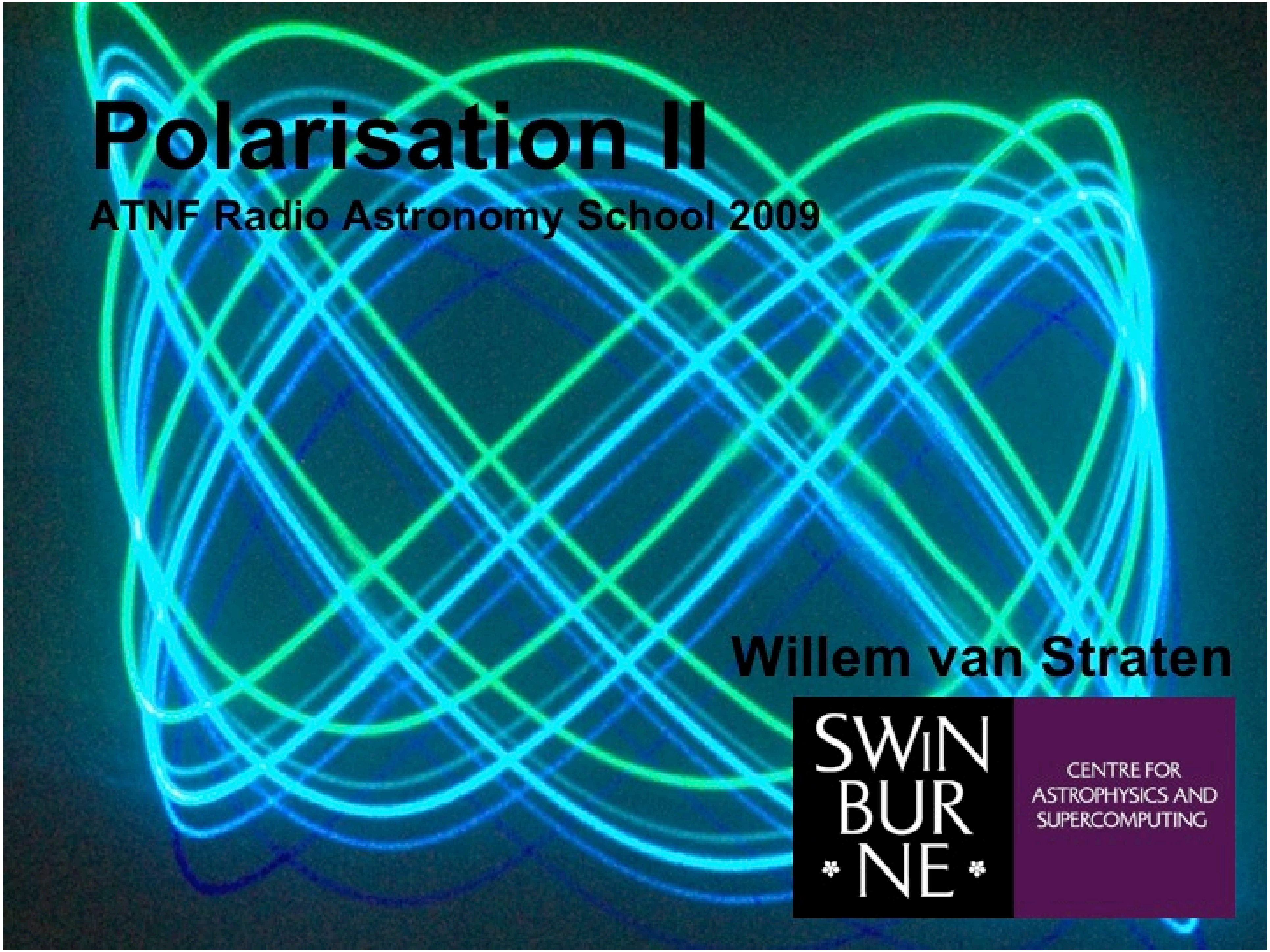
Polarisation II

ATNF Radio Astronomy School 2009



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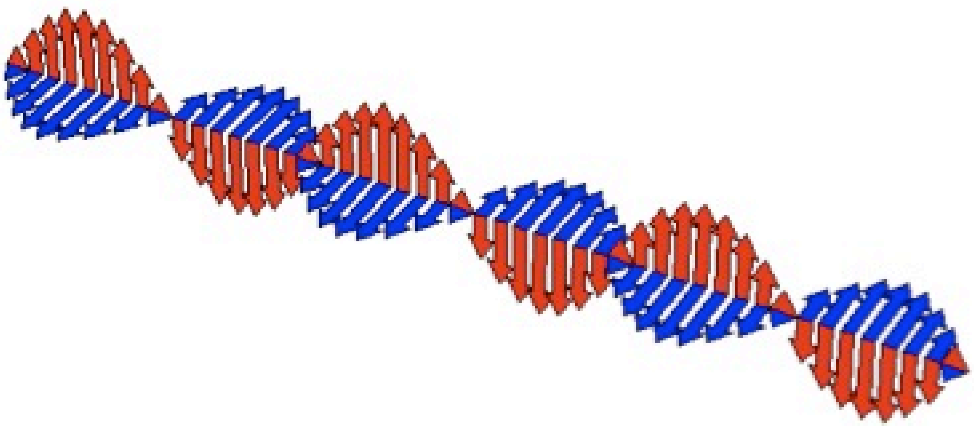
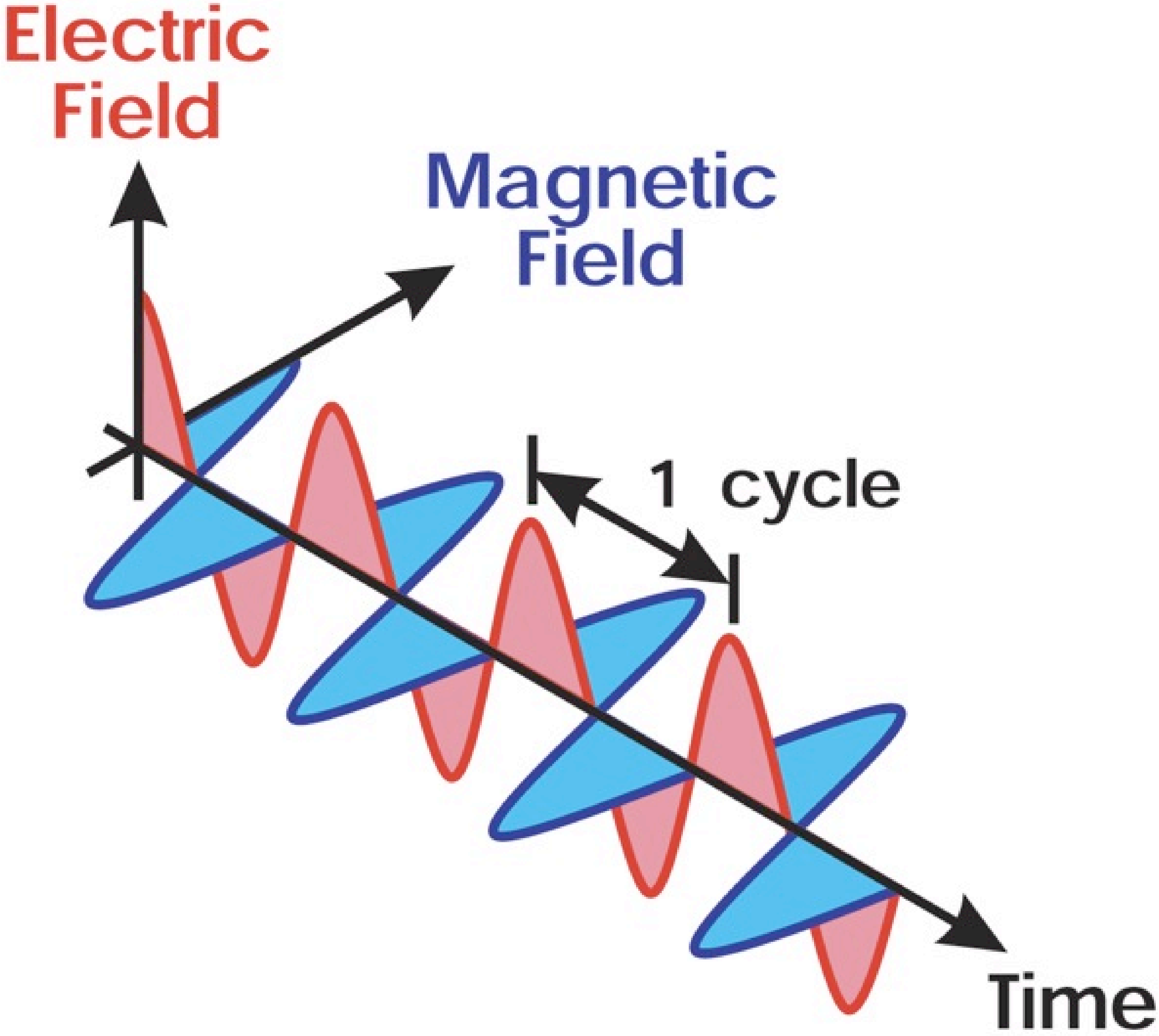
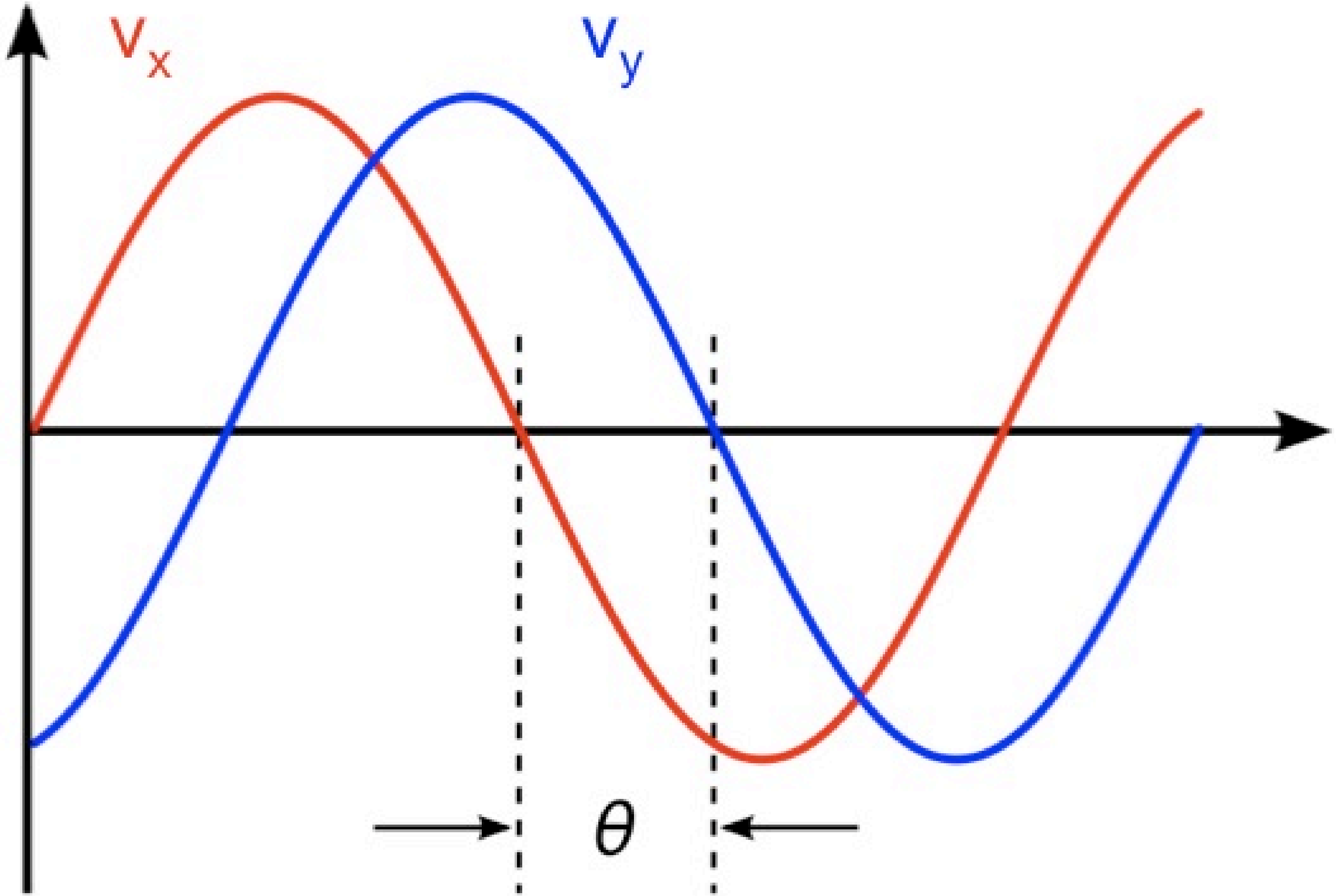
Outline

- Representations of polarisation
  - geometric and statistical
- Radio pulsar polarisation
  - rotating vector model, single pulses, Faraday rotation, high-precision timing
- Polarimetric calibration
- The Future

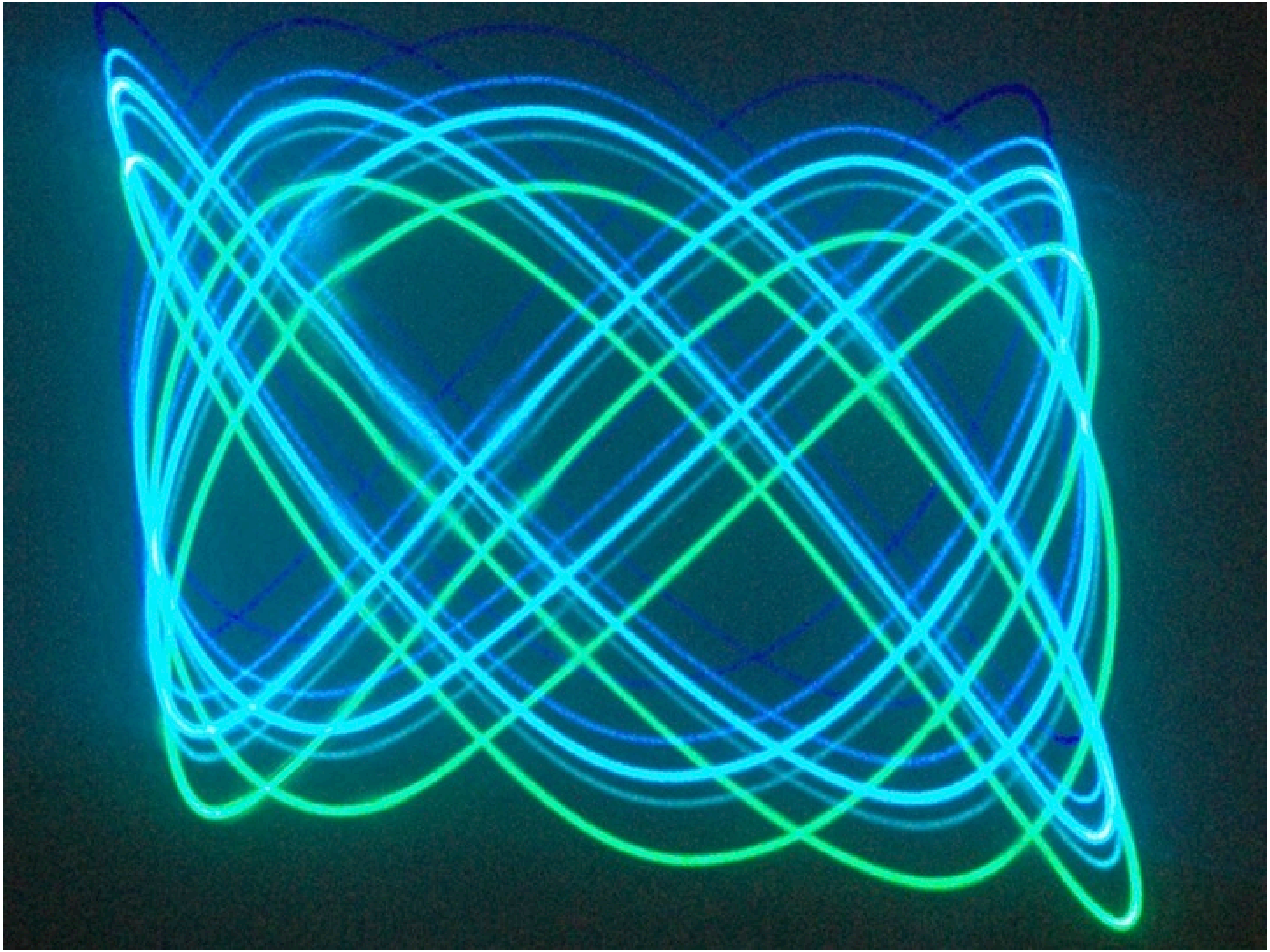
Electromagnetic Plane Wave



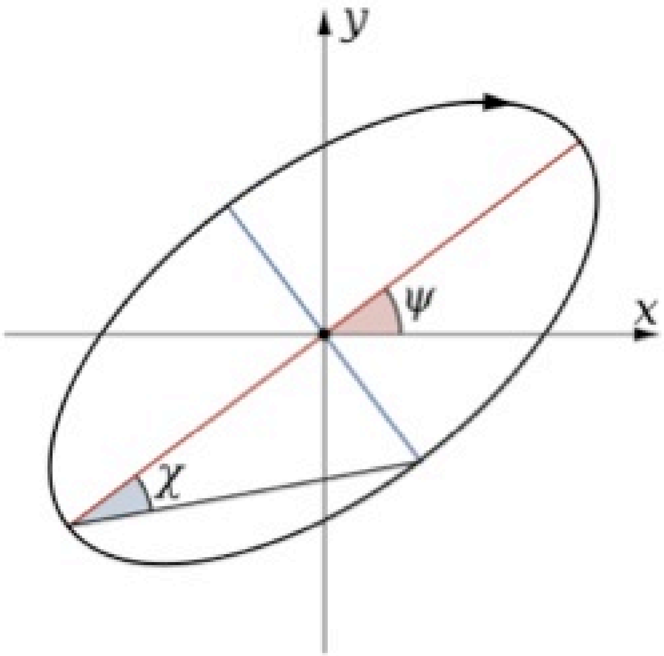

Electric
Field
Magnetic
Field
1 cycle
Time



v_x
v_y
θ

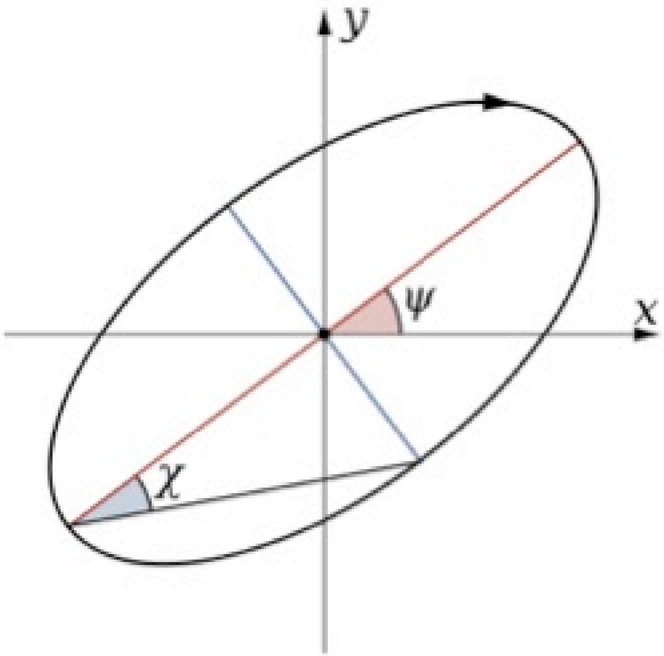


Polarisation Ellipse



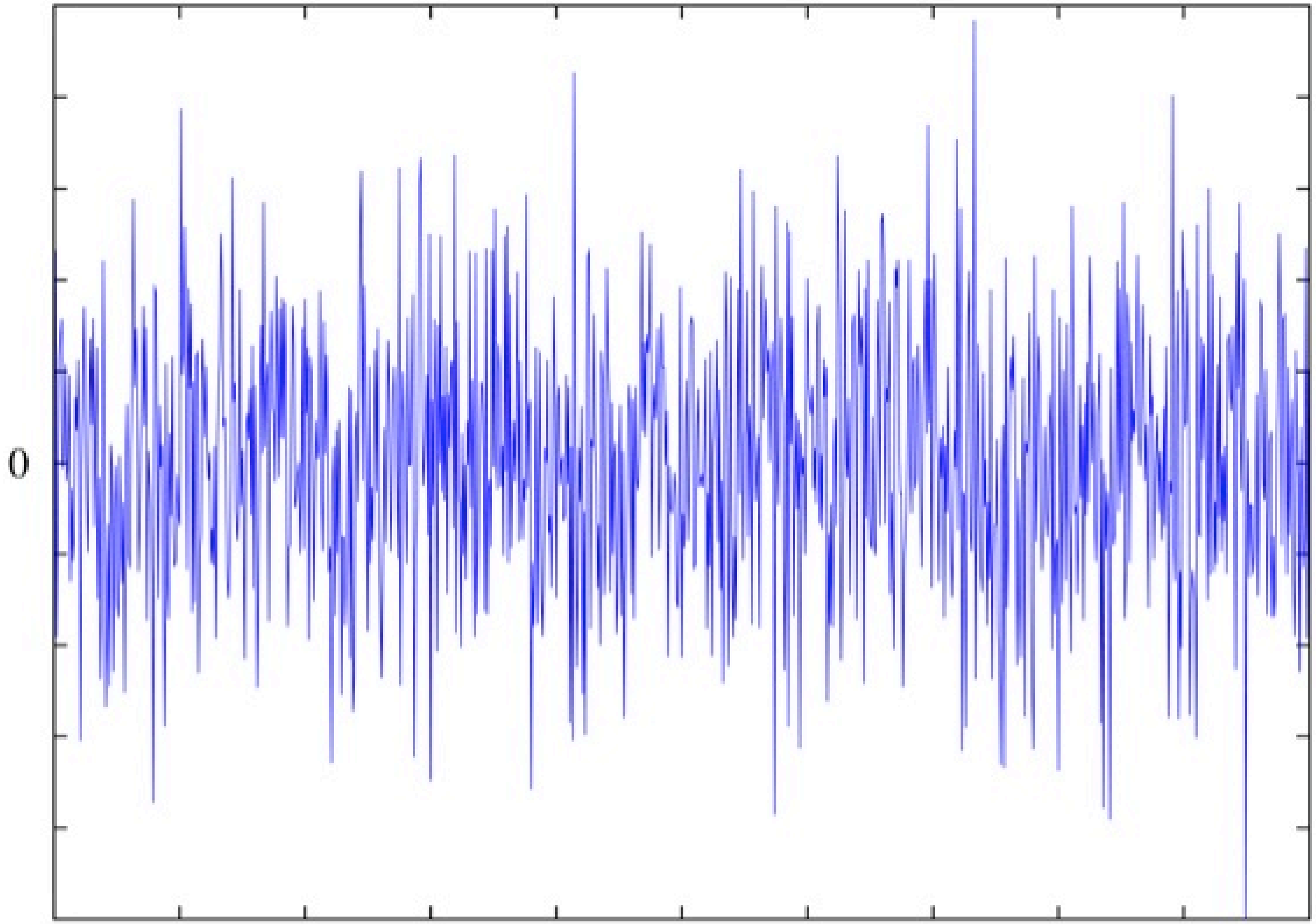
- 3 degrees of freedom
  - size, orientation ψ, and axial ratio $\tan\chi$
  - direction = sign of χ
- insensitive to absolute phase

