

# Cost Equation for the SKA

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1. General form of spreadsheet
2. Performance and cost parameters
3. Results: Cost vs antenna diameter and other parameters
4. Front-end model (Modified July 21, 2001)
5. Signal processing model

Appendix A - Notes on Signal Processing Costs for SKA (Added July 21,2001)

Appendix B - Cost Dependence When Field of View is Maintained (Added July 21, 2001)

**(Full Excel Spreadsheet is available as a file on SKA web sites: SKAcostEq1.xls)**



## SKA Cost Equation Input Parameters

Units: K\$US(2001), meters, GHz

Param	Array Performance Parameters	Default Value
Ns	Number of stations in array	100
M	Specified Figure of Merit, $M = A/T_{sys}$	20,000
B	Processed total continuum bandwidth	4.0
NI	Number of spectral line channels	16,000
D	Physical diameter of element (meters)	10.0
Ef	Aperature efficiency	0.70
Tant	Antenna noise temperature, $T_{ant} = 10$	14
Kln	Lna noise coefficient dependant upon	0.40
Tphy	Physical temperature of LNA	15
F	Frequency for system temperature sp	10
Nbn	Number of frequency bands	3
Kch	_ Number of separately digitized chan	4
Le	_ Average distance, element to station	0.50
Nbear	_ Beams per station	4

Param	Array Cost Parameters	Default Value
Cso	Fixed cost per station, land, civil,	300
Ka	Antenna cost coefficient,	0.10
X	Antenna cost exponent	3.0
Ccl	Cooling cost per antenna	20.0
Cfd	Average dual-polariz feed cost	2.0
Cln	Average LNA + mixer cost	0.8
Clo	LO cost	3.0
Cifo	Fixed IF cost per polarization	1.0
Kif	Dual IF cost per GHz of bandwidth	0.2
a1	_ Digitization coefficient	2.00
e	_ Digitization exponent	2.00
a2	_ Digitization constant	0.50
d	_ Tracking coefficient (per GHz)	0.72
f	_ Tracking constant	0.10
Kproc	_ Processing cost coefficient (per	0.48
Kmem	_ Memory cost (per word)	2.10E-04
kct	_ Corner turner cost coefficient (p	1.00E-03
Kcor	_ Correlator cost coef (per baselin	0.024
Kchip	Price per large FPGA chip	0.14

Color code:

Purple: correction of error since previous version.

Blue: change to value or formula since previous version.

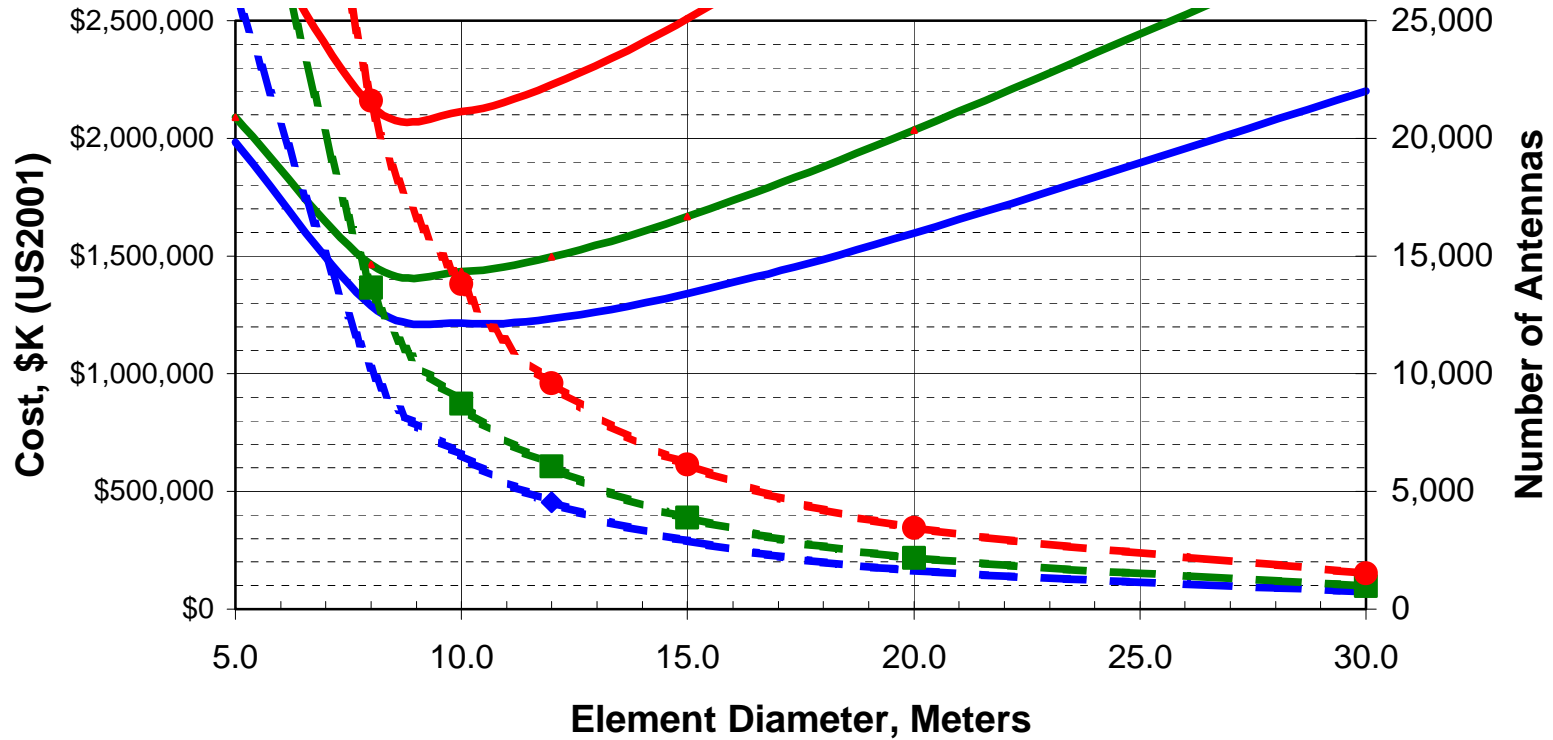
Red: variable whose value changes across this sheet

