POLARISATION I:
FOR THE LOVE OF STOKES

Jimi Green ATNF Radio Astronomy School 2009

Outline

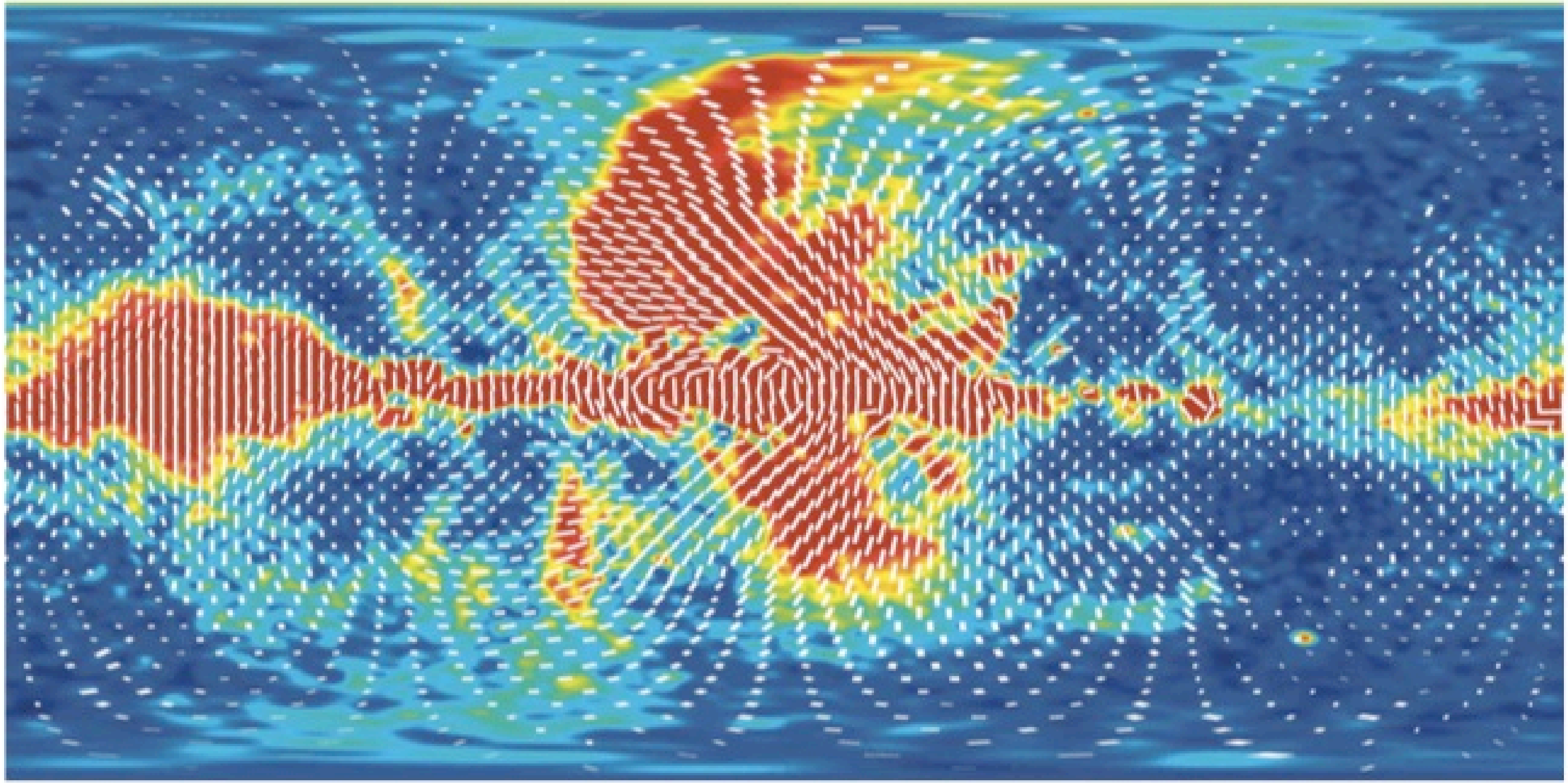
- Why study Polarimetry and what is Polarisation?
- Poincare and his spheres
- Jones and his vectors (and matrices)
- Stokes and his parameters
- How do we measure Polarisation?
- Leakages and Mueller's Matrix
- Polarised beam effects
- Science with Polarisation:
  - Masers and Zeeman splitting

Why Polarimetry?

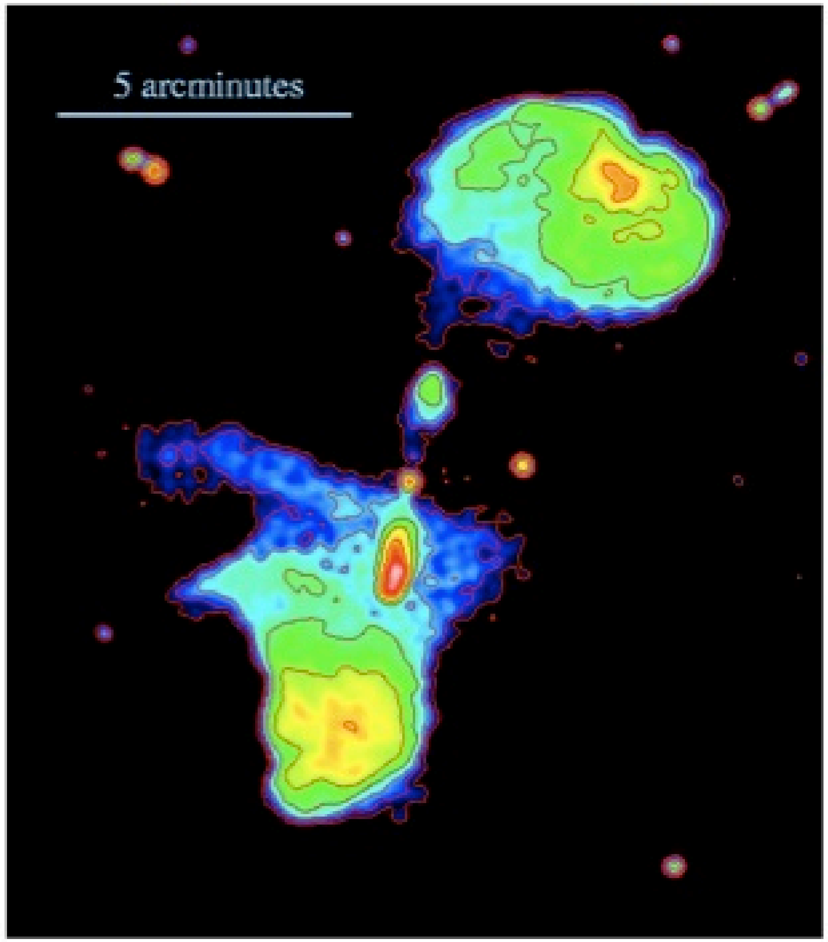
Why Polarimetry?

- Polarisation is fundamentally important to understanding the Universe
    - Provides insight into magnetic fields
- In optical astronomy, it's difficult to make polarimetric observations; in radio astronomy, they can be made easily, so why not use this to our advantage!
- (It's also very important to the Birds & the Bees, navigationally speaking, c.f. Rossel & Wehner,1984)

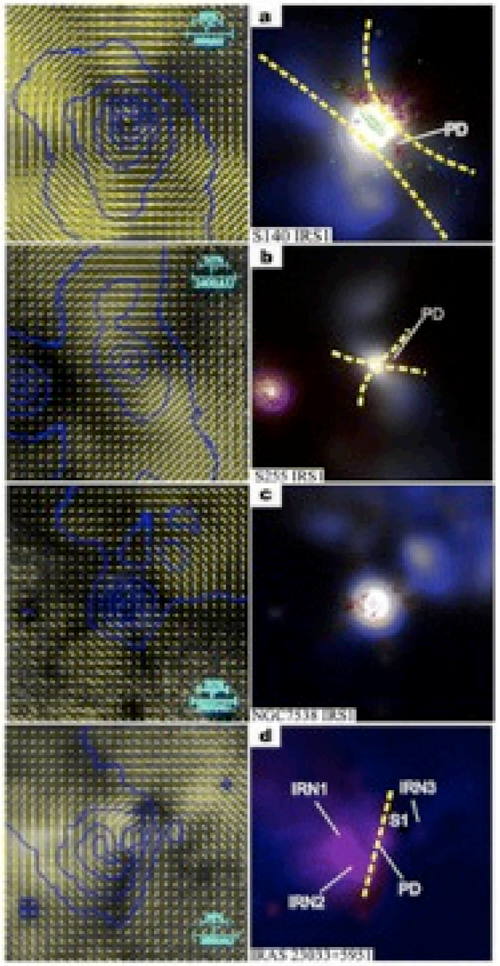
Why Polarimetry?