Polarisation Ellipse



- 0 < size < ∞
- $0 < \psi < \pi$
- $-\pi/4 < \chi < \pi/4$

Partially polarised light



Need for Complex Numbers

$$v_x(t) = a_x(t)\cos(\phi_x(t))$$
$$v_y(t) = a_y(t)\cos(\phi_y(t))$$

$$\langle \phi_x(t) - \phi_y(t) \rangle ?$$

Analytic Signal

$$e_x(t) = v_x(t) + i\hat{v}_x(t) = a_x(t)\exp(i\phi_x(t))$$
$$e_y(t) = v_y(t) + i\hat{v}_y(t) = a_y(t)\exp(i\phi_y(t))$$

$\hat{v}(t)$ is the Hilbert transform of $v(t)$

$e(t)$ is the analytic representation of $v(t)$

$$\langle e_x(t)e_y^*(t)\rangle = \langle a_x(t)a_y(t)\exp\left(i\left[\phi_x(t) - \phi_y(t)\right]\right)\rangle$$

Coherency Matrix

$$\boldsymbol{\rho} \equiv \langle \boldsymbol{e}\, \boldsymbol{e}^\dagger \rangle = \begin{pmatrix} \langle e_0 e_0^* \rangle & \langle e_0 e_1^* \rangle \\ \langle e_1 e_0^* \rangle & \langle e_1 e_1^* \rangle \end{pmatrix}$$

$$\boldsymbol{\rho} = \boldsymbol{\rho}^\dagger$$

Stokes Parameters - Definition

$$\boldsymbol{\rho} = S_k \, \boldsymbol{\sigma}_k / 2$$

$$S_k = \mathrm{Tr}(\boldsymbol{\sigma}_k \boldsymbol{\rho})$$

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \boldsymbol{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

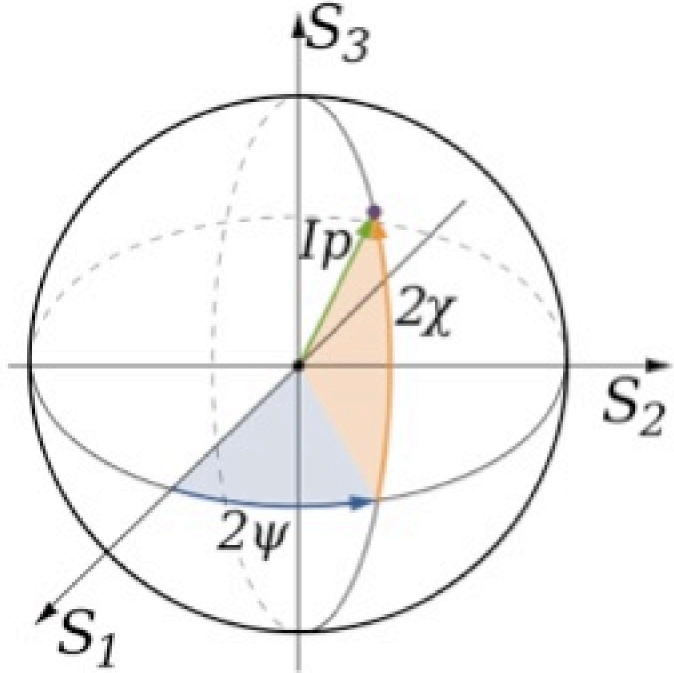
$$\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Eigen Decomposition

$$\boldsymbol{\rho}' = \mathbf{R}\boldsymbol{\rho}\mathbf{R}^{-1} = \begin{pmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{pmatrix}$$

$$S'_0 = \lambda_0 + \lambda_1$$
$$S'_1 = \lambda_0 - \lambda_1$$
$$S'_2 = S'_3 = 0$$

Stokes Parameters - 4 vector



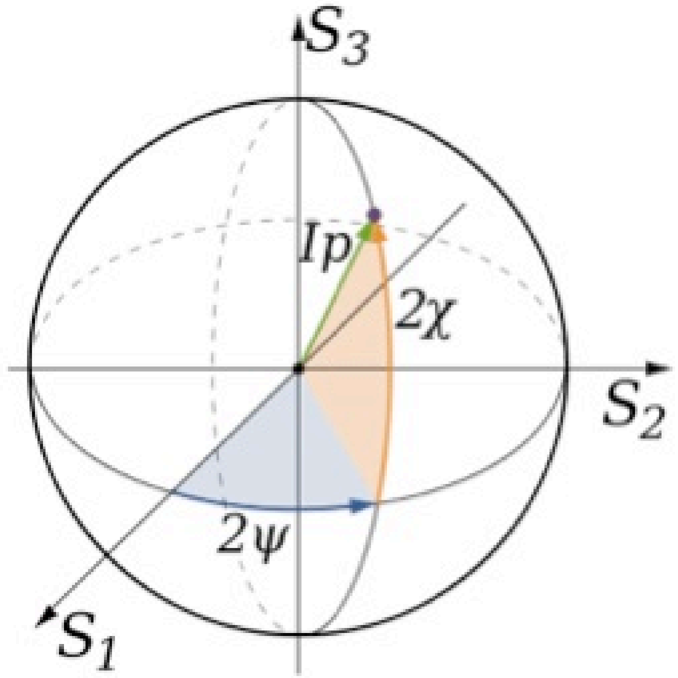
$S_0 = I$ total intensity

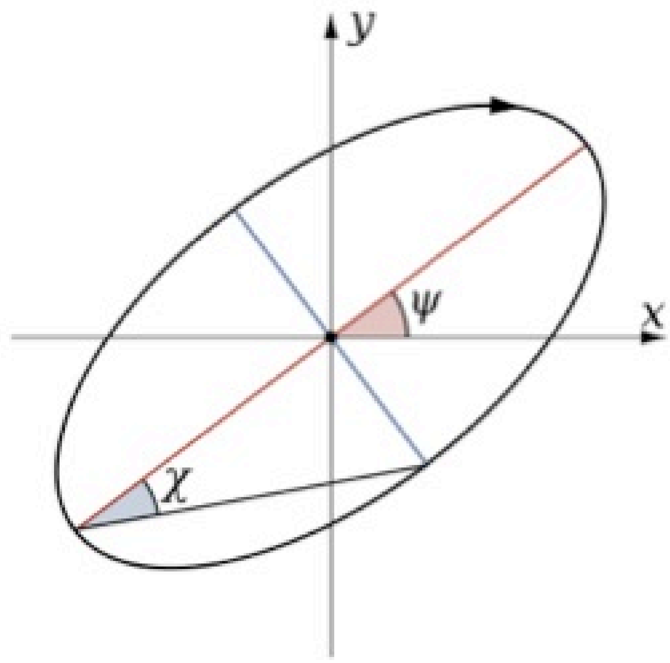
$\boldsymbol{S} = (S_1, S_2, S_3)$ polarisation vector

All have units of intensity

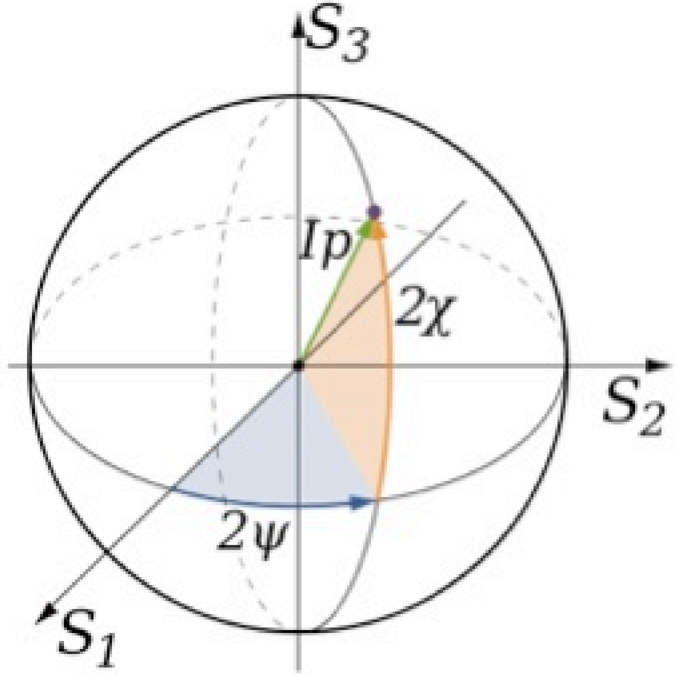
$p = |\boldsymbol{S}|/I$ degree of polarisation

Stokes Parameters - Ellipse





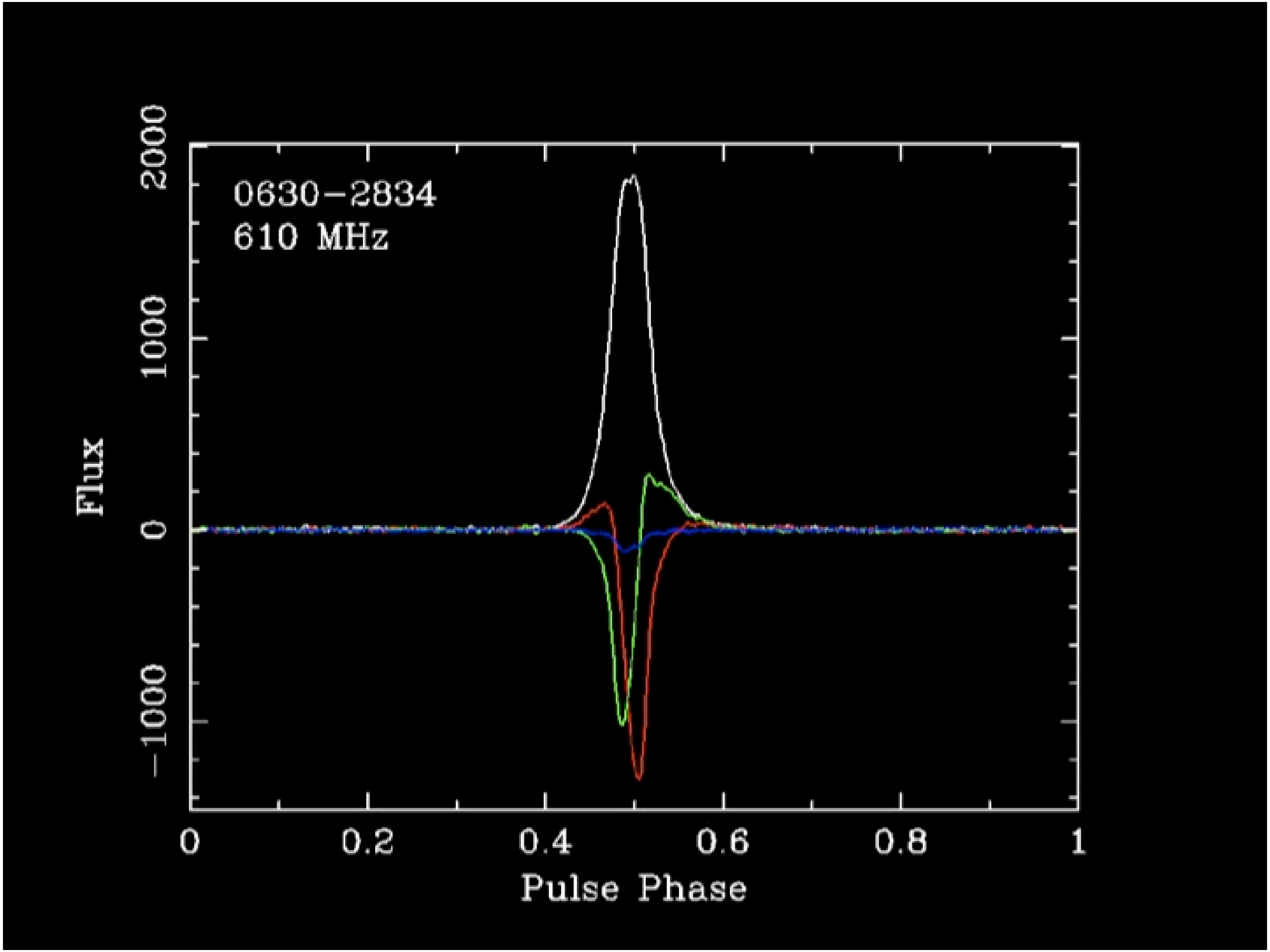
Stokes Parameters - Intensities



$$S_1 = \lambda_{\psi=0} - \lambda_{\psi=90}$$
$$S_2 = \lambda_{\psi=45} - \lambda_{\psi=135}$$
$$S_3 = \lambda_{\chi=45} - \lambda_{\chi=-45}$$

Stokes Parameters - Mnemonic

Animation Credit:
Andrew Jameson

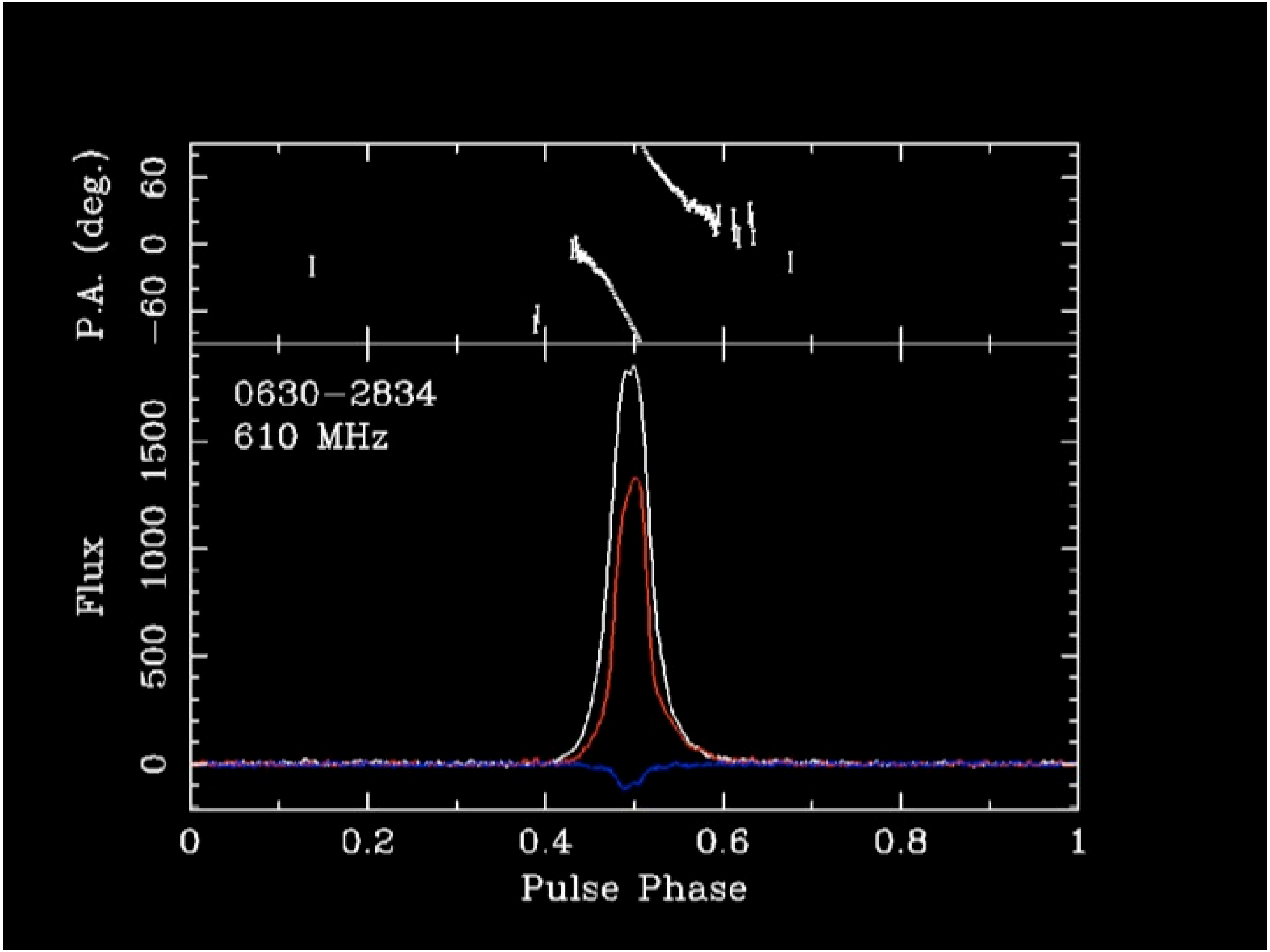


0630−2834
610 MHz
Flux
2000
1000
0
−1000
0
0.2
0.4
0.6
0.8
1
Pulse Phase

Stokes Parameters - Cylindrical

$$L = \sqrt{Q^2 + U^2}$$

$$\psi = \frac{1}{2} \tan^{-1} \frac{U}{Q}$$

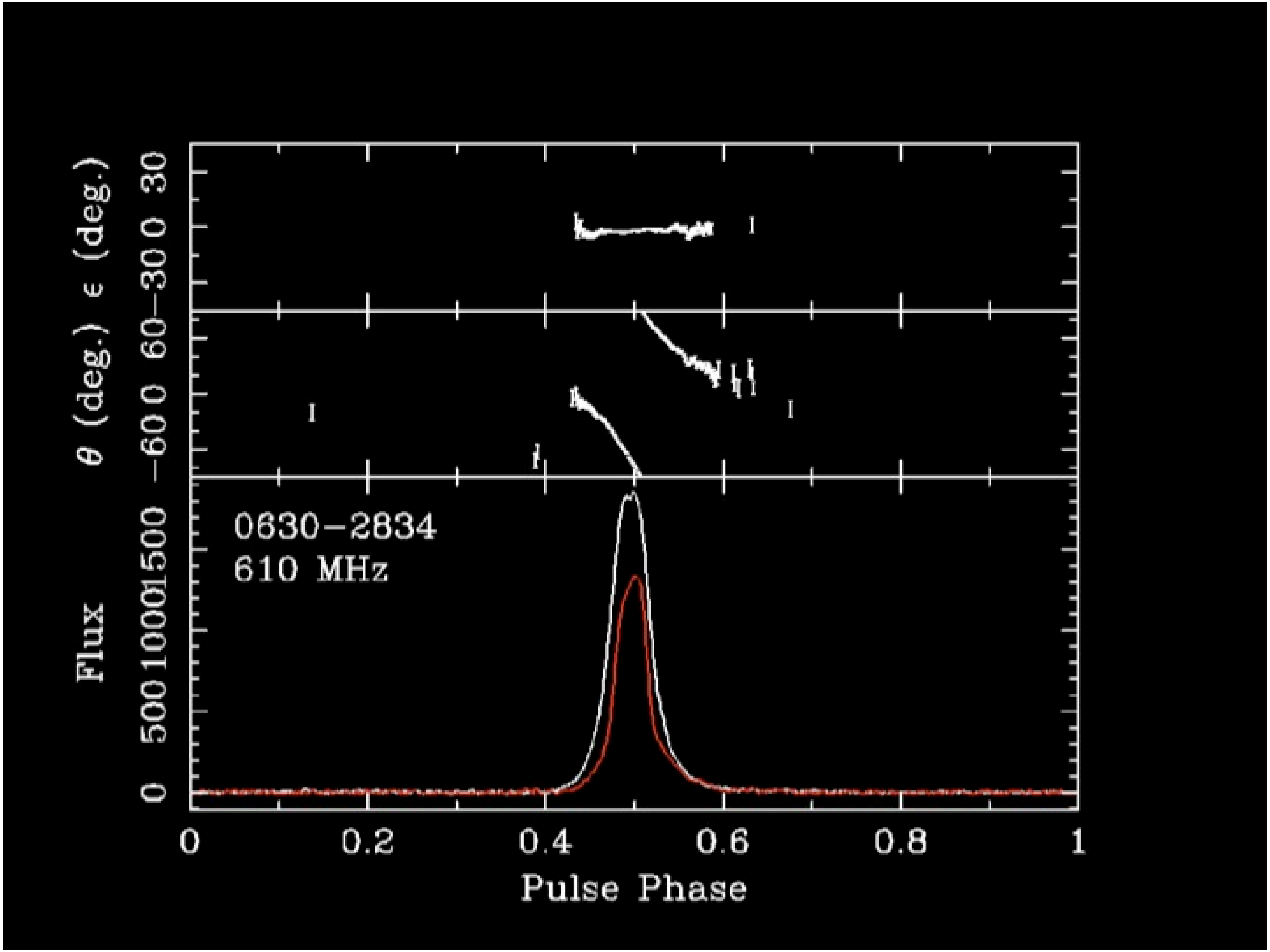

P.A. (deg.)
60
0
−60
Flux
1500
1000
500
0
0630−2834
610 MHz
0
0.2
0.4
0.6
0.8
1
Pulse Phase