Green: variable whose value is different from previous sheet

Bold type: input parameter; normal type means value is calculated.

### SKA Cost vs Antenna Diameter for 3 Cooling Temperatures

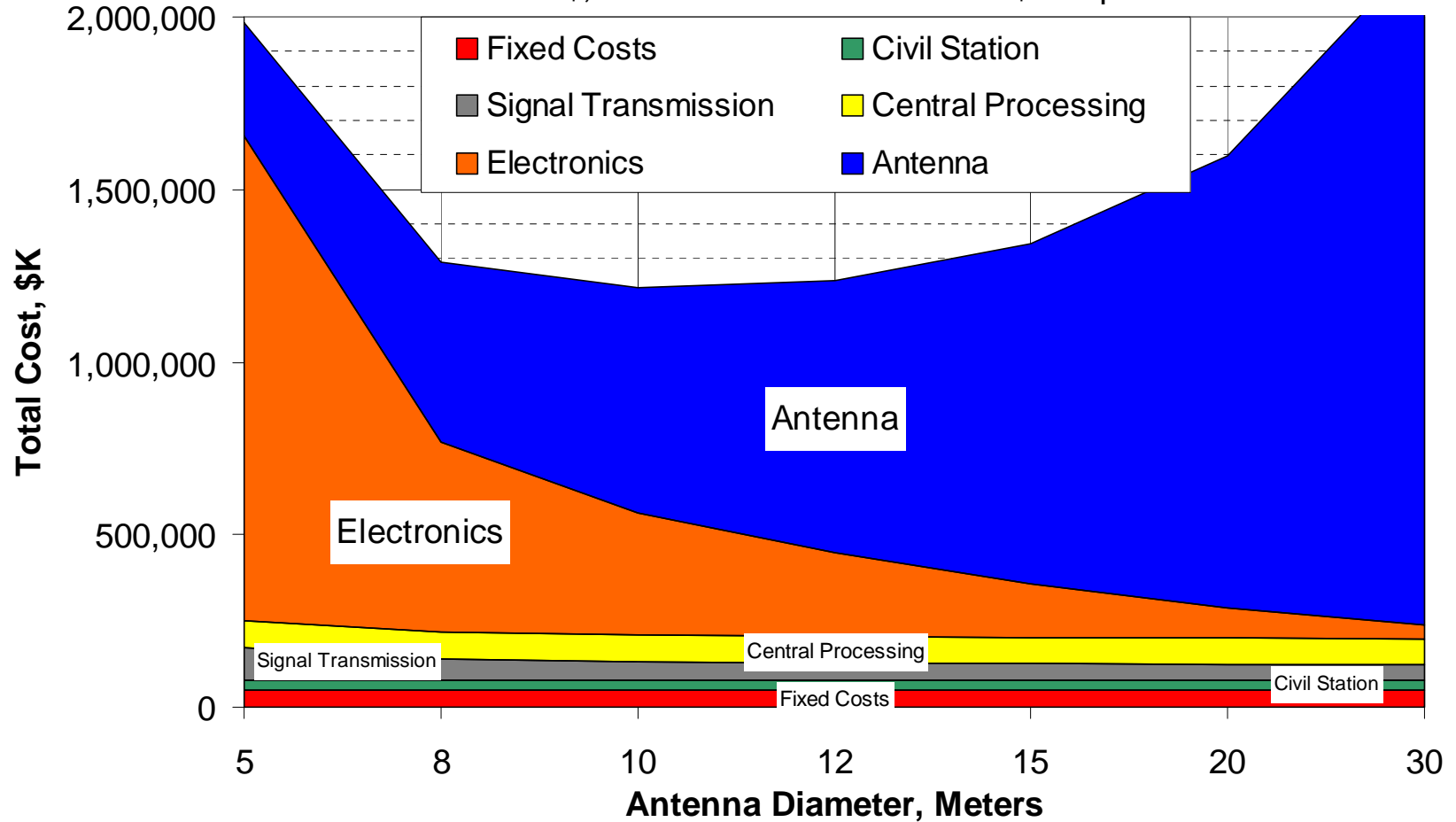
$A_{eff}/T_{sys} = 20,000$ ,  $A_{eff}=360,000$ ,  $T_{sys}=18K$ ,  $BW=4GHz$ ,  
 Antenna Cost =  $0.1D^3$  K\$, 2001 Electronics Cost = \$54K per Element