Why Polarimetry?



Why Polarimetry?

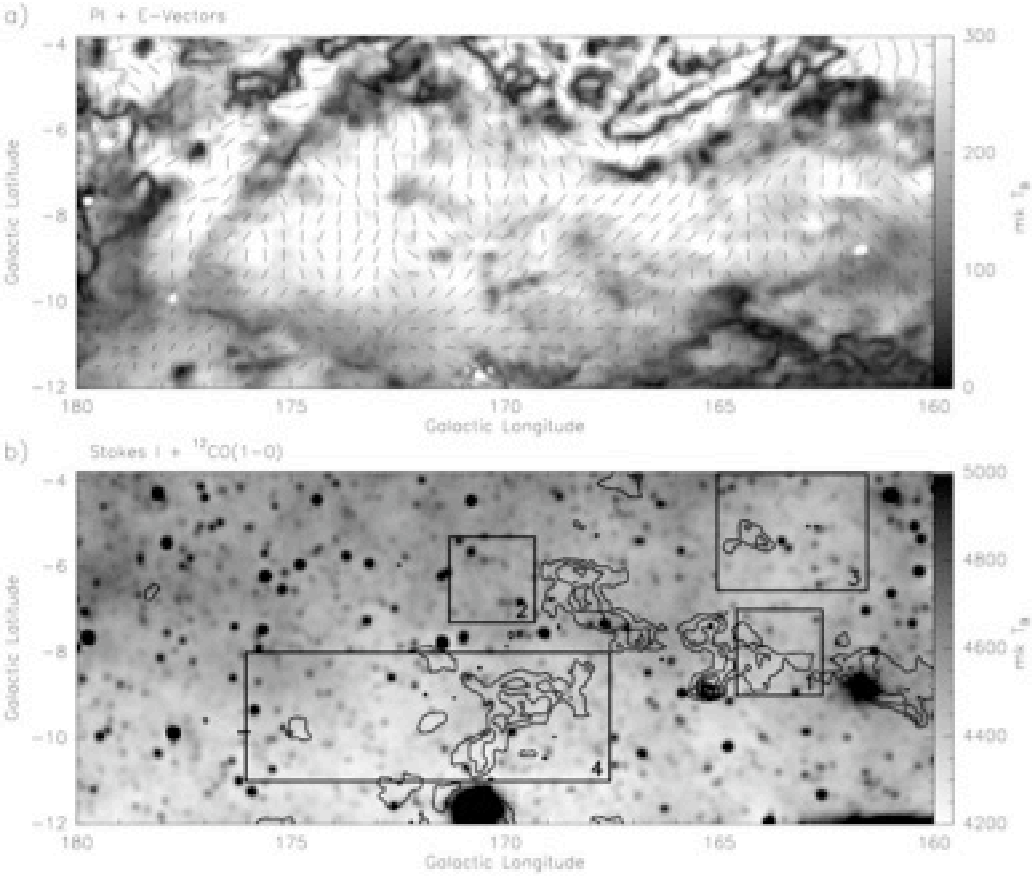


Zhibo et al. 2008

- *Left panels*: Polarization degree images overlaid by polarization vectors (*yellow dashes*) and total intensity contour (*blue curves*).
- *Right panels*: Pseudocolor images composed of pure brightness images (*red*) and polarized brightness images (*blue*).

Why Polarimetry

Faraday Screens have only a polarised structure, no standard (Stokes I) emission!



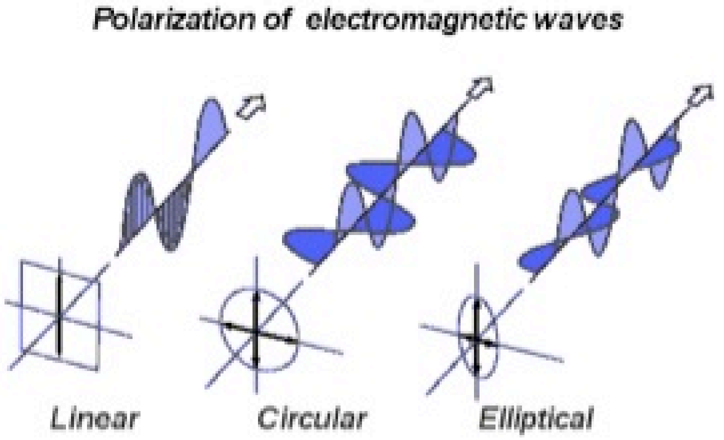
Haverkorn et al.

What is Polarisation?

What is Polarimetry & Polarisation?

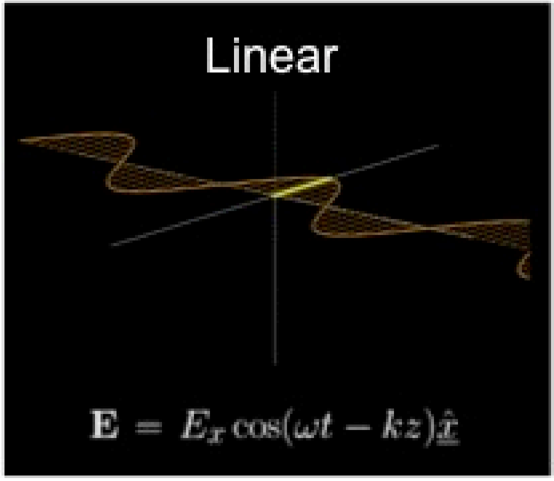
- Polarisation is the behaviour of the electric field with time.
- Natural radiation tends to be randomly polarised
  - The orientation of the electric field is completely random with respect to time.
- Astrophysical processes like synchrotron radiation can emit partially polarised emission, but *never* fully polarised.
- Interstellar matter can polarise random background emission or de-polarise polarised background emission.

What is Polarisation?



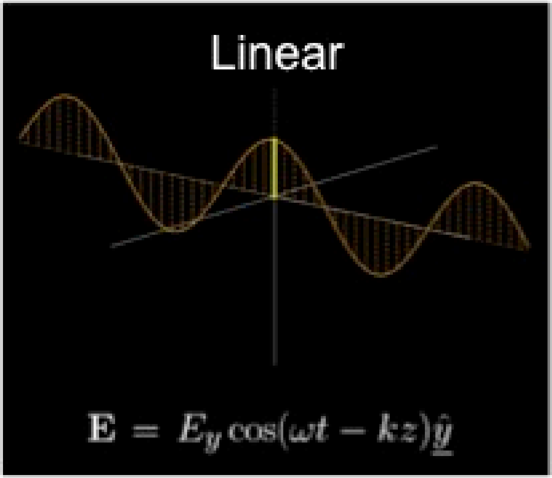
- Linear: orthogonal components in phase with constant ratio of strengths giving constant direction of electric vector.
- Circular: orthogonal components 90° out of phase with equal amplitudes – electric vector traces circle.

What is Polarisation?



