ASKAP Resolution for Continuum Surveys

Jim Condon

Abstract

One of the ASKAP science goals is surveying the southern sky to a detection limit of $50\,\mu$Jy beam^{-1} at 1.4 GHz. Fairly high angular resolution is needed to avoid confusion and measure accurate positions of faint sources, but lower resolution is desirable for detecting low-brightness sources and minimizing computing and construction costs. The best compromise resolution may be $\theta \approx 10$ arcsec FWHM.

1. Introduction

One of the ASKAP science goals is to make a 1.4 GHz continuum survey covering the southern sky with rms noise $\sigma_{\rm n} \approx 10\,\mu$Jy beam^{-1} and a detection limit near $50\,\mu$Jy beam^{-1}. Fairly high angular resolution is needed to avoid the confusion limit and to measure accurate positions of faint sources, but fairly low angular resolution is desirable for detecting sources having low surface brightnesses and to lower computing and construction costs. The analysis presented in this memo suggests that $\theta \approx 10$ arscec FWHM is the optimum resolution.

2. Confusion

Faint extragalactic sources too numerous to be clearly resolved produce "confusion" in sensitive survey images. The $P(D)$ distribution specifies the probability P that any point on a noiseless image will have intensity (e.g., Jy beam^{-1}) D. The $P(D)$ distribution can be calculated for power-law differential source counts $n(S) = kS^{-\gamma}$ (Condon 1974). In this scale-free case, the $P(D)$ distribution depends only on the "effective" solid angle $\Omega_{\rm e}$ of the point-source response, normally the restoring beam for a synthesis image. For a Gaussian point-source response having FWHM θ, the effective solid angle is

$$\Omega_{\rm e} = \frac{\Omega_{\rm b}}{\gamma - 1} \tag{1}$$

where

$$\Omega_{\rm b} = \frac{\pi \theta^2}{4 \ln(2)} \tag{2}$$

is the beam solid angle.

If $\gamma > 2$, the mean sky intensity is infinite (Olbers' paradox), so the distribution $P(D - \langle D \rangle)$ must be calculated relative to the mean intensity $\langle D \rangle$. If $\gamma < 2$, the distribution $P(D)$ above absolute zero can be calculated.

The variance of the confusion distribution formally diverges for power-law source counts, so the "rms confusion" σ_c can be defined only over a limited intensity range. If the confusion distribution is truncated at some intensity $D_{\rm max} = q\sigma_{\rm c}$, where q is a constant, then (Condon 1974)

$$\sigma_{\rm c} = \left(\frac{q^{3-\gamma}}{3-\gamma} \right)^{\frac{1}{\gamma-1}} (k\Omega_{\rm e})^{\frac{1}{\gamma-1}} \tag{3}$$

If $\gamma \approx 2$, a good approximation to the slope of the faint-source counts, the width of the $P(D)$ distribution (in Jy beam^{-1}) is proportional to $\Omega_{\rm e} \propto \Omega_{\rm b}$, so the confusion brightness (in K) is nearly independent of resolution. The "5σ" confusion limit $q = 5$ is often used to calculate the rms confusion because sources stronger than about 5σ can be detected individually and hence are not considered to be part of the confusion. The rms confusion defined by Equation 3 is fairly sensitive to the choice of q; e.g., it is directly proportional to q if $\gamma = 2$. Thus the widely used "rms confusion" is a simple but not very stable statistic of the $P(D)$ distribution.

Another statistic used to estimate the confusion limit of an image is the "number of beams per source." If the cumulative counts of sources stronger than S is:

$$N(>S) \equiv \int_S^\infty n(x)dx \ , \qquad (4)$$

then the number β of beam solid angles per source at flux density S is precisely defined by

$$\beta = [N(>S)\Omega_{\rm b}]^{-1} \ . \qquad (5)$$

For power-law source counts,

$$\beta = \frac{q^2}{3-\gamma} \ . \qquad (6)$$

The number of beams per source corresponding at $S = 5\sigma_{\rm c}$ ($q = 5$) ranges from $\beta = 25$ at $\gamma = 2$ to $\beta = 83$ at $\gamma = 2.7$. The former is appropriate for flux densities $S < 0.1$ Jy at 1.4 GHz, while the latter applies to higher fluxes encountered in early low-resolution surveys (e.g., 3C). Once again, this statistic is not very good because it is sensitive to the choice of q.