### SKA Cost Breakdown by Subsystem vs Antenna Diameter

$A_{eff}/T_{sys} = 20,000$ ,  $A_{eff}=360,000$ ,  $T_{sys}=18K$ ,  $BW=4GHz$ , 15K Cryogenics

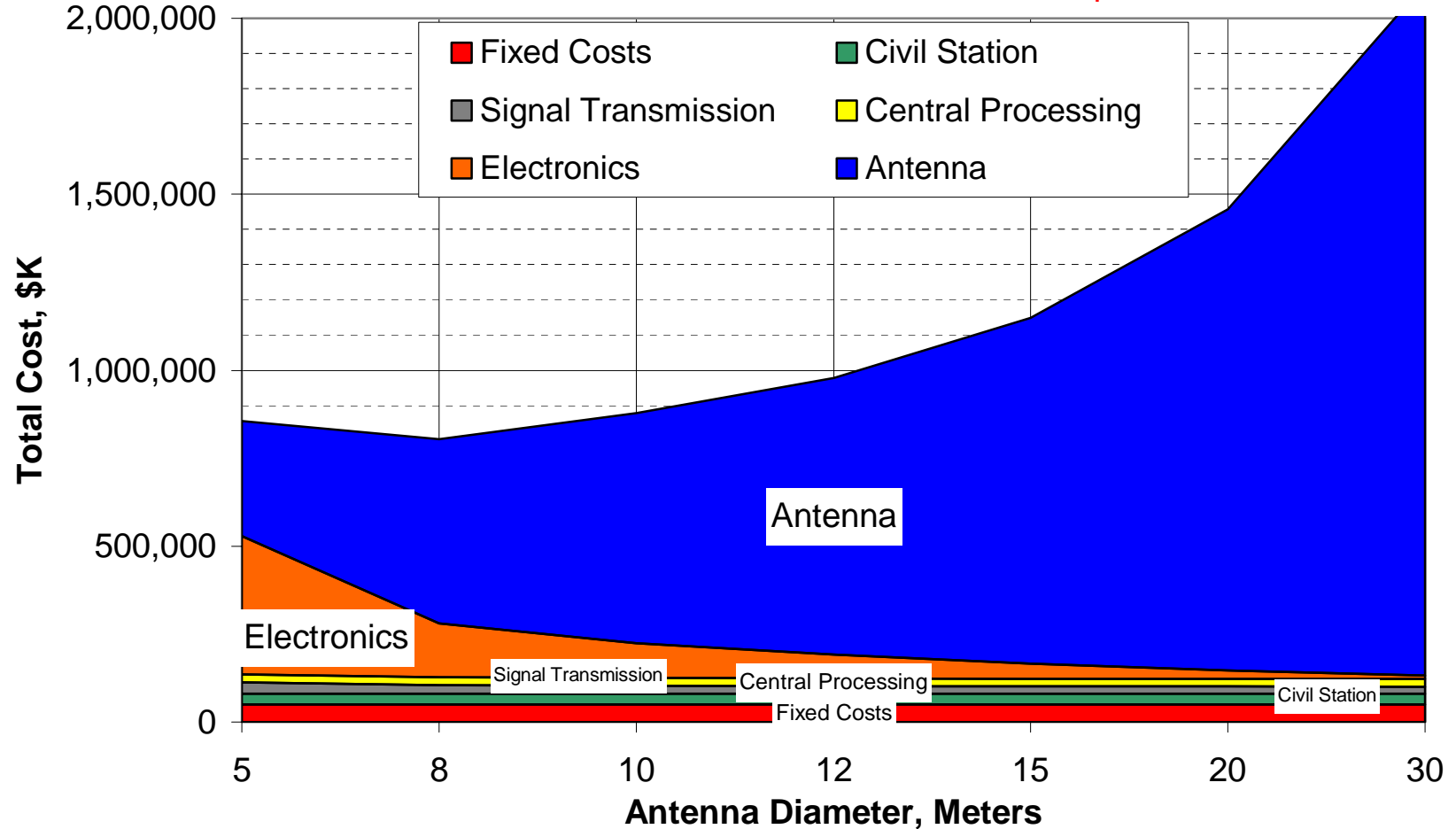
Antenna Cost =  $0.1D^3$  K\$, 2001 Electronics Cost = \$54K per Element



