# Cost of an Imaging Correlator for the SKA

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## *Introduction*

This memo analyses the cost of FX correlators that meet the performance required by the SKA in terms of field of view. It is assumed that an FX correlator will be used and the cost will have two main component: the frequency transform and the cross multiply accumulates. It will be shown that the cost of the frequency transform depends only on whether the antennas are arrayed or not and the cost of the cross multiply accumulate depends on the degree of mosaicing.

## *Reference correlator*

The specifications for the SKA specify a 1-square-degree imaging beam at 1.4GHz and total correlator bandwidth of 4 GHz dual polarisation. There are two cases to be considered:

1. Use filled aperture antennas such as phased arrays, LAR, KAST and cylindrical reflector that have a beam sizes less than or equal to the 1 square degree
2. Use arrays of antennas.

The one-degree field of view antenna can be treated as the simplest case of the filled aperture antenna. A 190m$^2$ parabolic reflector gives a 1-degree field-of-view at 1.4GHz[1]. An SKA built from these has 5250 antennas. One way of generating the data for an image is to correlate all 5250 station in a correlator. This can be considered as the reference cost of a correlator. It has 5250 signal that must undergo a frequency transformation and the cross-multiply accumulation (XMACs) must be formed on 13 million baseline.

## *Filled aperture antenna stations*

For filled aperture antenna with areas greater than 190m$^2$, the 1-degree image is generated by forming a mosaic of a number of subfields. If the antenna area is increased by a factor $k$ then the beam area decreases by $1/k$. To achieve the one square degree imaging area each antenna must generate $k$ beams. However, the increase in antenna size has reduced the number of antennas by the same factor. Thus the total number of signals into the correlator is unchanged but is now composed of $k$ signal from $5250/k$ antennas. For filled aperture antenna the data transmission and filterbank load is independent of the size of the antenna as long as its field-of-view is less than 1 degree. In contrast, the cost of XMACs decreases in direct proportion to $k$. A separate correlator is now needed for each beam but has only $5250/k$ inputs. Thus this correlator is smaller than the reference

[1] Derived by scaling from 1.4Ghz beam size of the 64m Parkes multibeam, 14.4 arcmin, and the 100m Effelsburg antenna, 9.4 arcmin.

correlator by a factor of $1/k^2$. To form the full 1 degree image there are $k$ of these correlates so the total cost of the correlator is $1/k$ that of the reference correlator.

## *Antenna station arrays*

For a regular array of antennas, the costs increases because for a given collecting area the area of the main beam of the array decreases. If grating lobes are ignored then the decrease in main beam area gives a proportionate increase in the number of beams that must be processed to generate the required 1 square degree image. The increase in cost depends on the minimum elevation and to a small extent on the number of antennas in an array. The equation defining this increase in cost is derived in the appendix. The results are plotted below for arrays of 10, 100 and 1000 antennas

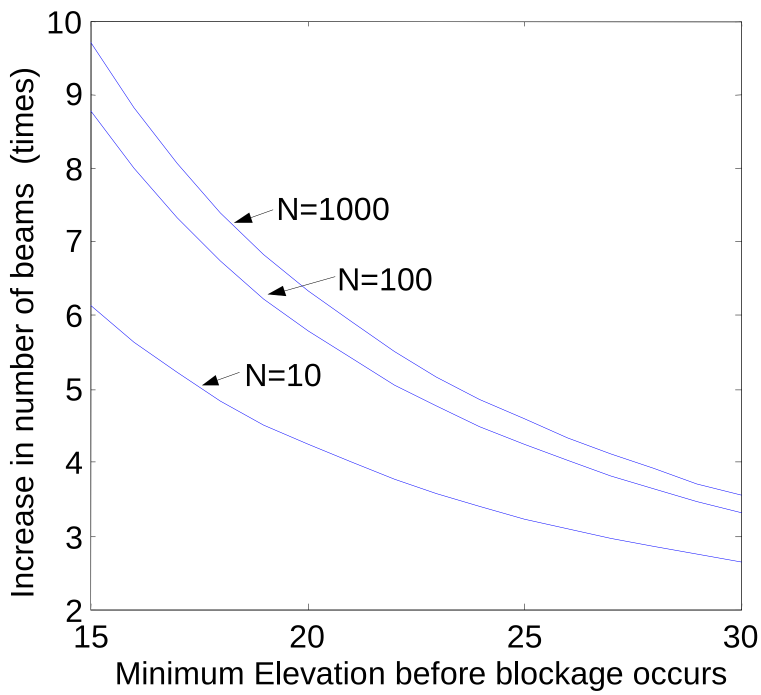


**Figure 1 Increase in number of beams needed to cover 1 square degree when a fully beamformed array is substituted for filed aperture antenna with an aperture efficiency of unity.**

It is seen that the cost of the correlator can increase by a factor of about 9 for minimum elevations of 15 degrees. If the effect of aperture efficiency η is included this increases to 11 for η=0.8. For a minimum elevation of 30 degrees the increase is about 4 after aperture efficiency is included. The increase in the number of beams that must be simultaneously processed means that the cost of the XMACs and filterbanks both increase by this factor.