Stokes Parameters - Spherical

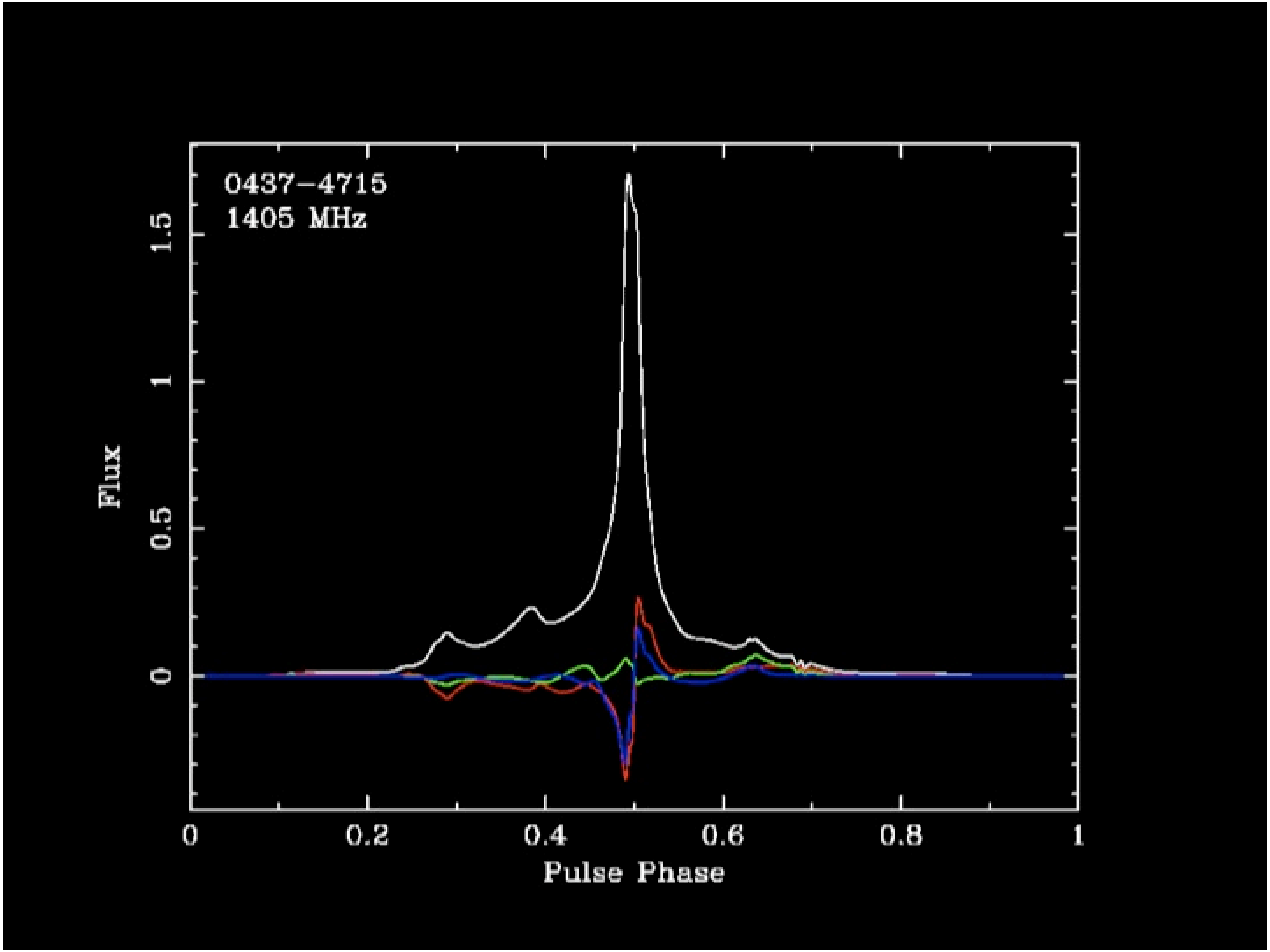
$$P = \sqrt{L^2 + V^2}$$

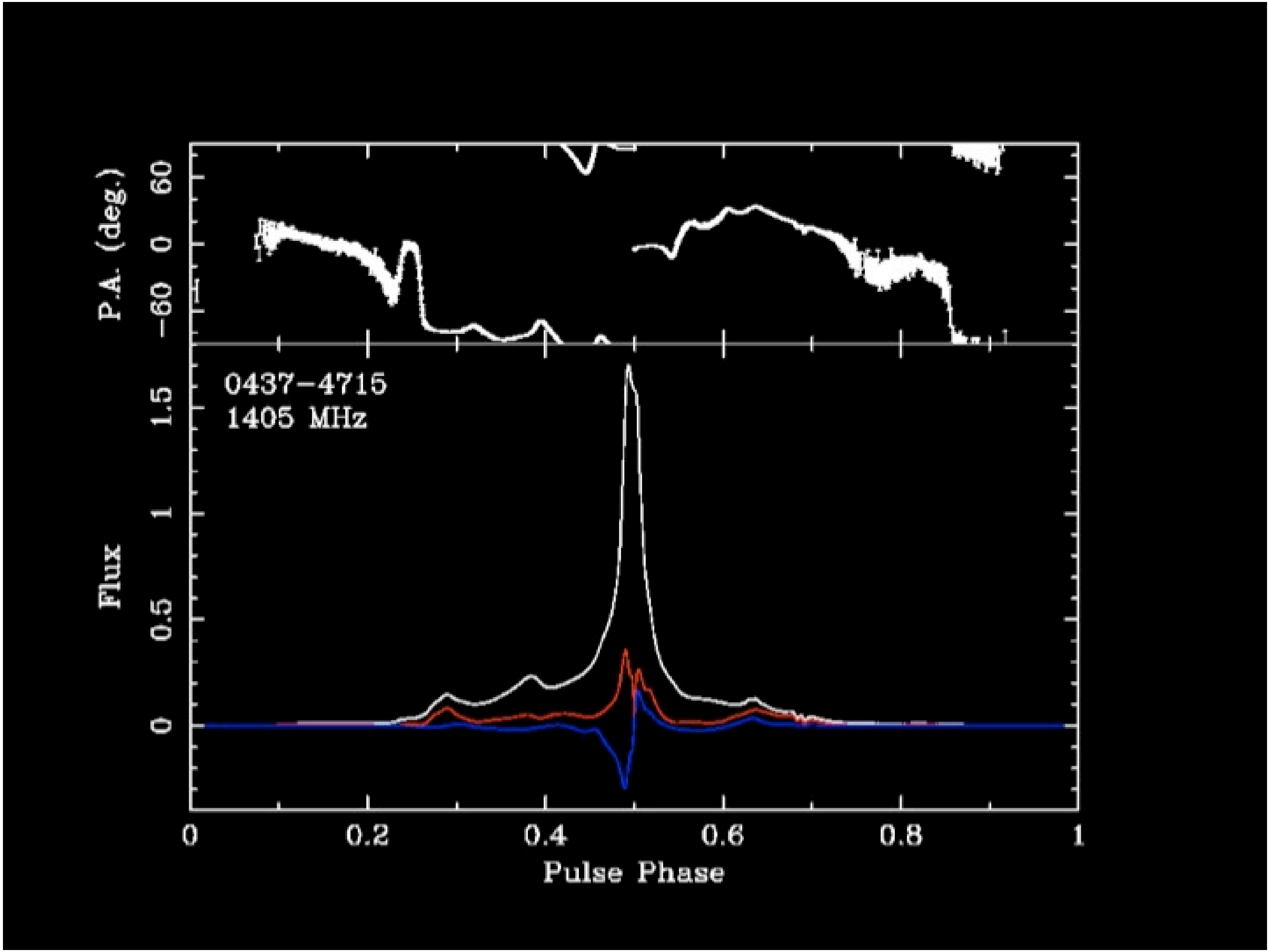
$$\psi = \frac{1}{2} \tan^{-1} \frac{U}{Q}$$

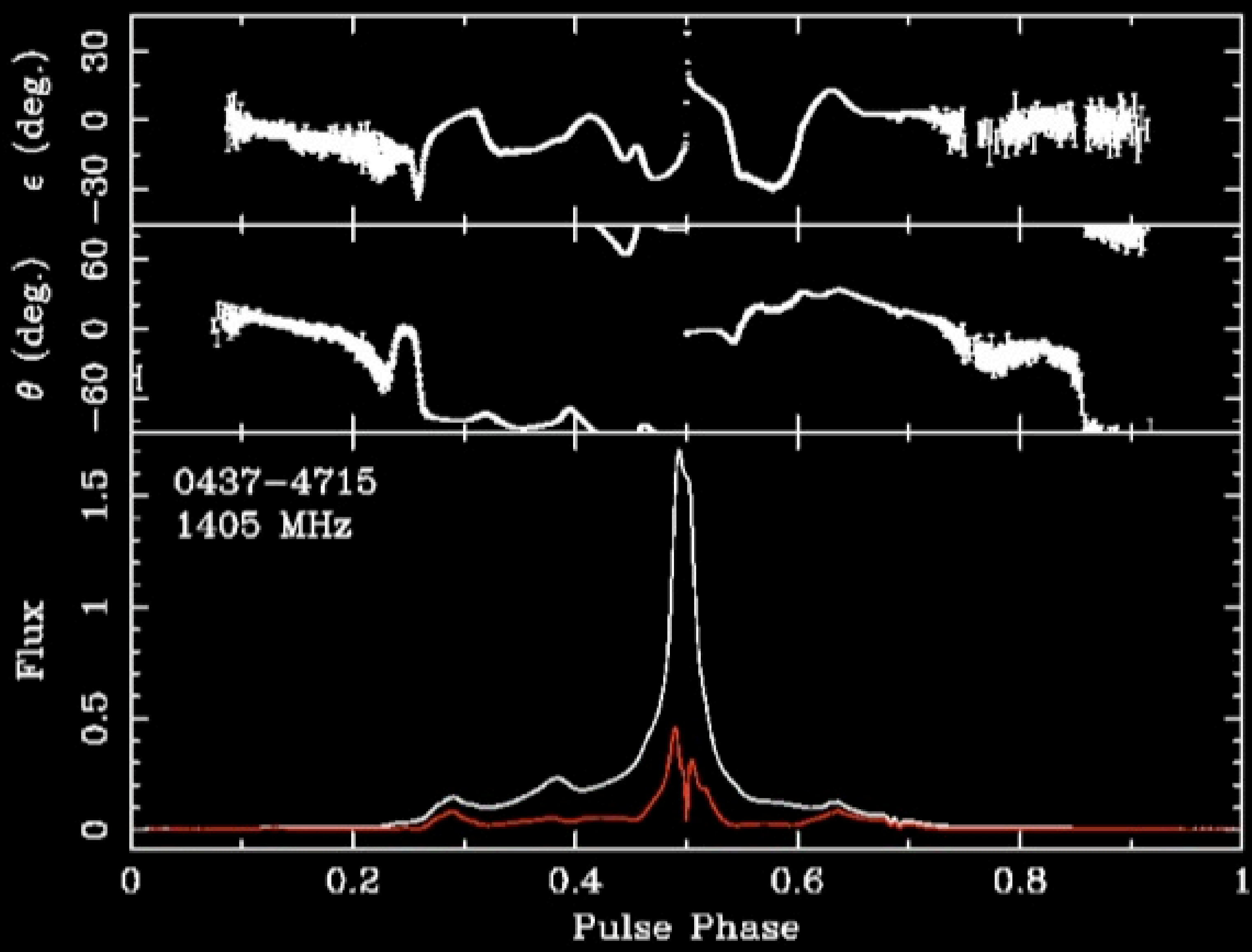
$$\chi = \frac{1}{2} \tan^{-1} \frac{V}{L}$$



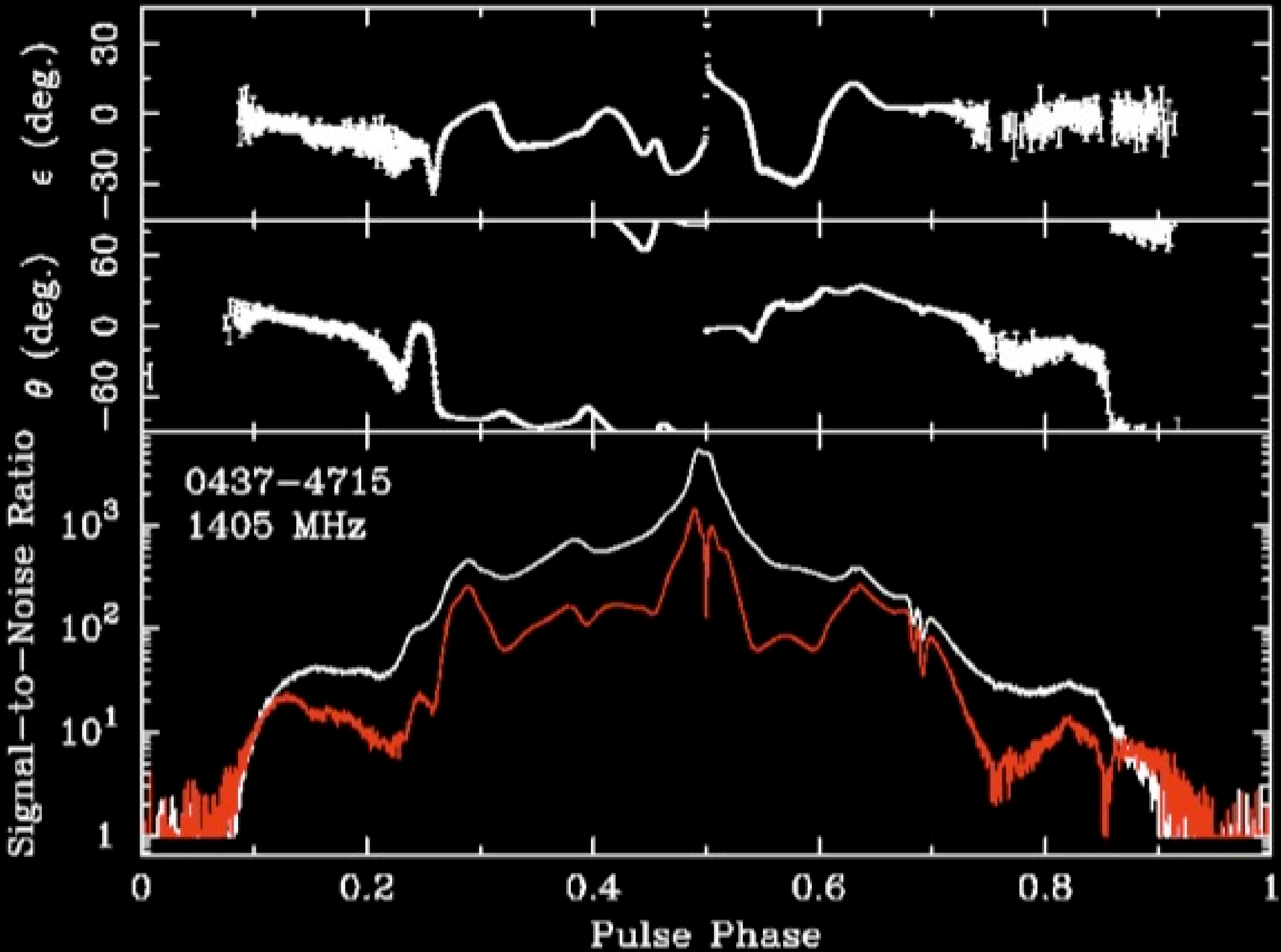
ε (deg.)
θ (deg.)
Flux
0630−2834
610 MHz
Pulse Phase


0437-4715
1405 MHz
Flux
Pulse Phase
0
0.5
1
1.5
0
0.2
0.4
0.6
0.8
1

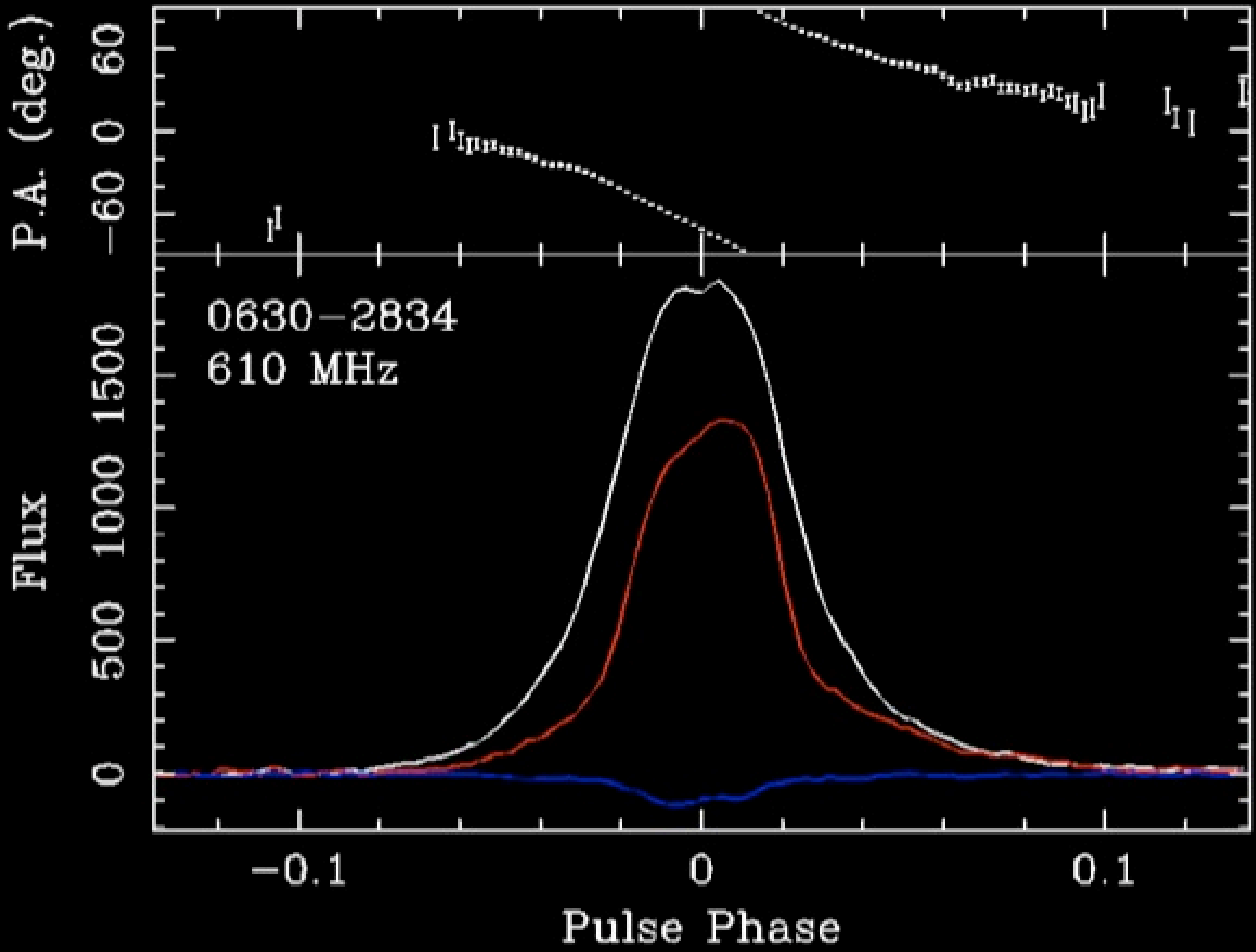

P.A. (deg.)
60
0
−60
0437−4715
1405 MHz
Flux
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
Pulse Phase



