Detection of the Incoherent Gravitational Wave Background with Pulsar Timing Arrays

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With credit to George Hobbs, Rick Jenet and Dick Manchester, plus Simon Johnson, David Champion and Barney Rickett, all of whom have tried to teach me something about pulsars.
Any failures of my understanding are their fault.

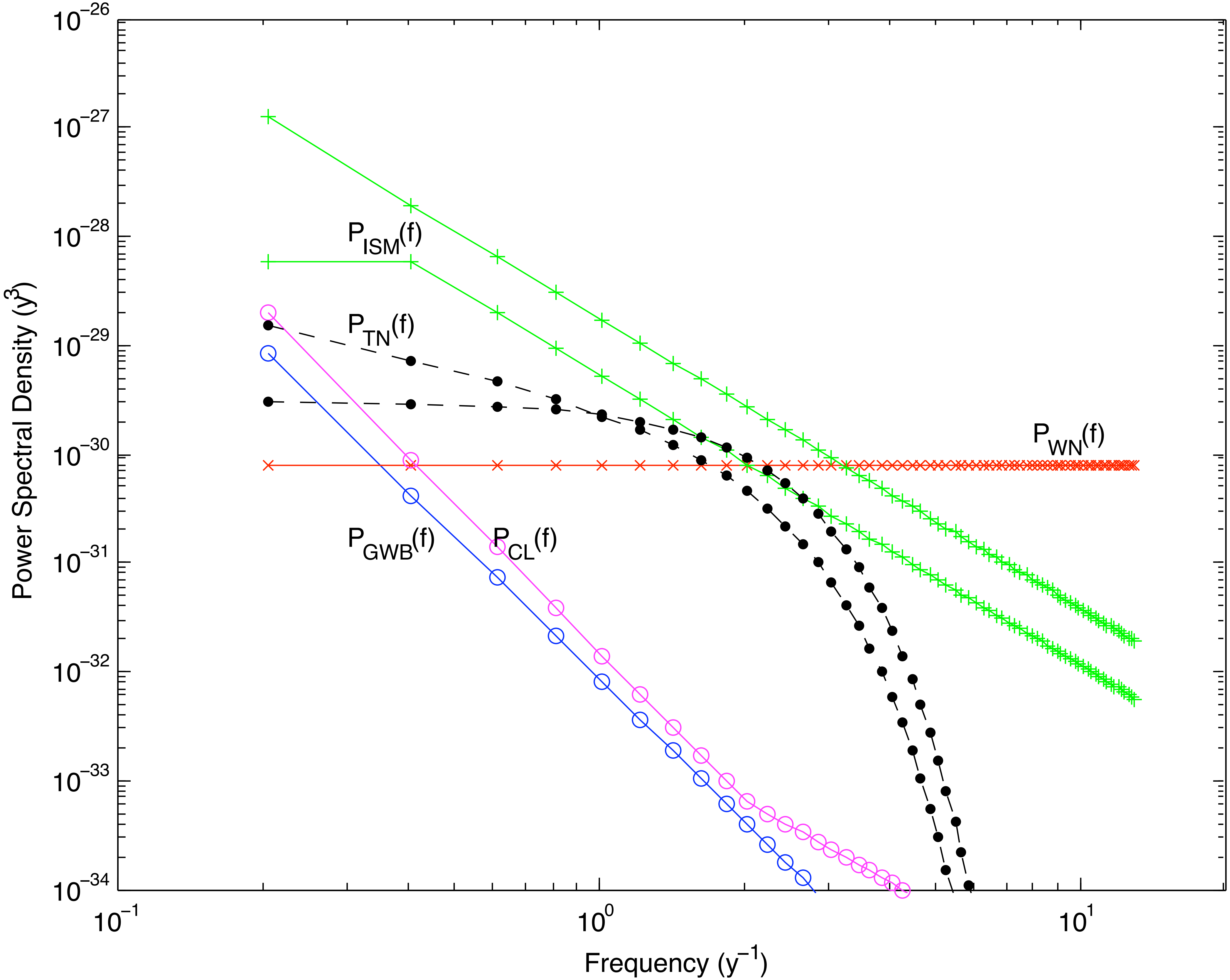
"Pulsar timing in a nutshell"

Measure the TOA, average over time and frequency, then correct for the motion of the Earth and the pulsar. The *residuals* are measurements of pulsar rotational phase. They contain:

- unmodeled physics of the Earth and the pulsar motion,
- receiver noise and RFI (white)
- clock errors (quite red),
- calibration errors, particularly polarization (mostly white),
- interstellar-plasma mean-density fluctuations (red),
- interstellar-plasma scattering (white),
- pulsar rotational noise (pink),
- the effects of gravitational waves (quite red).

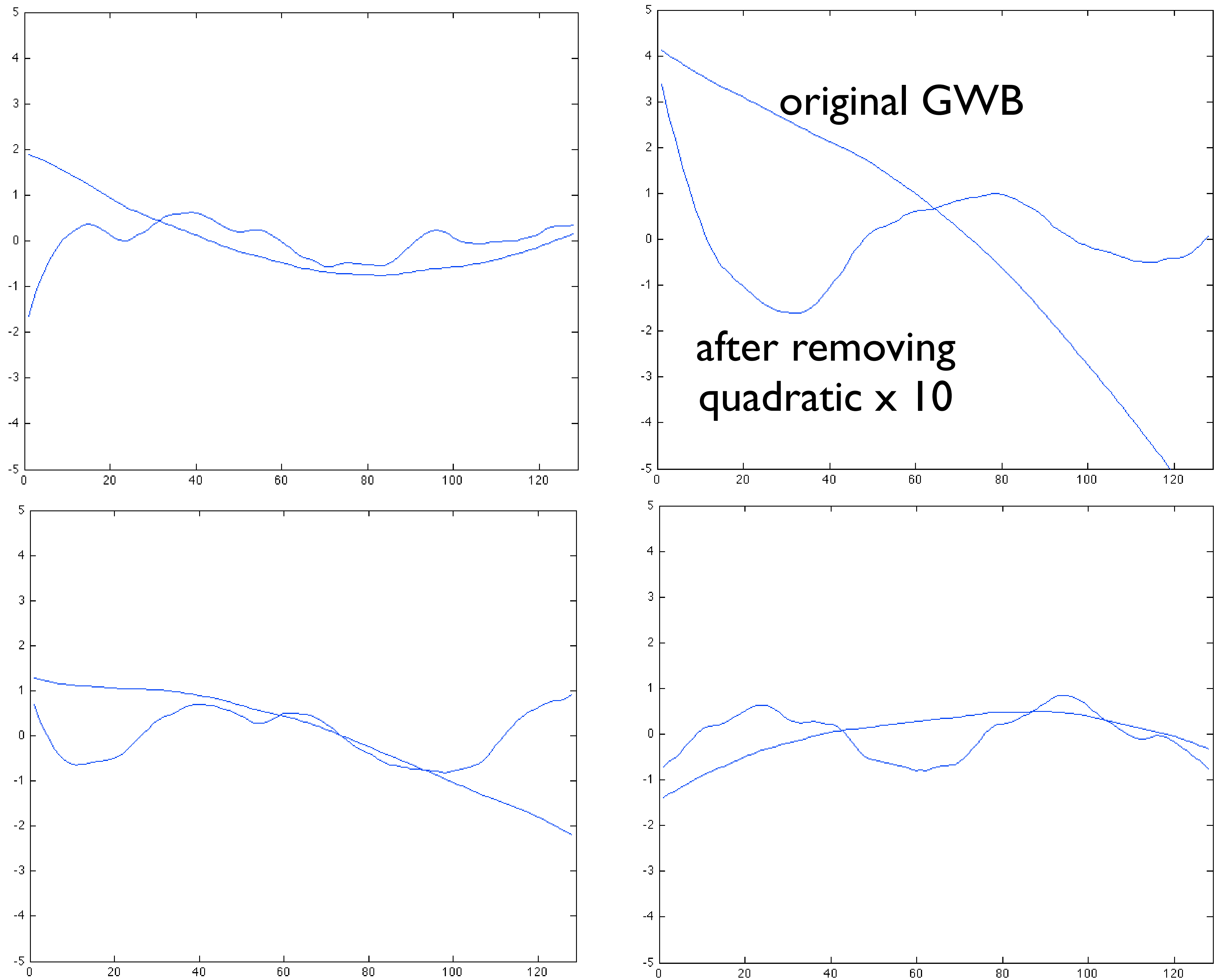
Incoherent GW background may be largest GW effect, its spectrum is $P_{GW}(f) = (A^2/12\pi^2)\ f^{-4.33}$.