### SKA Cost Breakdown by Subsystem vs Antenna Diameter

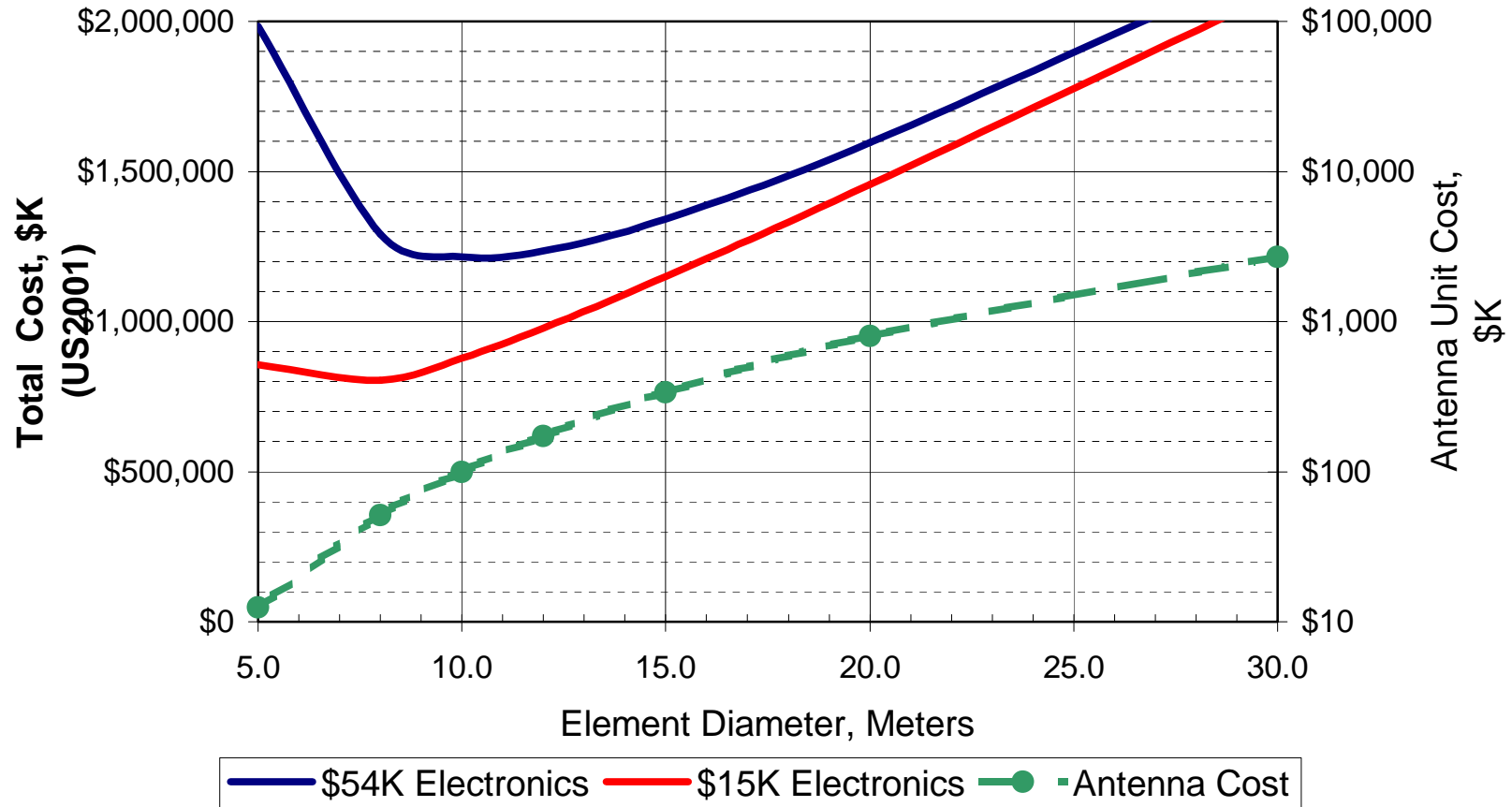
$A_{eff}/T_{sys} = 20,000$ ,  $A_{eff}=360,000$ ,  $T_{sys}=18K$ ,  $BW=4GHz$ , 15K Cryogenics