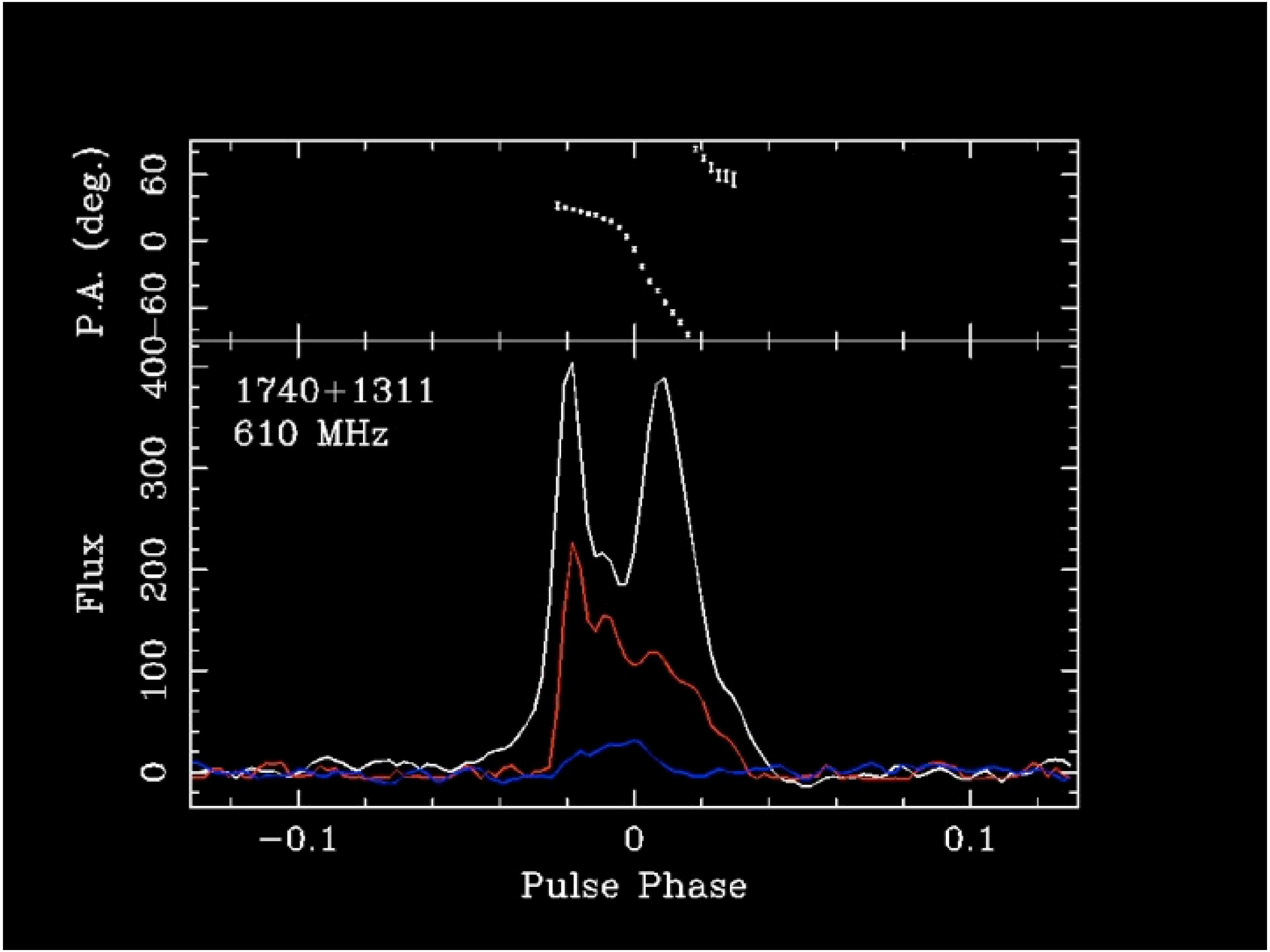
ε (deg.)
θ (deg.)
Flux
0437−4715
1405 MHz
Pulse Phase



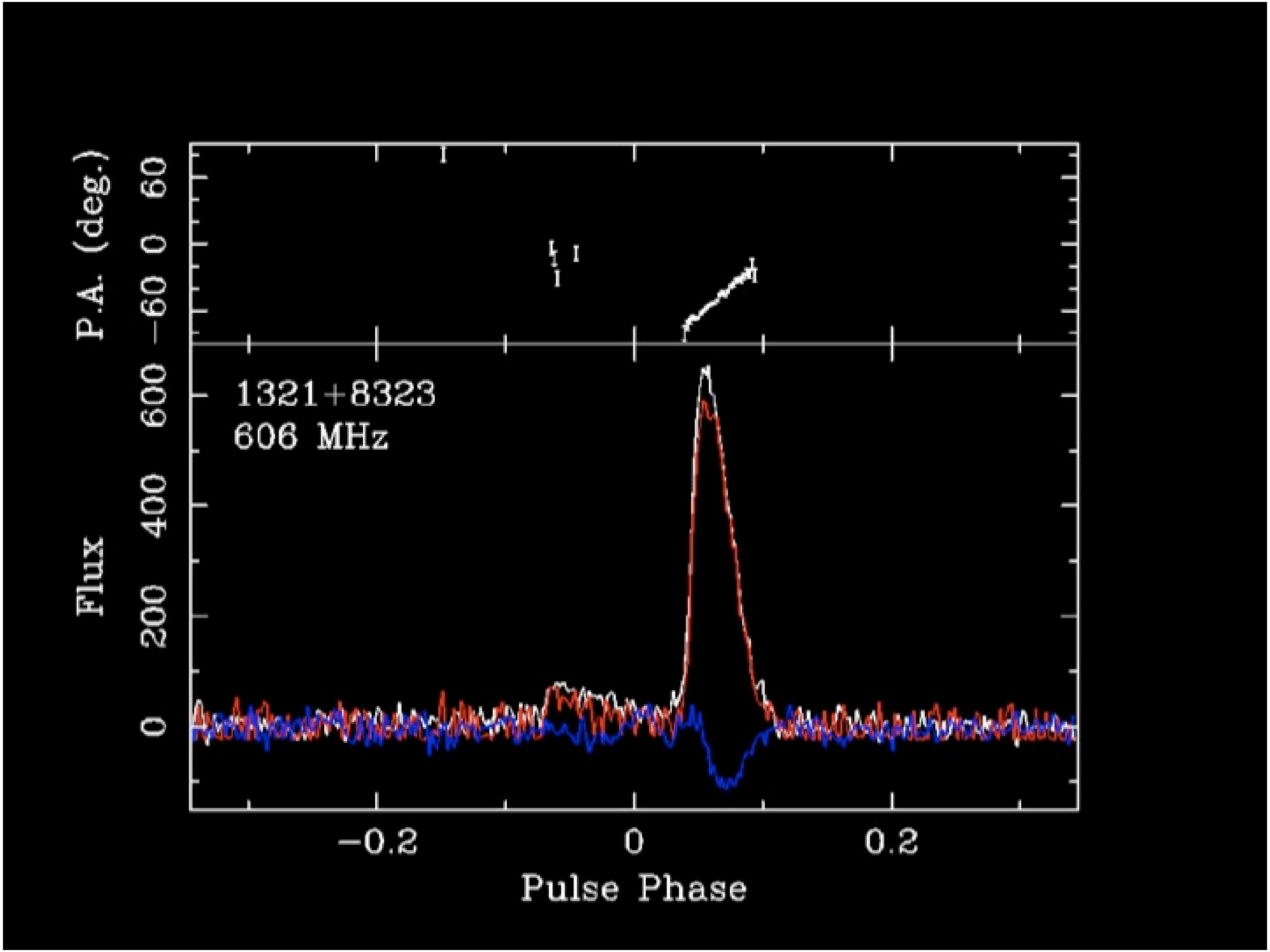
ε (deg.)
θ (deg.)
Signal-to-Noise Ratio
0437−4715
1405 MHz
Pulse Phase

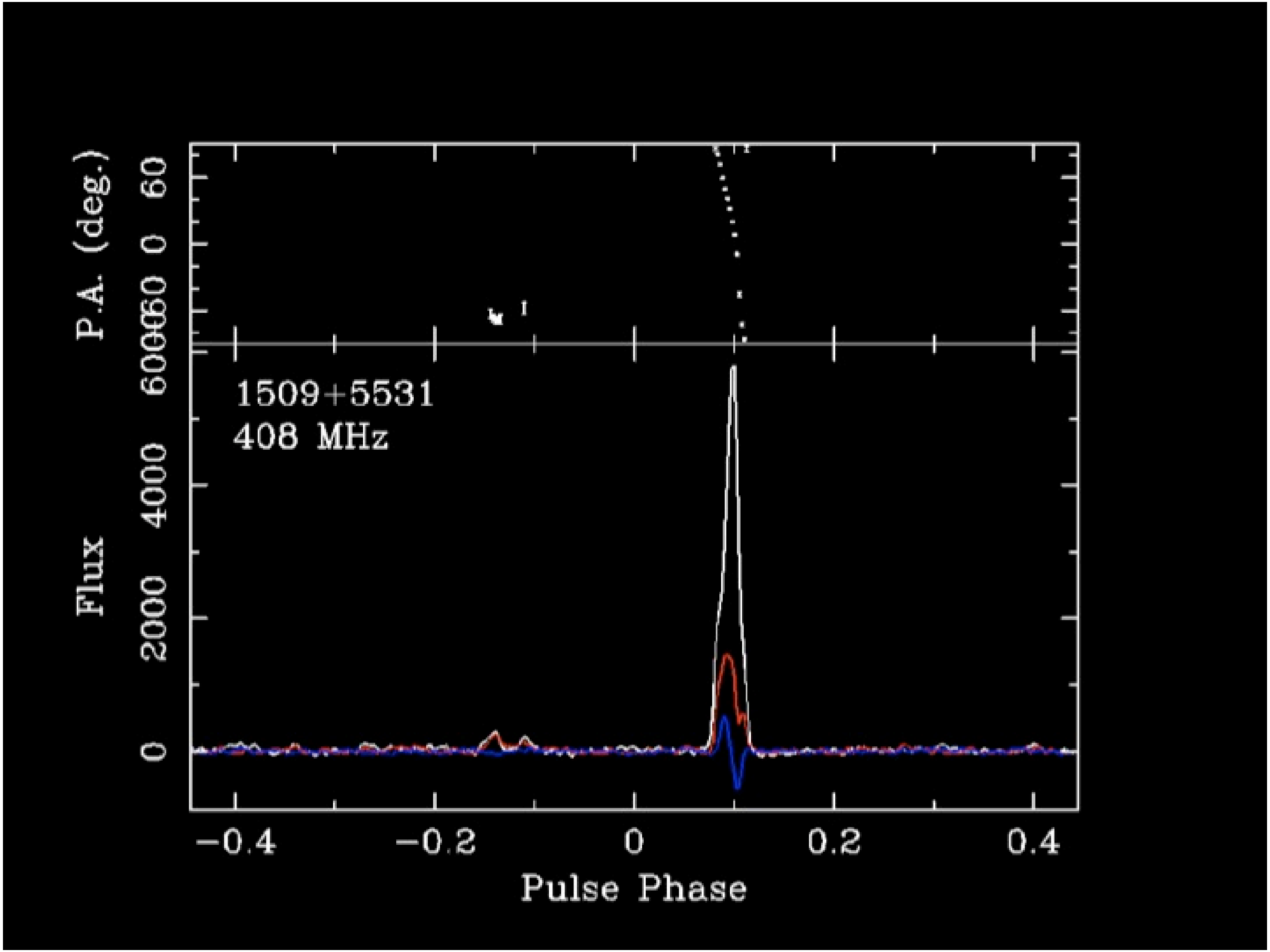


P.A. (deg.)
−60
0
60
Flux
0
500
1000
1500
0630−2834
610 MHz
−0.1
0
0.1
Pulse Phase

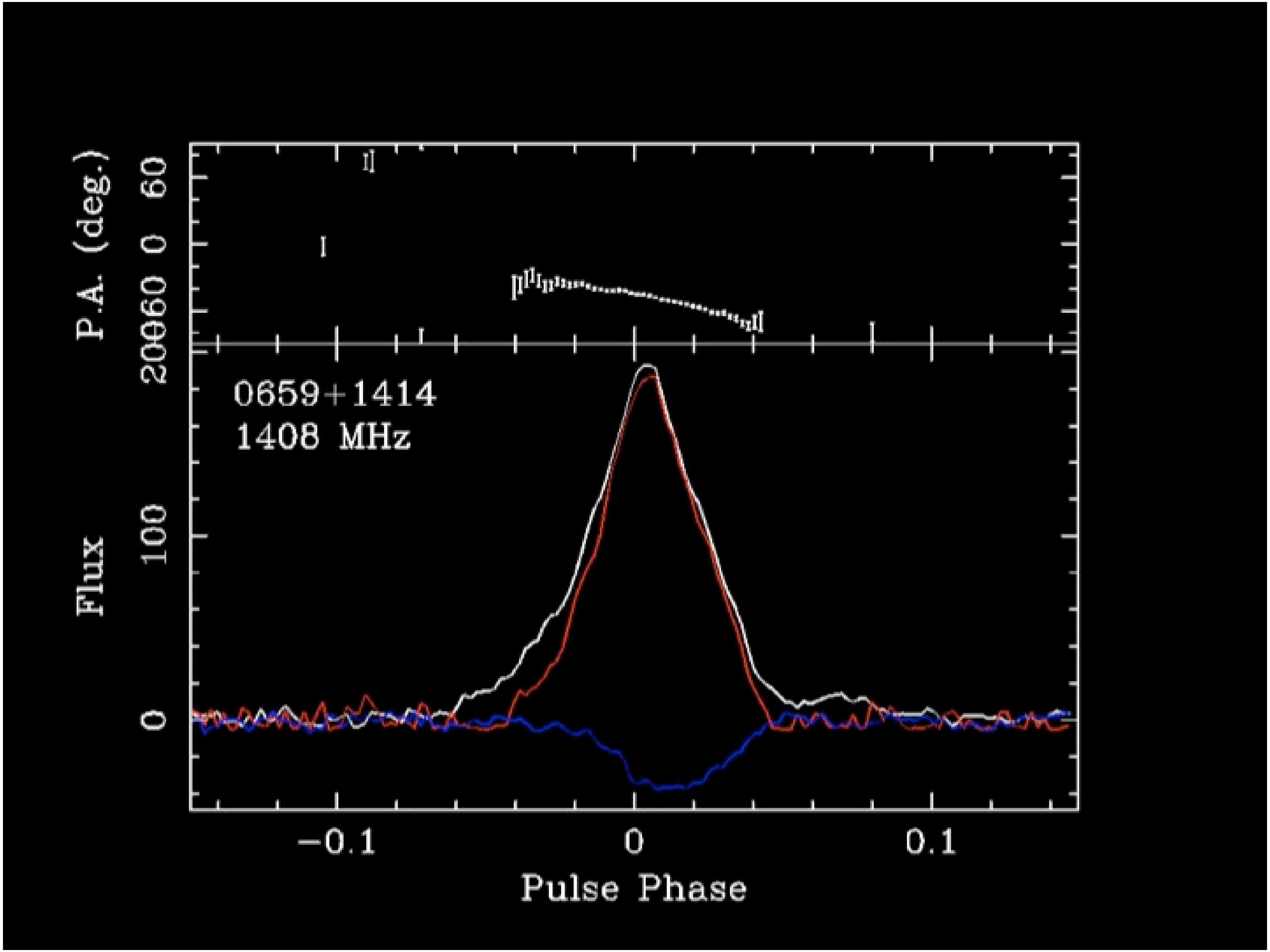


P.A. (deg.)
60
0
−60
Flux
400
300
200
100
0
1740+1311
610 MHz
−0.1
0
0.1
Pulse Phase

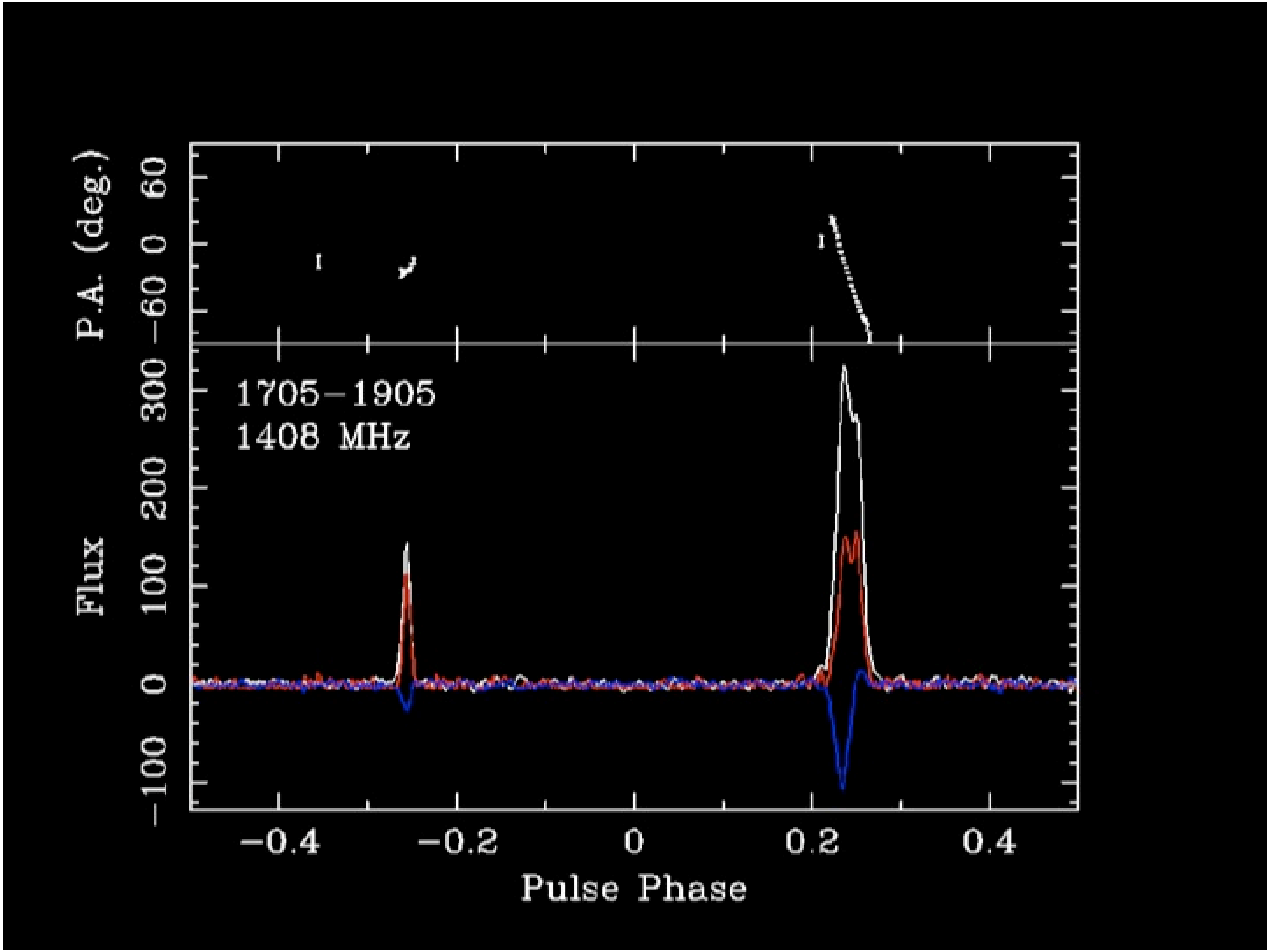

P.A. (deg.)
60
0
−60
1321+8323
606 MHz
Flux
600
400
200
0
−0.2
0
0.2
Pulse Phase



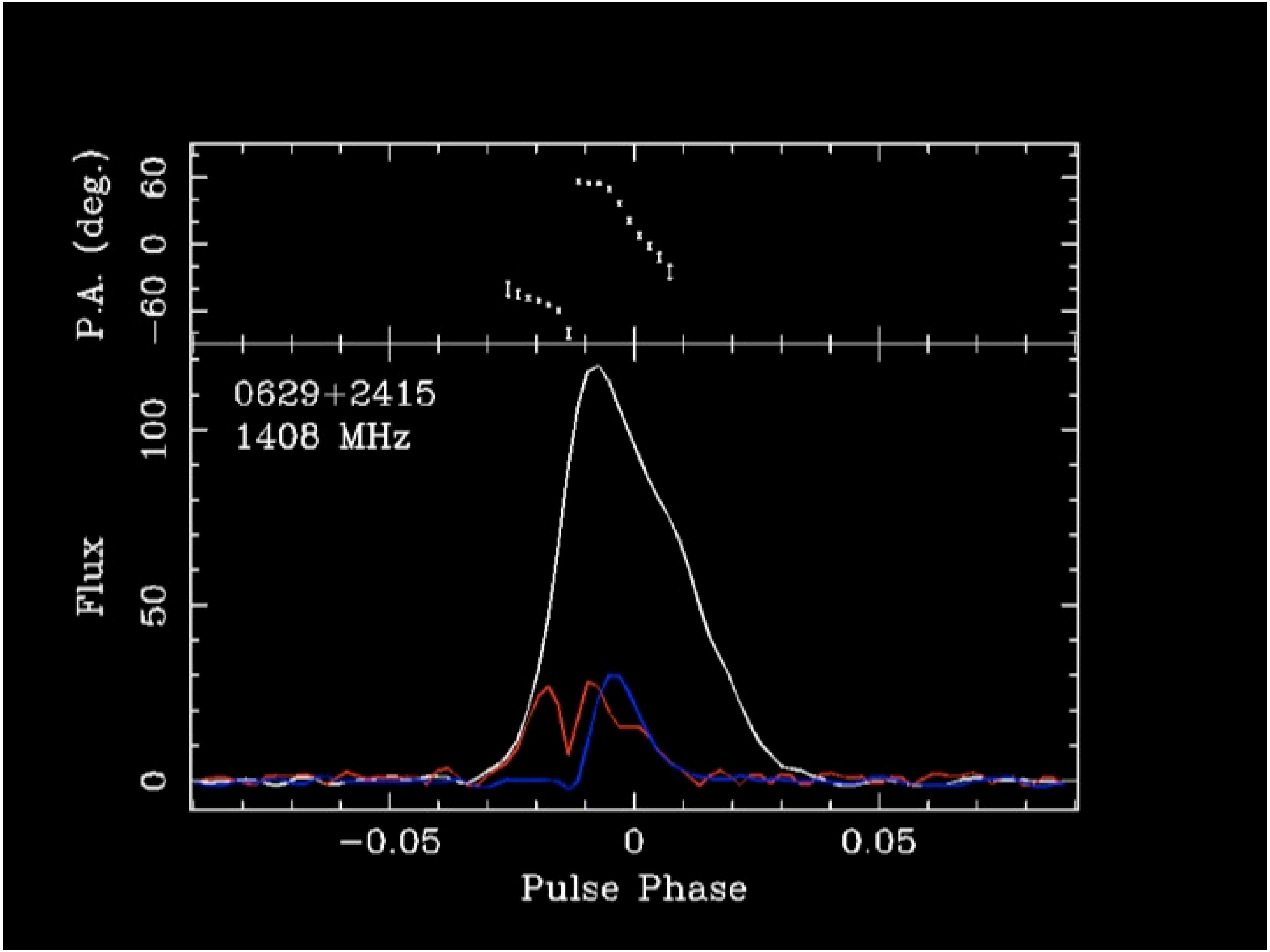
P.A. (deg.)
60
0
-60
Flux
0
2000
4000
6000
1509+5531
408 MHz
-0.4
-0.2
0
0.2
0.4
Pulse Phase