Spectra of GWB, ISM, Clock, Receiver and Timing Noise



Biweekly observations with A=10^{-15} y $^{-2/3}$ and 100 ns rms noise for 5 years, ISM spectra are model fit to observations of You et al for J1939+2134 (upper) and J0437-4715.

Fitting a timing model for the pulsar period and period derivative is equivalent to removing a quadratic polynomial from the TOAs



what does this do to the power spectra?

The Effect of Quadratic Removal

Quadratic removal is a linear operator. In fact *any* linear least squares modeling is a linear operator. The model is: $D = M P + E$ and the LSQ solution is $\tilde{P} = (M^T M)^{-1} M^T D$. The Residual is

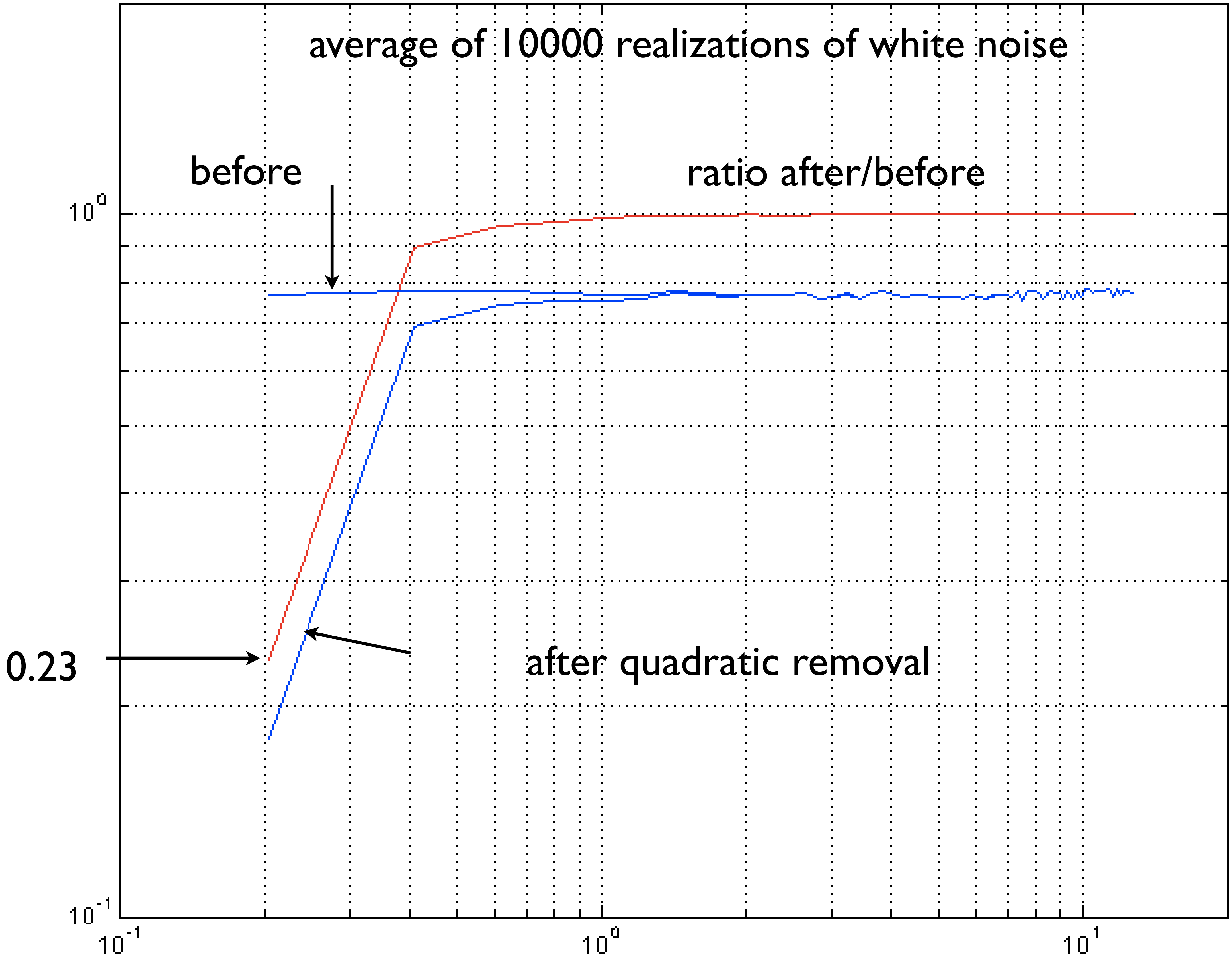
$$R = D - M \tilde{P} = (I - M (M^T M)^{-1} M^T) D = Q D$$

The matrix Q is not a filter, but it is linear. This is very important because superposition holds. For quadratic removal Q has a 3-dim null space because any function of the form $f(t) = a + b t + c t^2$ is reduced to zero. This null space cannot be recovered. e.g. if $n = 128$, then Q has rank = $125 < 128$ and thus does not have an inverse.