The angular resolution required of ASKAP is constrained by the requirement that a continuum survey reaching $S \approx 50\,\mu$Jy at 1.4 GHz will avoid the confusion limit. If $N(>50\,\mu\text{Jy}) = 1300\,\text{deg}^{-2}$ [Table 1 in Johnston (2006)], there is one source stronger than $50\,\mu$Jy per 1.00×10^4 arcsec^2. For randomly distributed sources stronger than $50\,\mu$Jy, the mean angular distance Δ between a source and its nearest neighbor is $\Delta = N^{-1/2}/2 \approx 50$ arcsec. This equals ten beamwidths ($\Delta = 10\theta$) if $\theta = 5$ arcsec is the FWHM beamwidth. Thus $\theta = 5$ arcsec is a very conservative estimate of the resolution needed to avoid confusion; it implies $\beta \approx 350$.

To determine just how large θ might be, I calculated the full $P(D)$ distributions for three different power-law approximations to the faint-source counts. The approximations for these three cases are shown as black lines in Figure 1.

(1) Kellermann (2000) extrapolated the 1.4 GHz counts of sources from deep VLA surveys extending to $S \approx 50\,\mu$Jy; his result is

$$n(S) = 8.23 S^{-2.4} \qquad (7)$$

where n is the differential number of sources per steradian per Jy and S is in Jy. Figure 1 shows that extrapolating the (Huynh et al. 2005) faint-source counts and evolutionary models gives a similar result.

(2) Mitchell & Condon (1985) estimated the sky density of sources as faint as $S \approx 10\,\mu$Jy from the observed confusion $P(D)$ distribution in an image restored with a $\theta = 17''.5$ Gaussian beam. Their result is

$$n(S) = 57 S^{-2.2} \ . \qquad (8)$$

(3) Condon (2007) estimated faint source counts consistent with the counts of individual sources down to $S \approx 50\,\mu$Jy, the Mitchell & Condon (1985) statistical result down to $S \approx 10\,\mu$Jy, and a

