INFLUENCE OF TURBULENCE ON THE SHAPE OF A SPECTRAL LINE: THE ANALYTICAL APPROACH

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ABSTRACT

We consider the propagation of a spectral-line radiation in a correlated turbulent atmosphere. The ensembles of realizations of turbulent velocities $\mathbf{u}(\mathbf{r},t)$ and optical depth τ_ν are assumed to be Gaussian. We investigate the explicit analytical solution of the stochastic radiative transfer equation for the intensity I_ν of radiation. The scattering term is not taken into account. It is shown that, in addition to the usual Doppler broadening of the spectral line, correlated turbulent motions of atoms and molecules give rise to considerable changes in the shape of a spectral line. It was found for the first time that the mean intensity $I^{(0)}_\nu$ ($I_\nu = I^{(0)}_\nu + I'_\nu$, $\langle I'_\nu\rangle = 0$) obeys the usual radiative transfer equation with renormalized extinction factor $\alpha^{\rm eff}_\nu$ if the correlation length R_0 of the turbulence is small as compared to a photon free path. A simple analytical expression for $\alpha^{\rm eff}_\nu$ is given. This expression integrally depends on the two-point correlation function of the turbulent velocity field.

1 INTRODUCTION

In turbulent atmospheres of stars and in interstellar clouds the optical depth τ_ν ($d\tau_\nu = \alpha_\nu ds$) of the radiation path is a stochastic quantity. As a result, the intensity $I_\nu(s)$ of the radiation, its polarization and the shape of the spectral line acquire a stochastic component.

As for the shape of a spectral line, it was found that, apart from thermal broadening, there exists additional Doppler broadening due to small-scale chaotic turbulent motions of atoms and molecules. The total line broadening $\Delta\nu_{\rm D}$ is determined by the expression:

$$(\Delta\nu_{\rm D})^2 = (\Delta\nu_{\rm Th})^2 + (\Delta\nu_k)^2 = \frac{2}{3}\cdot\frac{\nu_0^2}{c^2}(u^2_{\rm Th}+u^2_k), \qquad (1)$$

where $u^2_{\rm Th} = 2k_{\rm B}T/m$ and $u^2_k = \langle u^2(\mathbf{r},t)\rangle$ are rms values of thermal and turbulent velocities respectively, ν_0 is the central frequency of the spectral line, c is the speed of light, $k_{\rm B}$ is the Boltzmann constant, T is the temperature of the atmosphere. The brackets $\langle I\rangle$ denote the ensemble average of the stochastic quantity I.

Formula (1) has been derived under the assumption that both the thermal velocity of atoms or molecules and the velocity of turbulent motions are, so to say, "white noise", i.e., these velocities are not correlated even in the nearest space-time vicinity.

The real turbulent velocity $\mathbf{u}(\mathbf{r},t)$ is always correlated. There exist spatial interval $|\mathbf{r}-\mathbf{r}'| \equiv R \simeq R_0$ and time interval $|t-t'| \equiv \tau \simeq \tau_0$ where the velocity values are correlated. The case of uncorrelated velocities formally corresponds to the limits $R_0 \to 0$ and $\tau_0 \to 0$.

Turbulence gives rise to stochasticity of physical parameters of the atmosphere such as magnetic field, number density of scattering or absorbing particles, etc. As a result, the extinction coefficient α_ν is also a stochastic quantity. The radiative transfer equation

$$(\mathbf{n}\nabla)I_\nu(\mathbf{n},s) \equiv \frac{dI_\nu(\mathbf{n},s)}{ds} = -\alpha_\nu(\mathbf{n},s)I_\nu(\mathbf{n},s) + j_\nu(\mathbf{n},s) \qquad (2)$$

in a turbulent atmosphere acquires a stochastic meaning. Here $\mathbf{n}$ is the direction of the light propagation, s is the path along $\mathbf{n}$, j_ν is the source of radiation, α_ν is the extinction factor.