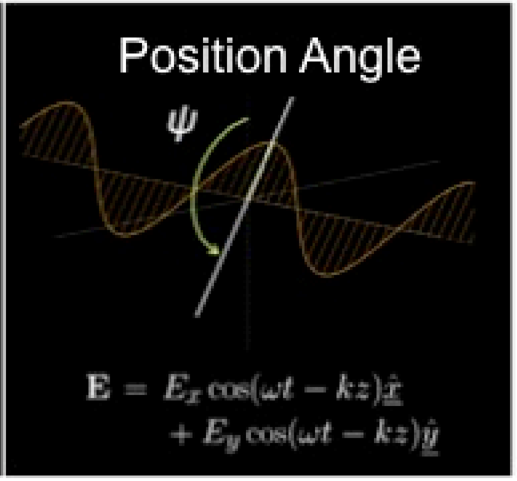
Linear

$$\mathbf{E} = E_x \cos(\omega t - kz)\hat{\underline{x}}$$



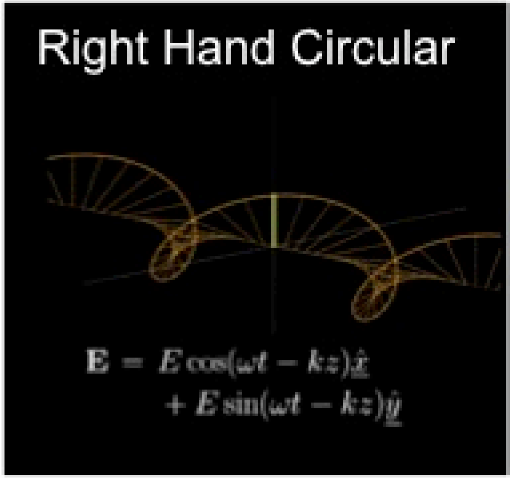
Linear

$$\mathbf{E} = E_y \cos(\omega t - kz)\hat{\underline{y}}$$



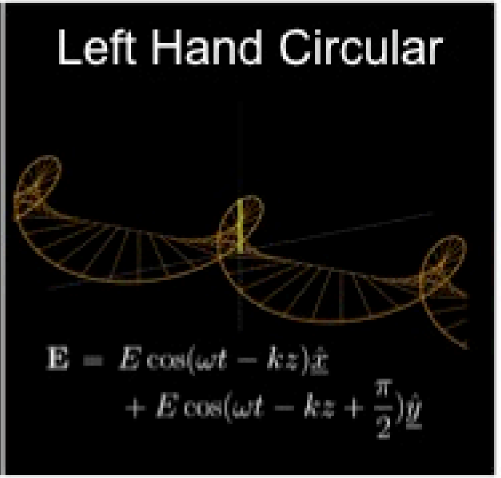
Position Angle

$$\begin{aligned}\mathbf{E} = {} & E_x \cos(\omega t - kz)\hat{\underline{x}} \\ & + E_y \cos(\omega t - kz)\hat{\underline{y}}\end{aligned}$$



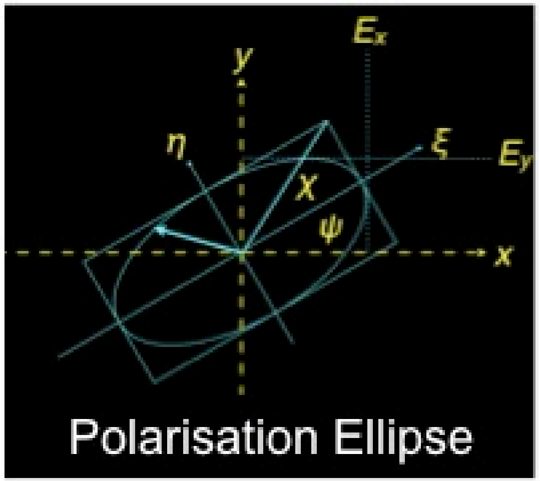
Right Hand Circular

$$\begin{aligned}\mathbf{E} = {} & E \cos(\omega t - kz)\hat{\underline{x}} \\ & + E \sin(\omega t - kz)\hat{\underline{y}}\end{aligned}$$



Left Hand Circular

$$\begin{aligned}\mathbf{E} = {} & E \cos(\omega t - kz)\hat{\underline{x}} \\ & + E \cos(\omega t - kz + \frac{\pi}{2})\hat{\underline{y}}\end{aligned}$$



Polarisation Ellipse

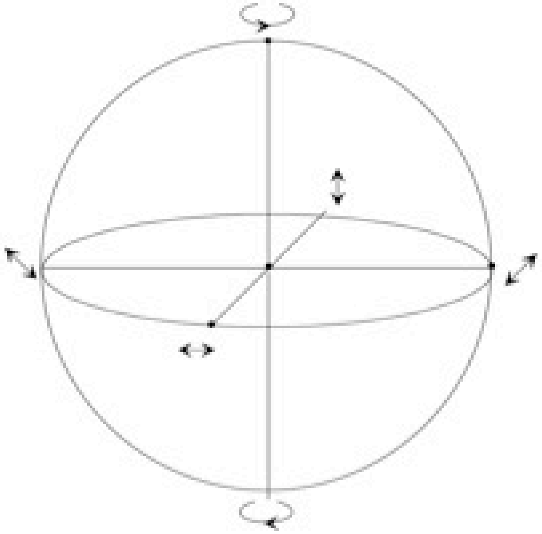
What is Polarisation?

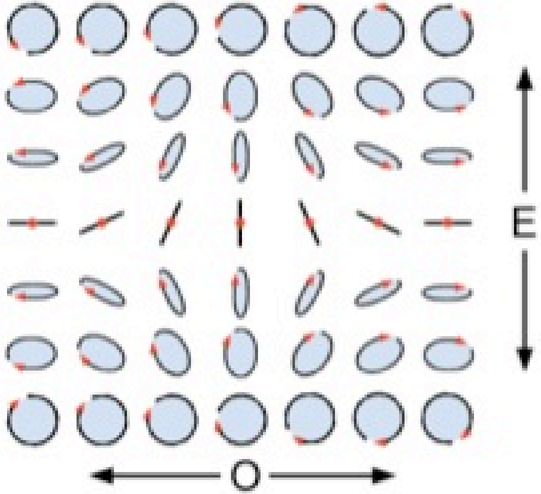
- Linearly polarised wave can be decomposed into two opposite handed circular waves
- Sum of two circular waves of unequal amplitude is elliptical.
- Sum of two orthogonal linears with a phase difference of between 0 and $\pi/2$ is also elliptical.

Jules Henri Poincaré

..and his sphere

- the spherical surface occupied by completely polarised states in the space of the vector
- Poles represent circular polarisations
  - Upper-hemisphere LHCP
  - Lower-hemisphere RHCP
- Equator represents linear polarisations with longitude representing tilt angle
- Latitude represents axial ratio





Now for the maths

Robert Clarke Jones

..and his vectors

- Jones calculus is a matrix-based means of relating observed to incident fields.
- Vectors describe incident radiation and matrices the response of the instrument.
- The Jones Vector: $\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}$
- Examples:
  - Linearly (x-direction) polarised wave: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
  - Left-Hand Circularly polarised wave: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$

Robert Clarke Jones

..and his matrices

- Effect of instrument described by 2x2 matrix:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_0 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

- Simple Examples:
  - Linear polariser: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
  - Left-Hand Circular polariser: $\frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$
- In practice matrix elements complex.
- Important: Only applicable to ***completely*** polarised waves.

Sir George Gabriel Stokes

..and his parameters

- Defined by George in 1852
- Adopted for astronomy by Chandrasehkar in 1947.
- Can be used for partially polarised radiation.
- Not a vector quantity! Deals with power instead of electric field amplitudes.
- The correlator can produce ALL Stokes parameters simultaneously (not so easy in optical astronomy!)

Stokes Parameters

- I – total intensity and sum of any two orthogonal polarisations
- Q and U – completely specify linear polarisation
- V – completely specifies circular polarisation

$$I = E_{0x}^2 + E_{0y}^2$$

$$Q = E_{0x}^2 - E_{0y}^2$$

$$U = 2E_{0x}E_{0y}\cos\delta$$

$$V = 2E_{0x}E_{0y}\sin\delta$$

$$I = \left\langle E_x E_x^* \right\rangle + \left\langle E_y E_y^* \right\rangle$$

$$Q = \left\langle E_x E_x^* \right\rangle - \left\langle E_y E_y^* \right\rangle$$

$$U = \left\langle E_x E_y^* \right\rangle + \left\langle E_x^* E_y \right\rangle$$

$$V = i\left(\left\langle E_x E_y^* \right\rangle - \left\langle E_x^* E_y \right\rangle\right)$$

(For linear feeds)

Fractional Polarisations

- The total linearly polarised intensity is defined as:

$$P = \sqrt{U^2 + Q^2}$$

[for native linear feed]

- A linearly polarised source will have an intrinsic position angle on the sky that is given by:

$$\Theta = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right)$$

[for native linear feed]

- The circular polarisation will be just Stokes V.
- Stokes parameters often presented as percentages of the total intensity.
- Since radio sources are *never* fully polarised, then the fractional linear and circular polarisation will always be <1

How do we measure it?

How do we measure it?

- Stokes parameters are the auto-correlation & cross-correlation products returned from the correlator, but input to the correlator can come from different feed types.
- Feeds normally designed to approximate pure linear or circular (known as ‘native linear’ or ‘native circular’)
  - Linear Feeds – intrinsically accurate & provide true linear response.
  - Circular Feeds – less accurate & frequency dependent response.

How do we measure it?

- Output of native linear feed is E_x and E_y field voltages, so:
  - I from XX+YY
  - Q from XX-YY
- Native circular adds 90° phase to X, so:
  - I from XX+YY
  - V from XX-YY

Stokes Parameters

- For circular feeds Q and V swap round..

