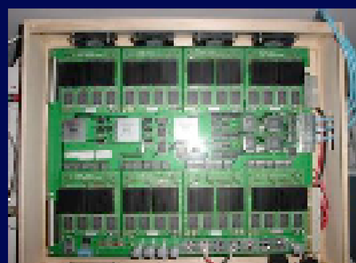
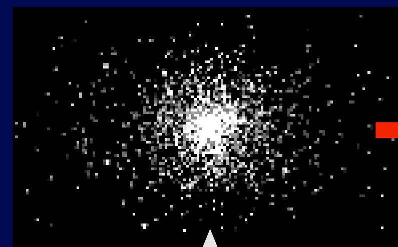
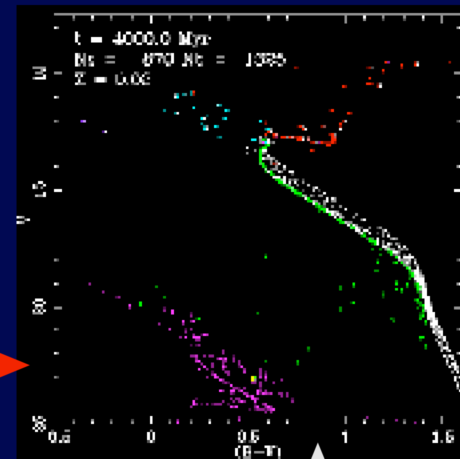
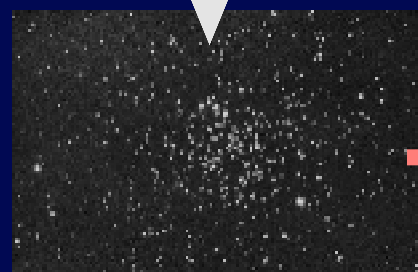
Simulating the Evolution of Stellar Clusters

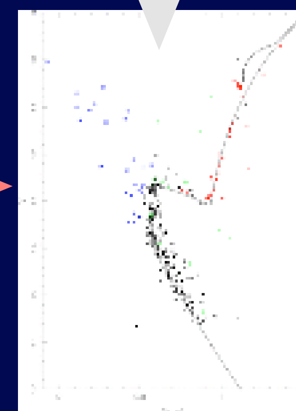






?





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Equation of Motion

$$\ddot{r}_i = -G\sum_{j=1}^{N}\frac{m_j(r_i - r_j)}{|r_i - r_j|^3}$$

Direct integration = $O(N^3)$ cost

- *N* large -> 10^6

+ density contrast
+ close encounters
+ binaries
+ long-lived system

- add softening parameter?

$$\ddot{r}_i = -G \sum_{j=1}^{N} \frac{m_j (r_i - r_j)}{(|r_i - r_j|^2 + \epsilon^2)^{3/2}}$$

  - prevents force singularity -> "collisionless"
  - reduces relaxation and mass-segregation
  - not accurate for relaxation dominated systems

- neighbour schemes to reduce cost -> $N^2 \log(N)$
- other methods: tree-code? Fokker-Planck?

require $\Delta E/E < 10^{-5}$

approximate
+ realism -> cpu

- Direct *N*-body is preferred method

Regularization for close encounters and binaries

3D -> 4D
+ t transformation

- improves efficiency
- greater accuracy
- extended to three or more bodies

1-D Regularization (Eulerian treatment):

Equation of motion (reduced mass) is

$$\ddot{x} = -\frac{M}{x^2} \,. \tag{1}$$

Take the time transformation

$$d\tau = dt/x \quad \rightarrow \dot{x} = x'/x \tag{2}$$

(primes for τ derivatives) in (1) to give

$$x'' = \frac{x'^2}{x} - M \,. \tag{3}$$

The energy integral is

$$\frac{1}{2}\dot{x}^2 = \frac{M}{x} + h \tag{4}$$

which combined with (3) gives

$$x'' = 2hx + M \,. \tag{5}$$

Make the coordinate transformation, $x = u^2$, to give

$$\begin{aligned} u'' &= \frac{1}{2}hu \\ t' &= u^2 \end{aligned}$$

Non-linear equation reduced to harmonic oscillator
$\Longrightarrow$ regular as $x \rightarrow 0$.