Usually, thermal Doppler broadening $\Delta\nu_{\rm Th}$ is much greater than the natural linewidth, and the extinction factor can be taken as

$$\alpha_\nu = \alpha_0 \exp\left[-\left(\frac{\nu-\nu_0-\mathbf{U_0 n}\frac{\nu_0}{c}-\mathbf{un}\frac{\nu_0}{c}}{\Delta\nu_{\rm Th}}\right)^2\right]. \qquad (3)$$

Here $\mathbf{U}_0$ is the regular velocity of the medium, $\mathbf{u}$ is the stochastic (turbulent) velocity ($\langle\mathbf{u}\rangle = 0$), $\alpha_0 = N_0\sigma_0\sqrt{\pi}/\Delta\nu_{\rm Th}$ is the extinction factor at the line center, N_0 is the number density of atoms or molecules at the resonant level, σ_0 is the quantum-mechanical cross section at $\nu = \nu_0$.

The conditions when the observed intensity coincides with the ensemble averaged value $I^{(0)}_\nu$ ($I_\nu = I^{(0)}_\nu + I'_\nu$, $\langle I'_\nu\rangle = 0$) were discussed in detail by Levshakov & Kegel ([1979]). Usually, the exposure time is less than the lifetime τ_0 of turbulent motions and the coincidence of $I^{(0)}_\nu$ with the observed intensity requires the existence of multiple turbulent cells in the observed region. In observations of stars this condition is implemented nearly always; it takes place more seldom in the observation of light propagation in turbulent interstellar clouds. In the latter case it is necessary to observe a very large part of a cloud.

As will be shown, the effect of correlated turbulence on the line shape increases with increasing optical depth of the light path. It seems that this effect can be observed most readily in the propagation of maser emission. Usually the regions with inverted populations of molecular levels are observed as small "spots" inside the large molecular clouds. Intense maser emission can be observed even after the passage of an optically thick layer. In this case the radiation penetrates many turbulent cells and the observed emission coincides with the average intensity $I^{(0)}_\nu$. The effect of turbulent correlations on the spectral line shape was investigated in a number of papers (see references).

All the features of the influence of turbulence on the spectral line shape exist in the presented below simple analytical theory of radiative transfer in turbulent atmospheres. Here we first derive the usual radiative transfer equation for the averaged intensity in turbulent atmospheres with renormalized (effective)

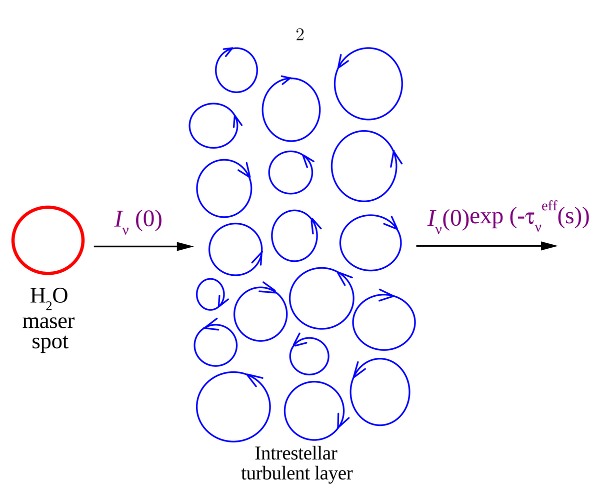

Figure 1: Propagation of the maser radiation trough the interstellar turbulent layer.

absorption coefficient $\alpha^{\rm eff}_\nu$. The simple analytical expression for $\alpha^{\rm eff}_\nu$ obtained in our paper (Silant'ev et al. [2006]) depends on the explicit form of the two-point turbulent velocity correlation function.

In this paper we investigate the effect of finite correlation length R_0 of turbulence on the spectral line shape using the assumptions that stochastic optical length τ_ν ($d\tau_\nu = \alpha_\nu(z)ds$) is a Gaussian stochastic quantity and the turbulence is statistically uniform and isotropic. This approach is simpler than the Fokker–Planck equation approach, because we use the exact explicit solution of the stochastic radiative transfer equation (2) without the source term:

$$I_\nu(s) = I_\nu(0)e^{-\tau_\nu(s)}, \qquad \tau_\nu(s) \equiv \tau^{(0)}_\nu(s) + \tau'_\nu(s) = \int_0^s ds'\alpha_\nu(s'). \qquad (4)$$

Below we study mainly the shape of the absorption line, which corresponds to the mean intensity $I^{(0)}_\nu(s)$ from Eq. (4) (See Figure 1). The analysis of this expression shows that for the case $s \gg R_0$ (s is the thickness of a turbulent layer) the averaged intensity $I^{(0)}_\nu(s)$ obeys the usual radiative transfer equation (2) with renormalized extinction factor $\alpha^{\rm eff}_\nu$.