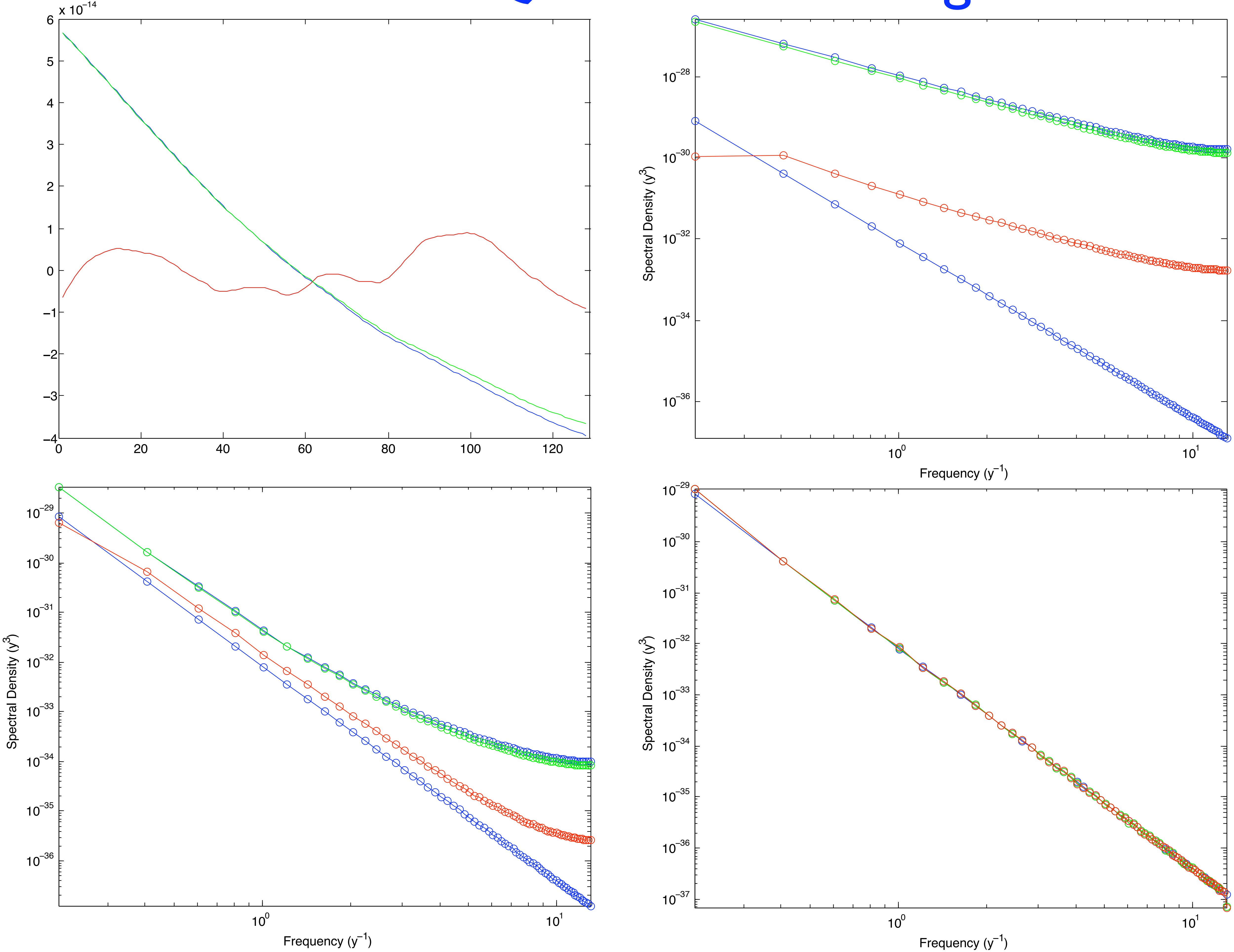
Thus a unique solution of the form $D = Q^{-1} R$ does not exist. But solutions for D^* which satisfy $R = Q D^*$ with various constraints exist. We need to examine the power spectra of these solutions.

We also need to study the effect of Q on the spectrum of white noise. Because of superposition we can study this independently of the effect on the signal.

The effect on white noise is easily seen. Since it is linear we can correct the spectrum to rewhiten the noise.



Effect of Q on the GWB signal



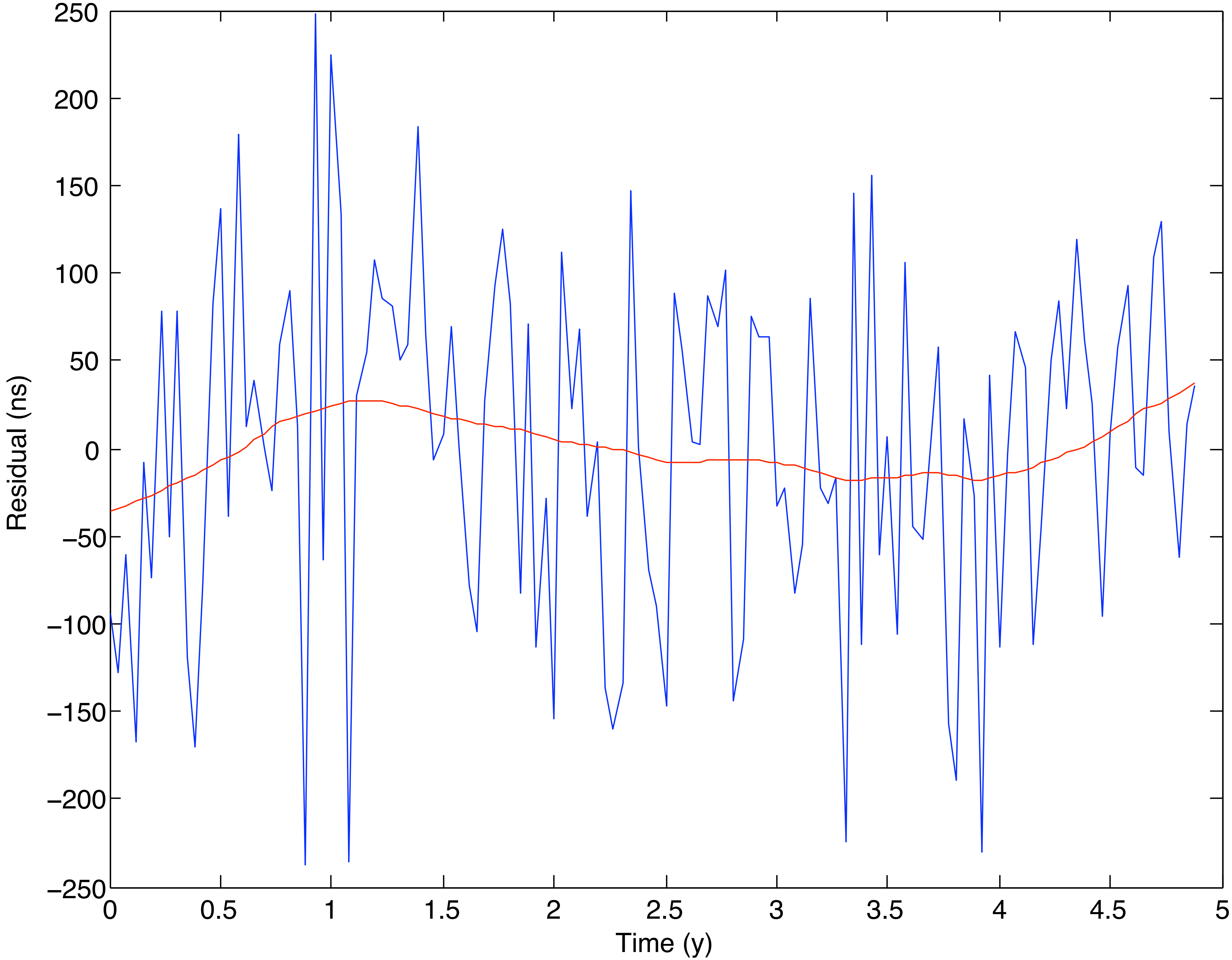
To Recover the Fourier Transform Exactly

Instead of using the Fourier tranform as usual, we perform a linear LSQ fit for the Fourier coefficients, i.e. D = F P + E where P is the vector of Fourier coefficients and the columns of F are dc, fundamental, first harmonic, etc.