Antenna Cost =  $0.1D^3$  K\$, 2010 Electronics Cost = \$15K per Element

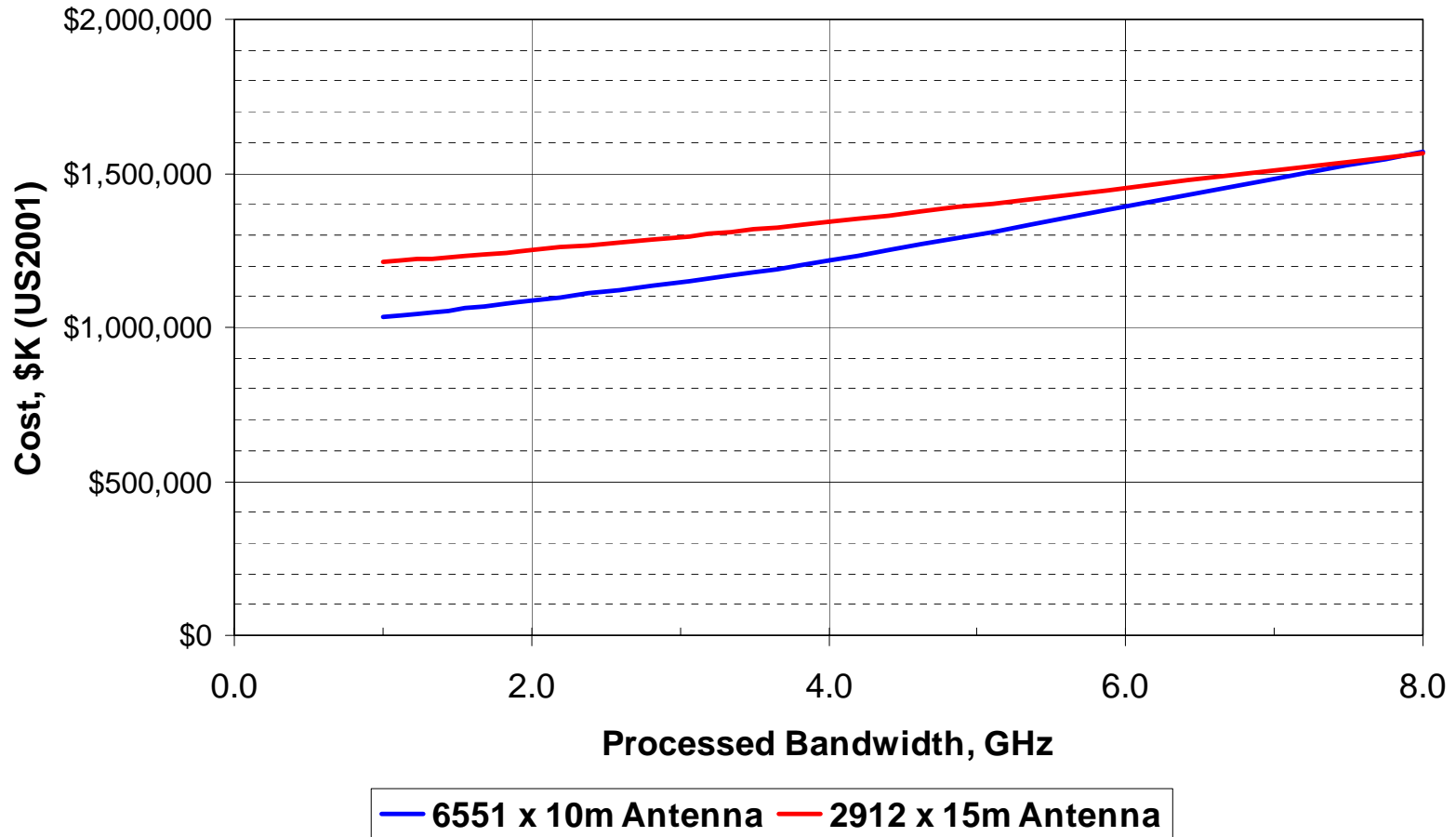


## SKA Cost vs Antenna Diameter

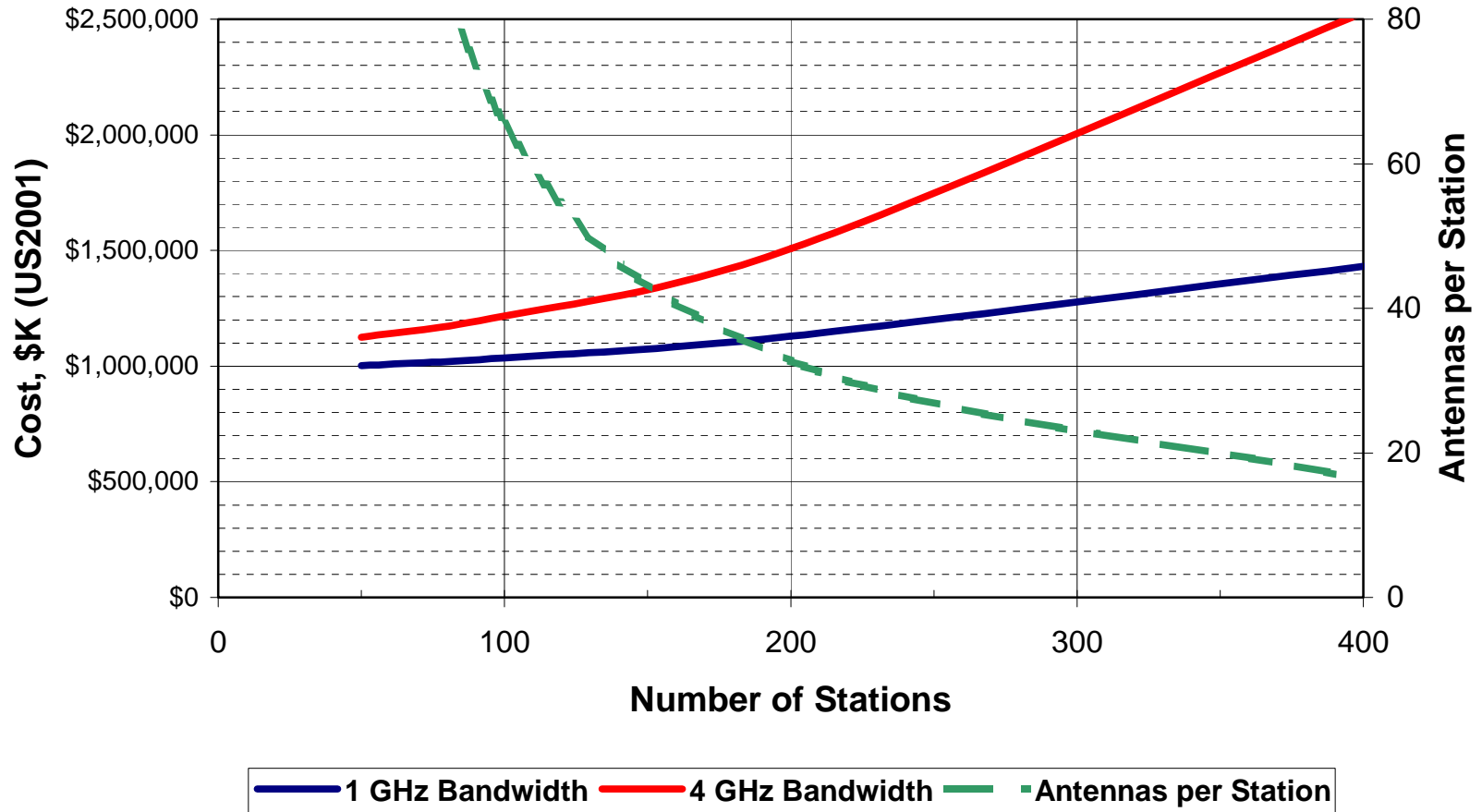
**Compares Current and Projected (2010) Electronics Costs**  
 All for 15K cryogenics, 4 GHz BW, 100 Stations, A/T = 20,000  
 Antenna Cost =  $0.1D^3$  \$K