Thus, averaging Eq. (4) yields the following expression for the mean intensity:

$$I^{(0)}_\nu(s) = I_\nu(0)e^{-\tau^{(0)}_\nu(s)}\, e^{\frac{1}{2}\langle\tau'^2_\nu(s)\rangle} \equiv I_\nu(0)e^{-\tau^{\rm eff}_\nu(s)} \qquad (5)$$

From this equation follows the definition of the effective optical depth $\tau^{\rm eff}_\nu(s)$:

$$\tau^{\rm eff}_\nu(s) = \tau^{(0)}_\nu(s) - \frac{1}{2}\langle\tau'^2_\nu(s)\rangle. \qquad (6)$$

According to this formula, $\tau^{\rm eff}_\nu \le \tau^{(0)}_\nu$, i.e., the turbulent atmosphere is effectively more transparent than the non-turbulent one for all the frequencies. This is a purely statistical effect. Remember that $\tau^{\rm eff}_\nu(s)$ describes the propagation of the mean intensity $I^{(0)}_\nu(s)$ in the atmosphere. In an atmosphere with inverted populations of resonant atoms or molecules $\tau^{(0)}_\nu < 0$, and the absolute value of $\tau^{\rm eff}_\nu$ is larger than $|\tau^{(0)}_\nu|$, i.e., a turbulent medium increases the intensity of resonant radiation more efficiently than a non-turbulent one.

Our explicit formulae enable us to estimate the correlation length R_0 of the turbulence, its characteristic velocity u_k and the ratio of thermal and turbulent velocities $u_{\rm Th}/u_k$. It is easier to analyze the usual radiative transfer equation for the mean intensity with the effective absorption factor, derived here for the first time, than the more complicated Fokker–Planck equation.

2 EXPLICIT FORMULA FOR THE MEAN INTENSITY

To obtain the explicit relationship for the mean intensity $I^{(0)}_\nu(s)$ (see Eq. (5)), we use the expressions (3) and (4), assuming that the turbulent velocity field $\mathbf{u}(\mathbf{r},t)$ is Gaussian one. As a result we derive the following explicit formulas for the mean intensity $I^{(0)}_\nu(s)$ and $\tau^{eff}_\nu(s)$:

$$I^{(0)}_\nu(s) = I_\nu(0)A(s,x-x_0), \qquad A(s,x-x_0) = e^{-\tau^{\rm eff}_\nu(s)}, \qquad (7)$$

where A_ν is transmission coefficient.

$$\tau^{\rm eff}_\nu(s) = \tau^{(0)}_\nu(s)\left\{1 + \frac{\tau_1 s}{2R_0}e^{-(x-x_0)^2} - \tau_1\int_0^{s/R_0}dy\left(\left(1-y\frac{R_0}{s}\right)\frac{\exp\left[-(x-x_0)^2\frac{1-\eta b_{||}(y)}{1+\eta b_{||}(y)}\right]}{\sqrt{1-\eta^2b^2_{||}(y)}}\right)\right\}. \qquad (8)$$

Here we have used the following dimensionless quantities:

$$y = \frac{R}{R_0}, \quad \tau_1 = \alpha_0\frac{\Delta\nu_{\rm Th}}{\Delta\nu_{\rm D}}R_0, \quad \tau_0 = \tau_1 s/R_0, \quad x = \frac{\nu-\nu_0}{\Delta\nu_{\rm D}}, \quad x_0 = \frac{\mathbf{U_0 n}}{c}\cdot\frac{\nu_0}{\Delta\nu_{\rm D}}.$$

$$\eta = \frac{\Delta\nu_k^2}{\Delta\nu_{\rm D}^2} = \frac{\Delta\nu_k^2}{\Delta\nu_{\rm Th}^2+\Delta\nu_k^2} \equiv \frac{1}{1+\xi^2}, \qquad \xi = \frac{\Delta\nu_{\rm Th}}{\Delta\nu_k} = \frac{u_{\rm Th}}{u_k}. \qquad (9)$$

