically the value of PA_{fixed} will be fixed at either zero or 180 degrees. Other values are possible but result in a rotation of the polarization state of the antenna, as discussed later.

Formed-Beams and Beam-Forming

Complex linear combinations of the PAF port voltages with specially chosen weights generate "formed beams", which can be steered to a particular direction with respect to the boresight axis. We may characterise the direction of each such beam in either of two ways – by the coordinates of the peak response of its diffraction pattern in the image plane, or by the coordinates of the beam's peak response on the sky, relative to the boresight direction. For the purposes of this preliminary discussion it is conceptually simpler to select the latter and defer considerations of diffraction patterns and image plane distortions for the time being.

An arbitrary location within the field-of-view may be specified relative to the boresight direction using the polar coordinate system (θ,PA) or equivalently in rectilinear form using the direction cosines (*l,m*), as defined in Figure 5;

$$l = \theta \sin(PA) \cong d\alpha \cos(\delta)$$

$$m = \theta \cos(PA) \cong d\delta$$

Where δ is the Declination of the boresight direction and *d*α, *d*δ are the approximate offsets in Right Ascension and Declination respectively. Note that the wide field-of-view (of several degrees) used in ASKAP these linear approximations in dα and dδ are generally inadequate.

Our goal is to characterize the offsets for a particular beam *b* by instead using a coordinate system *(l',m')* which is fixed with respect to the image plane, as shown in Figure 5.

$$l'_b = \theta \sin\left(PA - PA_{fixed}\right)$$

$$m'_b = \theta \cos\left(PA - PA_{fixed}\right)$$

Alternatively the beam offsets might be expressed in polar coordinates (θ_b,FA_b) , where;

$$\theta_b = \sqrt{l_b'^2 + m_b'^2} = \sqrt{l_b^2 + m_b^2}$$

$$FA_b = \text{atan}\,(\frac{l_b'}{m_b'})$$

as shown in Figure 5. This is something of a hybrid system, with the angular radius θ_b defined in the sky plane and angle FA_b in the focal plane. One could instead use linear distance r_b related to θ_b as follows, recasting an earlier relation;

$$r_b = \frac{1}{s} * \tan\,(\theta_b)$$

where *s* is the plate scale. However as noted earlier this can be considered only an approximate relation, leading into diffractive effects and aberrations.

Determining beam-weights from a calibration observation

We now proceed according the following scheme to determine beam-weights for an arbitrary position within the field;

- For intended target field with tracking centre (α_0,δ_0) choose coordinates (α_b,δ_b) at which to place formed-beam *b*.

- Calculate the corresponding offsets in the sky frame (l_b,m_b) as follows;

  $$l_b \ = \sin(\alpha_b - \alpha_0)\cos\,(\delta_b)$$

  $$m_b \ = \sin(\delta_b)\cos(\delta_0) - \cos(\alpha_b - \alpha_0)\cos\,(\delta_b)\sin\,(\delta_0)$$

- Choose the value of polarization tracking angle PA_{target} to be used. This would typically, but not necessarily, be either 0 or 180 degrees. For a long observation of a source South of $\delta_0 = -26^o$ that culminates on the Southern meridian a value of zero degrees is generally convenient as this can be tracked continuously within the limits of the ASKAP antenna's polarization axis. Conversely, for sources that culminate on the Northern meridian a tracking angle of PA_{target} = 180 degrees is typically more convenient.

- Calculate the beam offsets in the frame fixed to the focal plane;

$$l'_b = l_b \cos(PA_{target}) - m_b \sin(PA_{target})$$

$$m'_b = m_b \cos(PA_{target}) + l_b \sin(PA_{target})$$

These are the offsets that will be used to characterize beam b, as they are fixed relative to the focal plane and the PAF feed. Alternatively, polar coodinates (θ_b, FA_b) could be used. An arrangement or "footprint" of beam positions in a rectangular grid it would probably be more conveniently be expressed in rectangular coordinates. A footprint comprising a hexagonal pattern might be more clearly recognisable when expressed in polar coordinates.

- The proceeding steps may be skipped entirely and the beam offsets (l'_b,m'_b) defined *a priori* if, for example, a regular grid of beam positions ("footprint") is being constructed.

- Identify a suitable calibration source with coordinates (α_c,δ_c) to be observed to determine the beam-weights for beam *b*, and select an appropriate polarization tracking angle PA_{cal} to be used (typically either 0 or 180 degrees).

- Transform the required beam offsets (l'_b,m'_b) to equivalent offsets (l_c, m_c) in the sky frame to place the calibrator at the offset position for beam *b*;

$$l_c = l'_b \cos(PA_{cal}) + m'_b \sin(PA_{cal})$$

$$m_c = m'_b \cos(PA_{cal}) - l'_b \sin(PA_{cal})$$

Substituting the previous two equations into the above two we have;

$$l_c = l_b \cos(PA_{target} - PA_{cal}) - m_b \sin(PA_{target} - PA_{cal}) .$$

$$m_c = m_b \cos(PA_{target} - PA_{cal}) + l_b \sin(PA_{target} - PA_{cal}) .$$

Thus in the simple case where $PA_{target} = PA_{cal}$ the offsets (l_c, m_c) and (l_b, m_b) are identical. and for $PA_{target} = PA_{cal} + 180$ they are mutually inverted;

$$l_c = -l_b , \quad m_c = -m_b .$$

For the particularly simple case where $PA_{target} = PA_{cal} = 0$ we have;

$$l_c = l_b = l'_b , \quad m_c = m_b = m'_b$$

and for $PA_{target} = 0$, $PA_{cal} = 180$ (Southern target, Northern calibrator);