However we don't have the original data, we have R = Q D, so we model R = Q F P + QE and solve this for P which are the spectral estimates for D not R.

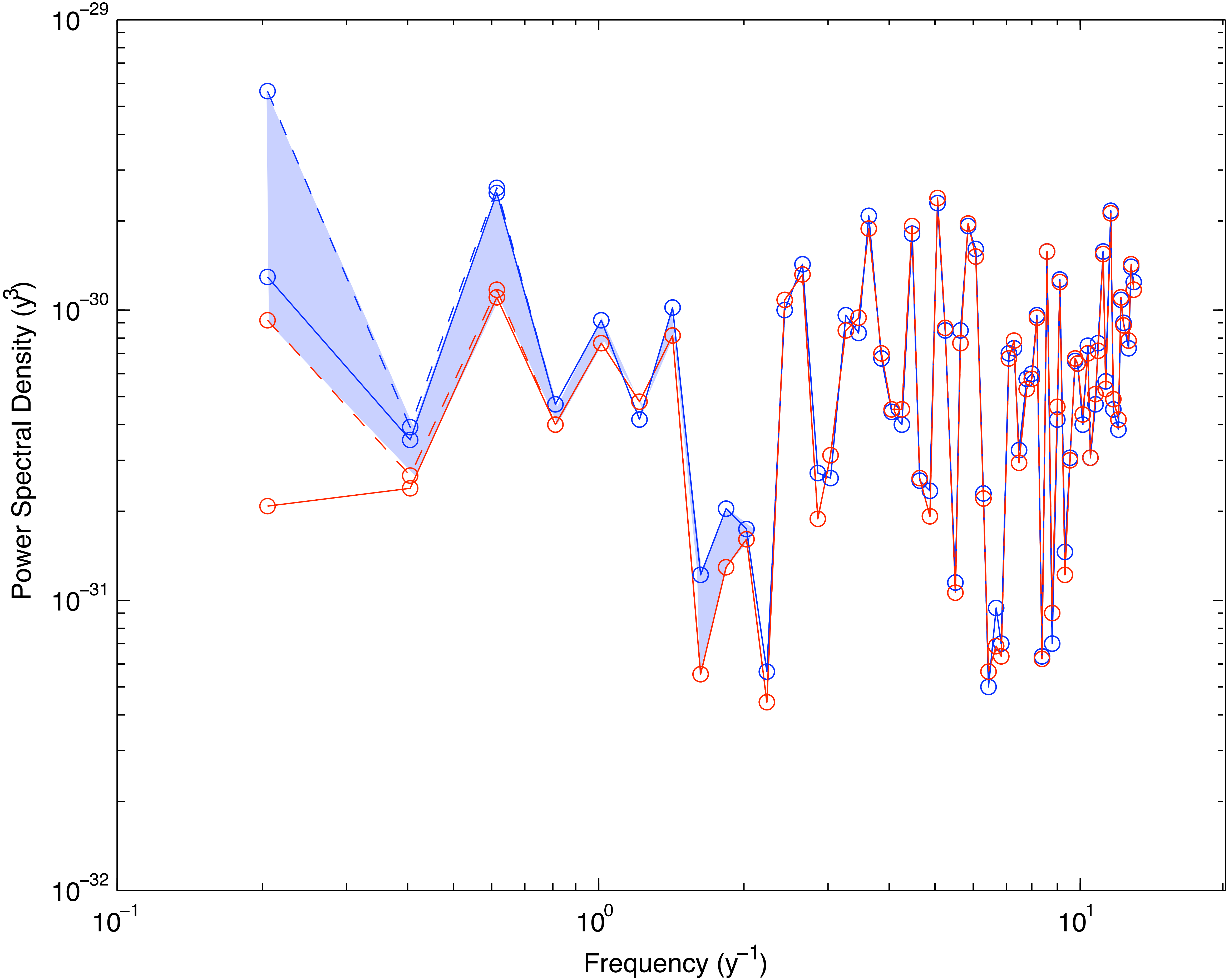
In this way we can solve for N-3 spectral estimates directly. We really only need the first few spectral coefficients so this is fine.

Example of Simulated Residual Including White Noise



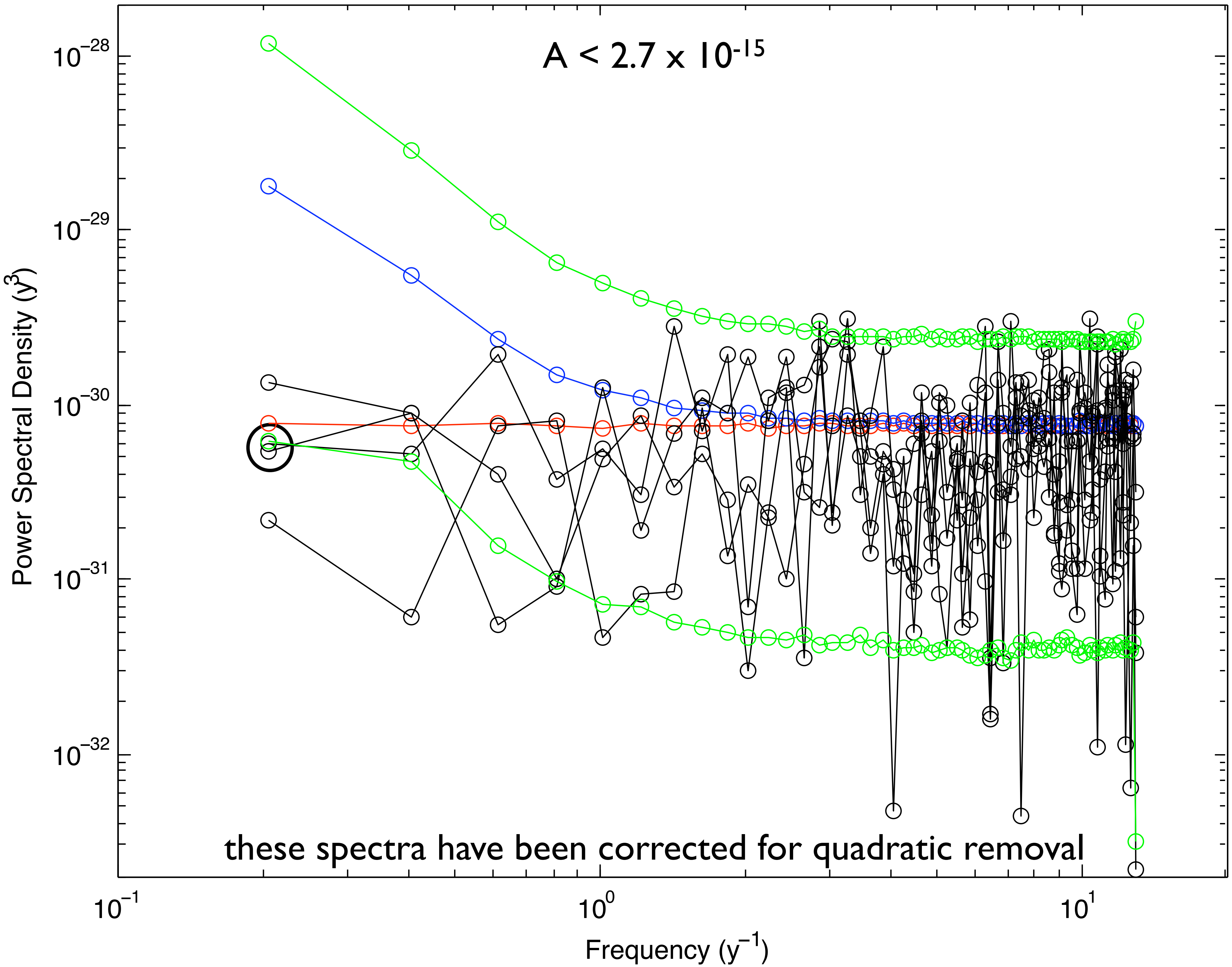
Biweekly observations for 5 y with A=10^{-15} y $^{-2/3}$, 100 ns rms noise and quadratic removal. Clearly we need to detect a weak signal buried in white noise.

