POLARISATION I:
FOR THE LOVE OF STOKES

Simon Ellingsen, CASS Radio School 2011
Thanks to Jimi Green
Outline

- Why study Polarimetry and what is Polarisation?
- Poincare and his spheres
- Jones and his vectors (and matrices)
- Stokes and his parameters
- How do we measure Polarisation?
- Leakages and Mueller’s Matrix
- Polarised beam effects
- Science with Polarisation:
  - Masers and Zeeman splitting
Why Polarimetry?
Why Polarimetry?

- Polarisation is fundamentally important to understanding the Universe
  - Provides insight into magnetic fields
- In optical astronomy, it’s difficult to make polarimetric observations; in radio astronomy, they can be made easily, so why not use this to our advantage!
- (It’s also very important to the Birds & the Bees, navigationally speaking, c.f. Rossel & Wehner, 1984)
Why Polarimetry?
Why Polarimetry?

Polarization and the Hidden Nucleus of NGC 1068

Intensity

Polarized intensity

Wavelength, Angstroms

Polarization map
Why Polarimetry?

- 15 GHz VLBA polarization observations of PKS 1502+106 (Abdo et al. 2010).
- E-field vectors shown on left, fractional linear polarization shown on right, contours are total intensity.
Why Polarimetry?

- **Left panels**: Polarization degree images overlaid by polarization vectors (*yellow dashes*) and total intensity contour (*blue curves*).

- **Right panels**: Pseudocolor images composed of pure brightness images (*red*) and polarized brightness images (*blue*).

Zhibo et al. 2008
Faraday Screens have only a polarised structure, no standard (Stokes I) emission!

Haverkorn et al.
What is Polarisation?
What is Polarimetry & Polarisation?

- Polarisation is the behaviour of the electric field with time.
- To simplify things we will start by considering monochromatic radiation.
- Astrophysical processes like synchrotron radiation can emit partially polarised emission, but never fully polarised.
- Interstellar matter can polarise random background emission or de-polarise polarised background emission.
What is Polarisation?

- **Linear**: orthogonal components in phase with constant ratio of strengths giving constant direction of electric vector.

- **Circular**: orthogonal components 90° out of phase with equal amplitudes – electric vector traces circle.
What is Polarisation?

**Linear**
\[ E = E_x \cos(\omega t - k z) \hat{x} \]
\[ E = E_y \cos(\omega t - k z) \hat{y} \]

**Position Angle**
\[ E = E_x \cos(\omega t - k z) \hat{x} + E_y \cos(\omega t - k z) \hat{y} \]

**Right Hand Circular**
\[ E = E \cos(\omega t - k z) \hat{x} + E \sin(\omega t - k z) \hat{y} \]

**Left Hand Circular**
\[ E = E \cos(\omega t - k z) \hat{x} + E \cos(\omega t - k z + \frac{\pi}{2}) \hat{y} \]

**Polarisation Ellipse**
What is Polarisation?

- Linearly polarised wave can be decomposed into two opposite handed circular waves.
- Conversely a circularly polarized wave can be decomposed into two orthogonal linear waves.
- Sum of two circular waves of unequal amplitude is elliptical.
- Sum of two orthogonal linears with a phase difference of between 0 and $\pi/2$ is also elliptical.
the spherical surface occupied by completely polarised states in the space of the vector

- Poles represent circular polarisations
  - Upper-hemisphere LHCP
  - Lower-hemisphere RHCP

- Equator represents linear polarisations with longitude representing tilt angle

- Latitude represents axial ratio
Now for the maths
Jones calculus is a matrix-based means of relating observed to incident fields.

Vectors describe incident radiation and matrices the response of the instrument.

The Jones Vector:

\[
\begin{pmatrix}
E_x(t) \\
E_y(t)
\end{pmatrix}
\]

Examples:

- Linearly (x-direction) polarised wave: \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \)
- Left-Hand Circularly polarised wave: \( \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \)
Robert Clarke Jones
..and his matrices

- Effect of instrument described by 2x2 matrix:
  \[
  \begin{pmatrix}
  E_x \\
  E_y 
  \end{pmatrix}
  =
  \begin{pmatrix} a & b \\
  c & d \end{pmatrix}
  \begin{pmatrix}
  E_x \\
  E_y 
  \end{pmatrix}_i
  \]

- Simple Examples:
  - Linear polariser: \[
  \begin{pmatrix}
  1 \\
  0 \\
  0 \\
  0 
  \end{pmatrix}
  \]
  - Left-Hand Circular polariser: \[
  \frac{1}{2}
  \begin{pmatrix}
  1 & -i \\
  i & 1 
  \end{pmatrix}
  \]

- In practice matrix elements complex.