Hierarchical time steps for density contrast

$$\Delta t_i = \left(\frac{\eta \,| F |}{| \ddot{F} |} \right)^{1/2} \rightarrow \quad \Delta t_n = \left(\frac{1}{2} \right)^{n-1} \qquad \eta \approx 0.02, \; n \leq 40$$

- individual timesteps
- advance “block” of particles together
- facilitates sub-system search

and works with ...

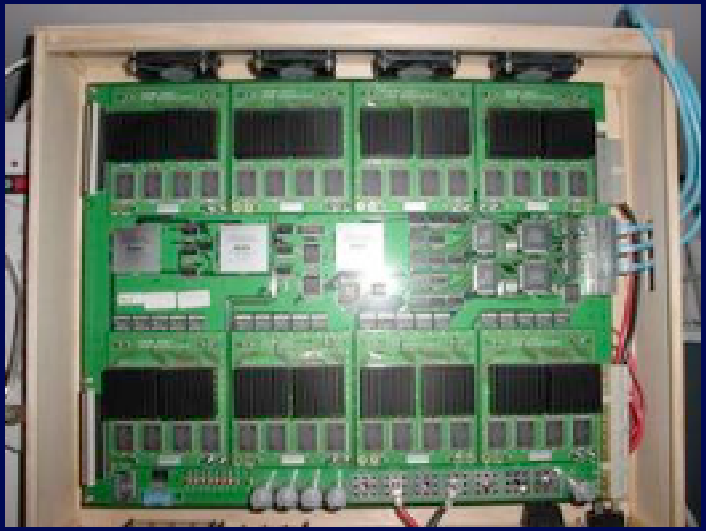
Hermite Integration Scheme

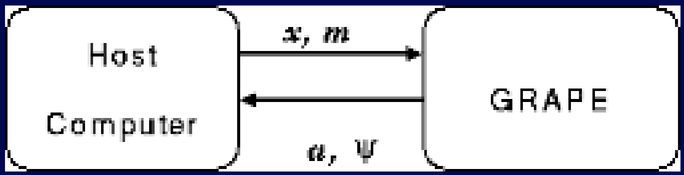
- 4th order force polynomial
- more accurate and less memory

... but direct *N*-body still expensive

... *N*-body saved by the GRAPE

- built by astrophysicists at University of Tokyo (1990 -)
  [Makino, Kokubo & Taiji 1993]
- GRAvity piPE
- “Newtonian” accelerator for the force calculation loop + prediction + neighbour list
- special-purpose hardware with hardwired logic
- GRAPE-4 available 1996
  -> Gflops performance
  -> open clusters of 10,000+ stars
- GRAPE-6 available 2001
  -> Tflops for $50k
  -> small globular clusters





NBODY4 software

- includes stellar evolution
  - fitted formulae as opposed to “live” or tables
  - done in step with the dynamics
- and a binary evolution prescription
  - tidal evolution, magnetic braking, gravitational radiation, wind accretion, RLOF: mass transfer, common-envelope, mergers
- and as much realism as possible
  - perturbed orbits (hardening & break-up), chaotic orbits, exchanges, triple & higher-order subsystems, collisions, etc.

    ... regularization techniques

    + external tidal field
    + Hermite integration with GRAPE
    + block time-step algorithm

Everything you need to know ...