Power Spectral Density Estimates of GWB+Noise and Noise

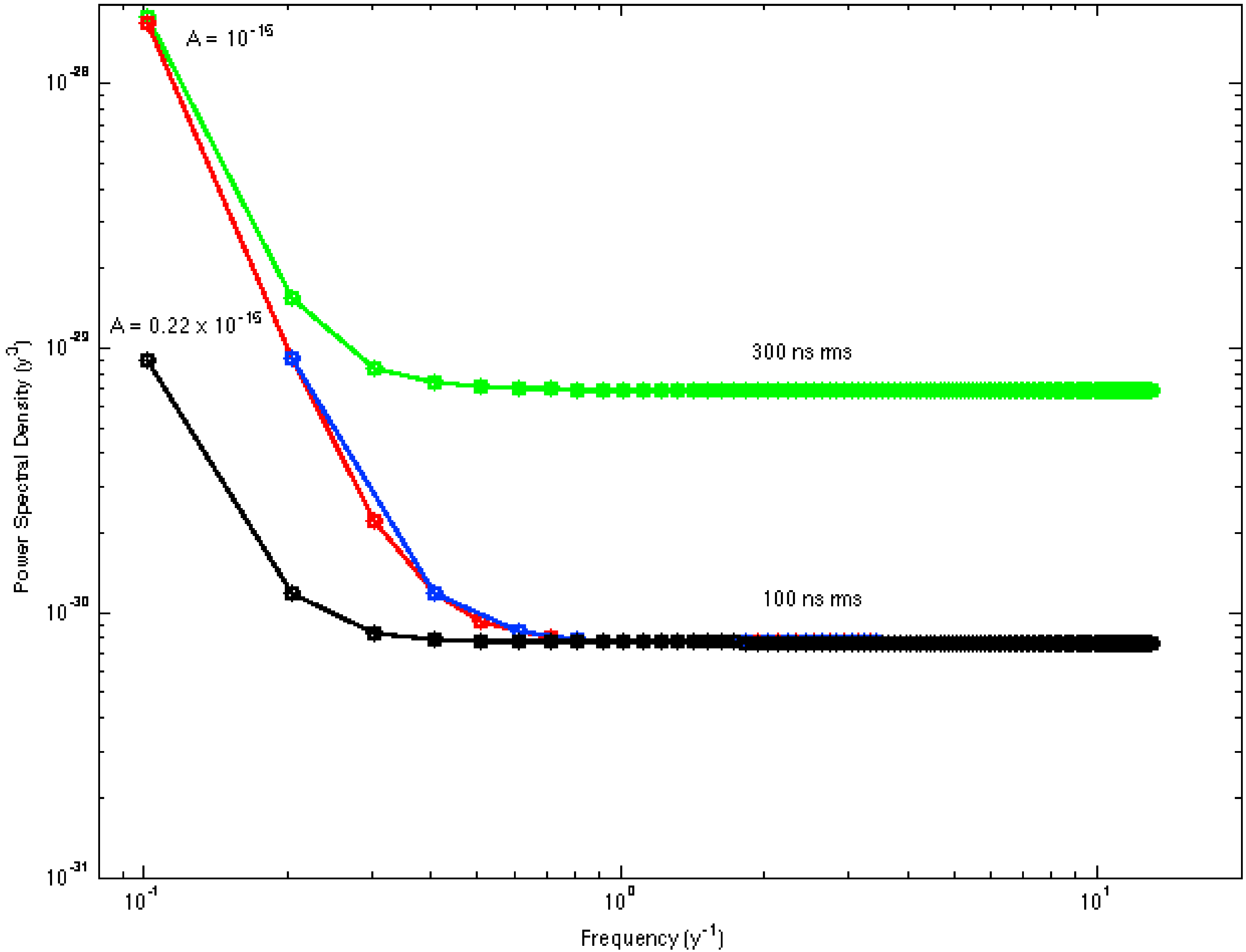


Obtaining an Upper Bound from Observations of a Single Pulsar

We can't make a *detection*, with a single pulsar, because we can't separate the *unknown noise processes* from the GWB spectrum.

