Constraints on SKA array configurations
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Introduction

It is a stated aim of the Square Kilometer Array (SKA) design that 40% of spacings between antennas should be 'short', 40-50% should be medium, and the remaining 20-10% should be 'long'. This note seeks to explore the consequences of these two requirements.

To explore this question, we adopt a definition of 'short' spacings, spacings $<D$, and a simple, possible model of the SKA array. In this model, compact configurations of antennas of diameter D, are arranged in a plane. Each compact group contains n antennas, and there are m of these groupings, all containing the same number of antennas (in order to reduce the dimensionality of the problem). In addition there may be p other antennas outside the compact groupings, spaced $\geq D$ apart from each other and from the compact groups. This model appears to be a possible way to ensure a high percentage of 'short' baselines. We shall see that there are severe limitations on this model's ability to deliver a high percentage of 'short' baselines.

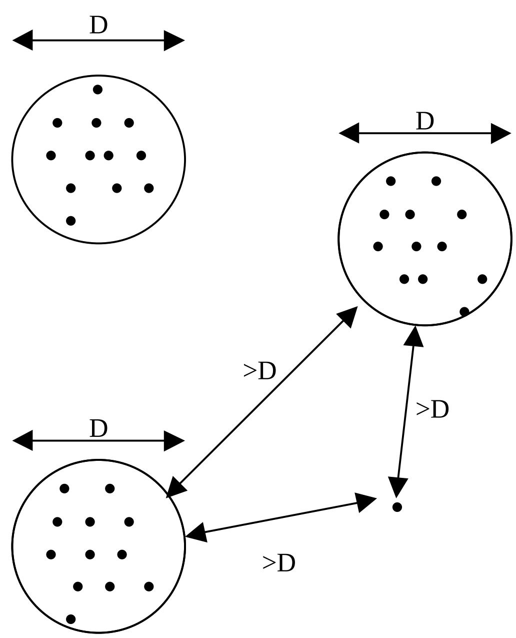

Figure 1: The model used here consists of compact groups of antennas of diameter D separated by >D, plus isolated antennas >D from other antennas. For simplicity the compact groups contain the same number of antennas.

Consequences of the first requirement with the model.

Assume, for the model described in the introduction, that there are *m* compact groups, each containing the same number, *n*, of antennas, and that there are *p* 'isolated' antennas. We can then calculate the number of 'short' baselines:

$$N_{<D} = m \cdot n(n-1)/2,$$

the total number of baselines:

$$N_{tot} = (m \cdot n + p)(m \cdot n + p - 1)/2,$$

and the fraction of 'short' baselines, *f*:

$$f = N_{<D} / N_{tot}.$$

The astronomers have specified that they want *f*=0.4. This gives us an equation which relates, *m*, *n*, and *p*. For example, if we specify *m*, *n* , we can solve for the value of *p* which yields the right value of *f* :

$$p = \left(-b + \sqrt{b^2 - 4c}\right)/2, \text{ where } b = 2mn - 1, \text{ and } c = mn(mn - 1 - (n-1)/f).$$

However, unless *c*<0, we will find that *p*<0, which cannot be realised. Thus for $n >> 1$, the model can only be realised for $m < \frac{1}{f}$; for *f*=0.4, we must have *m*≤ 2!

For large values of n, the expression for *p* can be written approximately:

$$p \approx mn\left(\frac{1}{\sqrt{mf}} - 1\right)$$

For $m = 1$ and $f = 0.4$, this yields $p = 0.58n$. In other words, 63% of the antennas should be inside the diameter *D*, and the remaining 37% outside that diameter

Figure 2 shows the resulting value of *p* for the two cases, *m*=1 and *m*=2, as a function of *n*.

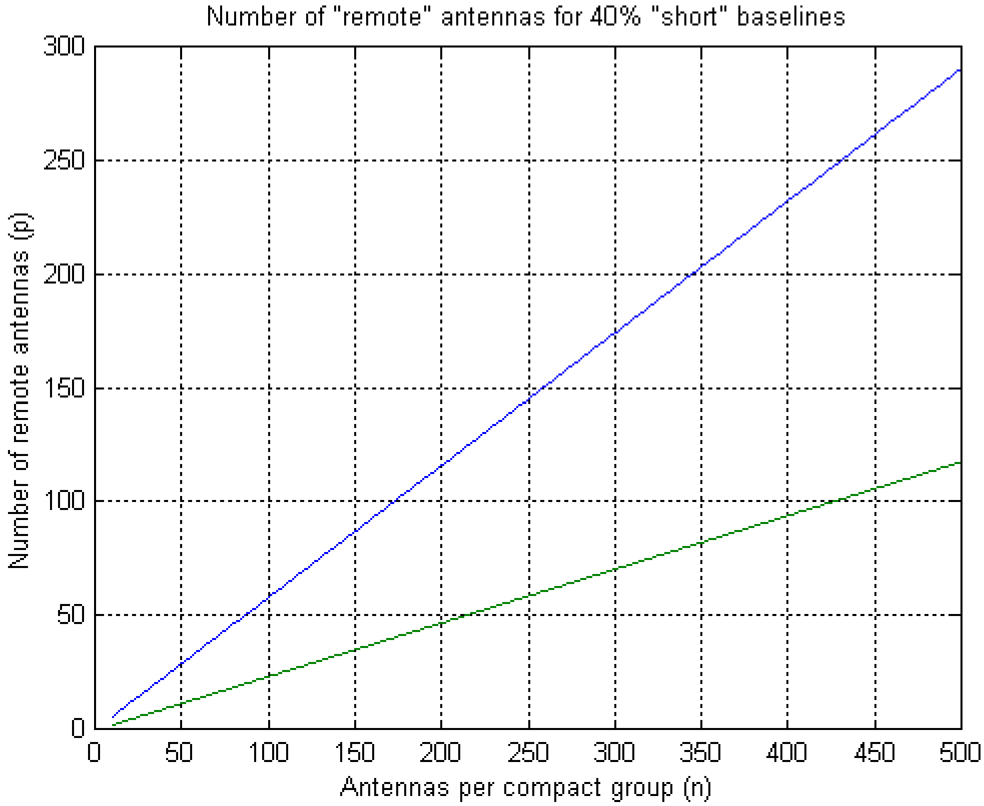

Figure 2: The consequent proportions of antennas inside and compact groups resulting from the requirement that 40% of baselines be short.

The consequences of the second requirement.

The model outlined earlier can be used to investigate the second requirement, that 80-90% of baselines should be short to medium. The same formalism is used, although the distance *D*, (who's value does not affect the argument) is much larger, and the fraction, *f*, of baselines longer than *D* is 0.8 or 0.9. The result is $m = 1$ and $p \approx 0.12n$ ($f = 0.8$) or $p = 0.06n$ ($f = 0.9$). In other words 89-95% of the antennas should lie inside this larger diameter, *D*, and only the remaining 11-5% outside.

Conclusion

If we consider a model of the SKA as described in the introduction, then we can only have a maximum of two compact groups of antennas if we respect the requirement that 40% of spacings should be short. Larger numbers of compact groups of antennas would result in fewer short spacings. The two-group model is probably not very attractive, because it concentrates intermediate spacings along the axis joining the two groups.

All this suggests a configuration for the SKA consisting of a single compact group of antennas containing 63% of the total number of antennas. Then a dispersed set of antennas out to a larger radius containing a further 26-32% of antennas, and finally beyond that radius, still more widely dispersed antennas, numbering only 11-5% of the total number of antennas.

More work needs to be done to investigate the effects of changing the details of the model, for example, by placing compact groups of antennas closer than *D.*