Gravitational N-body Simulations: Tools and Algorithms

Sverre Aarseth, 2003, Cambridge University Press

or

From NBODY1 to NBODY6: The Growth of an Industry

Sverre Aarseth, 1999, PASP, 111, 1333

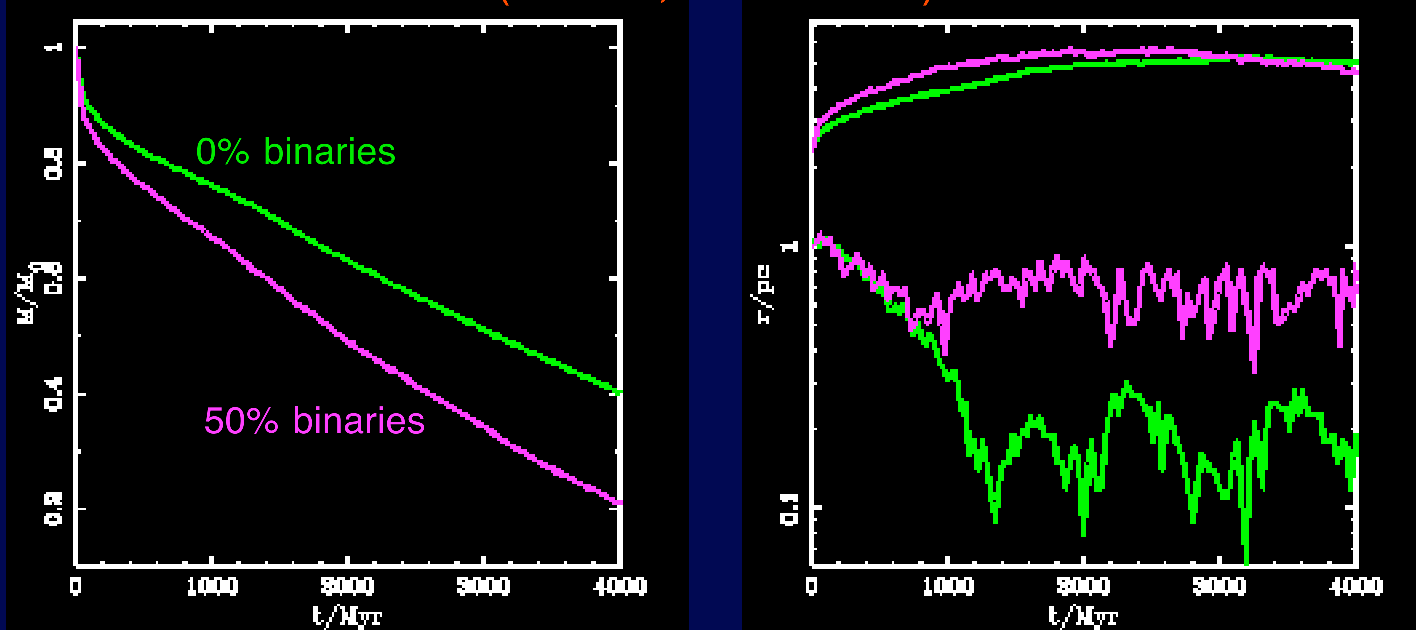
also

The Gravitational Million-Body Problem

Douglas Heggie & Piet Hut, 2003, Cambridge University Press

General Results: 1. The Effect of Binaries

(N = 30,000 models)