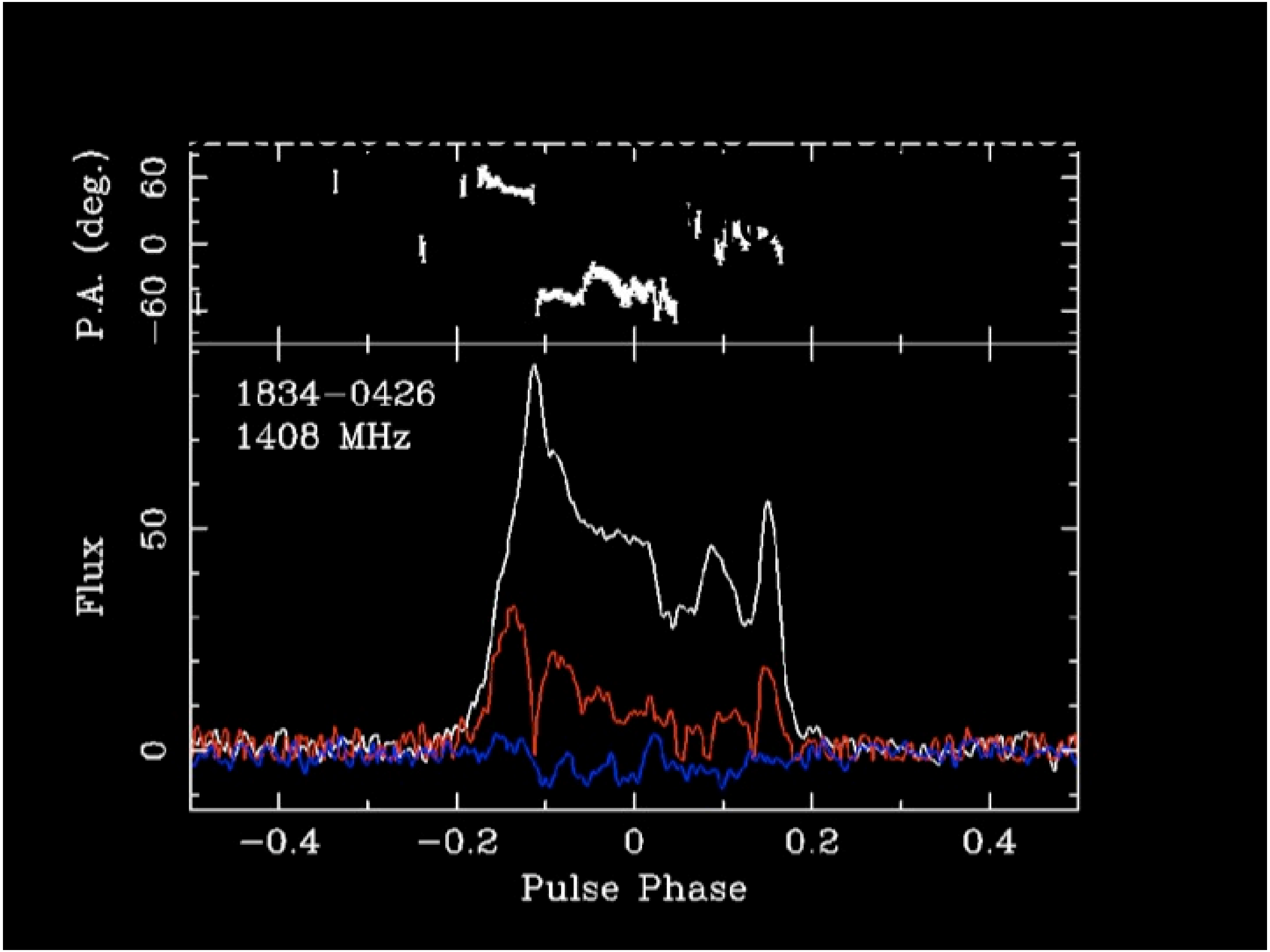
P.A. (deg.)
60
0
-60
200
Flux
100
0
0659+1414
1408 MHz
−0.1
0
0.1
Pulse Phase

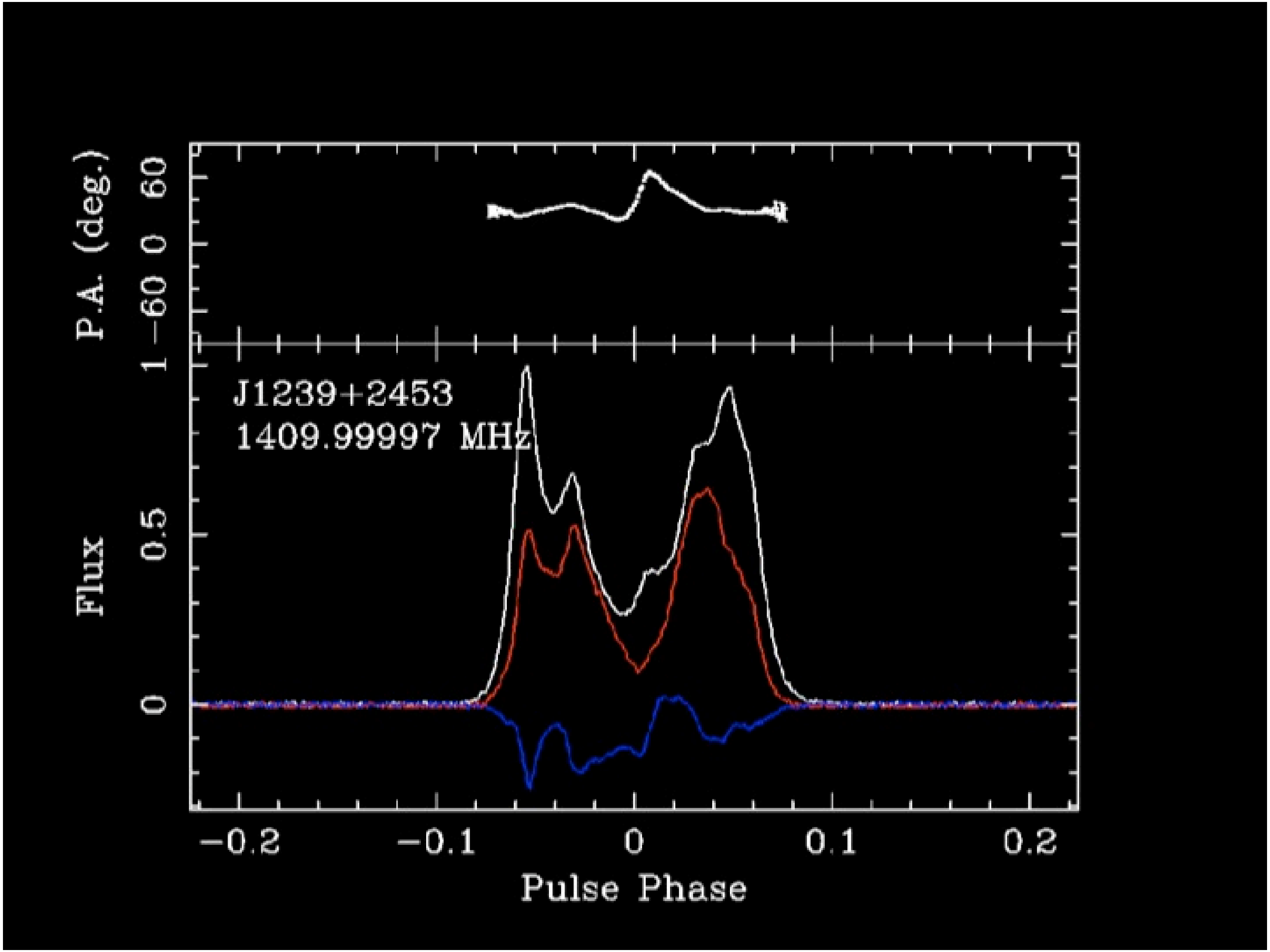


P.A. (deg.)
60
0
−60
Flux
300
200
100
0
−100
1705−1905
1408 MHz
−0.4
−0.2
0
0.2
0.4
Pulse Phase

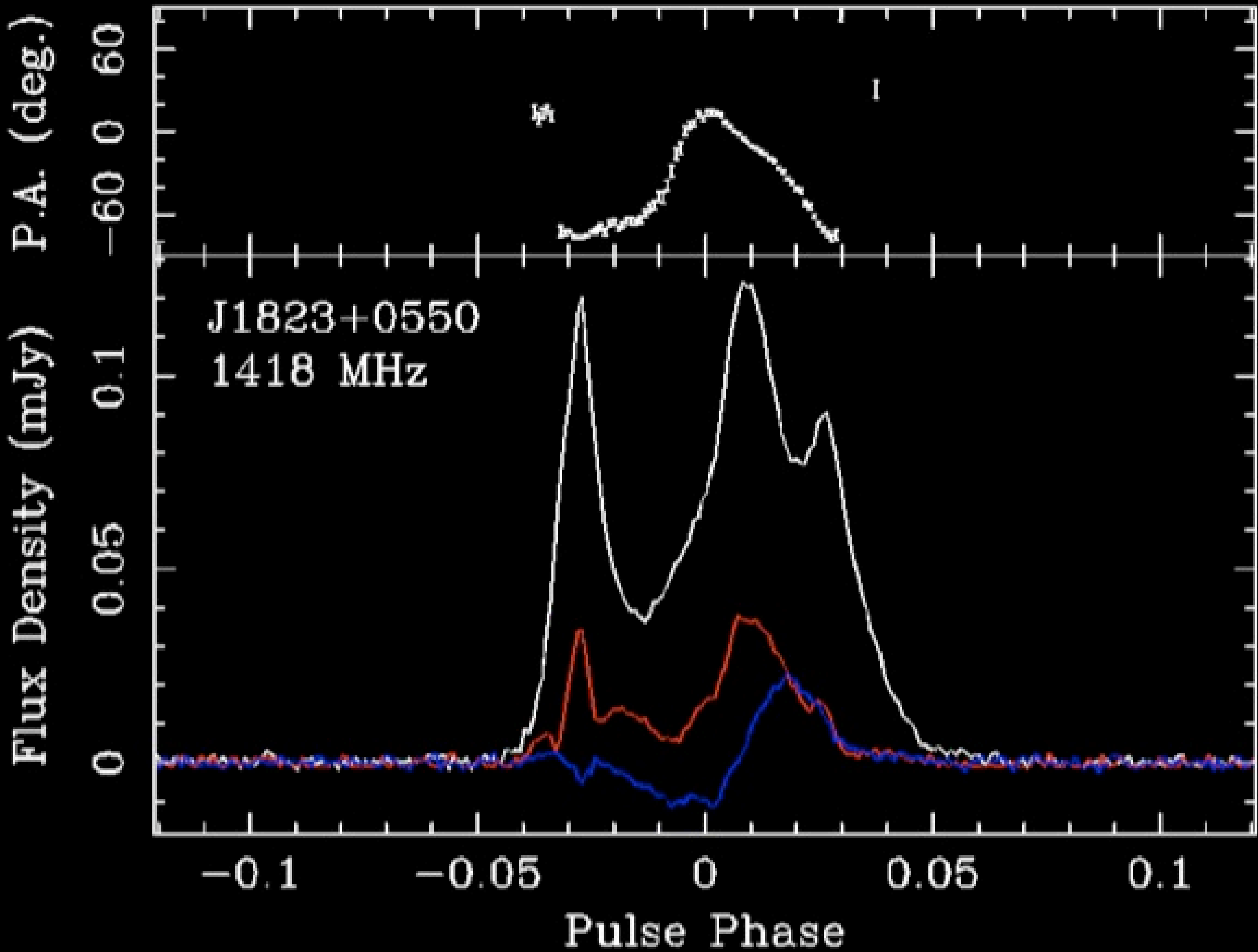


P.A. (deg.)
60
0
−60
Flux
100
50
0
0629+2415
1408 MHz
−0.05
0
0.05
Pulse Phase


P.A. (deg.)
60
0
−60
1834−0426
1408 MHz
Flux
50
0
−0.4
−0.2
0
0.2
0.4
Pulse Phase



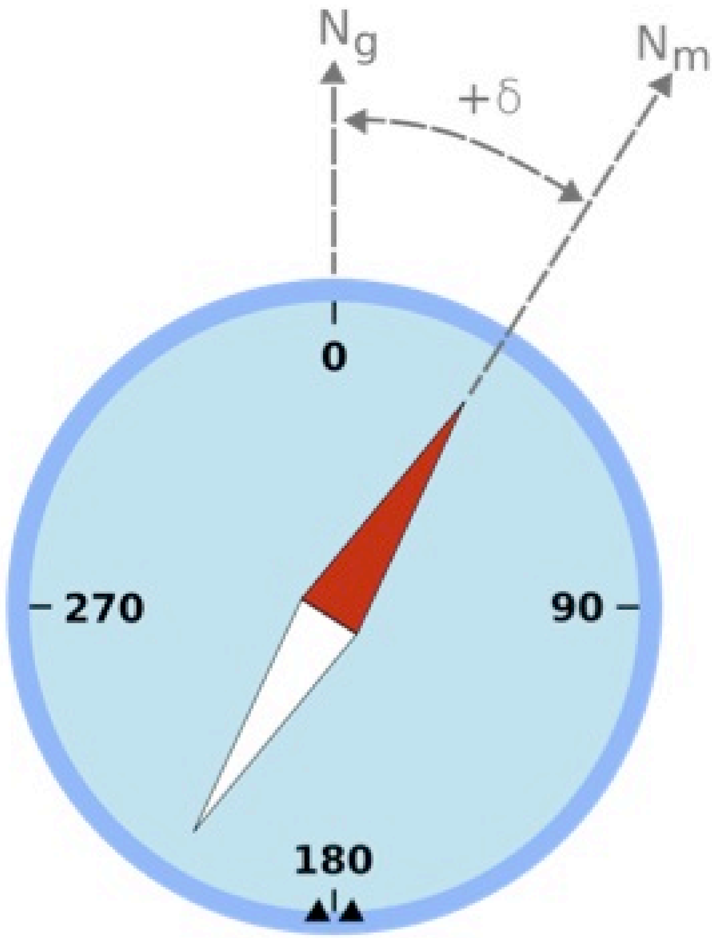
P.A. (deg.)
−60
0
60
1
0.5
0
Flux
J1239+2453
1409.99997 MHz
−0.2
−0.1
0
0.1
0.2
Pulse Phase



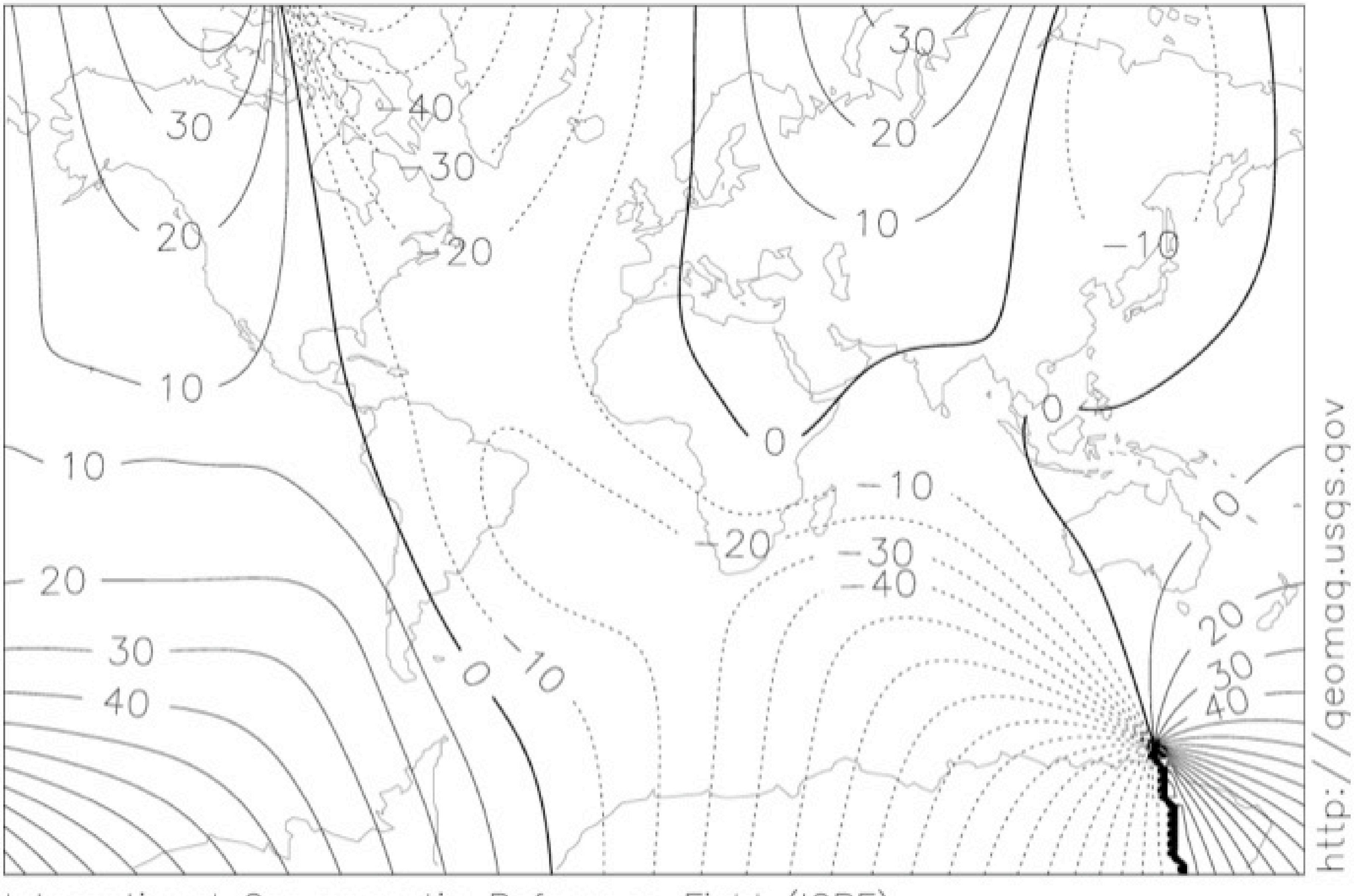
P.A. (deg.)
60
0
−60
J1823+0550
1418 MHz
Flux Density (mJy)
0.1
0.05
0
−0.1
−0.05
0
0.05
0.1
Pulse Phase