$$l_b = l_b' = -l_c \ , \ m_b = m_b' = -m_c$$

Calculate the sky coordinates of the tracking centre $(\alpha_{bc}, \delta_{bc})$ that places the calibrator at the offset position (l_c,m_c) relative to the boresight position;

$$\delta_{bc} = \arcsin(\sin\ \delta_c/\sqrt{(1-l_c^2)}) - \arctan\,(m_c/\sqrt{(1-\mathrm{l}_c^2-\mathrm{m}_c^2)}\,)$$

$$\alpha_{bc} = -\arcsin\,(l_c/\cos(\delta_c)\,) + \alpha_c$$

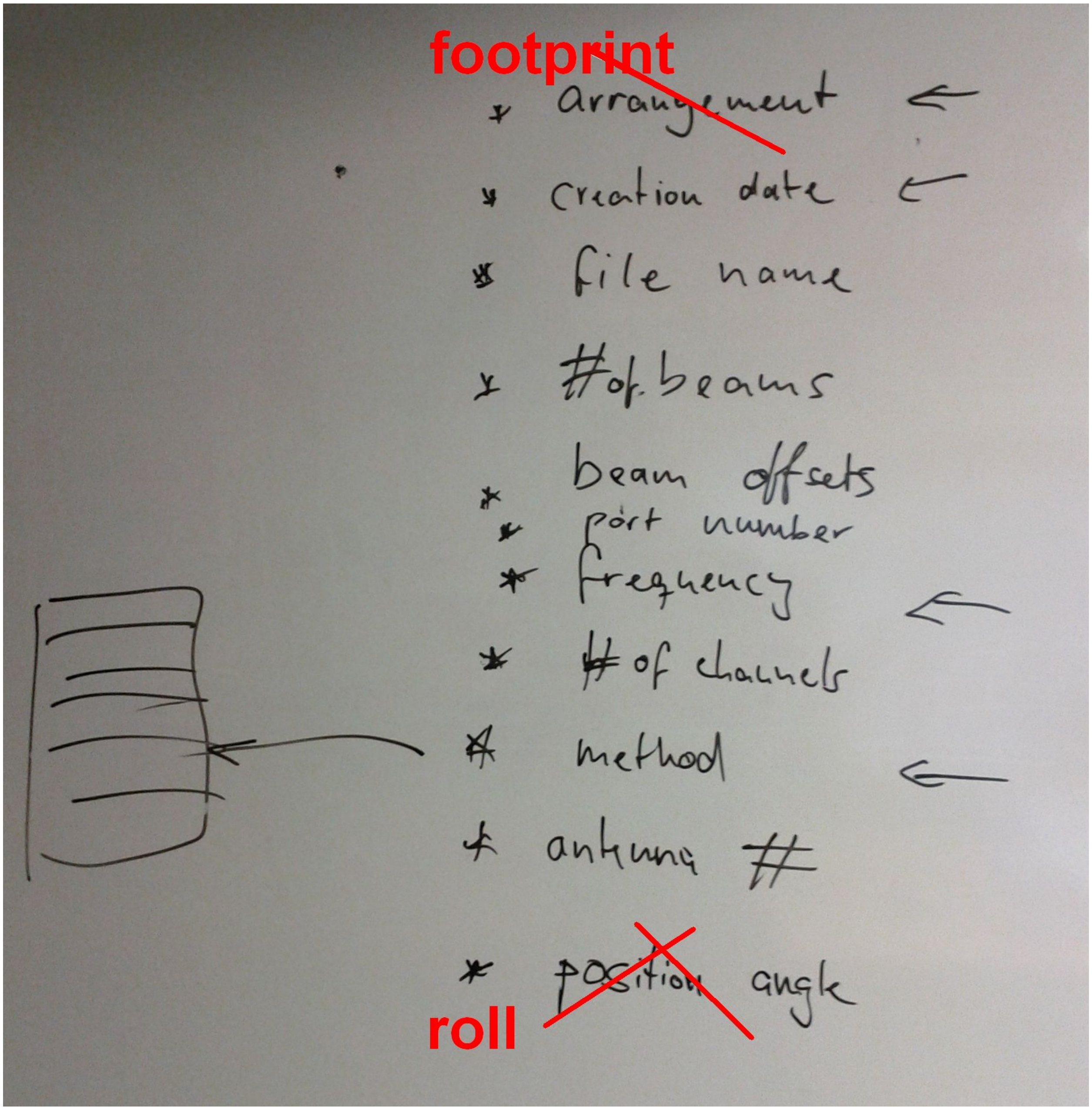
In the simple case $PA_{target} = PA_{cal}$ the above offsets are to first order just the inverse of the offsets of the beam *b* from the target field tracking centre;

$$d\alpha = \alpha_{bc} - \alpha_c \cong -(\alpha_b - \alpha_0) * \cos\,(\delta_0)/\cos\,(\delta_c)\,,$$

$$d\delta = \delta_{bc} - \delta_c \cong -(\delta_b - \delta_0)$$

as one would expect, but it is in general necessary to use the full great circle calculation to obtain the necessary precision.

Beam-weight format and meta-data



Description of instrumental polarization

Will follow the approach of van Straten et al., (2010), PASA, 27, 104;

FD_POLN = 'LIN' / LIN or CIRC

FD_HAND = 1 / +/- 1. +1 is LIN:A=X,B=Y, CIRC:A=L,B=R (I)

FD_SANG = -90. / [deg] FA of E vect for equal sig in A&B (E)

FD_XYPH = 0. / [deg] Phase of A^* B for injected cal (E)

BE_PHASE= -1 / 0/+1/-1 BE cross-phase:0 unknown,+/-1

Primary beam considerations