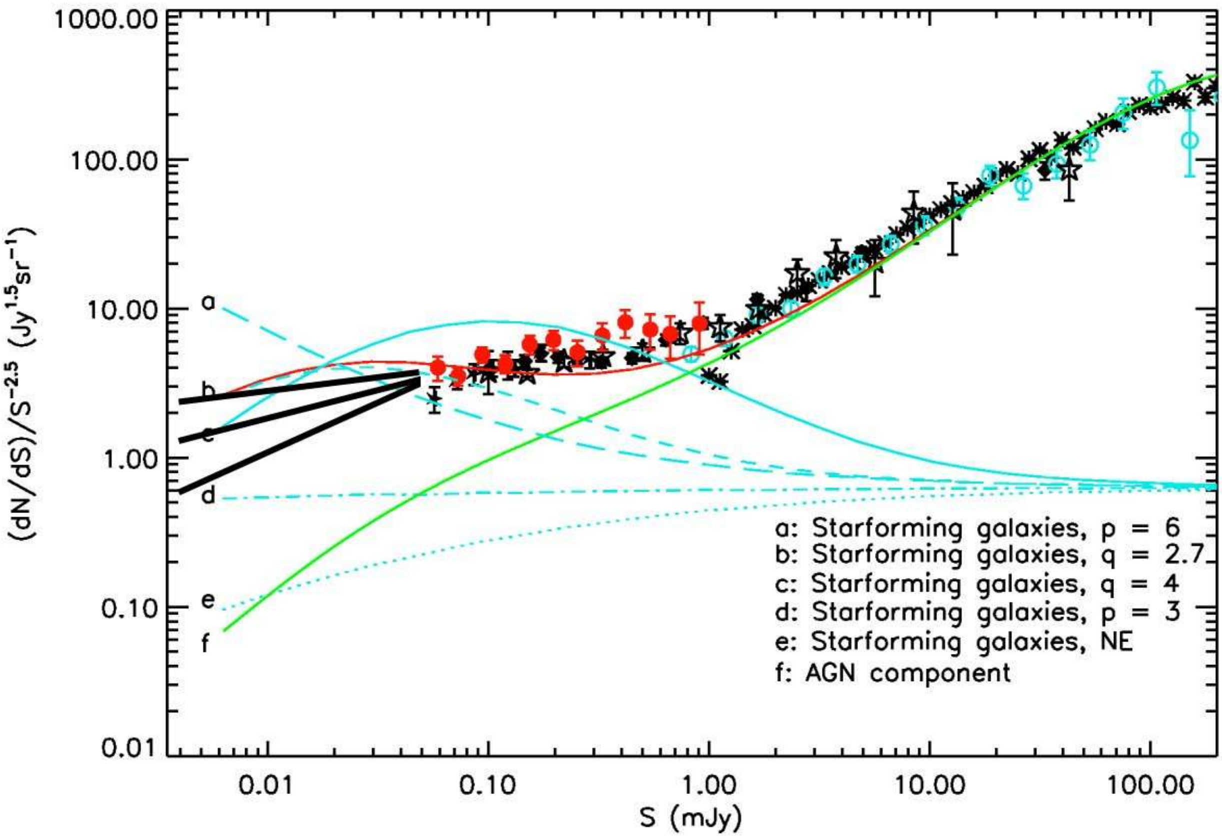

Fig. 1.— Observed (data points) faint-source differential counts $n(S) = dN/dS$ and evolutionary models (colored curves) normalized by a static Euclidean universe $S^{5/2}$ from Huynh et al. (2005). The black lines are power-law extrapolations of the 1.4 GHz source counts below $S = 0.05$ mJy from deep surveys (Kellermann 2000) (top line), statistical "counts" based on the confusion $P(D)$ distribution (Mitchell & Condon 1985) (middle line), and an evolutionary model consistent with the contributions of faint sources to the sky brightness Condon (2007) (bottom line). Abscissa: 1.4 GHz flux densiy (mJy). Ordinate: log differential counts normalized to a static Euclidean universe (Jy$^{3/2}$ sr^{-1}).

simple evolutionary model (Condon 1989) also constrained by the contribution of sources to the sky brightness at 1.4 GHz, $T \approx 0.1\,\mathrm{K}$. These counts are plotted in Figure 3. In the relevant flux-density range $-6 < \log(S) < -4$, they can be approximated by the power law

$$n(S) = 1000 S^{-1.9} \tag{9}$$

.

The confusion $P(D)$ distributions for cases (1), (2), and (3) are plotted in Figures 3, 4, and 5, respectively. These figures show the confusion distributions for both $\theta = 5$ arcsec and $\theta = 10$ arcsec. Figures 3 and 4 also show a Gaussian noise distribution with rms width $\sigma_{\mathrm{n}} = 10\,\mu\mathrm{Jy~beam}^{-1}$ for comparison. The total rms fluctuation in an image with both confusion and noise is the quadratic sum of the noise and confusion: $\sigma^2_{\mathrm{tot}} = \sigma^2_{\mathrm{c}} + \sigma^2_{\mathrm{n}}$. However, there are better measures of the "widths" of the $P(D)$ distributions. Since each of these confusion and noise distributions is normalized so that $\int P(D) dD = 1$, the height $P_{\max}$ of the peak is inversely proportional to the width of the core of the distribution. So long as $P_{\max}$ is higher than the peak $P_{\max} = (2\pi)^{-1/2} \sigma_{\mathrm{n}}^{-1} \approx 0.040\,(\mu\mathrm{Jy/beam})^{-1}$ of the Gaussian noise distribution, the image is limited more by noise than by confusion. Thus, even the worst case (1) observed with $\theta = 10$ arcsec resolution has $P_{\max} \approx 0.076\,(\mu\mathrm{Jy/beam})^{-1}$ and is noise limited by about a factor of $0.076/0.040 \approx 2$. The more realistic cases (2) and (3) are noise limited by factors of ≈ 3 and ≈ 5, respectively.