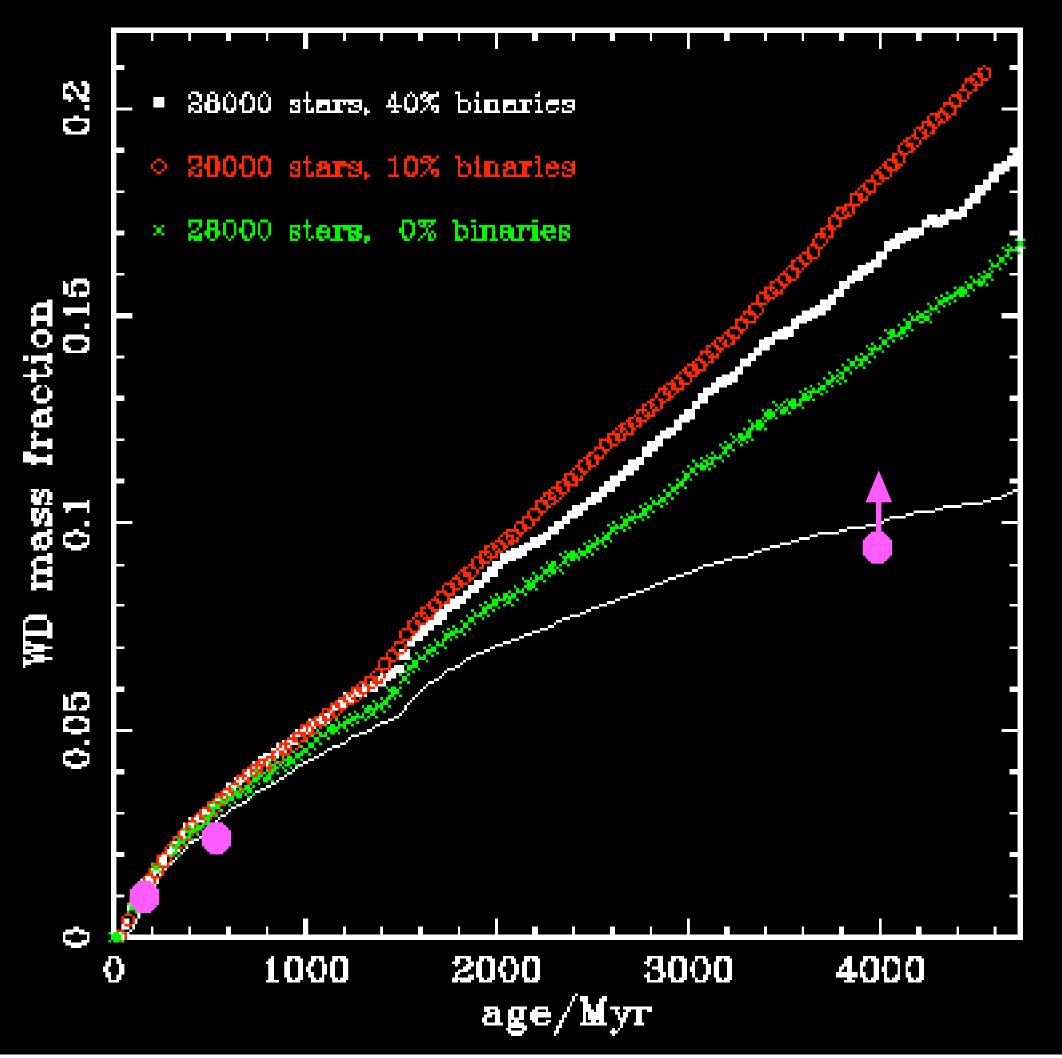


Inclusion of primordial binaries

- fraction of escaping stars increases by ~50%
- velocity of escaping stars increases by ~20%