The value $b_{||}(R)$ is the correlation function of the uniform and isotropic turbulence. It is determined by the expression:

$$\langle u_{||}(s)u_{||}(s')\rangle = \frac{u_k^2}{3}b_{||}(R), \qquad u_{||}(s) = \mathbf{un}. \qquad (10)$$

Here $R = |s-s'|$ is the distance along the line of sight $\mathbf{n}$. Function $b_{||}(R)$ at $R=0$ is equal to unity and it rapidly vanishes at $R > R_0$. Remember that R_0 is the characteristic length of turbulent eddies (cells).

Parameters τ_1 and τ_0 are the optical depths of the correlation length R_0 and of the layer s at the line center respectively; $\tau^{(0)}_\nu(s)$ is the mean optical depth of the distance s. The terms with τ_1 in Eq. (8) describe the contribution of turbulent motions to the effective optical depth $\tau^{\rm eff}_\nu(s)$ of the layer s. Of course, the choice of parameters must preserve $\tau^{\rm eff}_\nu(s) > 0$ for the usual medium with non-inverted population of atomic levels.

We calculated the transmission coefficients $A(s,x)$ for the different turbulent correlation functions. Here we present only the results for the case:

$$b_{||}(R) = 1 - \left(\frac{R}{R_0}\right)^2 \text{for} R \le R_0, \qquad b_{||}(R) = 0 \qquad \text{for } R > R_0, \qquad (11)$$

If the correlation length $R_0 \to 0$, i.e., $\tau_1 \to 0$ (but $\langle\mathbf{u}^2\rangle \ne 0$), then the terms with τ_1 vanishes for any correlation functions $b_{||}(R)$ and any values of η. This limit corresponds to the case of uncorrelated turbulence when the effect of turbulence is reduced to an additional line broadening only. In this case the transmission coefficient is

$$A_0(s,x-x_0) = e^{-\tau^{(0)}_\nu(s)}. \qquad (12)$$

Below we will compare coefficients $A(s,x-x_0)$ and $A_0(s,x-x_0)$.

3 THE RADIATIVE TRANSFER EQUATION FOR THE MEAN INTENSITY

The terms with τ_1 in Eq. (8) depends on the distance s, i.e., $\tau^{\rm eff}_\nu(s)$ is a nonlinear function of s, and the expression (7) is not interpreted as a solution of radiative transfer equation (2) with any effective extinction coefficient $\alpha^{\rm eff}_\nu$. However, in the case $R_0 \ll s$ this is possible. Indeed, in this case the term with τ_1 virtually does not depend on s, because correlation function $b_{||}(R)$ rapidly vanishes at $R \ge R_0$. In this case formula (7) can be considered as the solution of the radiative transfer equation for the mean intensity $I^{(0)}_\nu(s)$ with renormalized effective extinction factor:

$$\frac{dI^{(0)}_\nu(s)}{ds} = -\alpha^{\rm eff}_\nu I^{(0)}_\nu(s) + \langle j_\nu(s)\rangle, \qquad (13)$$

$$\alpha^{\rm eff}_\nu = \langle\alpha_\nu\rangle\left\{1 + \tau_1 e^{-(x-x_0)^2} - \tau_1\int_0^1 dy\frac{\exp\left[-(x-x_0)^2\frac{1-\eta b_{||}(y)}{1+\eta b_{||}(y)}\right]}{\sqrt{1-\eta^2b^2_{||}(y)}}\right\}. \qquad (14)$$

When deriving Eq. (14), we assume that R_0 corresponds to the distance at which the correlation of turbulent velocities almost disappears, i.e., $b_{||}(R_0) \simeq 0$. For this reason, in Eq. (14) the upper integration limit can be taken to be ∞; this makes the calculations simpler. In the radiative transfer theory the characteristic length is the unit optical depth. Therefore, the validity condition of the usual radiative transfer equation (13) is $\tau_1 \ll 1$, i.e., the optical depth at the center of the line of the correlation region is to be small.