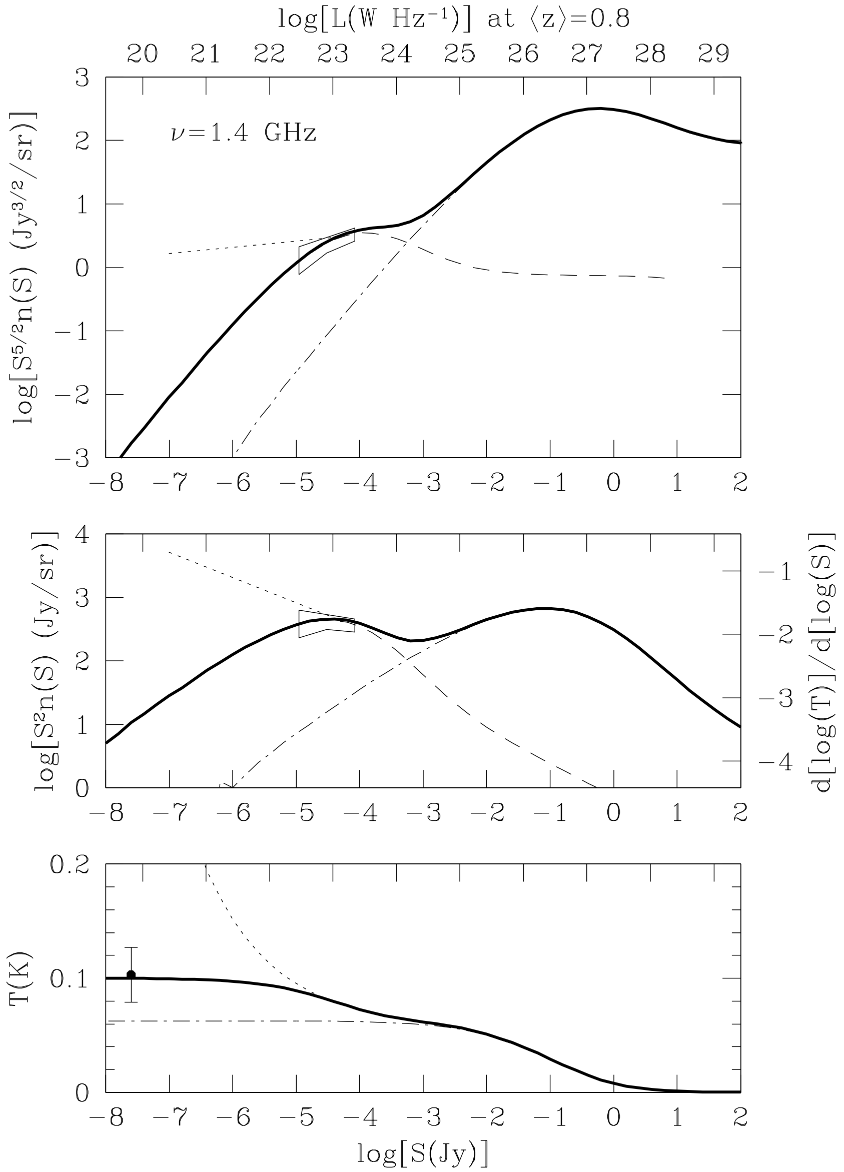

Fig. 2.— Differential source counts are usually normalized by a static Euclidean universe $S^{5/2}n(S)$ (top), but normalizing by $S^2 n(S)$ (middle) is better for showing the contributions of different logarithmic flux-density ranges to the cumulative sky brightness (bottom). The dot-dash curves indicate the contributions of AGNs, the dashed curves are the contributions of star-forming galaxies, and the heavy curves are their sums in a very simple evolutionary model (Condon 1989). The dotted curves below $\log(S) \approx -5$ are extrapolations of 1.4 GHz source counts from deep surveys (Kellermann 2000), and the polygon bounds statistical "counts" based on the confusion $P(D)$ distribution (Mitchell & Condon 1985). Abscissa: log flux densiy (Jy). Ordinates: log differential counts normalized to a static Euclidean universe ($\mathrm{Jy}^{3/2}/\mathrm{sr}$), log differential counts normalized to brightness per decade of flux density (Jy/sr), and cumulative sky brightness temperature (K).