- evidence for saturation of primordial binary effects above ~25%

2. WD Mass Fractions



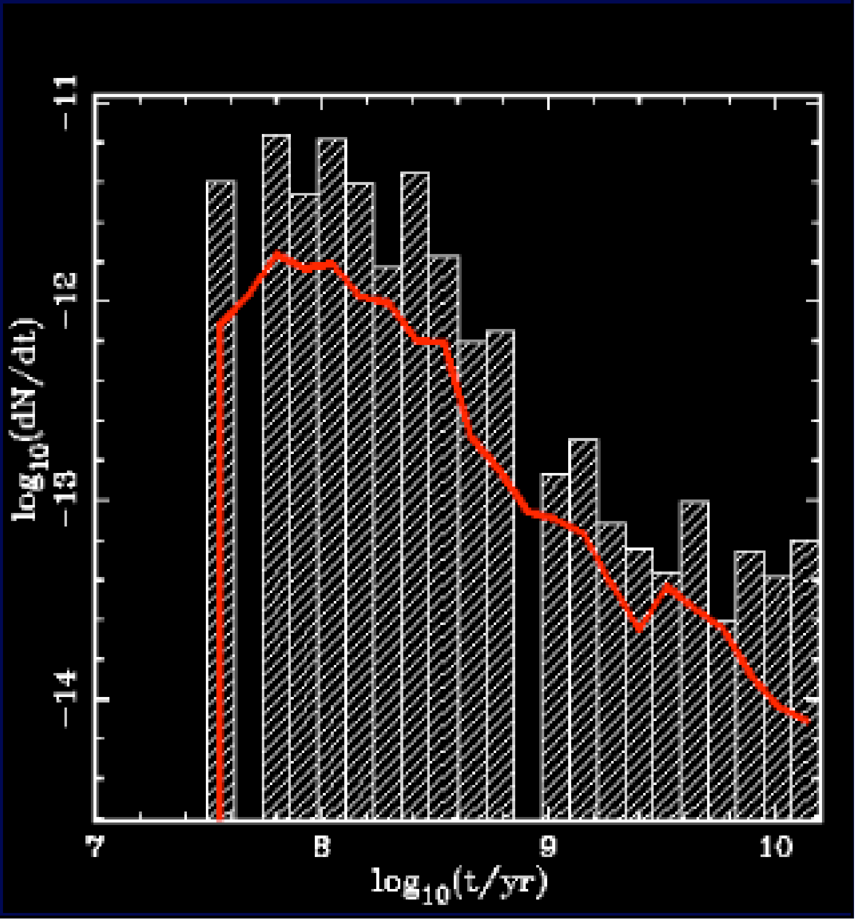
- at $t/t_{rh} = 10$

$f_{b,0}$	f_{WD}
0.4	0.15
0.1	0.17
0.0	0.17

- evaporation & binaries are important

3. Supra-Chandrasekhar DWD Merger Rate

- 2 WDs, $M_b > 1.44\ M_{sun}$, $T_{grav} < 12$ Gyr
  - 10x expected (non-dynamical) merger events
- Blame for enhancement shared equally between:
  - exchange interactions
  - pre-DWD perturbations
  - post-DWD perturbations
- Type Ia supernova?
- AIC collapse to NS?
  - interesting either way



An Example

Primordial Binary:

M_1 = 6.9 M_{sun}
M_2 = 3.1 M_{sun}
a = 4050 R_{sun}

After 60 Myr:

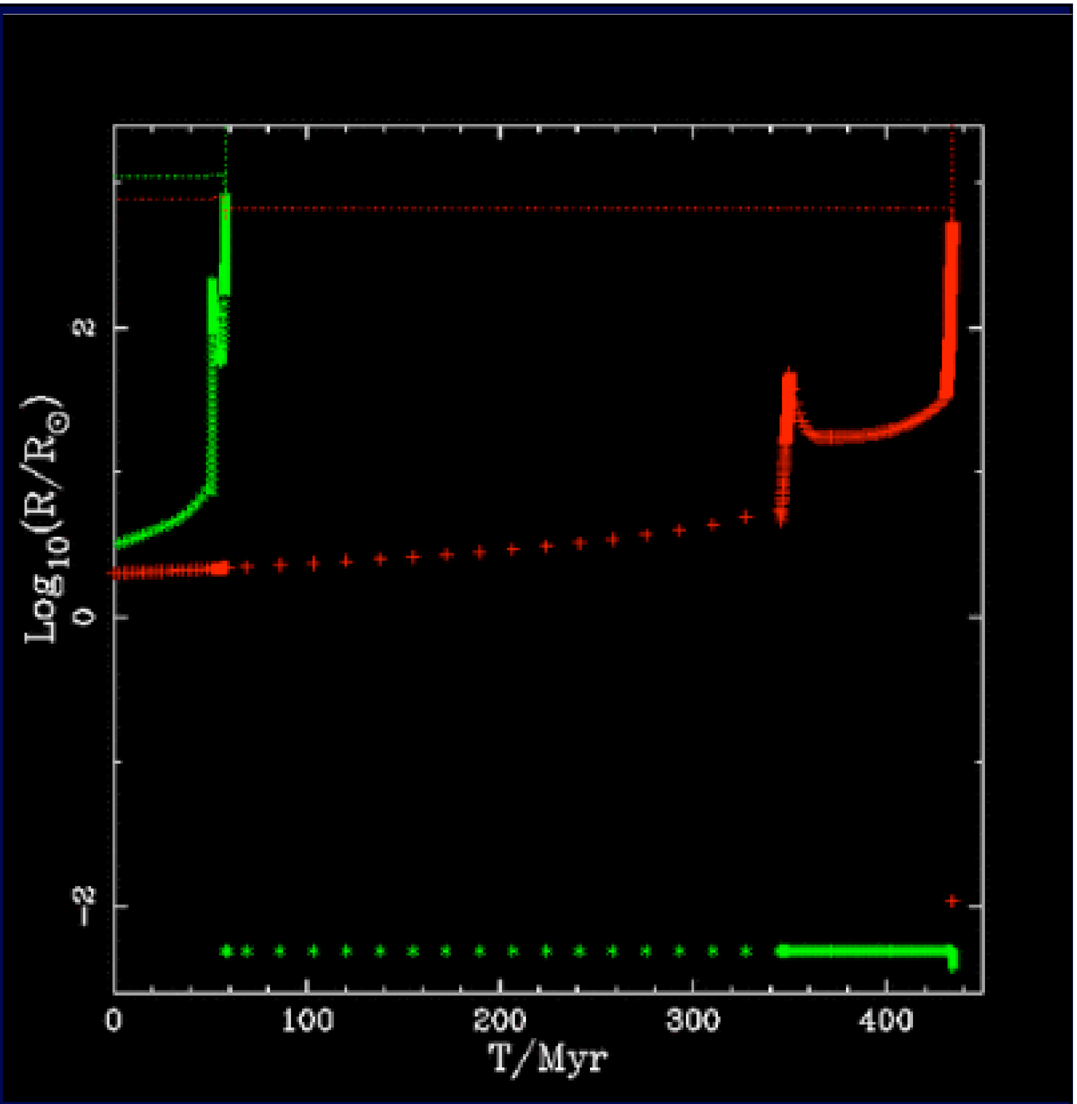
M_1 = 6.3 on AGB
e = 0.0 (tides)

RLOF => CE
M_1 = 1.25 ONeWD

After 430 Myr:

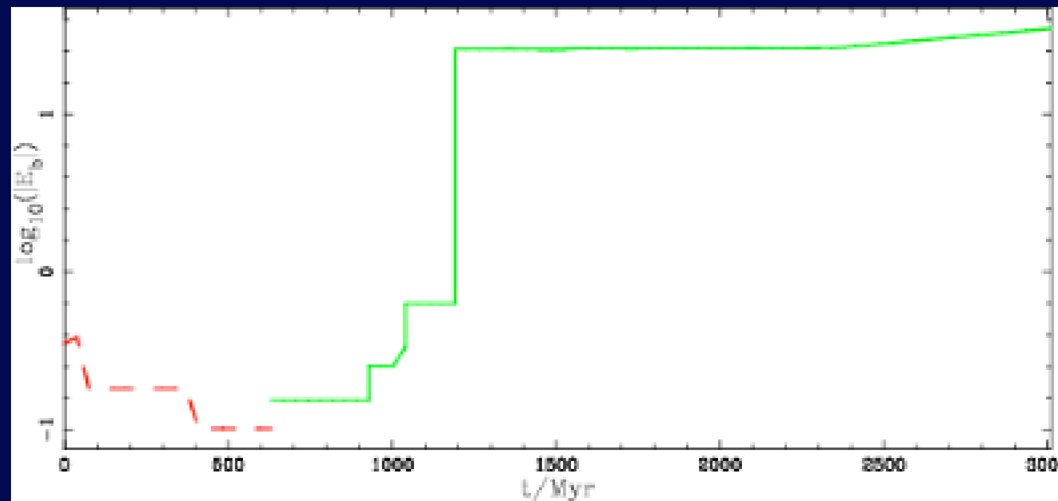
M_2 = 2.0 on AGB
M_1 = 1.30 (symbiotic)