**SKA Cost vs Processed Bandwidth for  
Two Antenna Diameters  
All for 15K Cryogenics**



**SKA Cost vs Number of Stations for  
Two Processed Bandwidths**  
All for 15K Cryogenics, 10m antennas. A/T = 20,000



## Front-End Cost and Performance Model

Cooler Type	Physical Temp	Cooler Cost, K\$	Noise @ 10 GHz	Noise @ 50 GHz	Noise @ F GHz
None	300	1	40	200	$4 \cdot F + 16$
Peltier	180	3	24	120	$2.4 \cdot F + 10$
Pulse or Klemenko	60	6	10	50	$1.0 \cdot F + 4$
Gifford-McMann	15	20	4	20	$0.4 \cdot F + 1$

### Total Front-End Cost, FE\$

$$\begin{aligned}
 \text{FE\$} &= \text{Cooler\$} + 3(\text{Bands}) \cdot 2(\text{Polarizations}) \cdot \text{LNA\$} \\
 \text{LNA\$} &= \$0.8\text{K} \cdot (\text{Fmax}/10)^{.3} \\
 \text{i.e. FE\$} &= \$11.9\text{K} \text{ for Pulse Cooling with Fmax} = 20 \text{ GHz}
 \end{aligned}$$

### Total Electronics Cost, EL\$, per Antenna

$$\begin{aligned}
 \text{EL\$} &= \text{FE\$} + 3 \cdot \text{Feed\$} + \text{LO\$} + \text{IF\$} + \text{Fixed\$} \\
 \text{Feed\$} &= \$1\text{K}, \quad \text{LO\$} = \$5\text{K}, \quad \text{IF\$} = 2 \cdot (0.5 + 2 \cdot \text{BW}), \quad \text{Fixed\$} = \$1\text{K} \\
 \text{i.e. EL\$} &= 11.9 + 3 \cdot 1 + 5 + 2 \cdot 4.5 + 1 = \$29.9\text{K}
 \end{aligned}$$

### Total System Noise, Tsys

$$\text{Tsys} = \text{Tlna} + [10 + 4 \cdot (\text{F}/10)] \quad \text{F} > 1 \text{ GHz, Not Including Antenna Losses}$$

# Signal Processing Cost Model

- **EACH ANTENNA**

1. Digitization

$$C_d = [k_d (B/N_{ch})^e + c_d] N_{ch} \quad (\text{exponent } e \text{ is approximately } 2)$$

Cost per channel increases *faster* than bandwidth => more channels

- Presently expensive -- total for array is more than the correlator!

2. Tracking

- Primarily delay and phase tracking of sources
- Per antenna per beam -- multi beaming is probably needed to mitigate small FOV of station beam.
- All-digital implementation assumed

- **EACH STATION**

1. Summation to form phased-array, per beam

- Multi-beaming probably needed due to small station beam

2. Transmission to central processing facility

- available bandwidth shared among beams.

- **CENTRAL PROCESSING (CORRELATION)**

1. Per antenna section

- filter bank for  $L$  spectral channels
- cost grows as  $B \log(L)$  if minimum-memory FFTs are used

2. Interconnection section

- can dominate cost for large  $N$
- natural architecture grows as  $B N^3$
- clever architecture grows as  $B N \log(N)$

3. Cross correlation section

- grows as  $B L N^2$  (note  $L$  dependence, even for FX architecture)

# Appendix A - Notes on Signal Processing Costs for SKA

L. D'Addario 2001-Jul-04; revised Jul-07, Jul-12

## ANTENNA-ELEMENT SIGNAL PROCESSING COSTS

### 1. Digitization

In present technology, this is an expensive element of the overall signal processing. We estimate about 10k\$ per element for 4 GHz bandwidth, which becomes 45M\$ for 4500 elements (100 stations of 45 elements each). This is about equal to the cost of the central signal processing (correlator) and 3% of the total cost of the array, based on our current estimates for one reasonable case.

The cost of digitization rises faster than the bandwidth of each separately-digitized channel, and appears to go approximately as the square over the range 10 to 1000 MHz for presently available commercial devices. Therefore, it is advantageous to break up the total bandwidth prior to digitization into as many channels as practical, provided that performance does not suffer. In our cost estimates, we have usually used at least 4 channels (to allow for 2 polarizations at 2 independent sky frequencies) and kept the channel bandwidth at 1 GHz or less.