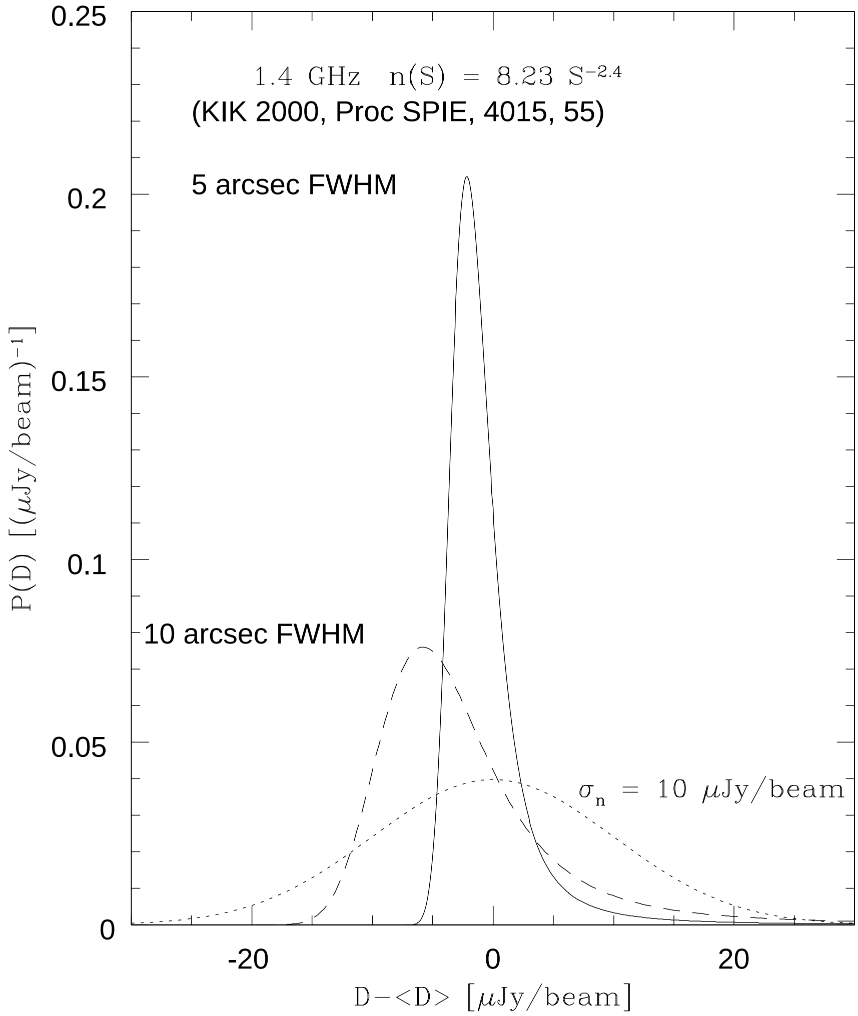

Fig. 3.— Confusion $P(D)$ distributions for images with Gaussian point-source responses of 5 arcsec FWHM (solid curve) and 10 arcsec FWHM (dashed curve) compared with a $10\,\mu$Jy rms Gaussian noise distribution (dotted curve). The source counts $n(S) = 8.23S^{-2.4}$ were obtained by extrapolating the observed counts from deep VLA surveys at 1.4 GHz (Kellermann 2000). These are similar to the counts and models of Huynh et al. (2005). Abscissa: Image "deflection" or intensity D relative to the mean $< D >$ (μJy beam^{-1}). Ordinate: Probability $P(D)$ [(μJy beam^{-1})$^{-1}$].