The solution methods for the usual radiative transfer equation (13) are known. It is easier to solve this equation for the calculation of the mean intensity $I^{(0)}_\nu(s)$ than to use the more complicated Fokker–Planck equation, derived by Gail et al. ([1974]).

4 RESULTS OF CALCULATIONS

The transmission coefficients $A(x)$ and $A_0(x)$ for a number of values of τ_0 and ξ are presented in Fig. 2. They correspond to the layer thickness of $10R_0$. The optical depth takes the values $\tau_0 = 1$, 2 and 3 (red, green and blue curves, respectively). Figure 2 corresponds to the model of turbulence (11).

Firstly, we see that for all the cases there exists inequality $A(x) > A_0(x)$. The existence of this inequality in correlated turbulence is a pure statistical effect (see Eq. (6)).

Secondly, the calculation demonstrate that the larger is τ_0 the greater is the contribution of the finite correlation length R_0.

Figure 2 demonstrate that the transparency effect is very sensitive to the value of $\xi = \Delta\nu_{\rm Th}/\Delta\nu_k$. From this figure it follows that at $\xi = 1$ the transparency effect is very small, but for $\xi = 0.1$ it is large.

Let us summarize the results of calculations. A very large number of turbulent cells ($n \ge 100$) within the layer is a sufficient condition for the existence of the short-correlated limit. If this condition is not valid, the short-correlated limit occurs when parameter $\xi = u_{\rm Th}/u_k \ge 1$. In the intermediate case ($n < 100$ and $\xi \le 1$) the effects of finite correlation length of turbulent velocities on the line shape can be large, especially if the optical depth $\tau_0 > 2$.

5 PROPAGATION OF MASER RADIATION

Very frequently a maser emission line has a Gaussian shape with the Doppler width $\Delta\nu_{\rm S}$. First, using our general formulae, we demonstrate the influence of turbulence on the shape of maser radiation. We obtain the following expression for the mean intensity:

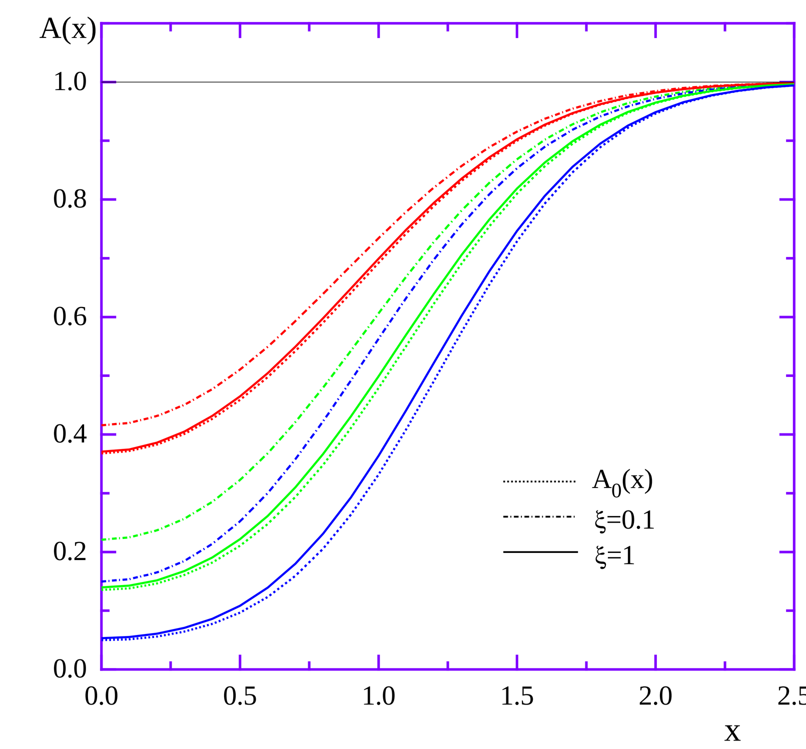

Figure 2: Transmission functions $A(x)$ and $A_0(x)$ for turbulence model (11) for optical depths of the layer at the line center $\tau_0 = 1$, 2 and 3 (red, green and blue curves, respectively). The layer thickness is $10R_0$.