### 2. Tracking

For each signal channel of each antenna element, it is necessary to provide for time-varying adjustment of the delay and phase so as to track a desired direction on the sky. In our cost estimates, we assume that this is accomplished digitally. The cost of computation and control for this purpose appears to be negligible (<1% of the total in all cases studied).

The elements are clustered into  $N_s$  stations and the signals are combined so that each station is a phased array. This minimizes the cost of the central processing, which varies as approximately  $N_s^2 \log N_s$ . However, it also reduces the field of view on the sky from the size of the element beam to that of the phase array (station) beam. To mitigate this effect, the phase and delay tracking can be duplicated at each element so as to produce several simultaneous station beams. If the bandwidth per beam is fixed, then the total bandwidth increases along with the cost of signal transmission and central processing. Whether the increased sky coverage is worth the extra cost depends on the scientific goals.

### 3. Signal Transmission.

The bandwidth  $B$  needs to be transmitted several times: Element to element's center; element center to control center; correlator F section to "corner turner"; and corner turner to correlator X section. In all cases we use the same coefficients for transmitter/receiver cost and installed fiber cost. Average fiber length is taken to be 0.5 km from elements within a station, 100 km from each station to the center, and zero ( $\ll 1$  km) within the correlator. Sharing of parts of the transmission routes among stations could reduce this, but the number of stations has to exceed 350 before the transmission cost exceeds 5% of the total; at 100 stations it is around 1.5%

## CORRELATOR COST DEPENDENCIES

We assume that it's important to cross-correlate the signals from all stations, over the full bandwidth of each. The cost of this is proportional to the total bandwidth, regardless of whether that bandwidth represents a single channel on the sky or several independent channels (beams). There is a weak dependence on the number of channels, but this is insignificant if that number is small compared with the final number of subchannels into which the total bandwidth will be analyzed.

The central processing separates naturally into a per-station part and a per-baseline part whose costs are proportional to  $BN$  and  $BN^2$ , respectively. Each cost term has additional factors that depend on the architecture and on the spectral resolution, as well as a coefficient that depends on the implementation technology. For the large- $N$  case, the baseline term dominates the station term. For sufficiently large  $N$ , the baseline electronics will not fit into a single chip, nor a single circuit board or small set of boards, and therefore it must be partitioned into some number  $K$  of separate packages, each of which accomplishes  $1/K$  of the processing. There are several ways to do the partitioning [1], and this determines the top-level architecture of the system. We concentrate here on the so-called "FX" architecture, where the bandwidth  $B$  from each antenna is partitioned by frequency into segments of bandwidth  $B/K$  and each element of the baseline part processes one such segment for all antennas.

The available technology determines how much processing can be fit into one package, and thus how many packages  $K$  are required and how small each bandwidth segment  $B/K$  must be. We set  $K = tBN^2/2$ , where  $t$  is a technology-dependent coefficient that gets smaller as circuitry becomes denser and/or faster.

The segmentation is done in the station part, which we assume to consist of one package per antenna. Therefore,  $C=NK$  interconnections are needed between different packages in order to get the signals from the station part to the baseline part. Substituting, we see that  $C = tBN^3/2$ . Because of this  $N^3$  dependence, for sufficiently large  $N$  the cost can easily be dominated by the interconnections, even if  $t$  is small. This situation can be substantially improved by imposing an additional device between the station part and the baseline part whose purpose is to re-order the data; it is called a "corner turner" [2]. Then the number of interconnections is reduced from  $NK$  to  $N+K$ , where  $N$  are needed from the station part to the corner turner, and  $K$  from the corner turner to the baseline part. Unfortunately, the corner turner itself is potentially a very large machine whose obvious implementation involves  $NK$  internal interconnections and data storage proportional to  $NK$ , hence its size grows as  $N^3$ . But a much more efficient implementation exists [2][3] that reduces it to a size proportional to  $N^2 \log N$ , in a manner similar to the FFT's reduction of the number of operations in a discrete Fourier transform of size  $L$  from  $L^2$  to  $L \log L$ .

In addition, a new architecture for the baseline part [4] allows it to be organized so that not all  $K$  segments need to be connected directly to the corner turner. This allows the corner turner size to be grow only as  $N \log N$ .

It may be desired to analyze the bandwidth into many more than  $K$  subchannels, say  $L \gg K$ , for spectroscopy. Then each of the  $K$  segments processes  $L/K$  subchannels. It is often suggested that the FX architecture allows  $L$  to be increased arbitrarily at negligible additional cost. In the station part, filter banks based on the FFT can be used so that the cost increases only as  $\log(L)$ ; then, unless  $L$  is extreme, the cost of the station part remains small compared to the interconnections and baseline part. The interconnection cost is independent of  $L$  for a given  $B$  and  $N$ . In the baseline part, the computation rate (for cross multiplication and accumulation) is also independent of  $L$ , since the data rate of each subchannel goes as  $1/L$ . However,  $L$  separate accumulators are needed per baseline, so storage requirements increase as  $L$ . Depending on the cost of storage compared with computation, this may be important. Usually it is not possible to take advantage of the low cost per bit of large RAM because efficient use of the computational hardware requires many multiply-add operations to be done in parallel at high speed, leading to many short memories rather than few large ones, and requiring them to be integrated on the same chip with the computational elements. This can cause the accumulator storage to be the dominant cost, so that the overall cost is nearly proportional to  $L$ . The dependence is then nearly the same as in the XF architecture, where the costs of both computation and storage are proportional to  $L$ . Indeed, this would make the absolute cost of the XF architecture smaller because the individual multipliers and accumulators are considerably less complex.