If there are a small number of elements in the array there is reduction the cost increase because the fill factor of the array increases. When the number of elements in the array decreases to two and three elements it is better to use the signals from the antenna as separate inputs to the correlator (assuming filterbank cost are approximately equal to XMAC costs). For four to six elements in the array all the relevant factors need to be considered and for seven or more elements arraying and beamforming will normally minimise the correlator cost.

## *Conclusion*

Correlator costs are minimised if the number of SKA antenna stations is minimised and they are made from a single filled aperture antenna. Doubling the number of antenna station doubles the cost of XMAC but leaves the cost of filterbanks unchanged. For a given antenna station size using an array instead of a filled aperture antenna significantly increases costs. The cost increase ranges from a factor of about 4 with a minimum elevation of 30 degrees to 11 and more for elevations below 15 degrees.

## *Appendix 1 Beam width and average beam area of an array*

Consider an array of circular aperture antennas with the same total area as a single circular filled aperture antenna of radius R. For observations at zenith the array of antennas can have their edges touching and the minimum area configuration corresponds to hexagonal packing. For the limiting case the area covered by the array is greater by a factor equal to the ratio hexagon to a circle inscribed within it: 1.1. As the minimum elevation without blockage is increased the distance between elements of the array increase in direct proportion to 1/sin(elevation). Thus the total area of the array of hexagonal areas is ~1.1/sin(elevation) greater than the area of the filled aperture antenna.

If the array has a large number of elements arranged within a circular region then the diameter of the array is. This can be though of as placing each element of the array within a hexagon that has a width of √1.1.(R/sin(elevation)]. At the edge these expanded hexagons includes an area that extends past the edge elements. Thus, to a first approximation, the radius of the array should be reduced by the width of the hexagon. But half of the antenna is in this removed area so to correct for this the actual radius of the dish must be added. An example helps to show this. Consider a hexagonal array of 7 elements unit elements, when close packed the radius is 3. If the spacing between the elements is increase by 10 the new radius, to the edge of the outer elements, is not 3x10=30 but 21. This can be calculated as 30 for the expansion less 10 for the radius of the hexagon around the outer unit elements plus 1 for the radius of the of a unit element. If there N antenna in the array then the radius of a single antenna in the array is R/√N and that of the hexagonal area that contains it 1/ sin(elevation)) greater. Thus if θ is the minimum elevation without blockage then:

$$\text{Radius of array} \approx \mathrm{R}\left[\frac{\sqrt{1.1}}{\sin(\theta)} - \frac{1}{\sqrt{N}}\left(\frac{1}{\sin(\theta)} - 1\right)\right]$$

Where R is the radius of a filled aperture antenna of the same total physical area. Note this is a large N approximation. For arrays with a small number of elements accurate results will require a separate calculation for each geometry. Even so, the equation gives a good first approximation with the error being about 10% for 7 elements.

For a filled aperture antenna the beam width is approximately 1.09 λ/(effective diameter) where the effective diameter is the diameter of a circle of area equal to that of effective aperture. If the filled aperture antenna has radius R then the effective diameter is $2R\sqrt{\eta}$, where η is the aperture efficiency. Thus the beam width of a circular filled aperture antenna is approximately 1.09 $\lambda/(2R\sqrt{\eta})$.

For an array the beam width at zenith is 1.02λ/diameter. The factor of 1.02 arises if a uniform grading is used across the array, for a filled aperture antenna there will be a taper across the aperture and this factor is approximately 1.09. Using the array radius calculated above it is found that the ratio of beamwidth at zenith for a filled aperture and array of the same total area is:

$$\frac{\text{Beam width filled}}{\text{Beam width array}} \approx \frac{1.09}{1.02\sqrt{\eta}}.\left[\frac{1.05}{\sin(\theta)}+\frac{1}{\sqrt{N}}\left(1-\frac{1}{\sin(\theta)}\right)\right]$$
$$\approx \frac{1.12}{\sin(\theta)\sqrt{\eta}}.\left[1-\frac{0.95}{\sqrt{N}}(1-\sin(\theta))\right]$$

For the array, change aperture efficiency does not decrease the effective radius only the effective collecting area. Thus, there is no increase in beamwidth as occurs with a filled aperture antenna as the aperture efficiency decreases.

At elevations other than zenith the array will be foreshortened and this increases the beam area by a factor equal to 1/sin(θ). Assuming this increased beam area can be fully utilised and that observations take place uniformly over the sky at elevation greater than θ then the average beam area of the array is increased by:

$$\text{Average beam area increase} = \frac{\int_{\theta}^{\pi/2}\frac{\cos(\varphi)}{\sin(\varphi)}.d\varphi}{\int_{\theta}^{\pi/2}\cos(\varphi).d\varphi}$$
$$= \frac{\ln(\sin(\theta))}{\sin(\theta)-1}$$
$$\approx \sin(\theta)^{-0.45}$$

The approximation is accurate to better than 2% for θ greater than 12 degrees. Combining the two equations it is found that

$$\frac{\text{Beam area filled}}{\text{Average beam area array}} \approx \frac{1.25}{\eta\sin(\theta)^{1.55}}.\left[1-\frac{0.95}{\sqrt{N}}(1-\sin(\theta))\right]^2$$