$$I^{(0)}_\nu(x) = I_0e^{-b^2x^2}A(x-x_0) \equiv I_0e^{-(b^2x^2+\tau^{\rm eff}_\nu(s))}. \qquad (15)$$

The term $I_0\exp(-b^2x^2)$ is the incident maser intensity from the source with the Doppler width $\Delta\nu_S$. Here x and x_0 are dimensionless frequencies (10) with the Doppler width $\Delta\nu_{\rm D}$ in the turbulent layer s, $b = \Delta\nu_{\rm D}/\Delta\nu_{\rm S}$.

In Figure 3 we present normalized intensities $I^{(0)}_\nu(x)/I^{(0)}_{\nu\max}$ for values $b = 0.5$, 1, 2 and mean optical

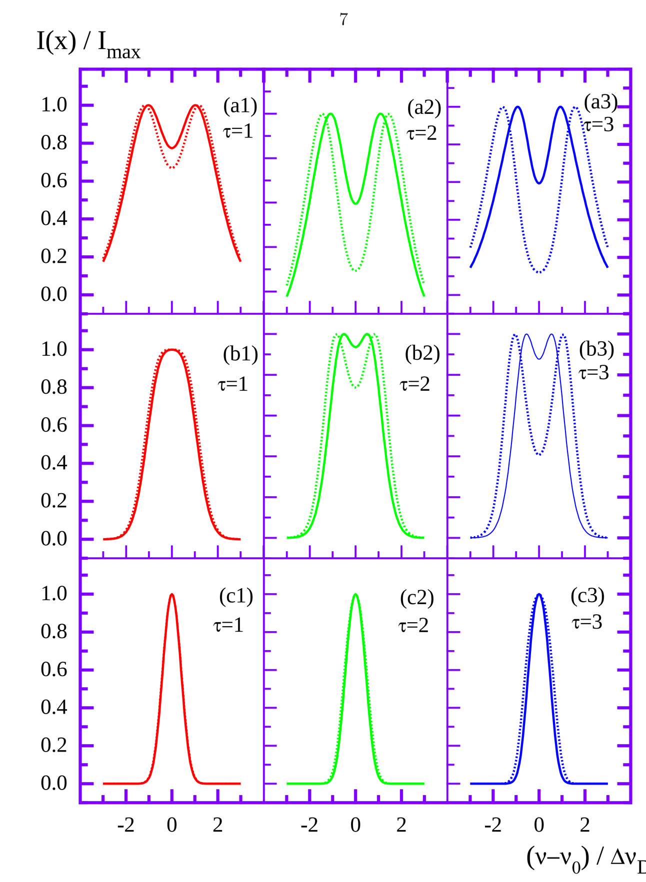

Figure 3: Normalized shape $I_\nu(x)/I_{\nu\max}$ of the maser line with a Gaussian initial profile after the passage of a turbulent cloud (solid lines). The dotted lines correspond to the usual model of short-correlated turbulence. The cases (an), (bn) and (cn) represent $\Delta\nu_D/\Delta\nu_S = 0.5$, 1 and 2, respectively; $\Delta\nu_D$ and $\Delta\nu_S$ are the Doppler widths in the cloud and maser source. The optical depth of the cloud τ at the line center takes the values 1, 2 and 3. Turbulence correlation length R_0 is taken to be one-fifth of the layer's depth, and parameter $\xi = u_{\rm Th}/u_k = 0.1$; $u_{\rm Th}$ and u_k-values are characteristic thermal and turbulent velocities in the cloud.

depths at the center of a line $\tau_0 = 1$, 2 and 3. Figures (an), (bn) and (cn) correspond to the values $b = 0.5$, 1 and 2, respectively. We have considered the simplest case with $x_0 = 0$, i.e., absence of a regular motion in the layer. Solid lines correspond to turbulence with $\xi \equiv u_{\rm Th}/u_k = 0.1$ and $s = 5R_0$. Dotted lines represent the transformation of the line shape when turbulence is short-correlated ($\tau_1 = 0$):

$$I^{(0)}_\nu(x) = I_0e^{-b^2x^2}A_0(x-x_0) \equiv I_0e^{-(b^2x^2+\tau^{(0)}_\nu(s))}. \qquad (16)$$