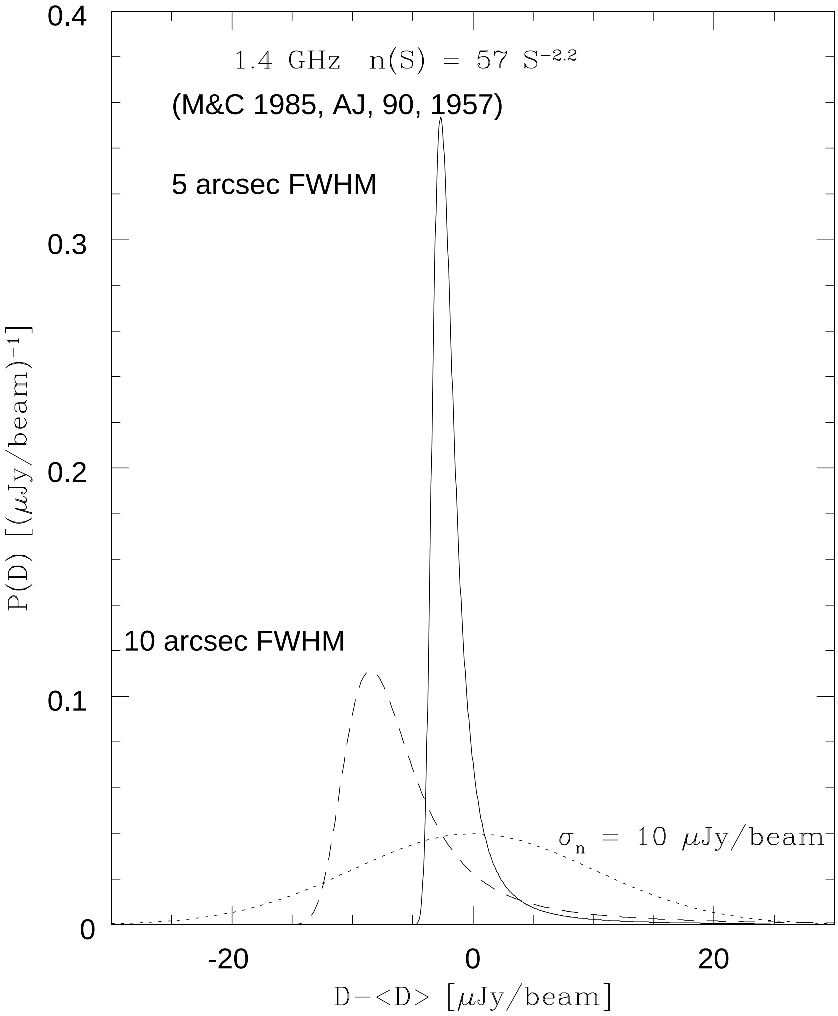

Fig. 4.— Confusion $P(D)$ distributions for images with Gaussian point-source responses of 5 arcsec FWHM (solid curve) and 10 arcsec FWHM (dashed curve) compared with a $10\,\mu$Jy rms Gaussian noise distribution (dotted curve). The source counts $n(S) = 57S^{-2.2}$ were obtained by extrapolating observations of the confusion at 1.4 GHz made with 17.5 arcsec FWHM resolution (Mitchell & Condon 1985). Abscissa: Image "deflection" or intensity D relative to the mean $< D >$ (μJy beam^{-1}). Ordinate: Probability $P(D)$ [(μJy beam^{-1})$^{-1}$].