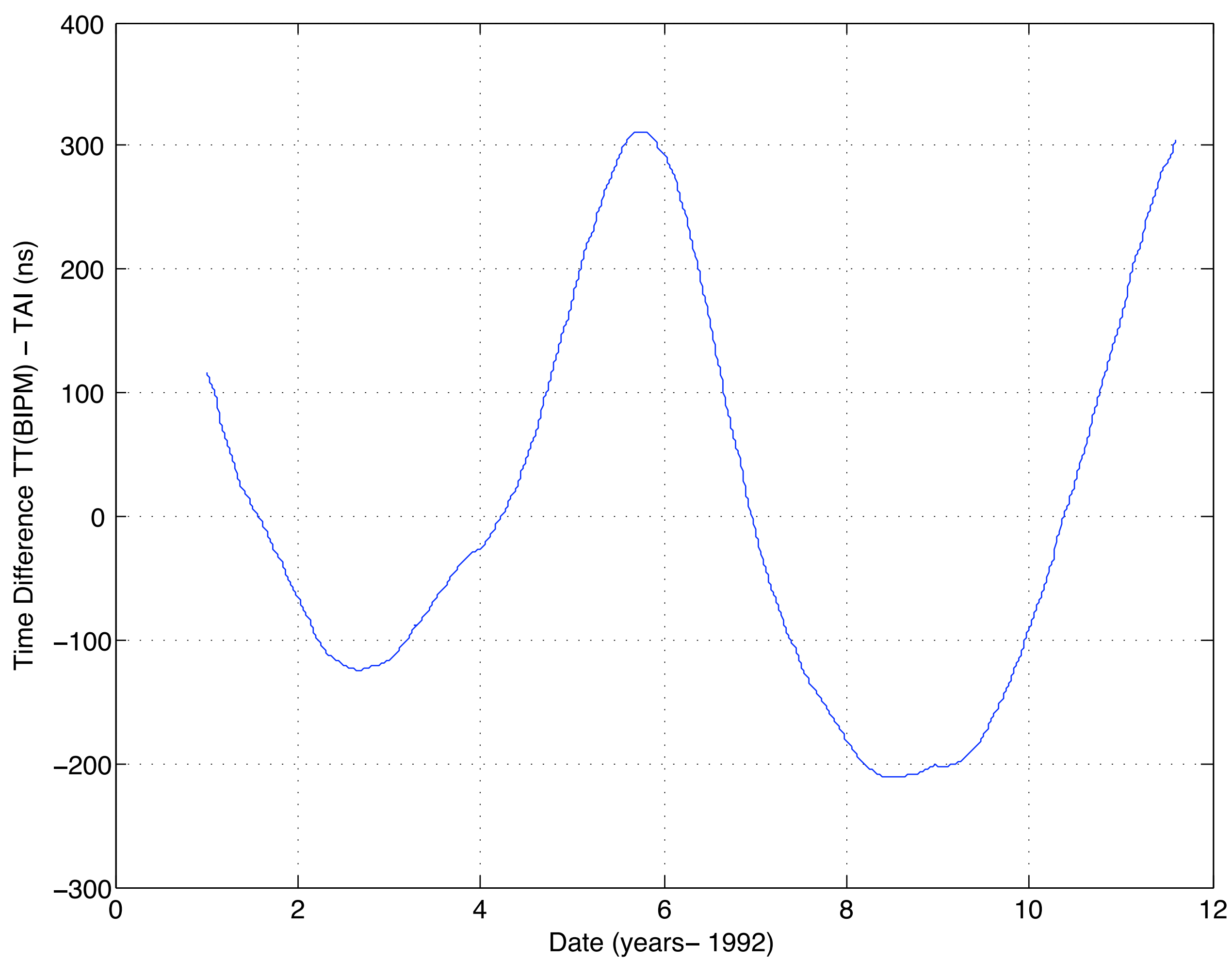


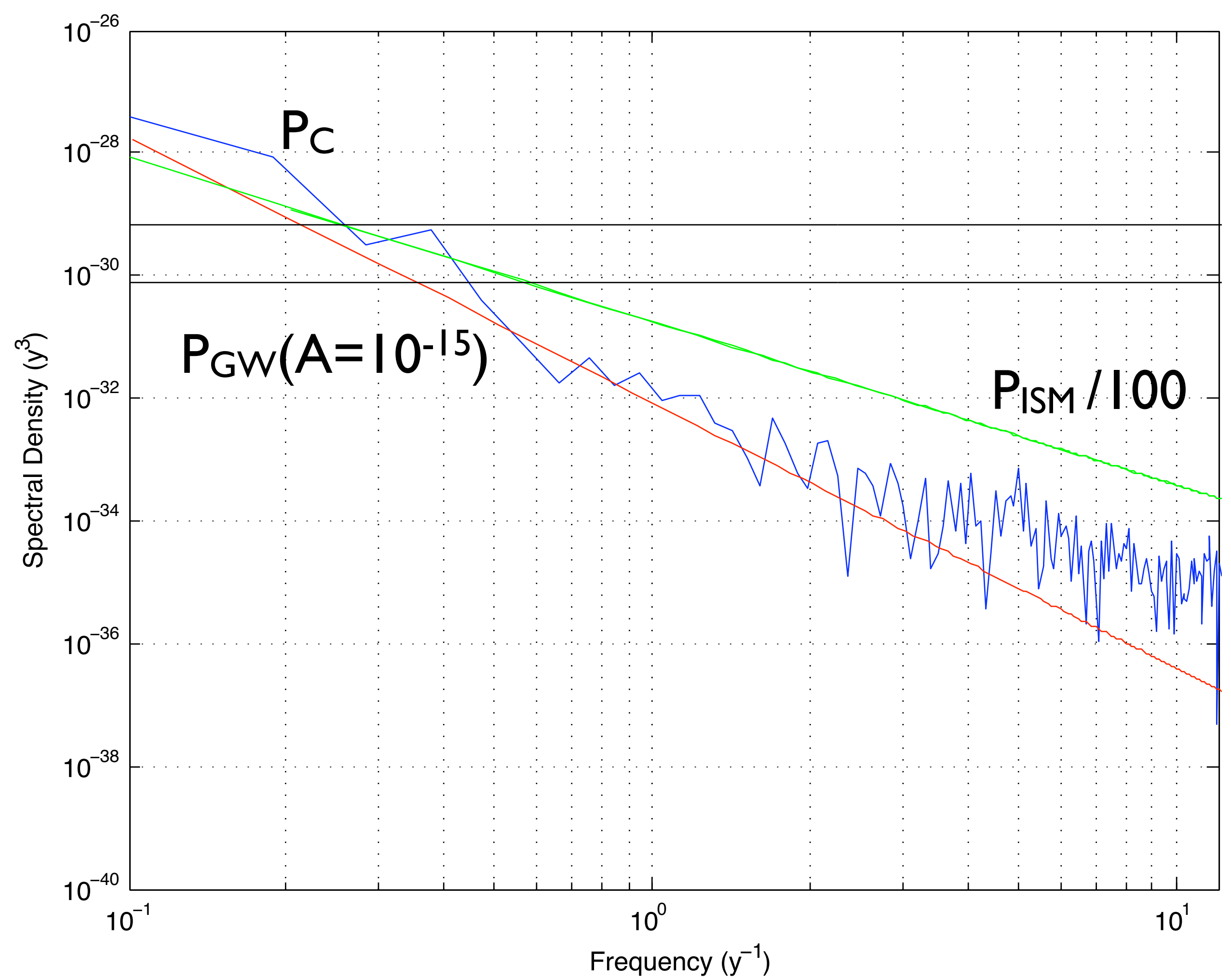
Effect of Duration of Observations



Doubling the duration will improve the sensitivity by a factor of 20, but the clock error spectrum and ISM spectrum will also have to be reduced by a factor of 20.