Rotating Vector Model

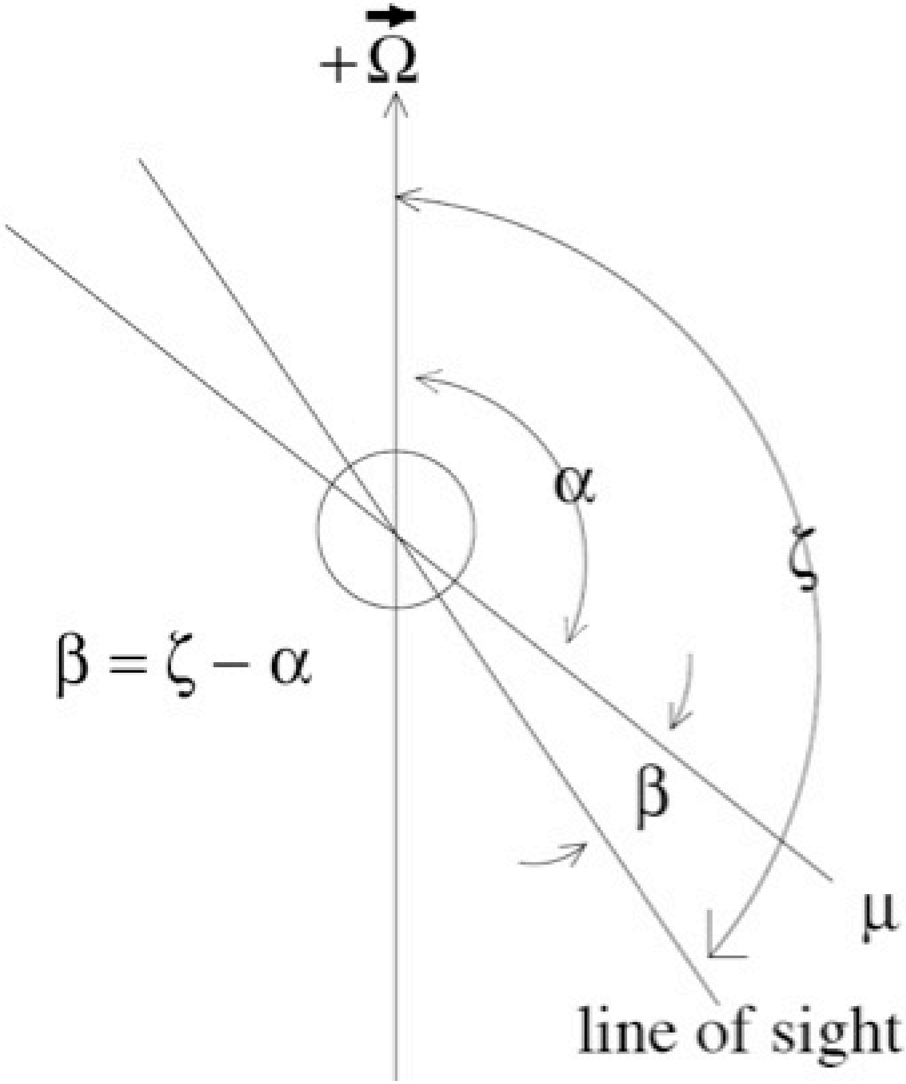
- Like magnetic declination on Earth



2000
Declination (degrees east)



International Geomagnetic Reference Field (IGRF)



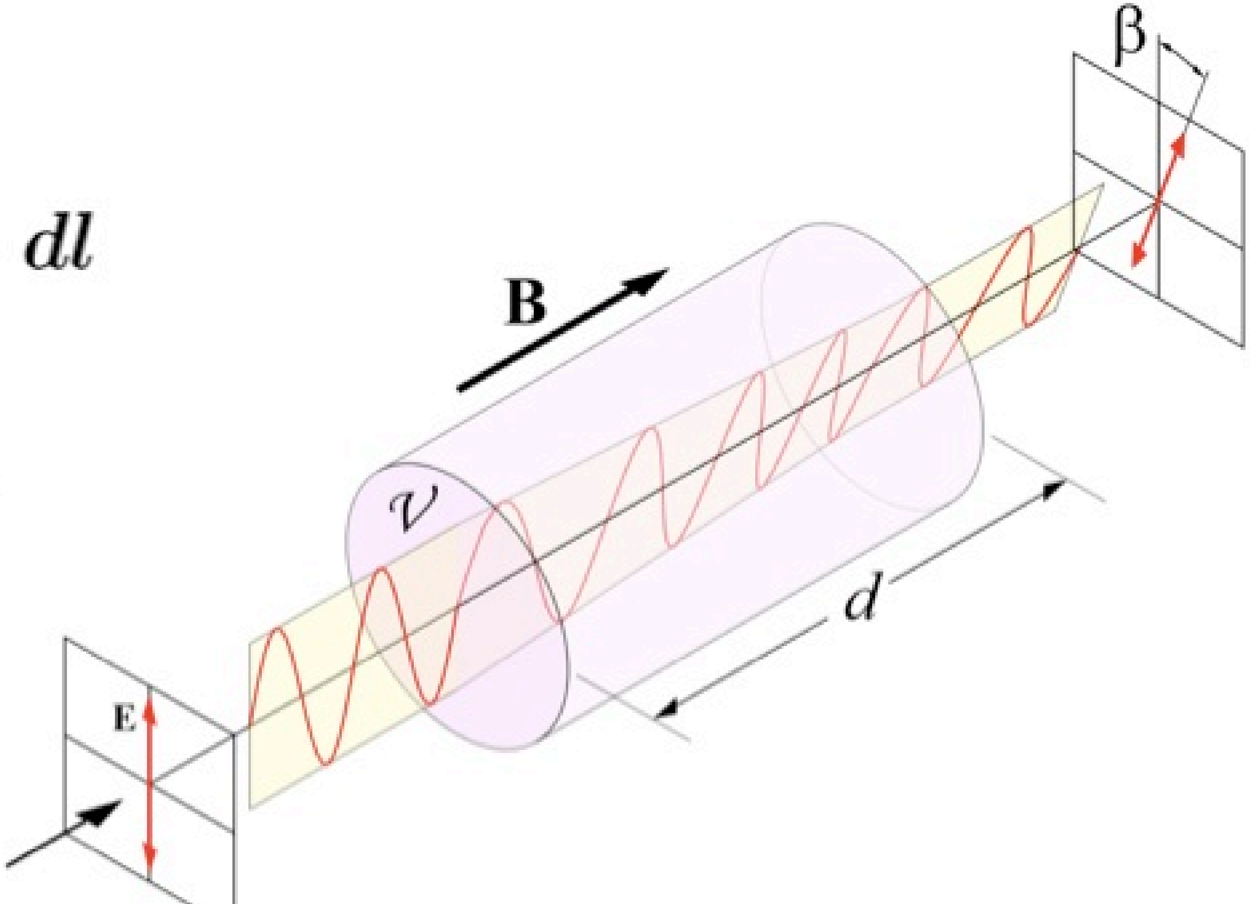
$$\tan\left(\psi' - \psi'_0\right) = \frac{\sin\alpha \sin\left(\phi - \phi_0\right)}{\sin\zeta\cos\alpha - \cos\zeta\sin\alpha\cos\left(\phi - \phi_0\right)}$$

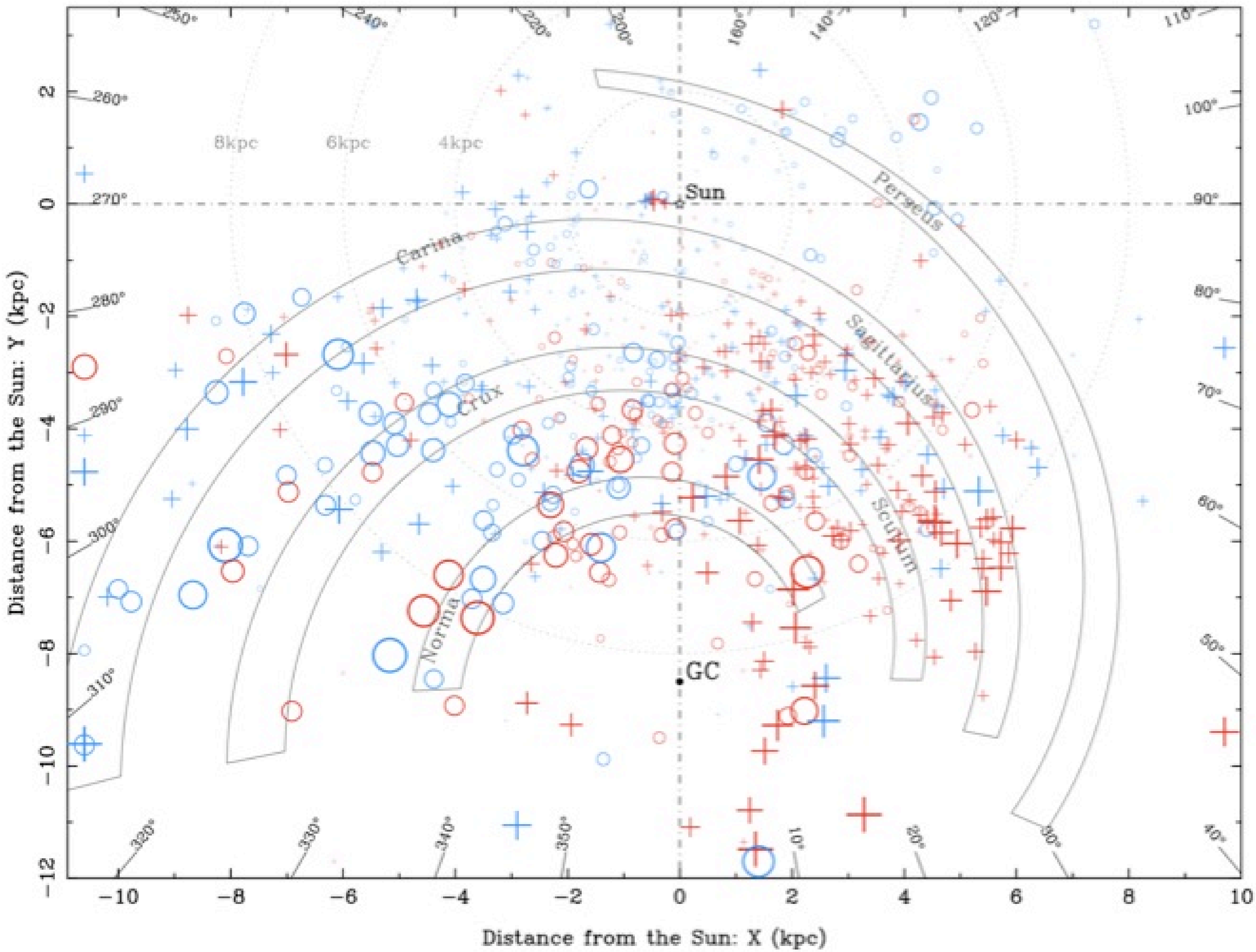
Faraday Rotation

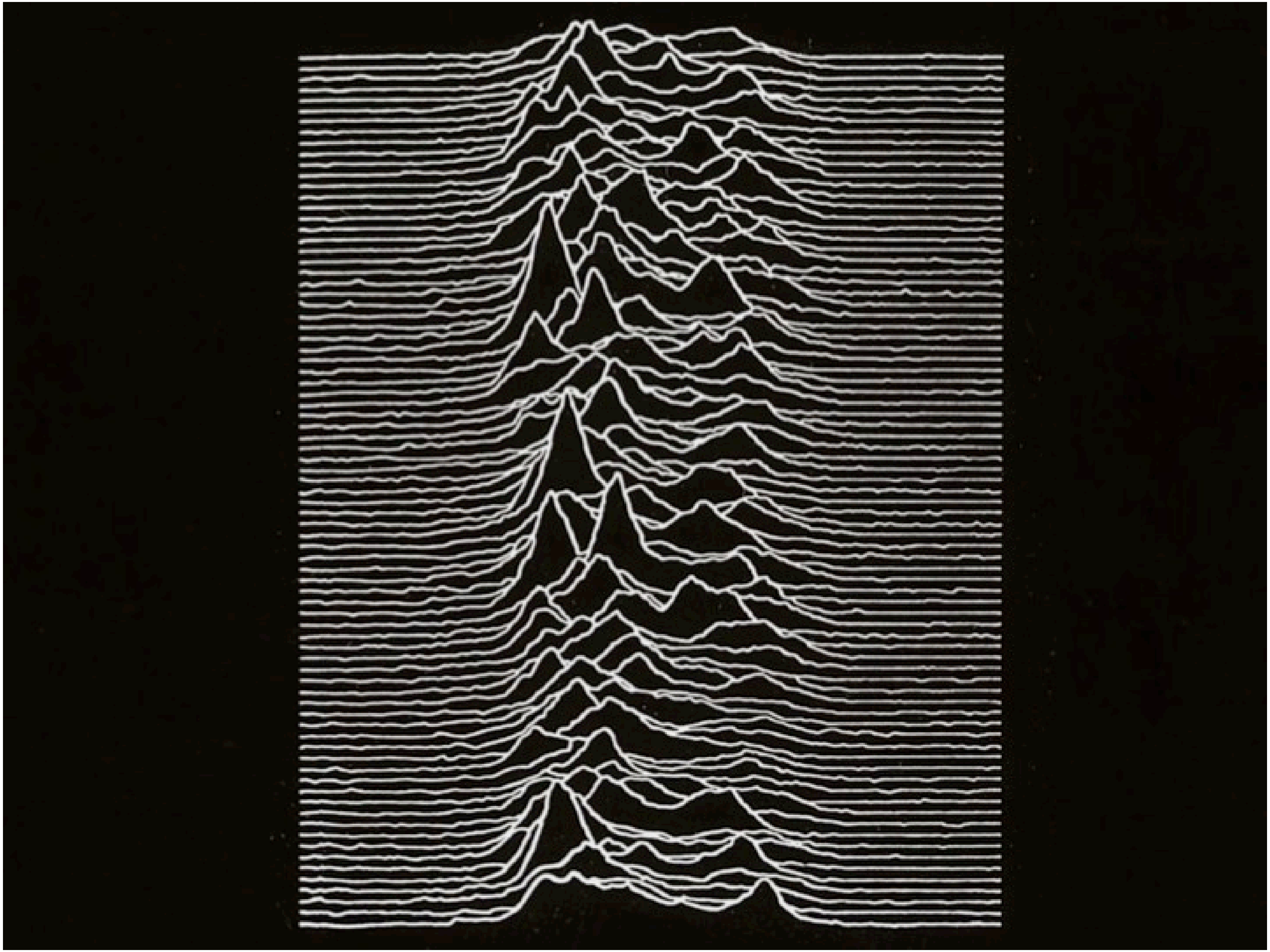
$$DM \propto \int_0^D n_e dl$$

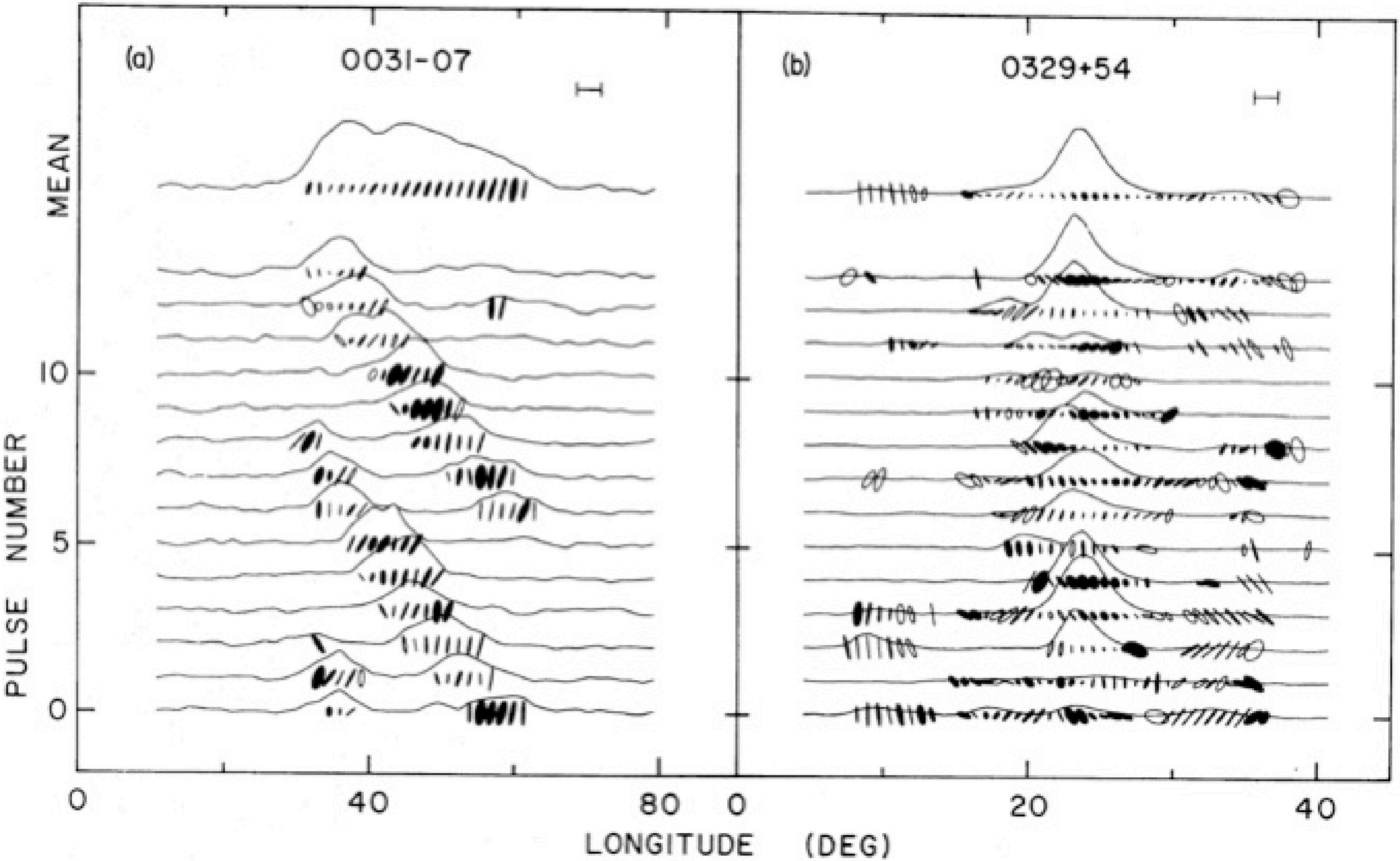
$$RM \propto \int_0^D n_e \boldsymbol{B} \cdot \boldsymbol{dl}$$