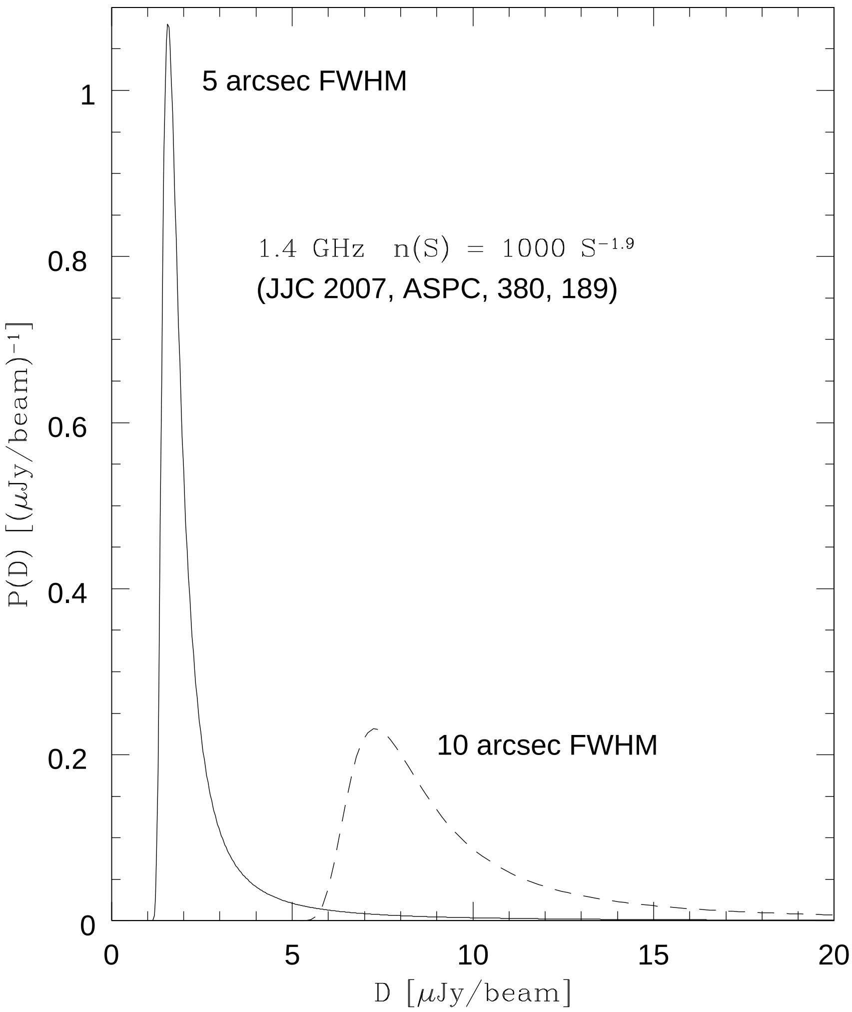

Fig. 5.— Confusion $P(D)$ distributions for images with Gaussian point-source responses of 5 arcsec FWHM (solid curve) and 10 arcsec FWHM (dashed curve). The source counts $n(S) = 1000S^{-1.9}$ were obtained by fitting the solid curvey in Figure 1 in the range $-6 < \log(S) < -4$. Abscissa: Image "deflection" or intensity D relative to the mean $< D >$ (μJy beam^{-1}). Ordinate: Probability $P(D)$ [(μJy beam^{-1})$^{-1}$].