Linear	Circular
$XX = I + Q$	$RR = I + V$
$YY = I - Q$	$LL = I - V$
$XY = U + iV$	$RL = Q + iU$
$YX = U - iV$	$LR = Q - iU$

But is it really that simple?

- Do we just plug in our computer and get {I,Q,U,V} out of the correlator?
- No, there are leakages!
  - The total intensity can leak into the polarised components (I into {Q,U,V}).
  - The linear polarisation can leak into the circular ({Q,U} into V).
  - … and all combinations and permutations are allowed!
- Without correcting for leakage, you're not going to get proper Stokes parameters!

Hans Mueller

..and his matrix

- The leakage of each polarisation into the other can be measured and quantified in a 4x4 matrix first proposed by Mueller in 1943.

$$M = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{ii} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix}$$

The Mueller Matrix

Correlator Output Incoming Radiation

$$\begin{bmatrix} XX+YY \\ XX-YY \\ XY \\ YX \end{bmatrix} = M \bullet \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Example (simple) Mueller Matrices

- If feeds were perfect:
  - Dual linear feed: M is unitary
  - Dual linear feed rotated 45°: Q and U interchange and sign change for rotation:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - A dual linear feed rotated 90°: signs of Q and U reversed:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- As Alt-Az telescope tracks source, feed rotates on sky by the parallactic angle (PA):

$$M_{sky} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2PA & \sin 2PA & 0 \\ 0 & -\sin 2PA & \cos 2PA & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The more general Mueller Matrix

- For a (realistic) dual linear feed:

$$M = \begin{bmatrix} 1 & \left(-2\varepsilon\sin\phi\sin 2\alpha + \frac{\Delta G}{2}\cos 2\alpha\right) & 2\varepsilon\cos\phi & \left(2\varepsilon\sin\phi\cos 2\alpha + \frac{\Delta G}{2}\sin 2\alpha\right) \\ \frac{\Delta G}{2} & \cos 2\alpha & 0 & \sin 2\alpha \\ 2\varepsilon\cos(\phi+\varphi) & \sin 2\alpha\sin\varphi & \cos\varphi & -\cos 2\alpha\sin\varphi \\ 2\varepsilon\sin(\phi+\varphi) & -\sin 2\alpha\cos\varphi & \sin\varphi & \cos 2\alpha\cos\varphi \end{bmatrix}$$

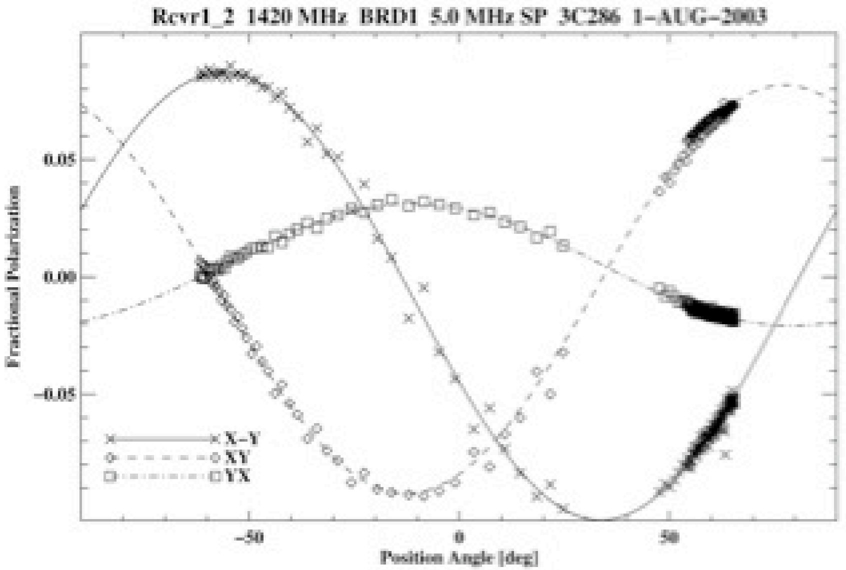
- The Mueller matrix has 16 elements, but ONLY 7 INDEPENDENT PARAMETERS. The matrix elements are not all independent.

Calculating the Mueller Matrix

- For a perfect system, as we track a polarised source across the sky the parallactic angle changes and this should produce:
  - For XX-YY: $\cos 2(PA_{az}+PA_{src})$, centred at zero.
  - For XY: $\sin 2(PA_{az}+PA_{src})$, centred at zero
  - For YX: zero (most sources have zero circular polarisation)

Calculating the Mueller Matrix

- ☐ But, what we find is:



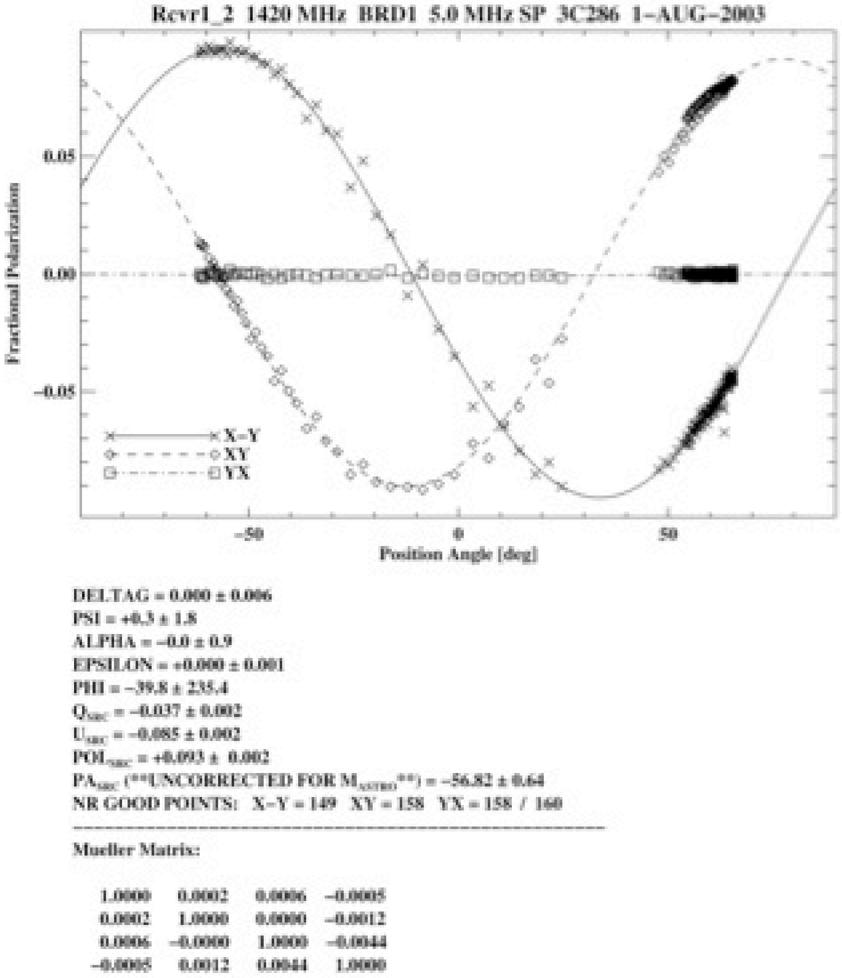
DELTAG = −0.017 ± 0.006
PSI = −16.7 ± 1.8
ALPHA = +0.4 ± 0.9
EPSILON = +0.004 ± 0.001
PHI = +152.3 ± 23.0
Q_{SRC} = −0.037 ± 0.002
U_{SRC} = −0.085 ± 0.002
POL_{SRC} = +0.093 ± 0.002
PA_{SRC} (**UNCORRECTED FOR M_{ASTRO}**) = −56.81 ± 0.64
NR GOOD POINTS: X−Y = 149 XY = 158 YX = 160 / 160

Mueller Matrix:

```
  1.0000  -0.0083  -0.0065   0.0033
 -0.0083   0.9999   0.0001   0.0145
 -0.0053  -0.0042   0.9581   0.2865
  0.0052  -0.0139  -0.2865   0.9580
```

Calculating the Mueller Matrix

- ☐ Which enables the matrix to be calculated and the observations corrected to give what we expect:



DELTAG = 0.000 ± 0.006
PSI = +0.3 ± 1.8
ALPHA = −0.0 ± 0.9
EPSILON = +0.000 ± 0.001
PHI = −39.8 ± 235.4
Q_{SRC} = −0.037 ± 0.002
U_{SRC} = −0.085 ± 0.002
POL_{SRC} = +0.093 ± 0.002
PA_{SRC} (**UNCORRECTED FOR M_{ASTRO}**) = −56.82 ± 0.64
NR GOOD POINTS: X−Y = 149 XY = 158 YX = 158 / 160