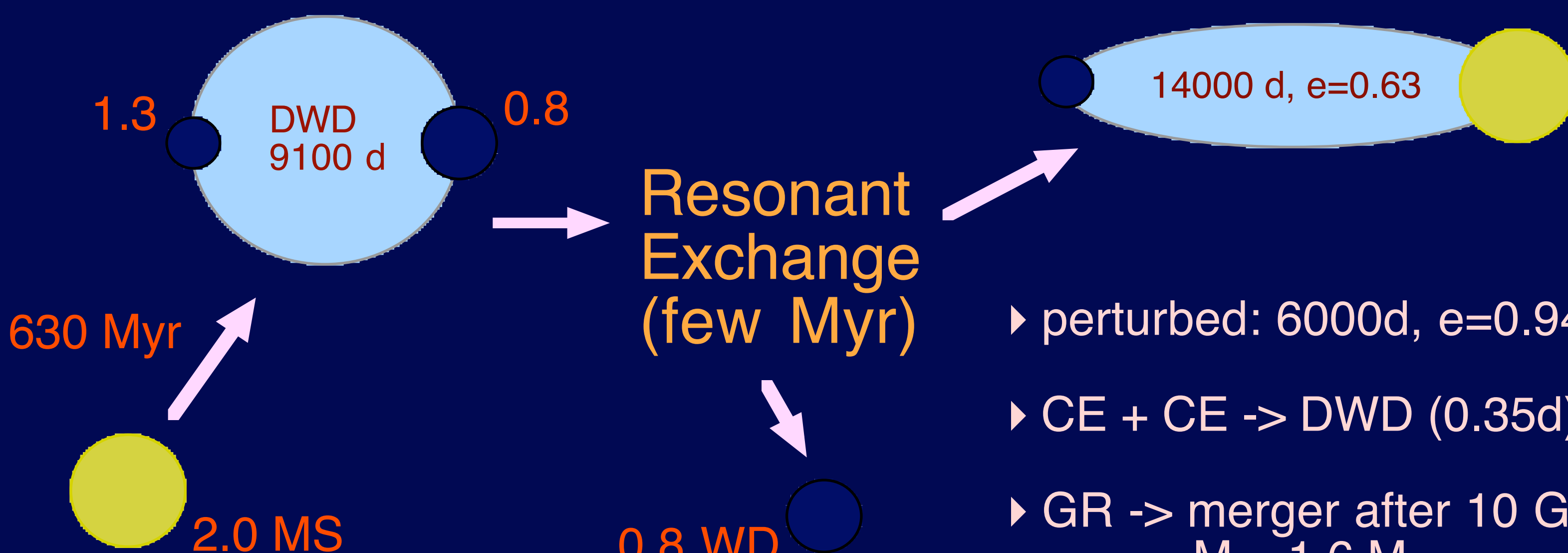
RLOF => CE
M_2 = 0.8 COWD
a = 2500 R_{sun}



DWD with $t_{grav} = 10^{22}$ yr

and then ...