Table 1 lists the rms confusion values (for $q = 5$) for all three source counts and $\theta = 5''$, $10''$, and $17''.5$. For all source counts, the rms confusion in a $\theta = 5$ arcsec beam is far below the expected $\sigma_{\rm n} = 10\,\mu$Jy beam^{-1} noise. The $P(D)$ distribution for Case (1) is too broad to be consistent with the distribution observed by Mitchell & Condon (1985), which requires $\gamma < 2.4$. The extrapolated counts for Case (2) are also likely to be too high at the low flux densities corresponding to confusion in $\theta = 5$ arcsec and $\theta = 10$ arcsec beams. I believe that Case (3) is the most realistic predictor for the planned ASKAP survey. Even for $\theta = 10$ arcsec, the rms confusion should be a factor of two below the rms noise, and the total rms (with $\sigma_{\rm n} = 10\,\mu$Jy beam^{-1}) is only $\sigma_{\rm tot} = 11\,\mu$Jy beam^{-1}.

Table 1. Source Counts and Confusion at 1.4 GHz

Differential Source Counts	Reference	$\sigma_{\rm c}$(μJy beam^{-1}) $\theta = 5''$	$\theta = 10''$	$\theta = 17''.5$
$8.23S^{-2.4}$	Kellermann (2000)	2.8	7.6	17.0
$57S^{-2.2}$	Mitchell & Condon (1985)	2.0	6.3	14.7
$1000S^{-1.9}$	Condon (2007)	1.0	4.6	16.0

3. Brightness Sensitivity

The brightness-temperature detection limit of a survey having point-source flux-density limit S and restoring-beam solid angle $\Omega_{\rm b}$ is

$$T = \frac{\lambda^2 S}{2k\Omega_{\rm b}} , \qquad (10)$$

where λ is the wavelength and here $k \approx 1.38 \times 10^{-23}$ J K^{-1} is the Boltzmann constant. The survey will miss all sources with lower brightnesses, no matter how nearby, extended, or strong in total flux. For the proposed ASKAP survey with $S = 50\,\mu$Jy and $\theta = 5$ arcsec, $T = 1.25\,$K. Doubling the beamwidth to $\theta = 10$ arcsec would lower the brightness limit to $T = 0.31\,$K. By comparison, the surface-brightness limit of the NVSS is $T \approx 0.70\,$K (Condon et al. 1998).

Going from $T = 1.25\,$K to $T = 0.31\,$K is important because the median brightness temperature of a nearby face-on spiral galaxy similar to our own, averaged over the optical disk, is about 1 K at 1.4 GHz (Condon et al. 1998). Furthermore, the brightness temperature of a synchrotron source whose flux density declines as the $\alpha \approx 0.7$ power of frequency falls with redshift z as $T \propto (1+z)^{-3-\alpha}$. The NVSS can only detect typical spiral galaxies out to $z \sim 0.04$, so this dimming is not important for the NVSS. However, the flux sensitivity of the proposed ASKAP survey is high enough to detect them out to $z \sim 0.3$, at which point their median face-on brightness temperature is only $T \sim 0.4\,$K. Lowering the resolution of the ASKAP survey from $\theta = 5$ arcsec to $\theta = 10\,$arcsec is both necessary and sufficient to follow the evolution of star formation in typical spiral galaxies out to $z \sim 0.3$.

4. Position Accuracy

Radio sources can be identified by position coincidence with sources detected in other wave bands. Noise limits the rms position uncertainty $\sigma_{\rm p}$ in each fitted sky coordinate (RA or DEC) of a faint point source found on an image having rms brightness fluctuation $\sigma_{\rm tot}$ and FWHM resolution θ to

$$\sigma_{\rm p} \approx \frac{\sigma_{\rm tot}\theta}{2S} . \qquad (11)$$

The faintest detectable source has $S \approx 5\sigma_{\rm tot}$, so $\sigma_{\rm p} \lesssim \theta/10$. Going from $\theta = 5\,$arcsec to $\theta = 10\,$arcsec increases the maximum $\sigma_{\rm p}$ from $0\overset{''}{.}5$ to $1\overset{''}{.}0$. However, this is still accurate enough for most identification programs (Condon et al. 1998).

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