Mueller Matrix:

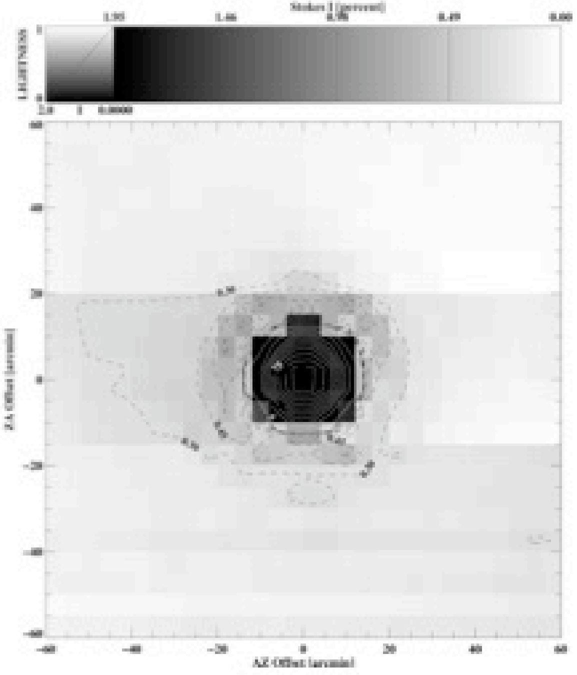
```
  1.0000    0.0002    0.0006   -0.0005
  0.0002    1.0000    0.0000   -0.0012
  0.0006   -0.0000    1.0000   -0.0044
 -0.0005    0.0012    0.0044    1.0000
```

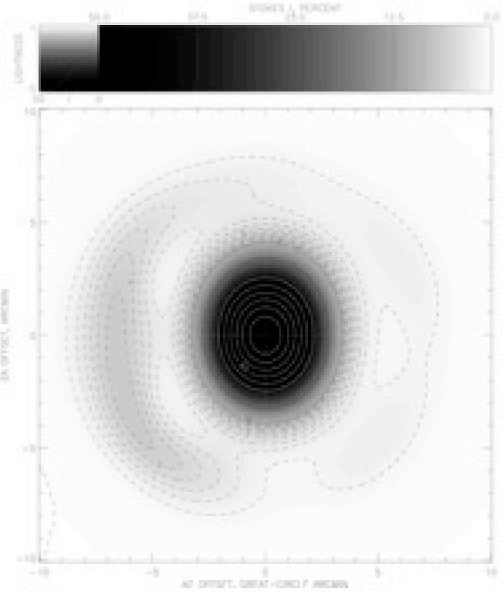
Beam Effects

Beam Effects

- For point sources, all of the previous is fine.
- What if the source you're looking at is extended compared to the telescope beam?
- There are instrumental beam effects that can confuse the measurement of extended polarised signals. They are…
  - Squint
  - Squash