Clock Errors



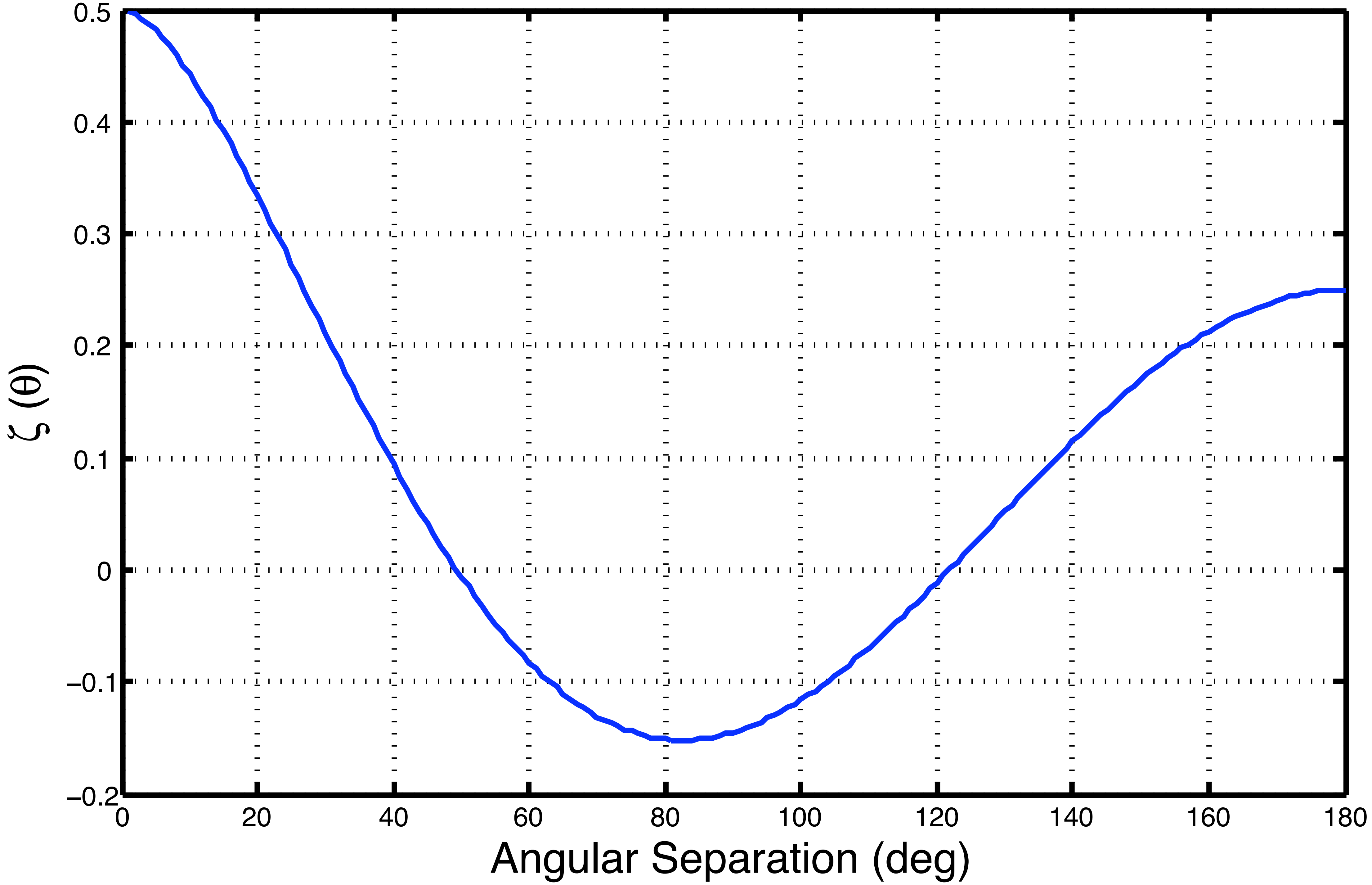


We may have four roughly equal contributions to the spectral density. However they can be separated by the cross correlation between different pulsars:

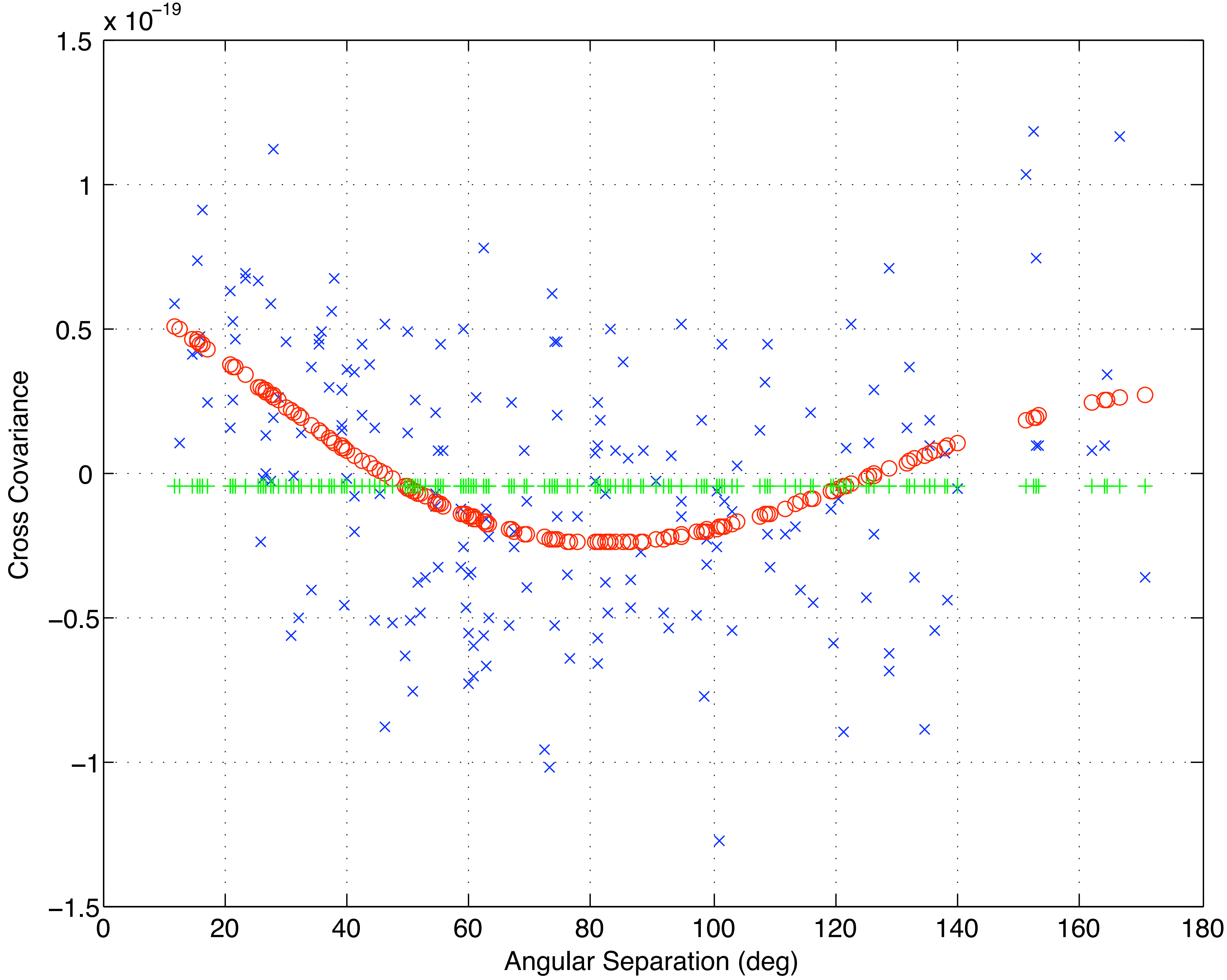
- clock errors are 100% correlated;
- ISM and white noise are completely uncorrelated;
- GWB is about half correlated with a distinctive signature.