- Important: Only applicable to completely polarised waves.
Sir George Gabriel Stokes
..and his parameters

- Defined by George in 1852
- Adopted for astronomy by Chandrasehkar in 1947.
- Can be used for partially polarised radiation.
- Not a vector quantity! Deals with power instead of electric field amplitudes.
- The correlator can produce ALL Stokes parameters simultaneously (not so easy in optical astronomy!)
Stokes Parameters

- I – total intensity and sum of any two orthogonal polarisations
- Q and U – completely specify linear polarisation
- V – completely specifies circular polarisation

\[
\begin{align*}
I &= E_{0x}^2 + E_{0y}^2 \\
Q &= E_{0x}^2 - E_{0y}^2 \\
U &= 2E_{0x}E_{0y}\cos\delta \\
V &= 2E_{0x}E_{0y}\sin\delta
\end{align*}
\]

\[
\begin{align*}
I &= \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle \\
Q &= \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle \\
U &= \langle E_x E_y^* \rangle + \langle E_x^* E_y \rangle \\
V &= i\left(\langle E_x E_y^* \rangle - \langle E_x^* E_y \rangle\right)
\end{align*}
\]

(For linear feeds)
Fractional Polarisations

- The total linearly polarised intensity is defined as:
  \[ P = \sqrt{U^2 + Q^2} \]  
  [for native linear feed]

- A linearly polarised source will have an intrinsic position angle on the sky that is given by:
  \[ \Theta = \frac{1}{2} \tan^{-1}\left( \frac{U}{Q} \right) \]  
  [for native linear feed]

- The circular polarisation will be just Stokes V.

- Stokes parameters often presented as percentages of the total intensity.

- Since radio sources are never fully polarised, then the fractional linear and circular polarisation will always be < 1
How do we measure it?
How do we measure it?

- Stokes parameters are the auto-correlation & cross-correlation products returned from the correlator, but input to the correlator can come from different feed types.

- Feeds normally designed to approximate pure linear or circular (known as ‘native linear’ or ‘native circular’)
  - Linear Feeds – intrinsically accurate & provide true linear response.
  - Circular Feeds – less accurate & frequency dependent response.
How do we measure it?

- Output of native linear feed is $E_x$ and $E_y$ field voltages, so:
  - I from $XX+YY$
  - Q from $XX-YY$

- Native circular adds $90^\circ$ phase to X, so:
  - I from $XX+YY$ (or $LL+RR$)
  - V from $XX-YY$ (or $LL-RR$)
Stokes Parameters

- For circular feeds $Q$ and $V$ swap round.

<table>
<thead>
<tr>
<th>Linear</th>
<th>Circular</th>
</tr>
</thead>
<tbody>
<tr>
<td>$XX = I + Q$</td>
<td>$RR = I + V$</td>
</tr>
<tr>
<td>$YY = I - Q$</td>
<td>$LL = I - V$</td>
</tr>
<tr>
<td>$XY = U + iV$</td>
<td>$RL = Q + iU$</td>
</tr>
<tr>
<td>$YX = U - iV$</td>
<td>$LR = Q - iU$</td>
</tr>
</tbody>
</table>
But is it really that simple?

- Do we just plug in our computer and get \{I,Q,U,V\} out of the correlator?
- No, there are leakages!
  - The total intensity can leak into the polarised components (I into \{Q,U,V\}).
  - The linear polarisation can leak into the circular (\{Q,U\} into V).
  - … and all combinations and permutations are allowed!
- Without correcting for leakage, you’re not going to get proper Stokes parameters!
The leakage of each polarisation into the other can be measured and quantified in a 4x4 matrix first proposed by Mueller in 1943.

\[
M = 
\begin{bmatrix}
  m_{II} & m_{IQ} & m_{IU} & m_{IV} \\
  m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\
  m_{UI} & m_{UQ} & m_{UU} & m_{ii} \\
  m_{VI} & m_{VQ} & m_{VU} & m_{VV}
\end{bmatrix}
\]
The Mueller Matrix

\[
\begin{bmatrix}
XX + YY \\
XX - YY \\
XY \\
YX
\end{bmatrix}
= M \cdot \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}
\]
Example (simple) Mueller Matrices

- If feeds were perfect:
  - Dual linear feed: $M$ is unitary
  - Dual linear feed rotated 45°: $Q$ and $U$ interchange and sign change for rotation:
  - A dual linear feed rotated 90°: signs of $Q$ and $U$ reversed:

As Alt-Az telescope tracks source, feed rotates on sky by the parallactic angle (PA):

$$M_{\text{sky}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2PA & \sin 2PA & 0 \\ 0 & -\sin 2PA & \cos 2PA & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
The more general Mueller Matrix