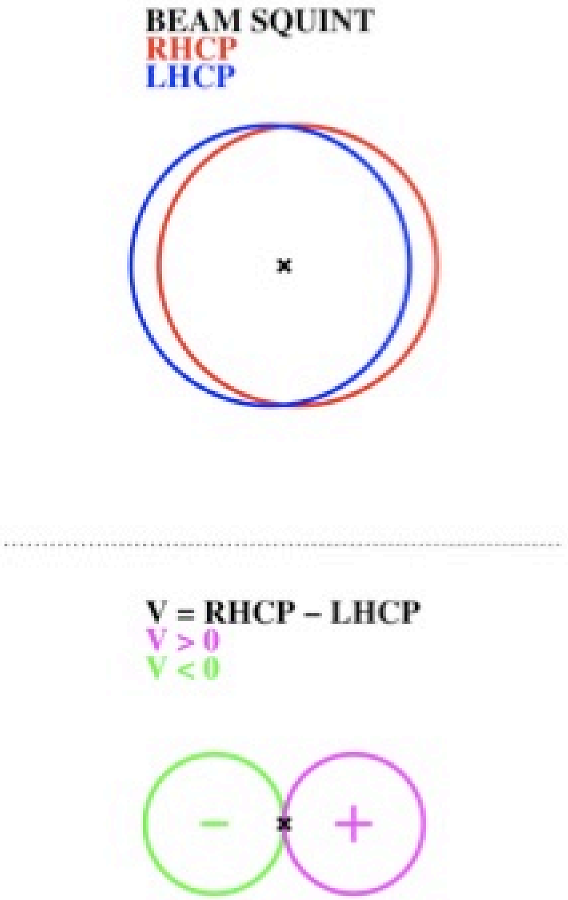
Stokes I response

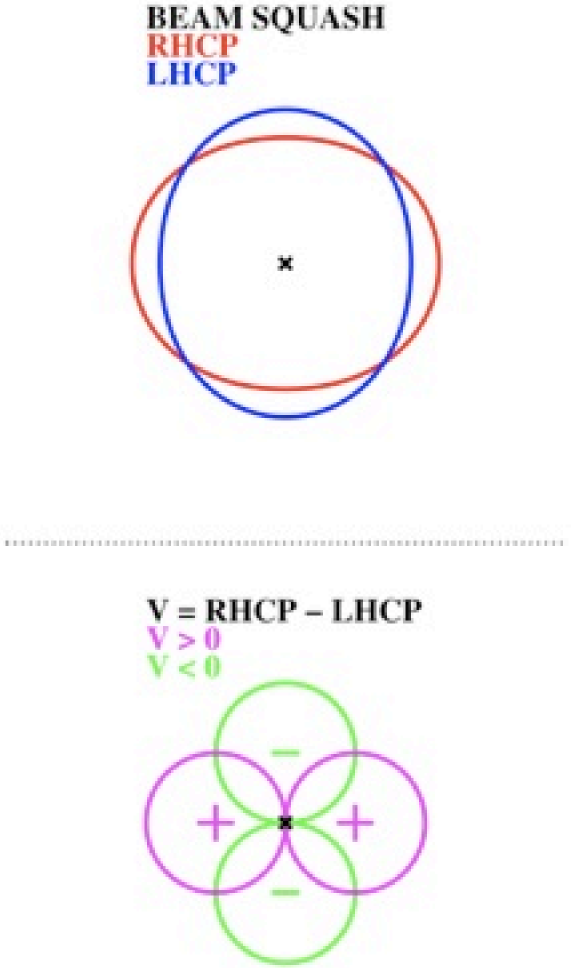




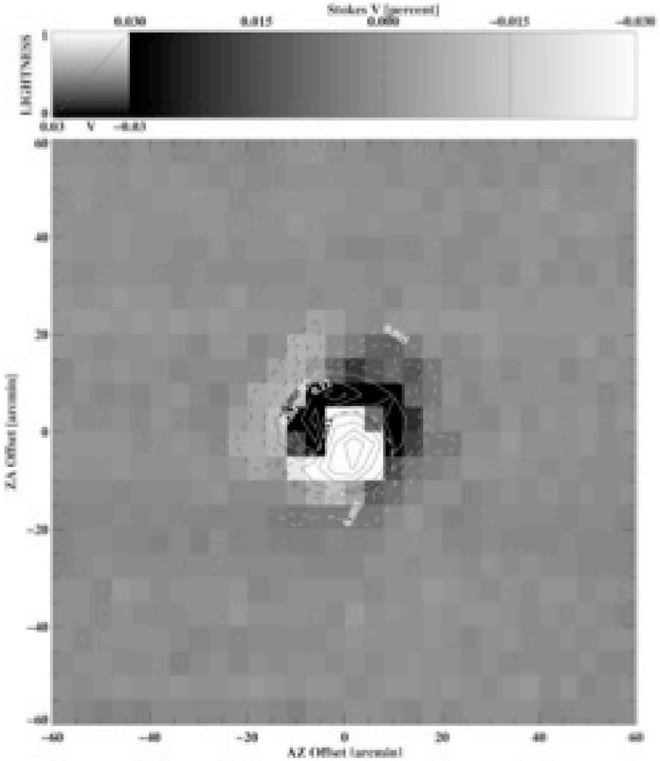
Heiles et al. 2001

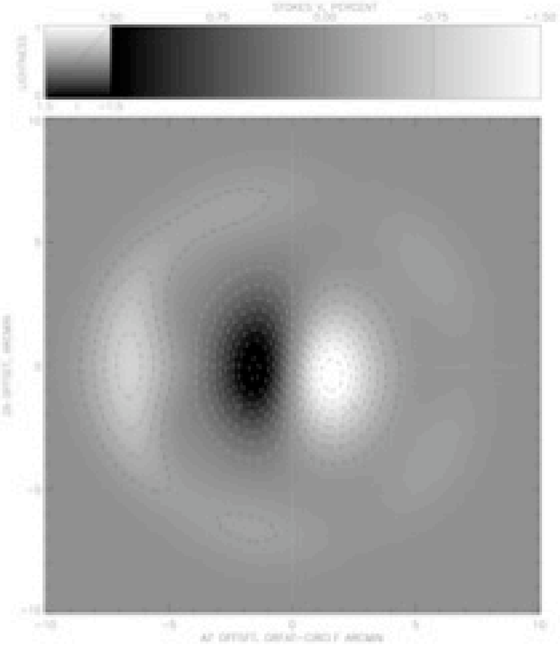
Beam Squint & Squash





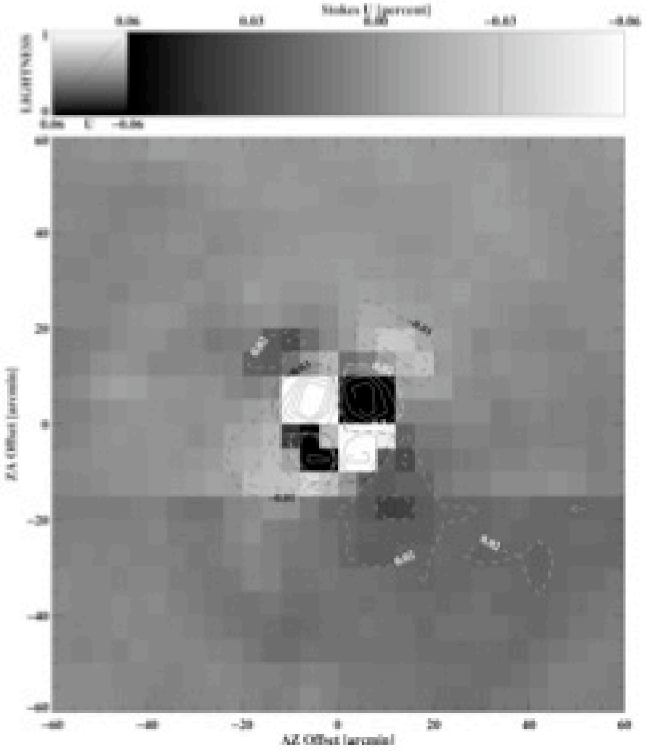
Squint in action

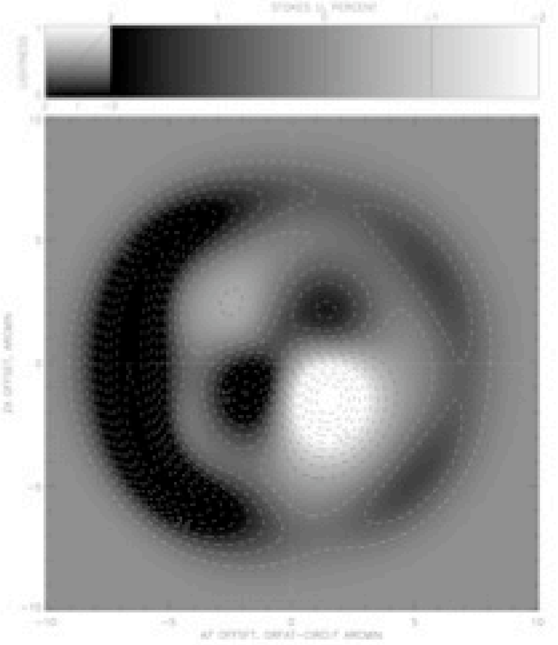




Heiles et al. 2001

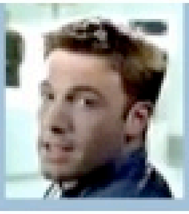
Squash in action



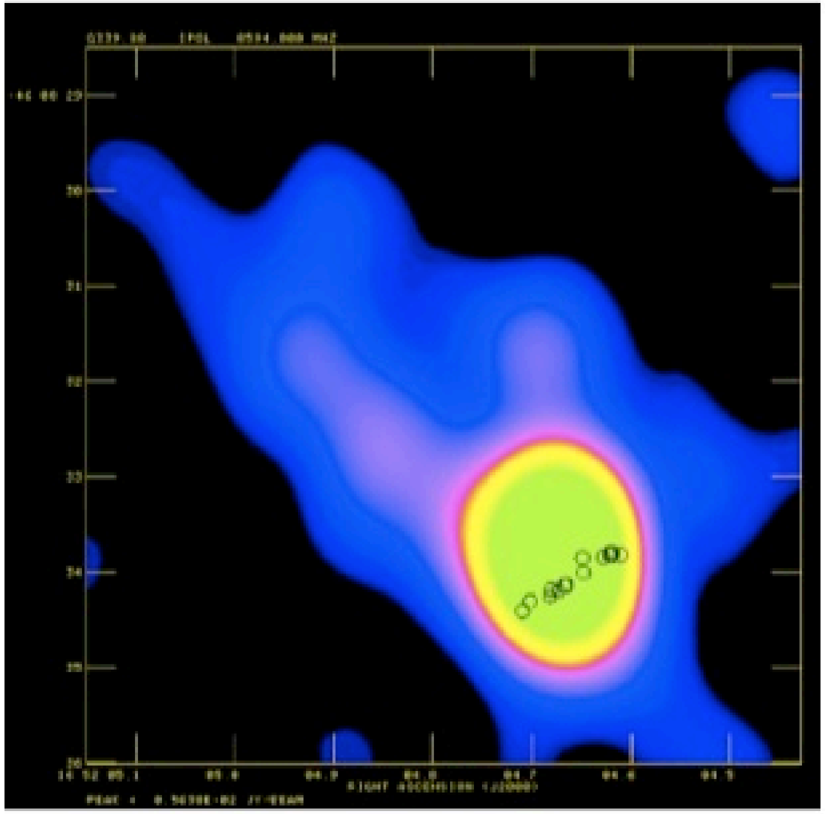


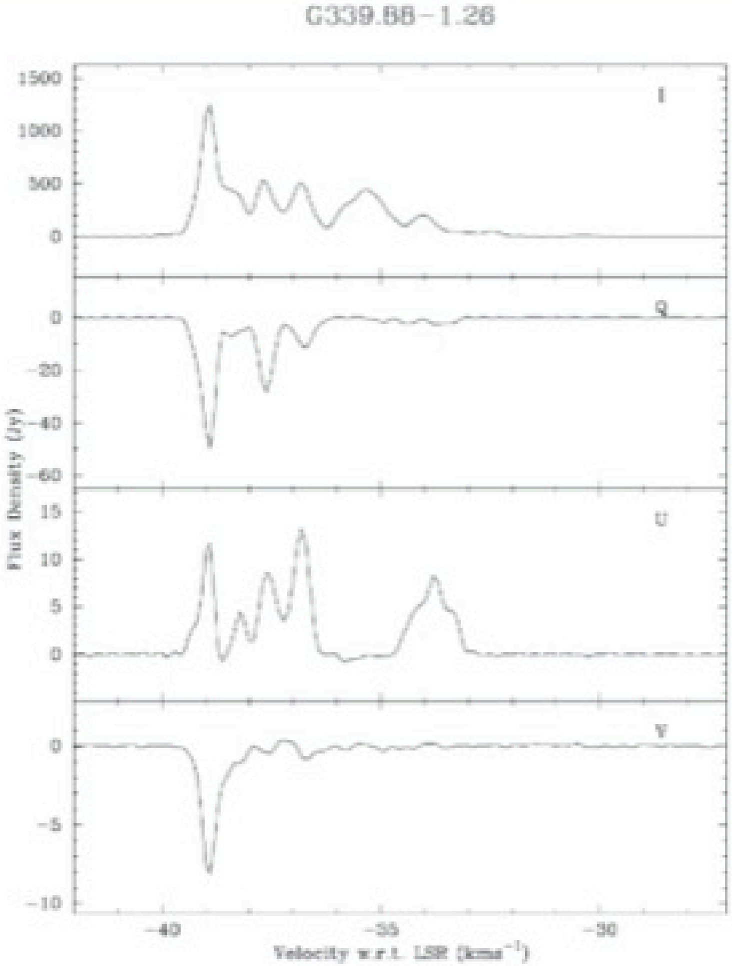