All of this is summarized in the chart on the last page of this document.

## REFERENCES

[1] L. D'Addario, "Correlators: General design considerations." ATA Memo No. 24, 2001-Mar-13.

[2] L. Urry, "A corner turner architecture." ATA Memo No. 14, 2000-Nov-17.

[3] L. D'Addario, "Generalization of the memoryless corner turner to the non-square case." ATA Memo No. 22, 2001-Mar-16.

(All of the above are temporarily available via  
<http://astron.berkeley.edu/ral/ATAImagerDocs.html> .)

[4] L. Urry, this conference.

## JUSTIFICATIONS OF COST COEFFICIENTS

- a1,e digitizer cost parameters, fit to prices of commercial ADC chips
- a2 digitizer cost constant: guess at costs of downconversion, anti-alias filtering, circuit board, \$500. This can't be too far off.

d,e tracking coef, const: One XV300E should do 2 channels at 100 MHz, costs \$144; based on preliminary design for ATA. So 722\$/GHz. Constant term, \$100, is just a guess to cover packaging and power.

Kproc Xilinx XV300E, 3072slices/1929perFFT1024inPlace -> 1.5 radix4 butterflys=3 radix2, \$144/3= \$48 ea., @100 MHz -> 480 \$/butterfly/GHz

Kmem Xilinx XV300E:

$144\$/ (2\text{mem}) * (32\text{b/word}) / (131072\text{b block RAM}) / 100\text{MHz} = 0.0001758 \text{ k\$/word/GHz}$   
where half of chip cost is allocated to memory

Kcorr Xilinx XV300E: 3072slices/51perMAC = 60 MACs, 100 MHz each, \$144/chip  
 $144\$/ (2\text{comp}) / 60\text{MAC} / 100\text{MHz} = .0120 \text{ k\$/MAC/GHz}$   
where half of chip cost is allocated to computation

Kinfr Total chip cost/\$144 -> chip count. 16 chips/board, \$160 for board, rack, power.

Check: Total central processing for B=0.2, Ns=350, NI=2048 is 5.6M\$ -- ATA case

B=16, Ns= 64, NI=8192 is 58.7M\$ -- ALMA case

The ATA result is about 4x higher than the independent cost estimate (hardware only) made for the proposal. That proposal used future FPGA prices that are indeed about 4x lower per unit performance than the present prices used here; with this correction, the agreement is excellent. The ALMA result is also about 4x higher than its presently projected actual cost, but its architecture is quite different than assumed here.

# Correlator Cost Model: FX Architecture

	<u>Filtering</u>	<u>Interconnection</u>	<u>Correlation</u>
<b>Computation</b>	$\alpha B N \log L$	$\beta B N \log N$	$\alpha B N^2 / 2$
<b>Storage</b>	$\chi B N L$	(control only)	$\delta B L N^2 / 2$

Where:

$B$  is the total processed bandwidth

$N$  is the number of stations

$L$  is the number of spectral channels

Greek symbols are cost coefficients, dependent on technology.

Correlation does not include cross-polarized products.

## Appendix B - Cost Dependence When Field of View is Maintained

LRD, 2001-Jul-12

The analyses presented at the conference considered the cost dependence on various parameters, including antenna size, front end cooling temperature, and electronics costs, while keeping the number of stations constant. This required varying the number of antennas per station in order to have enough collecting area to meet the  $A_e/T_{sys}$  requirement. Formally, the number of phased array beams was kept constant at 4. But the cost would be nearly unaffected if the number of beams were larger, provided that the processed bandwidth  $B$  is shared among all the beams.

Here we consider relaxing that restriction.

If the antennas are clustered into stations at  $N_s$  each, and if they are close-packed within a station, then each station beam is about  $1/N_s$  the size of the primary antenna (element) beam in solid angle. So if we still want to image the whole primary beam, we will need  $N_s$  phased-array beams. I put this into the spreadsheet to look at the dependence on  $N_s$ . Rather than sharing  $B$  among the beams, I fixed the bandwidth/beam  $b$ , making the total bandwidth  $B = 2 b N_s$  (2 for polarization). Naturally this can lead to very large  $B$  and high cost for transmission and central processing, so I considered values of  $b$  only from 0.1 to 0.5 GHz.

The result is given in the accompanying figure. It can be seen that a shallow minimum in the total cost occurs at  $N_s = 200$ . Above this, the slow increase in total cost is caused by rapid increase in central processing (correlation) cost, which reaches 33% of the total at 400 antennas per station and 500 MHz bandwidth. Below the minimum, the correlator cost continues to decrease (in spite of the larger number of beams required), but the total cost is strongly affected by the processing required at each station to form many beams, including both digitization (assumed separate for each beam) and tracking.

**SKA Cost vs Number of Stations for  
Beam-filled case, 0.1 and 0.5 GHz/beam**  
All for 15K Cryogenics, 12m antennas, 4550 total, A/T = 20,000

