Tunable Multi Notch Digital Filters
A MATLAB demonstration using real data

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1 Introduction

Many people are investigating a wide range of interference suppression techniques. Many of these:

- Adaptive filters
- Null Steering and Beam forming
- Parametric Signal Modelling
- Sub space tracking
- Eigen filtering
- Post-correlation cancellation

may make it possible to successfully remove interference that resides at the same frequency as a desired spectral line, without being harmful to the spectral line. Associated with this exciting possibility are some technical requirements:

- Extensive computational power
- Enough lags to completely sample the wavefront across the physical extent of the telescope array

For wide band observations these constraints can become prohibitive and there may be more appropriate and sensible ways to solve the problem.

2 Tunable Multi Notch Filters

Notch and band blocking filters provide an excellent way to deal with wide band applications, because in principle they only process the small part of the bandwidth that has interference. With the advent of digital signal processing it is now possible to implement filters for which the frequency and attenuation can be dynamically tuned. Figure 1 shows an example of the excellent attenuation that can be achieved. An added advantage is that the filters can be implemented with many bits per sample, allowing very strong signals to be removed, before reaching the 1 or 2-bit coarse quantisation used in most radio astronomy applications.

3 Application to real data

In order to test this out on some real data we used some base band data recorded at the CSIRO ATCA at a frequency of 2.4 GHz. The interfering signals present in this data are from a LEO satellite and are roughly 1000 times stronger than the receiver noise. Using this data in MATLAB we applied an algorithm that actively searches for strong interfering signals and tunes notch filters to the appropriate frequencies before applying them to the data. The data are processed in blocks of 1024 samples and notch filter frequencies are measured and retuned every 10 blocks which is every 64 microseconds. This should be fast enough to track any Doppler effects resulting from the fast motion of the LEO satellite.
The simple frequency finding algorithm works in the following way:
1. FFT a block of 1024 samples
2. Form power spectrum
3. Find maximum of power spectrum
4. Check if power is greater than threshold
5. Record frequency of maximum
6. Blank region of spectrum around maximum
7. Repeat steps 3-6 until frequencies for all signals above the threshold have been determined.

Three different implementations the notch filters and the results are discussed in sections 4-6.

Comments and suggestions welcome. Possible areas of application include:

- Continuum observations at ATCA – the 13cm band would be an ideal test case
- Dick Manchester’s wide band pulsar receiver

4 FIR notch filters at the full bandwidth

A simple FIR filter can be implemented as follows:

1. Find zeros on the unit circle in the Z-plane by

   \[ \text{zeros} = \text{radius} \times \exp(2\pi i (F_{\text{int}}/F_s)) \]

   where \( F_{\text{int}} \) is the frequency of the interference and \( F_s \) is the sample rate.

2. Find the complex conjugates of the zeros
3. Select the appropriate numbers of poles to be placed at the origin
4. Apply an inverse Z-transform to find the impulse response, which by definition gives the coefficients of the desired FIR filter.
5. Apply a scaling factor to ensure that the maximum gain is zero dB.

Figure 2: Block diagram of tunable multi notch implementation

This type of FIR notch filter has excellent attenuation at the desired frequency, but also has significant attenuation across other parts of the band. Moving the poles to a radius just inside the unit circle at the same angle as the zeros substantially reduces the attenuation at frequencies away from the desired notch. The example shown in Figure 1 fixed the poles at a radius of 0.9.

Figures 3 and 4 show power spectra of the data before and after application of these filters and the results are very encouraging.

Figure 3: Power Spectra of ATCA data on a linear scale. Top curve: Raw data, Bottom curve: After application of tunable multi notch filter. The top curve is raised by 10 for clarity.
5 Signal Matching Notch filters at the full band width

One of the problems with the notch filters derived in the previous section is that they do not necessarily match the spectral signature of the interference very well. An alternative approach is to derive a notch filter that is based on the spectral signature of the interference. This can be done as shown in Figure 5. For each interferer the data are mixed with a local oscillator at the frequency of the interferer, so that the interferer is moved to zero frequency. A low pass filter is then applied to remove all signals except the interferer. The data are then mixed back up to the nominal frequencies to provide an estimate of the interferer, which can then be subtracted from the raw data. As shown in Figure 6, the signal to be subtracted is an excellent match to the spectral response of the interfering signal. Figures 7 and 8 show the result of applying this filter to one of the interfering signals in the real data and averaging over 50 blocks of data. The suppression is not as good as the previous example and some noise is added into channels adjacent to the interference. Some further optimisation of this algorithm is required.
Figure 5: Block diagram of frequency matching notch filter implementation

Figure 6: Comparison of power spectra of raw data (top curve - raise by 5 dB for clarity) and signal to subtract.
Figure 7: Power Spectra of ATCA data on a linear scale. Top curve: Raw data, Bottom curve: After application of a single signal matching notch filter. The top curve is raised by 10 for clarity.

Figure 8: Same as Figure 7, but with a log scale. The top curve is raised by 10dB for clarity.
6 Multi-rate implementation of signal matching notch filters

The previous solution is quite expensive in terms of the number of multiplications required. The number of multiplications required may be substantially reduced by using multi-rate DSP techniques and the down-sampling (noble) identity, which allows the interchange of filtering and down-sampling operations. Figure 9 shows how the full band-width mixers and low pass filters of the previous implementation are replaced with a down sampler and low pass filters and half the number of mixers working at the down sampled data rate. Further reductions in computations may be achieved using polyphase structures, which we discuss later.

Figure 9: Block diagram of multi-rate implementation of a signal matching multi notch filter

How do we derive the tweaked LPF filter need here ?????