Heiles et al. 2001

Here comes the science



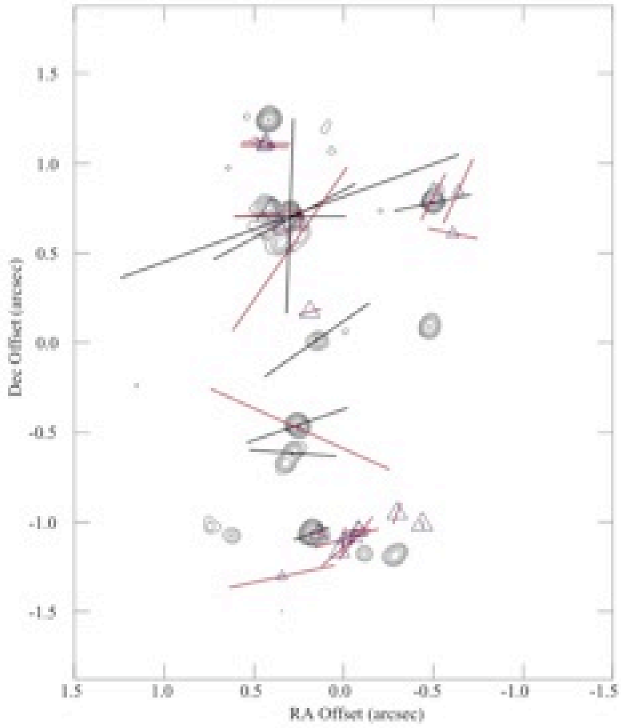
Polarisation of Masers





S. Ellingsen

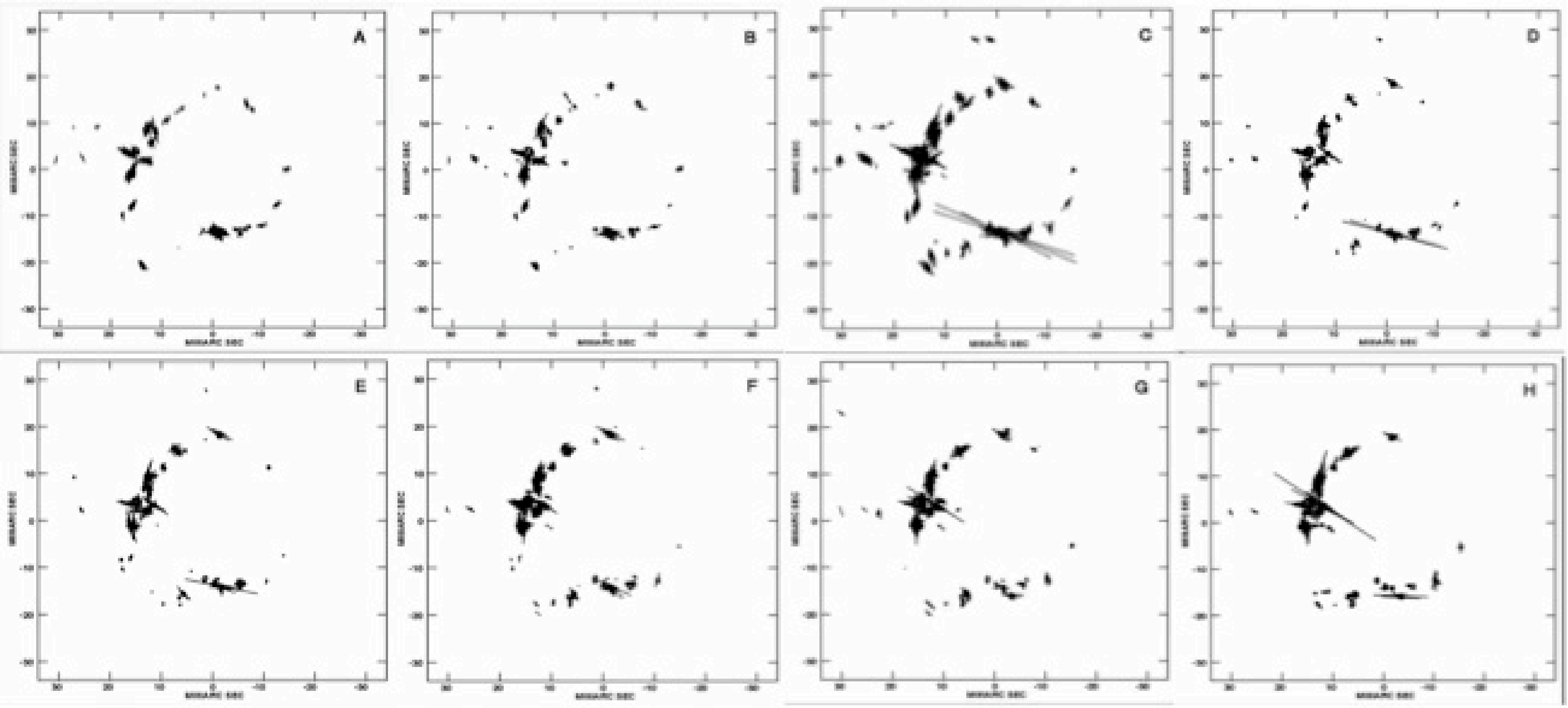
Polarisation of Masers



W3(OH) Harvey-Smith & Cohen 2006, Vlemmings et al 2006.

- Filamentary maser structure.
- Linear polarisation up to 8%.
- Polarisation angles indicate north-south structure, and are consistent with OH.

Linear Polarisation of Masers



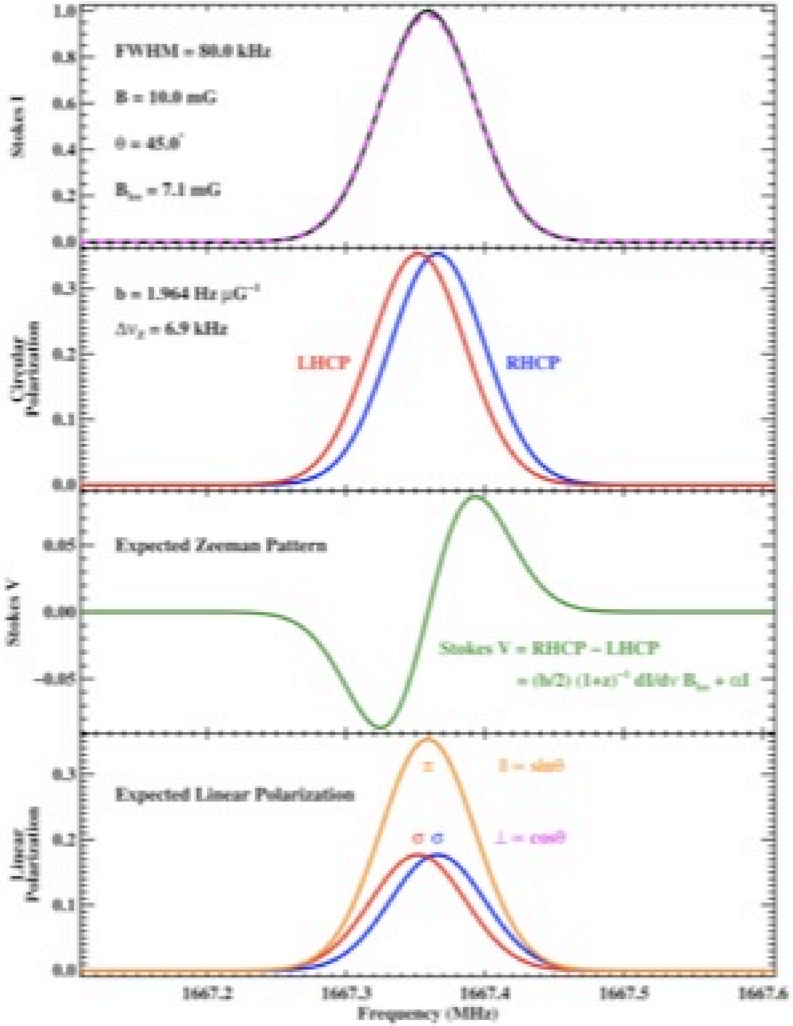
Kemball et al. 2009

Make like a banana:

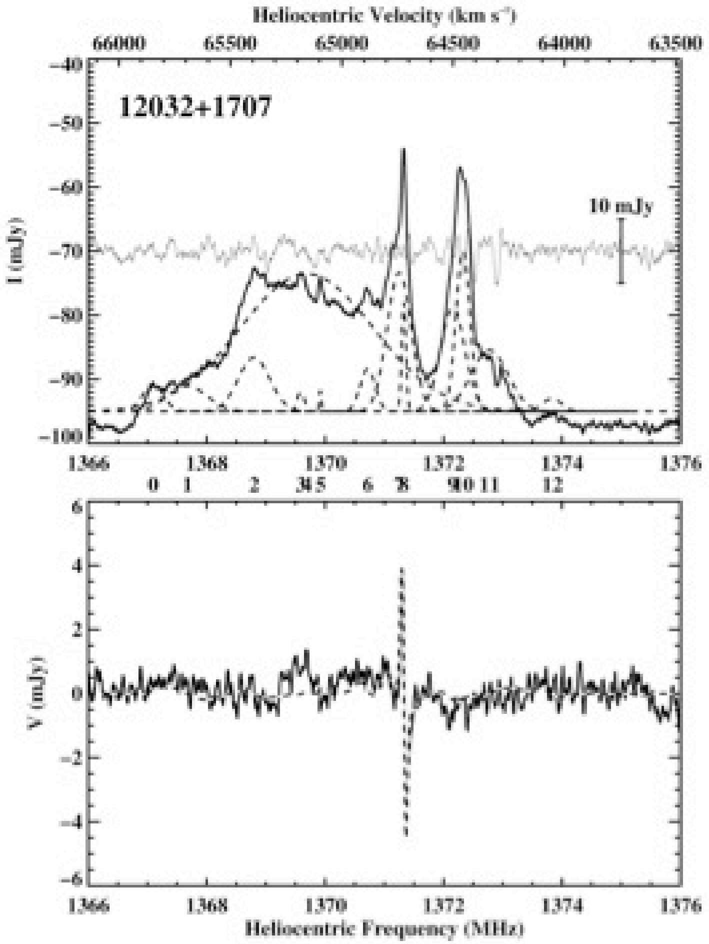
Zeeman Splitting

- Atoms & molecules in net magnetic moment will have their energy levels split in the presence of a magnetic field.
- Detected through frequency shift between right and left circularly polarised emission

$$V = RHCP - LHCP \propto B_{los}$$



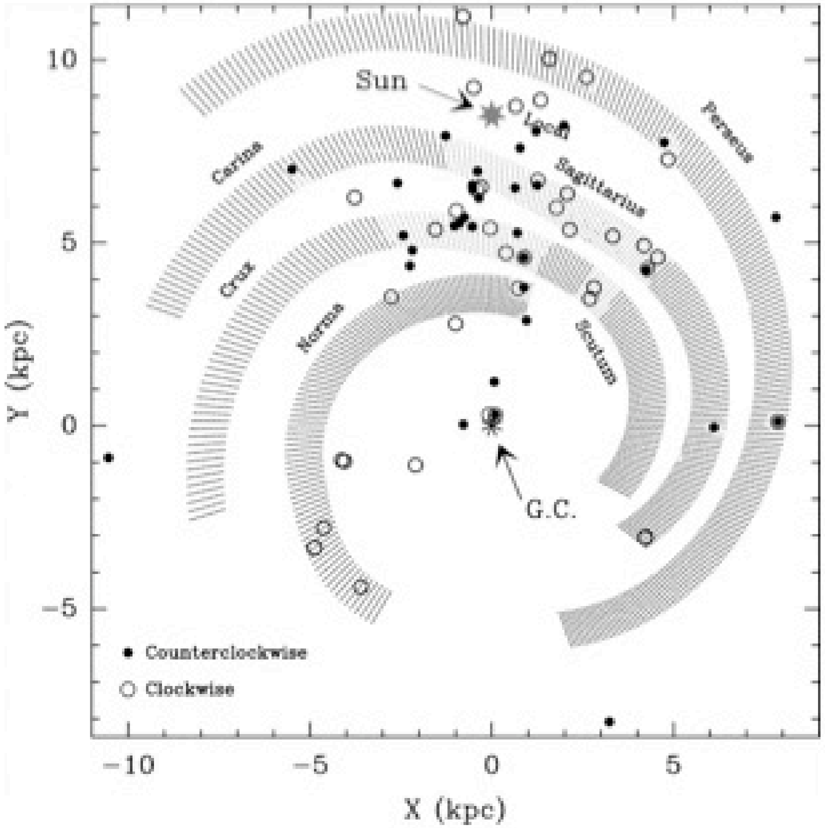
Zeeman Splitting of Masers



- ☐ OH Gigamaser (luminosity greater than 10^4 solar luminosities)

MYSTERIUM

Zeeman Splitting of Masers



Fish et al 2003

- Line-of-sight magnetic field directions deduced from OH maser Zeeman splitting.
- 74 star-forming regions:
  - 41 with an overall magnetic field oriented in a clockwise sense.
  - 33 with field oriented counterclockwise as viewed from above the Galactic center.
- Field consistency within 2-kpc of Sun.

In summary..

Summary

- Polarisation in radio astronomy very important to improving our knowledge & understanding.
- Can describe polarisation with the Polarisation Ellipse and the Poincare Sphere.
- Dr. Jones offers a vector representation for ideal cases of completely polarised emission.
- Mueller and his matrices are the best option for real situations.
- There are Linear and Circular feed types, must account for which you are using.
- **Understanding the polarisation properties of your dish is fundamental to successful observations!**
- (Masers offer exciting science opportunities!)

Useful References

- Heiles, C. 'A Heuristic Introduction to Radioastronomical Polarisation' (2002) ASP 278
- Tinbergen, J. 'Astronomical Polarimetry' (1996), Cambridge University Press (Cambridge, UK)
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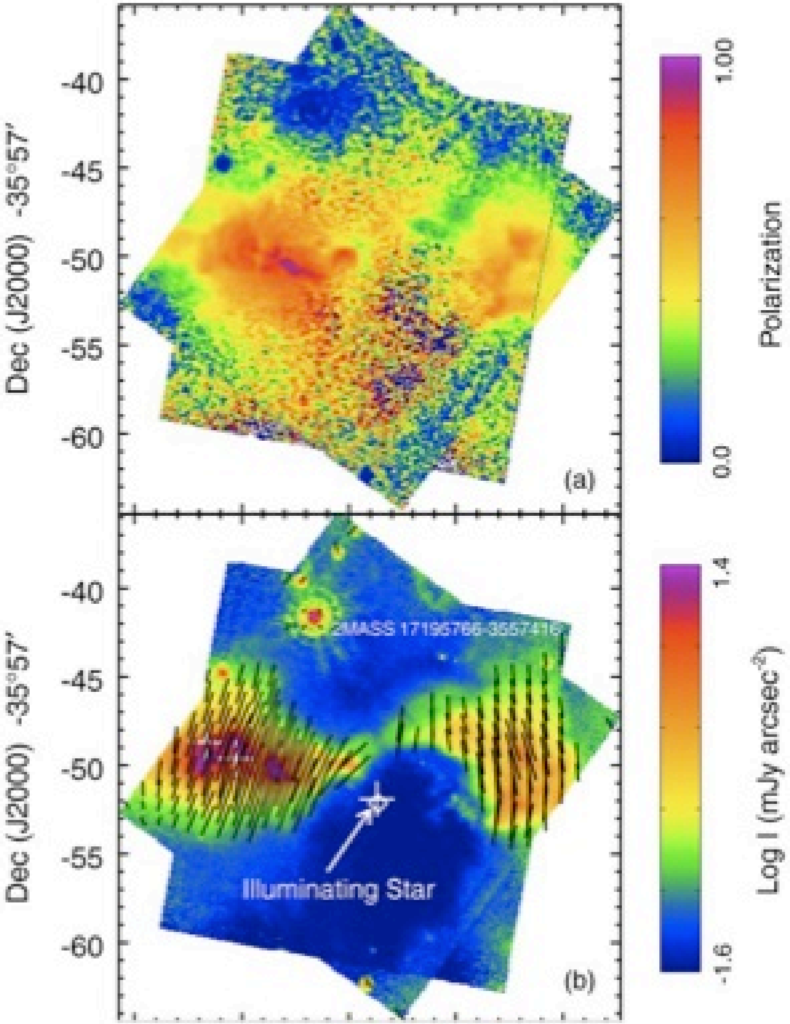
Why Polarimetry?



- *HST* NICMOS image of massive young stellar object NGC 6334 V.
  - (a) Fractional polarization.
  - (b) Log intensity with polarization vectors.

Michael Faraday

..and his rotations

- Magnetised plasma is birefringent – the refractive indices for the two circular modes are different due to the parallel component of the magnetic field (and dependent on electron density and frequency of radiation).
- Hence phases of two modes changes along the propagation path and so does the position angle of the resultant linearly polarised radiation.