First of all, we see that turbulence with a finite correlation size makes lines narrower than short-correlated turbulence. This is due to the fact that transmission coefficient $A(x)$ is narrower than the $A_0(x)$-coefficient (see Figure 2). The case $b = 0.5$ corresponds to broadband incident emission as compared to the characteristic width of the transmission coefficients $A(x)$ and $A_0(x)$. This produces central minima in the line shape. Naturally, layers with a large optical depth give rise to more profound minima than layers with lesser thickness. The case $b = 3$ corresponds to narrowband incident emission. In this case the line shape virtually does not change. The intensity decreases only by its value proportionally to $A(0)$ and $A_0(0)$.

Thus, the presented figures demonstrate that the shape of a maser line changes quite substantially after the passage of an optically thick turbulent layer with a finite length of correlation as compared to the model of short-correlated turbulence.

6 ESTIMATION OF THE PARAMETERS OF THE H_2O MASER SOURCE IN S252A

To illustrate our theory, we have chosen a flare of the H_2O maser in the source S252A at $V_{\rm LSR}$=10 km s^{-1}, not a single two-peak profile, because this one may be produced by a simple overlap of two physically unassociated components with similar velocities $V_{\rm LSR}$. In addition, it is important to stress that during strong flares the maser operation mode may change, resulting in a variation in linewidths of individual features. However, in such cases maximum linewidth variations are a few times less than the linewidth itself. For the flare in S252A at V = 10 km s^{-1} the minimum separation between the components was 0.5 km s^{-1}, which is close to the linewidth. The maximum separation reached 0.8 km s^{-1}. Hence it is clear that the changes in the line structure are not related to the maser operation mode. Finally, important evidence favouring our choice of the flare were time-correlated flux variations of both components.

Thus, the two-peak line profile at 10 km s^{-1} in S252A, in which the components have similar radial velocities and their fluxes change in phase can be a proof of physical association of the components, i.e., the source of emission at 10 km s^{-1} is a single maser feature.

As all the masers in star-forming regions H_2O maser in S252A is located inside a dense molecular cloud. The fragments of the observed spectra are presented in Fig. 4a (Berulis et al. [1996]). In the reduction of these spectra they were represented as a sum of two Gaussian profiles (Lekht et al. [1996]). It was found

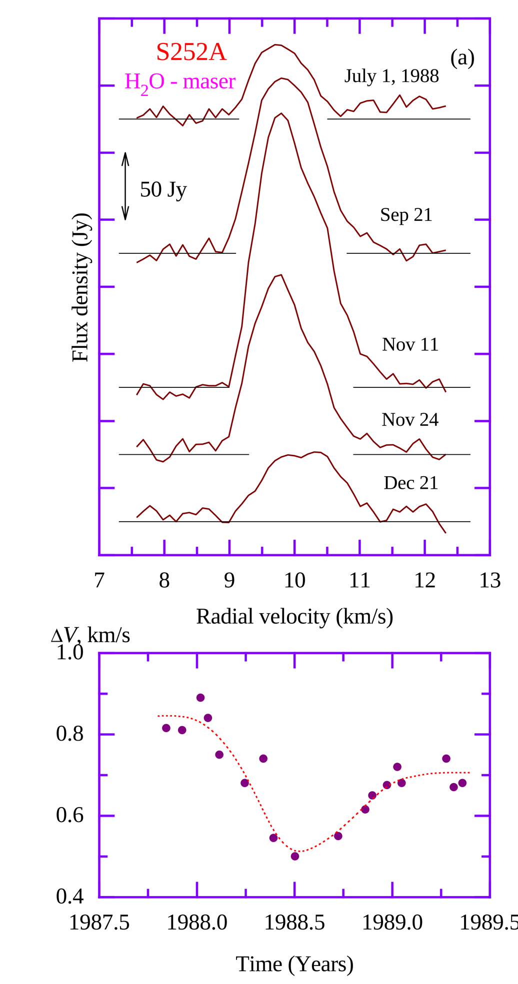

Figure 4: (a) – Fragments of the H_2O maser spectra of S252A in 1988; (b) – variations in the difference between the centers of the approximating Gaussians.

that after a burst of the emission at the beginning of 1988 the distance between these two Gaussian profiles decreased systematically up to May 1988, and then the distance increased to a new level, lower than the initial one. This is clearly seen in Fig. 4b. What is the cause of this effect? We have seen from Fig. 3 that propagation of maser emission through a turbulent layer with a finite correlation length decreases the width of the spectral line. Thus, it is natural to explain the effect by this mechanism.