$$\langle B_{||} \rangle \propto RM/DM$$

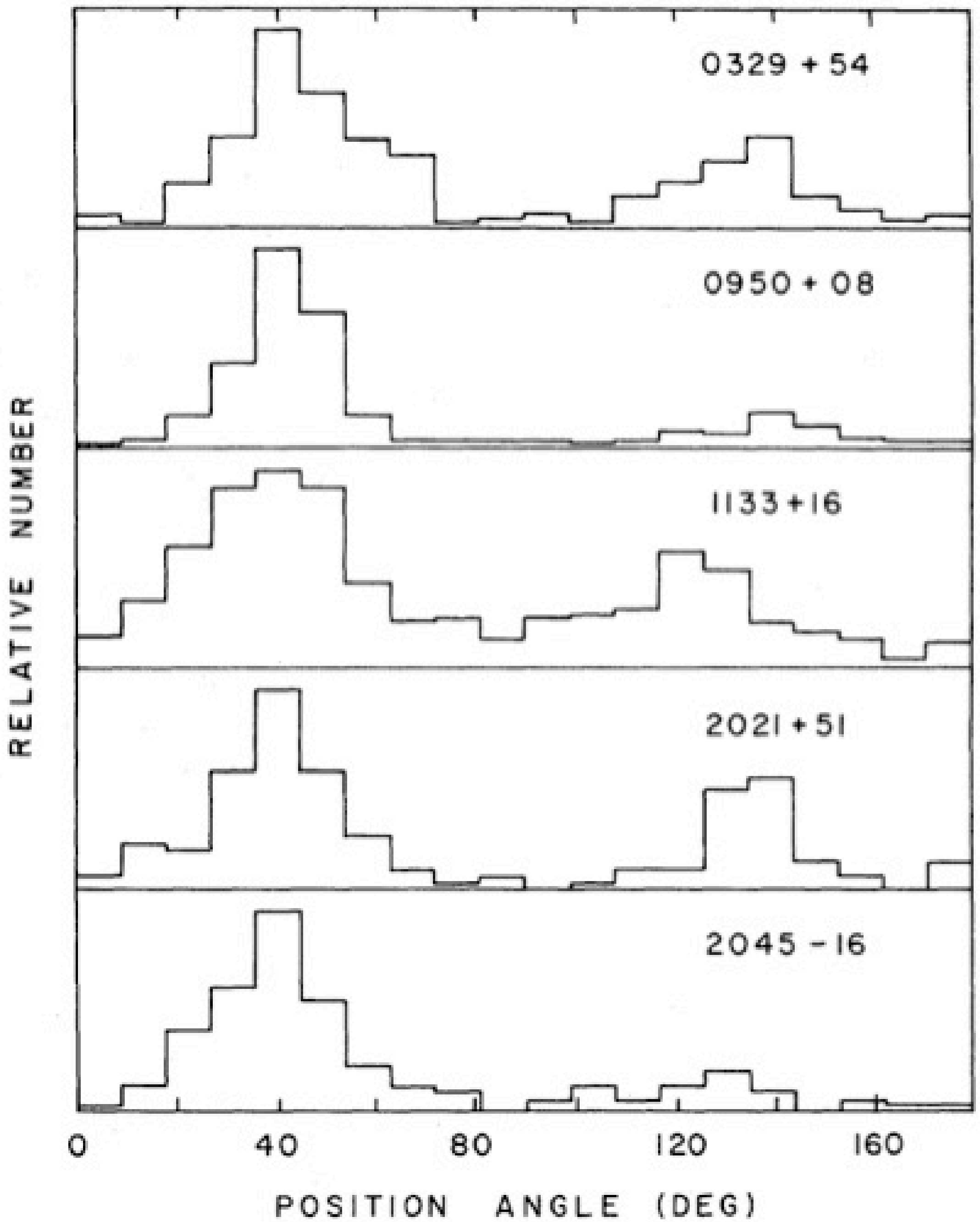


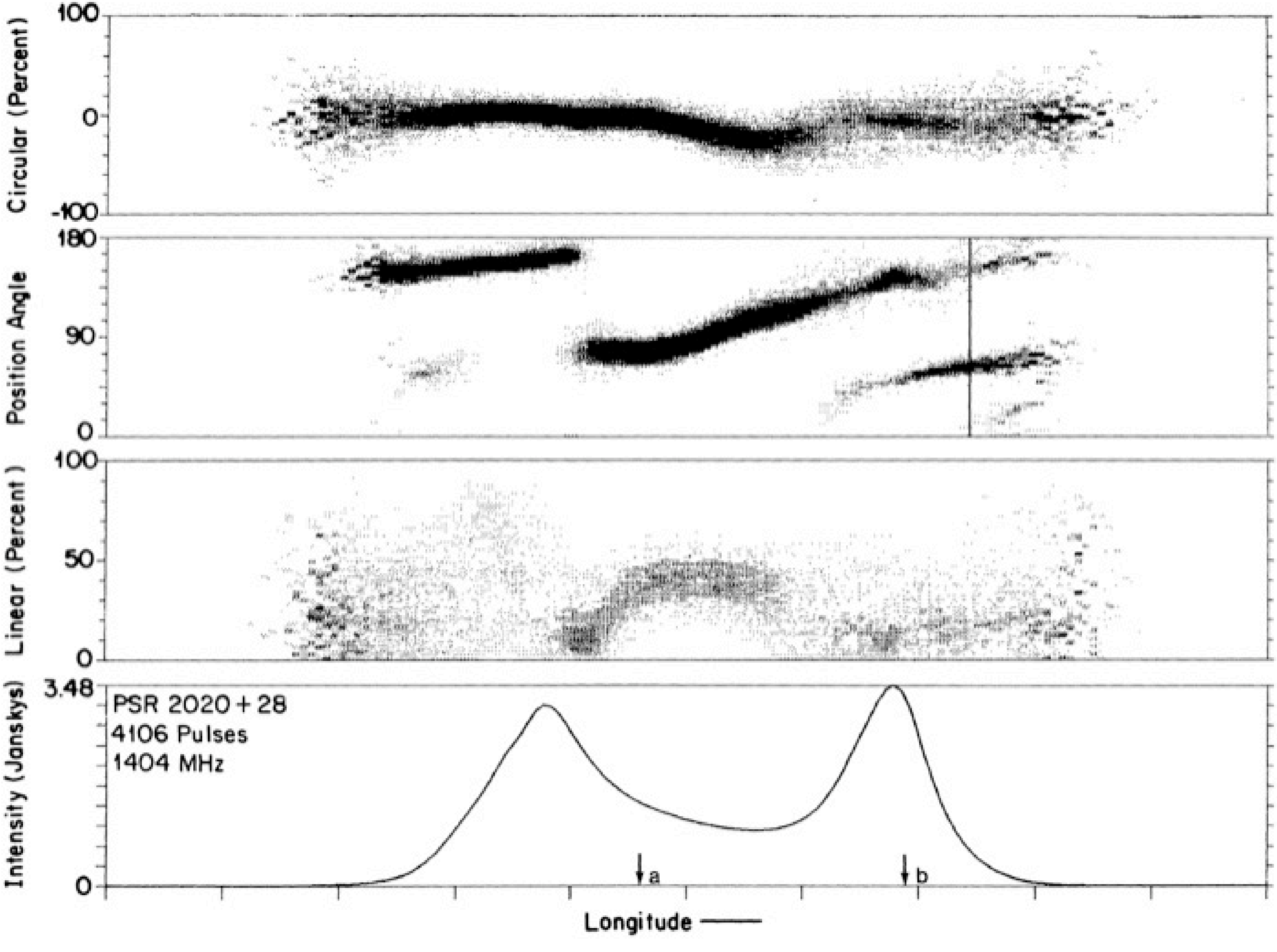

Distance from the Sun: X (kpc)
Distance from the Sun: Y (kpc)
Sun
GC
Perseus
Sagittarius
Scutum
Carina
Crux
Norma
8kpc
6kpc
4kpc



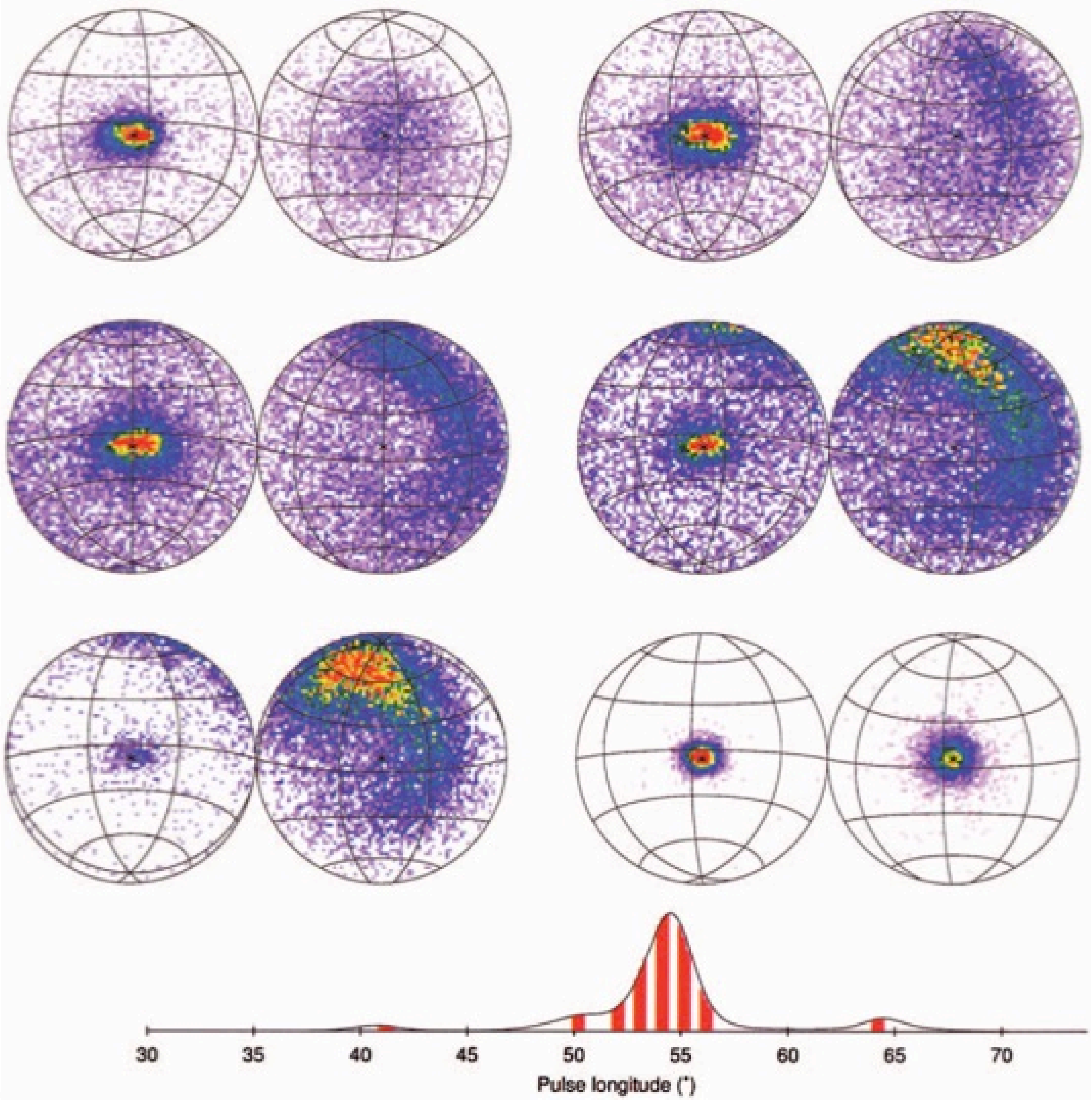


(a)
0031-07
(b)
0329+54
MEAN
PULSE NUMBER
0
5
10
0
40
80
0
20
40
LONGITUDE (DEG)

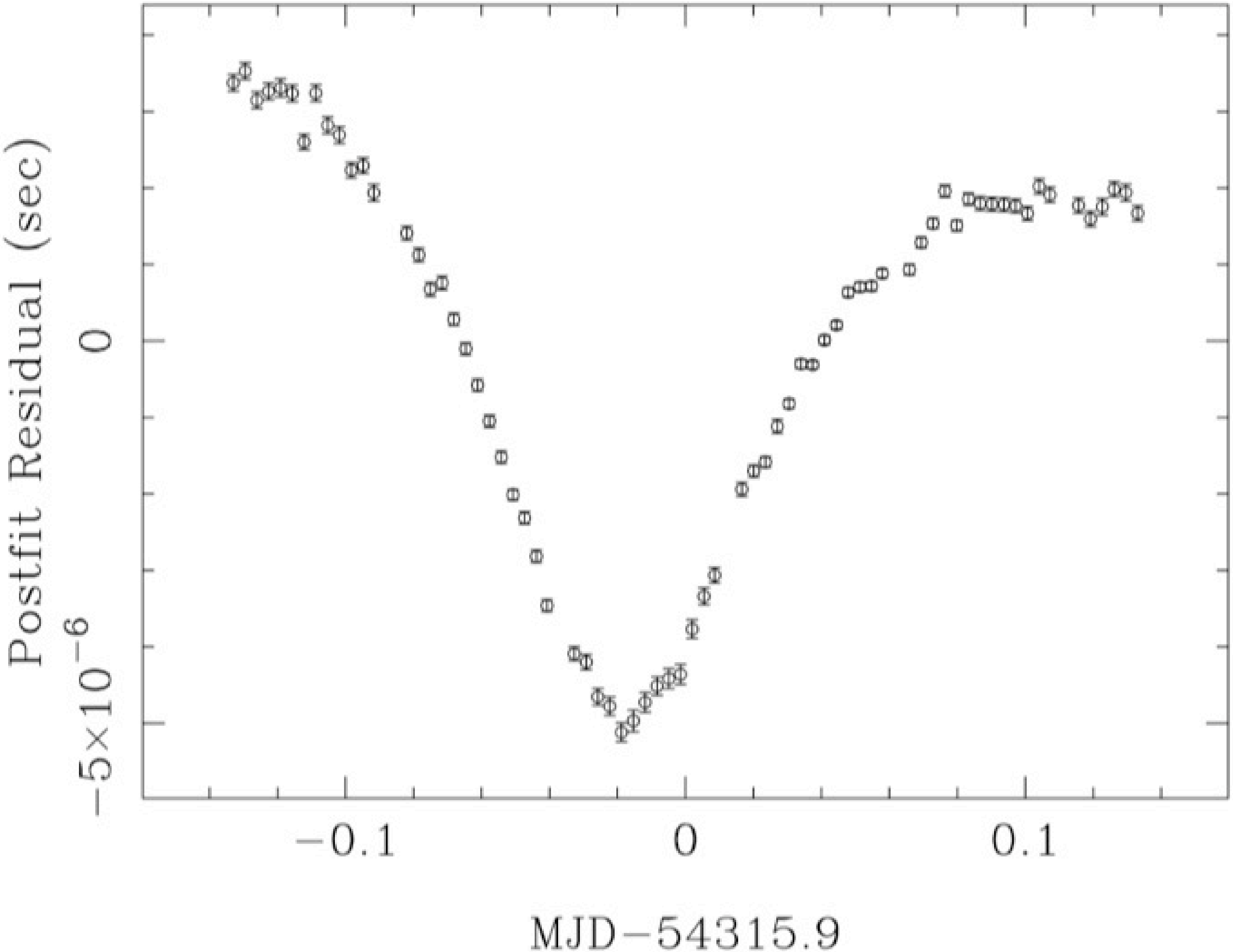

0329 + 54
0950 + 08
1133+16
2021 + 51
2045 -16
RELATIVE NUMBER
0
40
80
120
160
POSITION ANGLE (DEG)



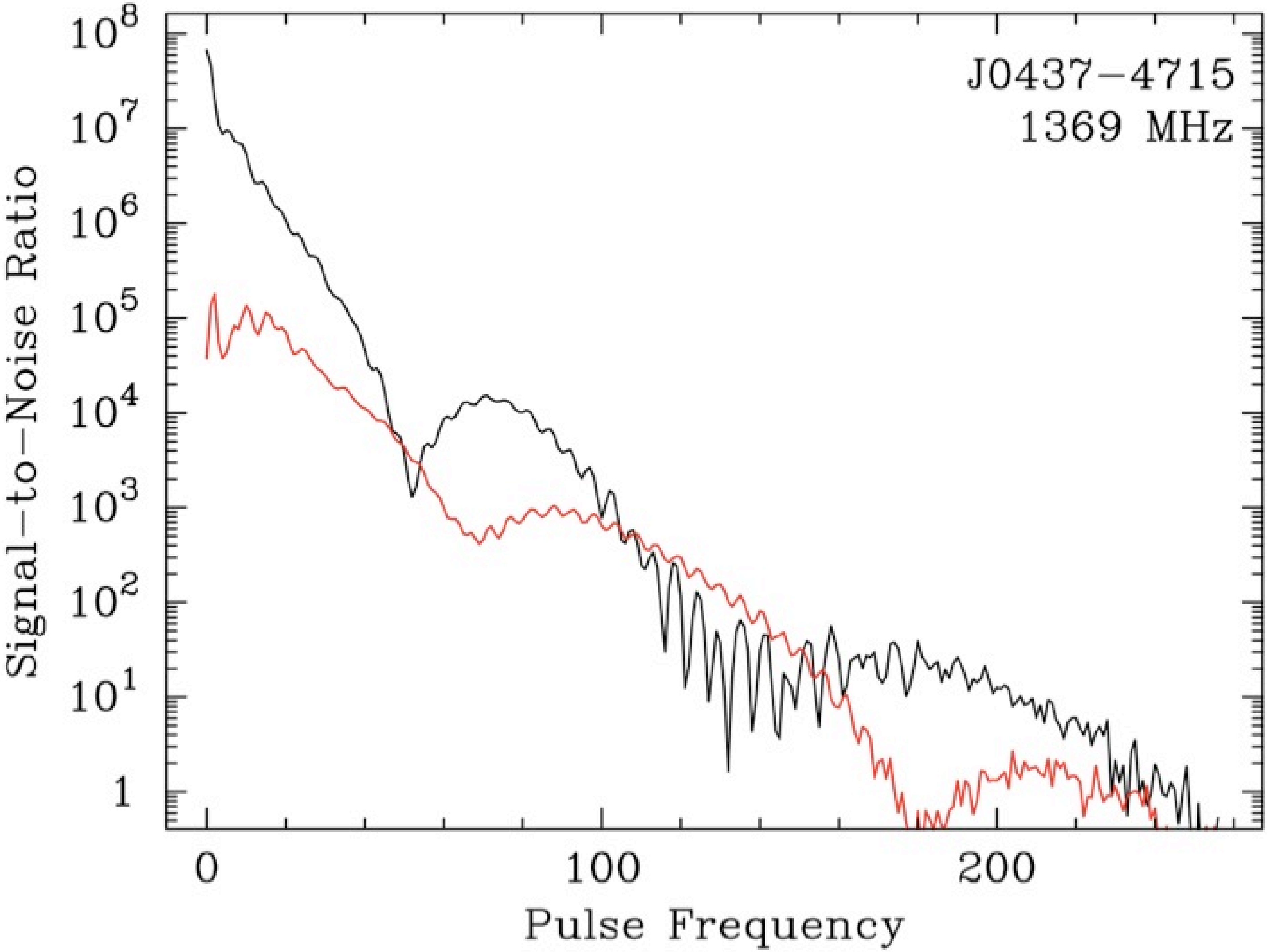
100
0
-100
Circular (Percent)
180
90
0
Position Angle
100
50
0
Linear (Percent)
3.48
0
Intensity (Janskys)
PSR 2020+28
4106 Pulses
1404 MHz
a
b
Longitude



30
35
40
45
50
55
60
65
70
Pulse longitude (°)



Postfit Residual (sec)
0
−5×10⁻⁶
−0.1
0
0.1
MJD−54315.9



J0437−4715
1369 MHz
Signal-to-Noise Ratio
Pulse Frequency
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
1
0
100
200

Calibration

$$\boldsymbol{e}' = \mathbf{J}\boldsymbol{e}$$

Jones matrix, $\mathbf{J}$

$$\boldsymbol{\rho}' = \mathbf{J}\boldsymbol{\rho}\mathbf{J}^\dagger$$

$$S'_i = M_i^k S_k$$

Mueller matrix, $\mathbf{M}$

$$M_i^k = \frac{1}{2}\mathrm{Tr}(\boldsymbol{\sigma}_i\,\mathbf{J}\,\boldsymbol{\sigma}_k\,\mathbf{J}^\dagger)$$

7 degrees of freedom

$$\hat{\mathbf{J}} = \mathbf{J}e^{i\phi}$$

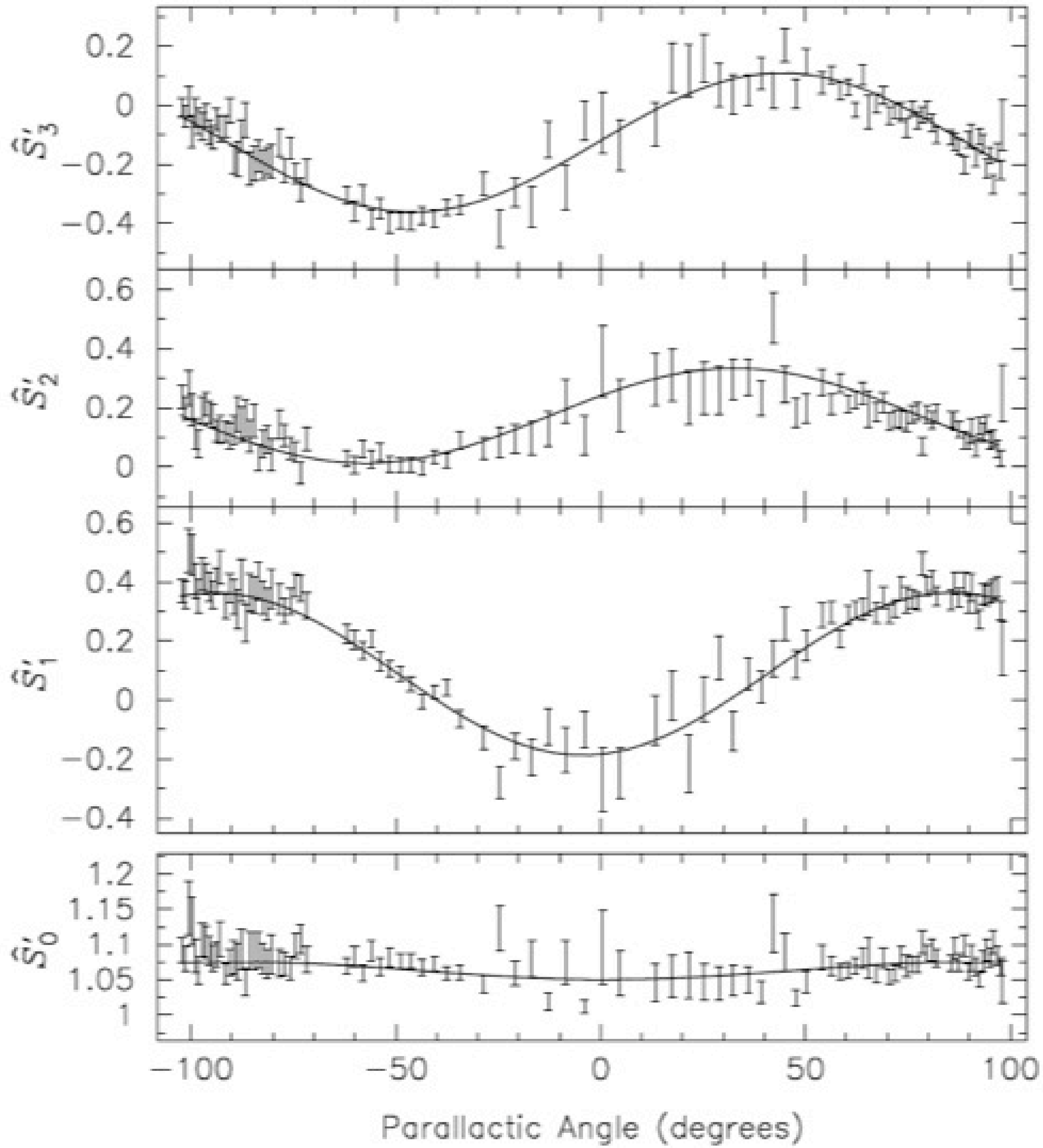
$$\begin{aligned}\boldsymbol{\rho}' &= \hat{\mathbf{J}}\boldsymbol{\rho}\hat{\mathbf{J}}^\dagger \\ &= \mathbf{J}e^{i\phi}\boldsymbol{\rho}e^{-i\phi}\mathbf{J}^\dagger \\ &= \mathbf{J}\boldsymbol{\rho}\mathbf{J}^\dagger\end{aligned}$$