- For a (realistic) dual linear feed:

\[
M = \begin{bmatrix}
1 & \left(-2\varepsilon\sin\phi\sin2\alpha + \frac{\Delta G}{2}\cos2\alpha\right) & 2\varepsilon\cos\phi & \left(2\varepsilon\sin\phi\cos2\alpha + \frac{\Delta G}{2}\sin2\alpha\right) \\
\frac{\Delta G}{2} & \cos2\alpha & 0 & \sin2\alpha \\
2\varepsilon\cos(\phi + \varphi) & \sin2\alpha\sin\varphi & \cos\varphi & -\cos2\alpha\sin\varphi \\
2\varepsilon\sin(\phi + \varphi) & -\sin2\alpha\cos\varphi & \sin\varphi & \cos2\alpha\cos\varphi
\end{bmatrix}
\]

- The Mueller matrix has 16 elements, but ONLY 7 INDEPENDENT PARAMETERS. The matrix elements are not all independent.
Calculating the Mueller Matrix

- For a perfect system, as we track a polarised source across the sky the parallactic angle changes and this should produce:
  - For XX-YY: \( \cos^2(\text{PA}_{\text{az}} + \text{PA}_{\text{src}}) \), centred at zero.
  - For XY: \( \sin^2(\text{PA}_{\text{az}} + \text{PA}_{\text{src}}) \), centred at zero.
  - For YX: zero (most sources have zero circular polarisation)
Calculating the Mueller Matrix

- But, what we find is:
Calculating the Mueller Matrix

- Which enables the matrix to be calculated and the observations corrected to give what we expect:

\[
\begin{array}{cccc}
1.0000 & 0.0002 & 0.0006 & -0.0005 \\
0.0002 & 1.0000 & 0.0000 & -0.0012 \\
0.0006 & -0.0000 & 1.0000 & -0.0044 \\
-0.0005 & 0.0012 & 0.0044 & 1.0000 \\
\end{array}
\]
Putting it all together

- In the end what we are trying to do is relate products from our correlator to the intrinsic polarized radiation from the source.

- So we need to correct the raw correlator outputs for
  - Imperfections in the receiver (leakages).
  - The orientation of the receiver with respect to the telescope structure (maybe).
  - The changing parallactic angle (maybe).
  - Any measured propagation related polarization effects (e.g. Faraday rotation).
Beam Effects
Beam Effects

- For point sources, all of the previous is fine.
- What if the source you’re looking at is extended compared to the telescope beam?
- There are instrumental beam effects that can confuse the measurement of extended polarised signals. They are...
  - Squint
  - Squash
Stokes I response

Heiles et al. 2001
Beam Squint & Squash

**Beam Squint**
- RHCP
- LHCP

**Beam Squash**
- RHCP
- LHCP

\[ V = \text{RHCP} - \text{LHCP} \]
- \( V > 0 \)
- \( V < 0 \)
Squint in action

Heiles et al. 2001
Squash in action

Heiles et al. 2001
Depolarization!

- 15 GHz VLBA polarization observations of PKS 1502+106 (Abdo et al. 2010).

- E-field vectors shown on left, fractional linear polarization shown on right, contours are total intensity.
Here comes the science
Polarisation of Masers

Ellingsen (2002)
Polarisation of Masers

- Filamentary maser structure.
- Linear polarisation up to 8%.
- Polarisation angles indicate north-south structure, and are consistent with OH.

Make like a banana: Zeeman Splitting

- Atoms & molecules in net magnetic moment will have their energy levels split in the presence of a magnetic field.
- Detected through frequency shift between right and left circularly polarised emission

\[ V = RHCP - LHCP \propto B_{\text{los}} \]
Zeeman Splitting of Masers

- OH Gigamasers
  (luminosity greater than $10^4$ solar luminosities)
Zeeman Splitting of Masers

Zeeman splitting in a Galactic 6.7 GHz methanol maser (Stack & Ellingsen 2011).
Zeeman Splitting of Masers

- ATCA Zeeman splitting measured simultaneously in 6 different OH maser transitions shows variations of nearly an order of magnitude on sub-pc scales in high-mass star formation regions.
Zeeman Splitting of Masers

- Line-of-sight magnetic field directions deduced from OH maser Zeeman splitting.
- 74 star-forming regions:
  - 41 with an overall magnetic field oriented in a clockwise sense.
  - 33 with field oriented counterclockwise as viewed from above the Galactic center.
- Field consistency within 2-kpc of Sun.

Fish et al 2003
In summary..
Summary

- Polarisation in radio astronomy very important to improving our knowledge & understanding.
- Can describe polarisation with the Polarisation Ellipse and the Poincare Sphere.
- Dr. Jones offers a vector representation for ideal cases of completely polarised emission.
- Mueller and his matrices are the best option for real situations.
- There are Linear and Circular feed types, must account for which you are using.
- Understanding the polarisation properties of your dish is fundamental to successful observations!
- (Masers offer exciting science opportunities!)
Useful References

- Stutzman, W. ‘Polarisation in Electromagnetic Systems’ (1993), Artech House (Norwood, MA, USA)
- Radhakrishnan. Polarisation. URSI proceedings (1990) pp. 34
- Born and Wolf: ‘Principle of Optics’, Chapters 1 and 10