Detection of GWB Using a Pulsar Timing Array

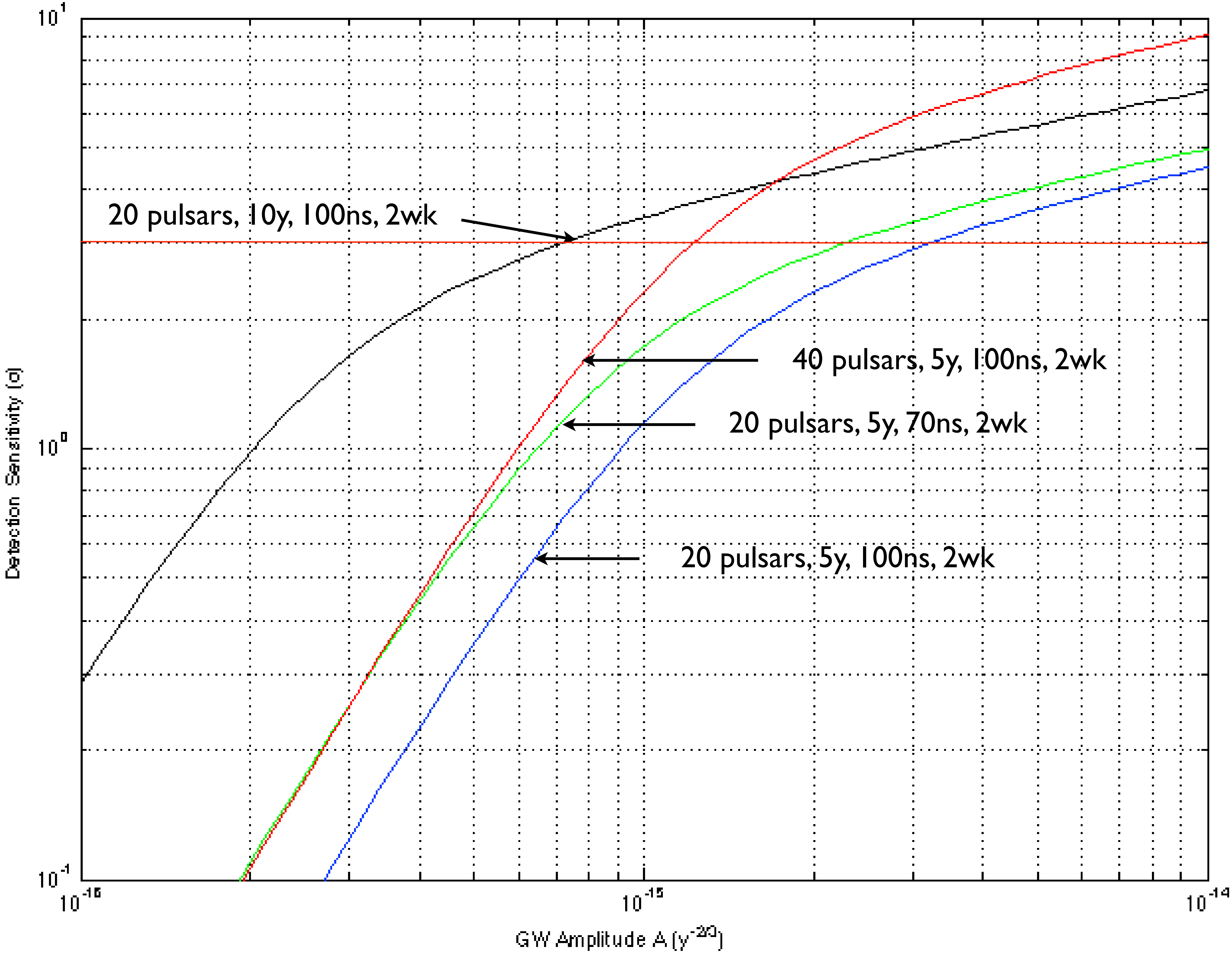
The GWB effect on different pulsars is *correlated* allowing reliable detection. Peak cross correlation is 50%



Individual covariance estimates are, like spectral estimates, very noisy, but there are a lot of them

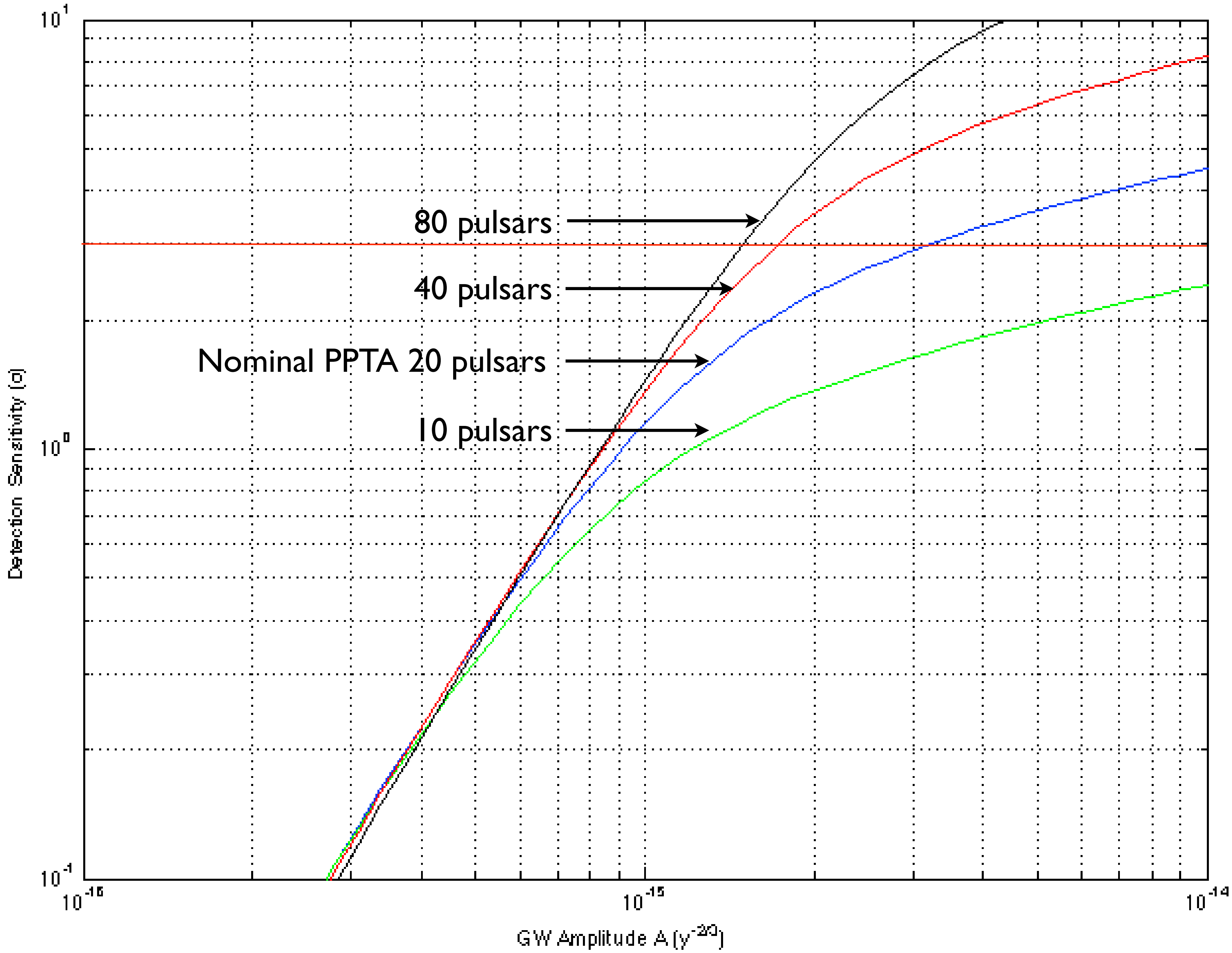


Theoretical Detection Sensitivity



Effect of Number of Pulsars

keeping observing time constant



Comparison of PPTA and SKA Detection Sensitivity