Receptor parameterisation

$$e = \boldsymbol{r}^\dagger \boldsymbol{e}$$

$$\boldsymbol{r} = g e^{i\phi} \begin{pmatrix} \cos\theta\cos\epsilon + i\sin\theta\sin\epsilon \\ \sin\theta\cos\epsilon - i\cos\theta\sin\epsilon \end{pmatrix}$$

$$\mathbf{J} = (\boldsymbol{r}_0\ \boldsymbol{r}_1)^\dagger$$



$\hat{S}'_3$
$\hat{S}'_2$
$\hat{S}'_1$
$\hat{S}'_0$
0.2
0
−0.2
−0.4
0.6
0.4
0.2
0
0.6
0.4
0.2
0
−0.2
−0.4
1.2
1.15
1.1
1.05
1
−100
−50
0
50
100
Parallactic Angle (degrees)

No Unique Solution

$$S' = \mathbf{M}\,\mathbf{R_v}(\Psi)\,S$$

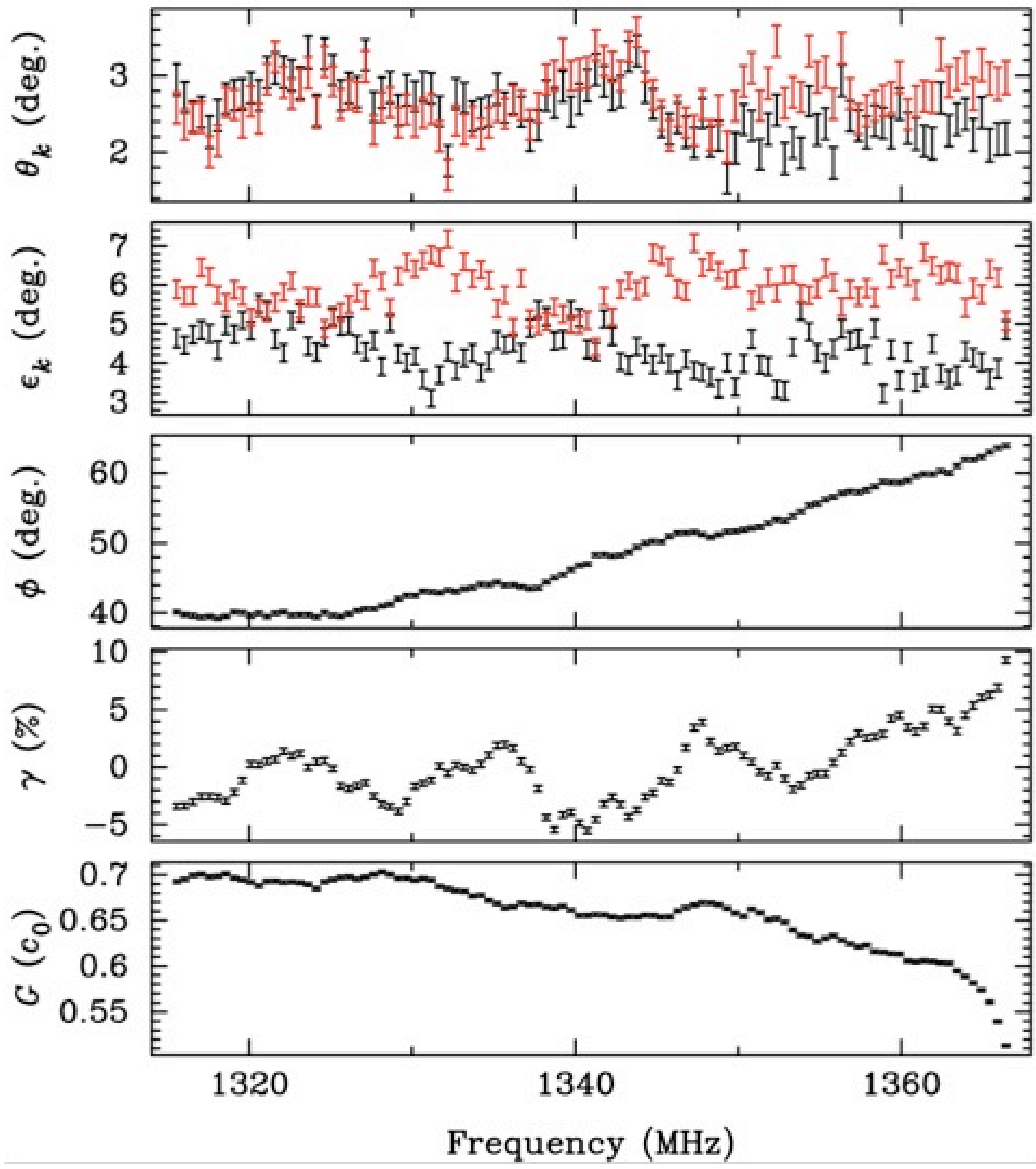
$$(cf.\ const = xy)$$

$$S' = \mathbf{M}^{u}\,\mathbf{R_v}(\Psi)\,S^{u}$$

$$\mathbf{M}^{u} = \mathbf{M}\,\mathbf{U_v}^{\mathbf{-1}}$$

$$S^{u} = \mathbf{U_v}\,S$$

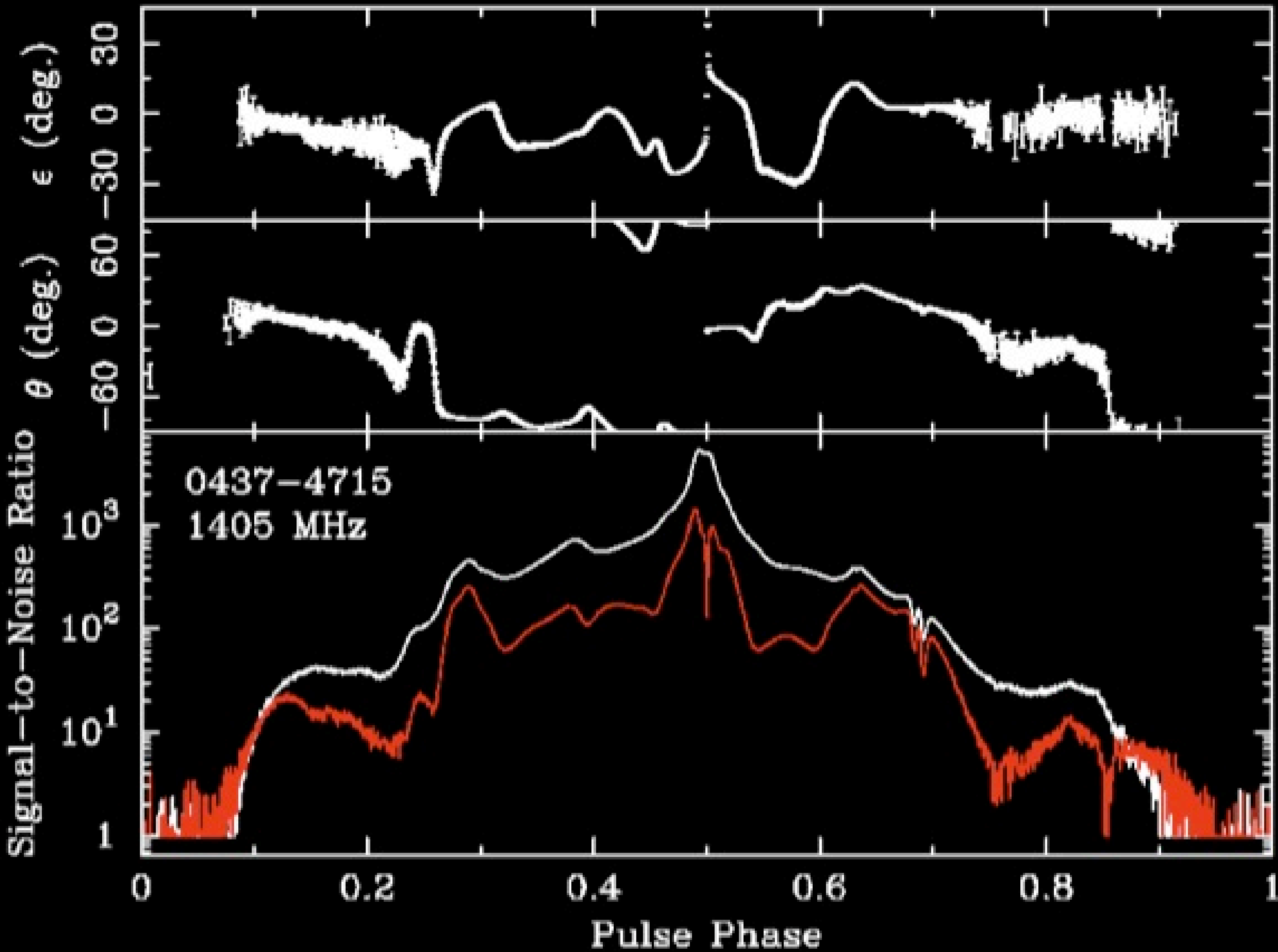
$$\mathbf{U_v}\,\mathbf{R_v}(\Psi) = \mathbf{R_v}(\Psi)\,\mathbf{U_v}$$



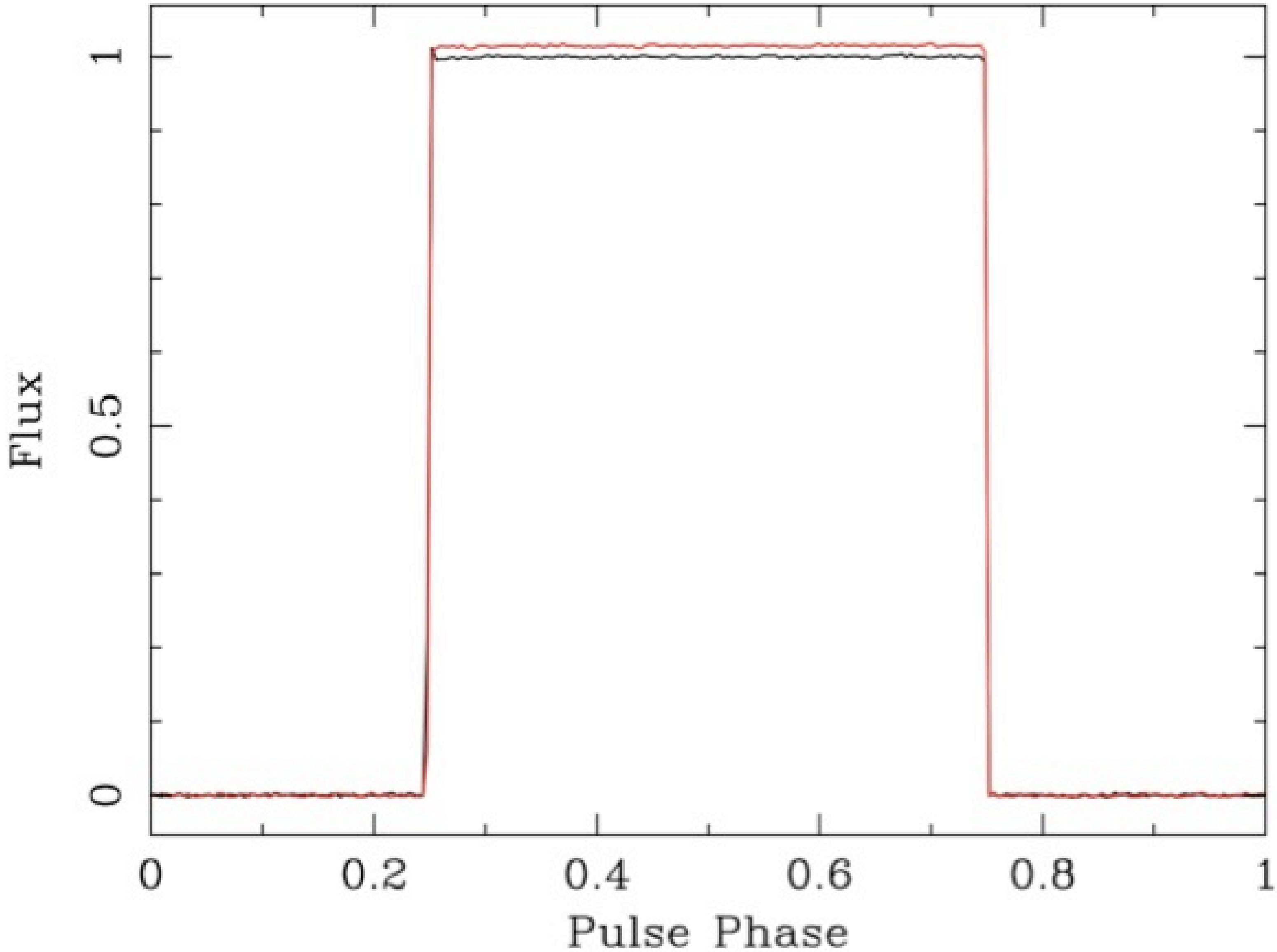
θ_k (deg.)
3
2
ϵ_k (deg.)
7
6
5
4
3
ϕ (deg.)
60
50
40
γ (%)
10
5
0
−5
G (c_0)
0.7
0.65
0.6
0.55
1320
1340
1360
Frequency (MHz)

Future Calibration Challenges

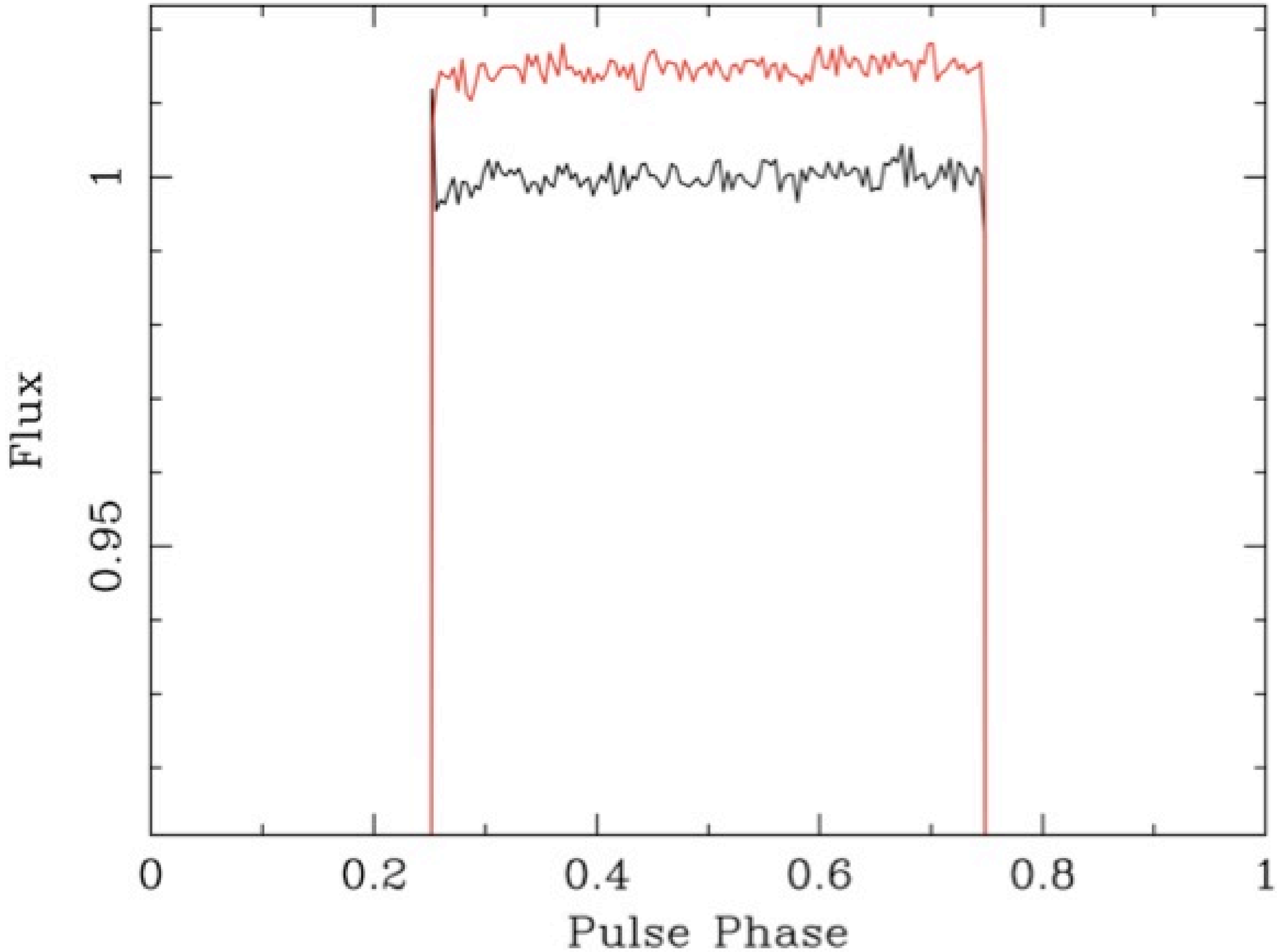
- Phased Arrays, $\mathbf{J} = \Sigma e^{i\phi(t)}\mathbf{J}_i$
- Off-axis polarization
- Non-linear effects
  - analog: amplifiers, etc.
  - analog-to-digital: quantization distortion



ϵ (deg.)
30
0
−30
θ (deg.)
60
0
−60
Signal−to−Noise Ratio
0437−4715
1405 MHz
10^3
10^2
10^1
1
0
0.2
0.4
0.6
0.8
1
Pulse Phase



Flux
0
0.5
1
0
0.2
0.4
0.6
0.8
1
Pulse Phase



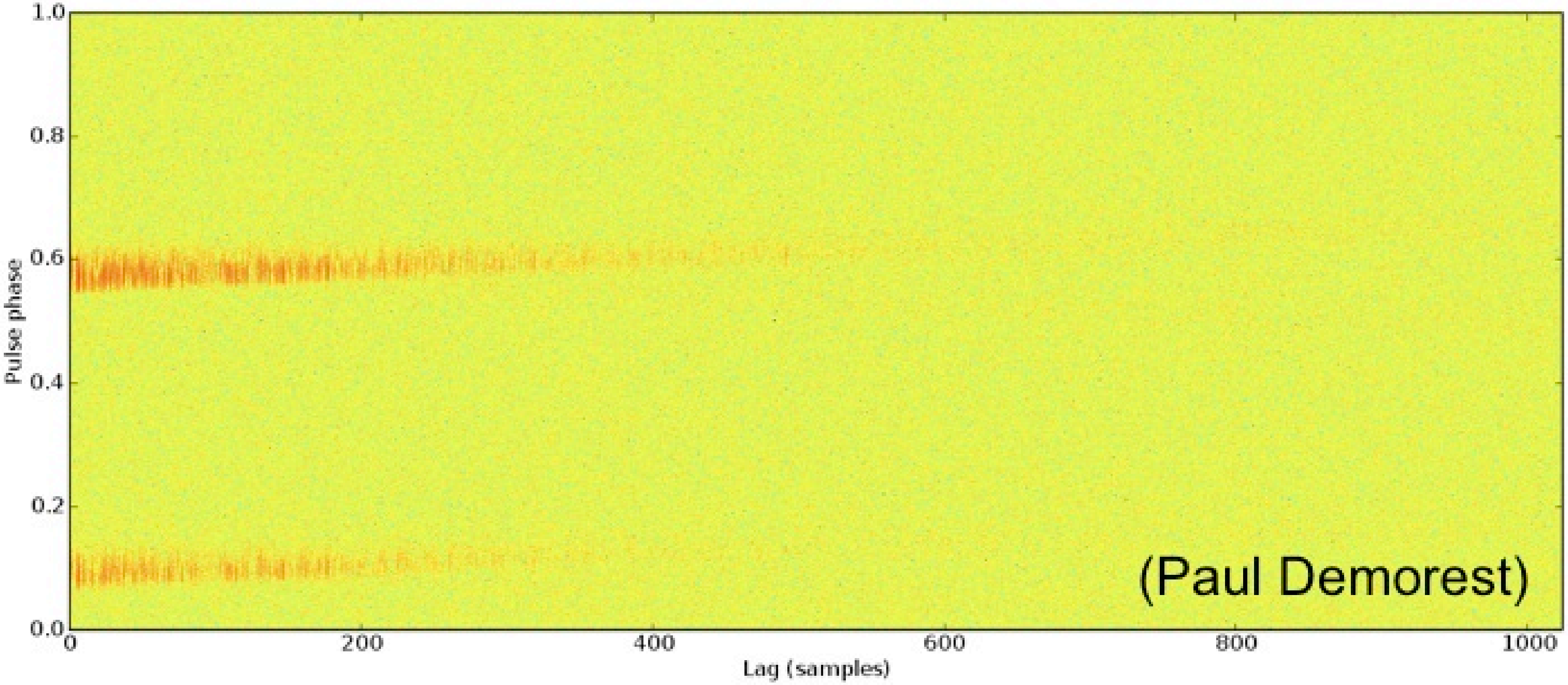
Flux
1
0.95
0
0.2
0.4
0.6
0.8
1
Pulse Phase

Future Directions

- Polarisation of transient events
  - high time resolution $\Rightarrow$ small number statistics
  - spectral coherence
- Scintillation of polarised radiation
  - vector diffraction theory
  - differential Faraday rotation
  - cyclostationary coherency matrix

Periodic coherency

$$\boldsymbol{\rho}(\phi, \tau) = \langle \boldsymbol{e}(t_\phi + \tau/2)\boldsymbol{e}^\dagger(t_\phi - \tau/2)\rangle$$



Thank you!

Questions?