Here we suppose that after the burst the existing turbulent layer has acquired an additional regular-velocity motion and, possibly, other parameters have also changed. Here we present only a preliminary solution of this problem. Our main goal will be to demonstrate that the simple analytical theory developed in this paper allows us to find very quickly the basic parameters of turbulence by the "hit and fit" method. For this purpose we use general formula (15) with unknown parameters τ_0, x_0, a^2 and b^2. We take the two initial Gaussian curves with $x_0 = 0$ and $x_0 = 0.8$ and seek at what parameters the distance between the centers of these curves takes the value $x_0 = 0.5$. The calculation of $\tau^{\rm eff}_\nu$ was done for correlation function (11). Of course, in such a simple statement of the problem the solution is not unique. But we restrict our solution by the condition that the final linewidth is about the observed value, 1–1.1 km s^{-1}. It seems that this restriction does not make the solution unique, but anyway it makes the range of possible values of the parameters narrower.

As a result, we have found that the parameters characterizing the layer are: $\tau_0 = 3$, optical depth in center of a line, regular velocity along the line of sight $U_0 \simeq 1$, mean turbulent velocity $u_k \simeq 0.5$, and the thermal velocity $u_{\rm Th} \simeq 0.1$ km s^{-1}. The obtained preliminary results can be used in the construction of a physical model of the maser source in S252A.

7 CONCLUSION

Let us summarize the results. First of all, we have developed the analytical theory of radiative transfer of a resonant line in turbulent atmospheres with a finite length of correlation. We have found for the first time that mean intensity $I^{(0)}_\nu \equiv \langle I_\nu\rangle$ obeys the usual radiative transfer equation with renormalized extinction factor $\alpha^{\rm eff}_\nu$ if correlation length R_0 is small as compared to the photon free path. Effective absorption coefficient $\alpha^{\rm eff}_\nu$ does not coincide with mean absorption factor $\alpha^{(0)}_\nu \equiv \langle\alpha_\nu\rangle$. The transition $\alpha^{\rm eff}_\nu \to \alpha^{(0)}_\nu$ takes place if the optical depth at the line center τ_1 of correlation length R_0 tends to zero (the case of short-correlated turbulence), or characteristic thermal velocity $u_{\rm Th}$ is equal to or greater than characteristic turbulent velocity u_k (parameter $\xi = u_{\rm Th}/u_k \ge 1$).

Assuming that the ensemble of turbulent velocities is Gaussian, we have obtained an explicit analytical formula for $\alpha^{\rm eff}_\nu$, which integrally depends on the correlation function of turbulent velocities. It is found that $\alpha^{\rm eff}_\nu \le \alpha^{(0)}_\nu$. It means that statistically a turbulent layer with a finite correlation length is more transparent then a layer of short-correlated turbulence.

It is shown that averaged spectral line intensity $I^{(0)}_\nu$ is narrower than that described by radiative transfer equation with averaged extinction factor $\alpha^{(0)}_\nu$. This distortion of the shape increases with increasing optical depth τ_0 of the turbulent layer at the line center. An increase in the parameter τ_1 and decrease in ξ also give rise to an increase in the distortion of the shape.

These effects were mentioned earlier in various papers as the results of purely numerical calculations, without a simple physical explanation. The simple analytical formulae derived in this paper allow us to understand the expected effects of turbulence without complicated numerical calculations.

As an illustrative example, we have estimated the possible parameters of the H_2O maser source in S252A. We hope that the obtained analytical formulae will be useful for a detailed explanation of various observational data (see Silant'ev et al., [2006]).

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Table 1:

Fig. 1.—

Fig. 2.—

Fig. 3.—