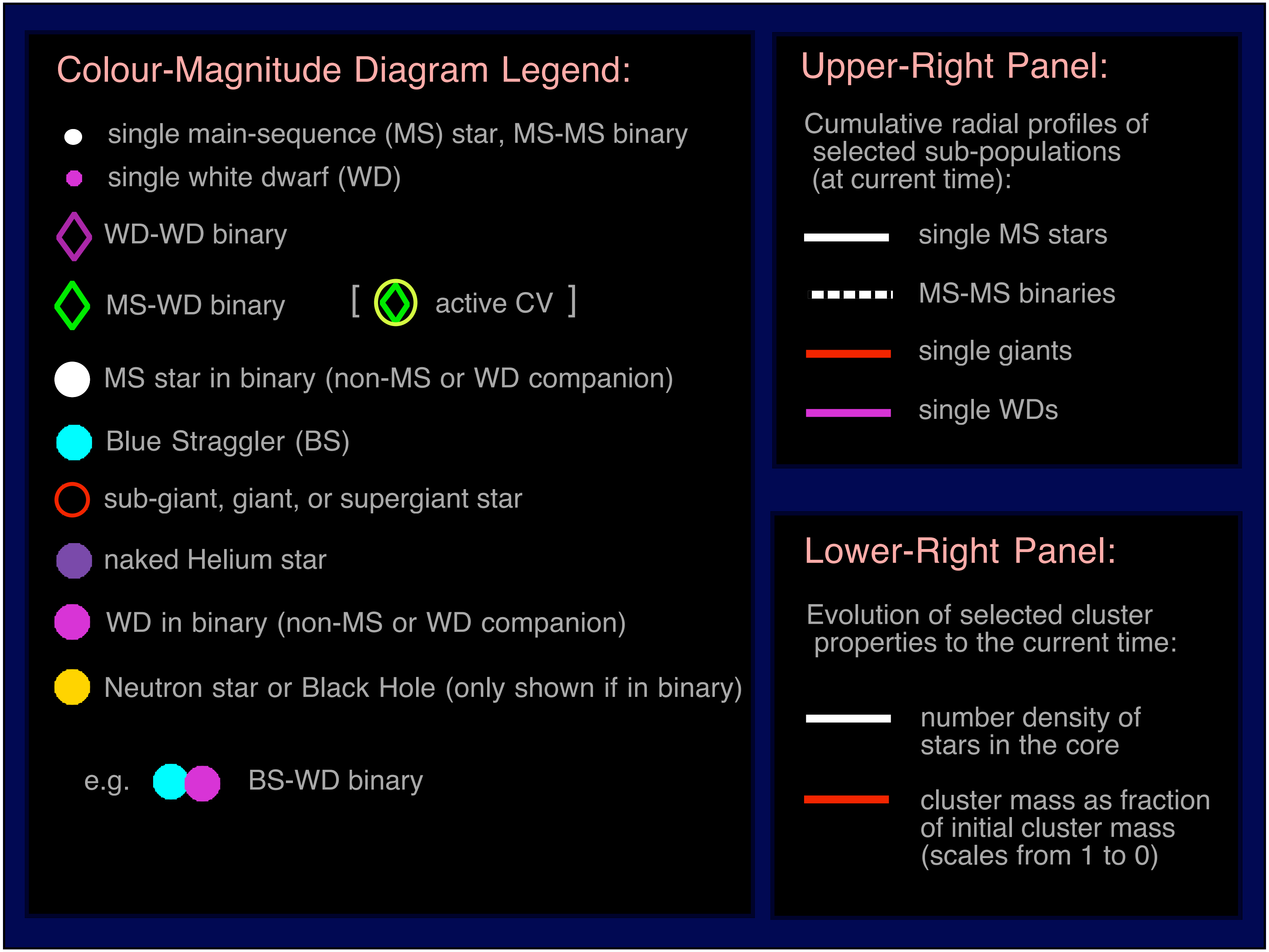


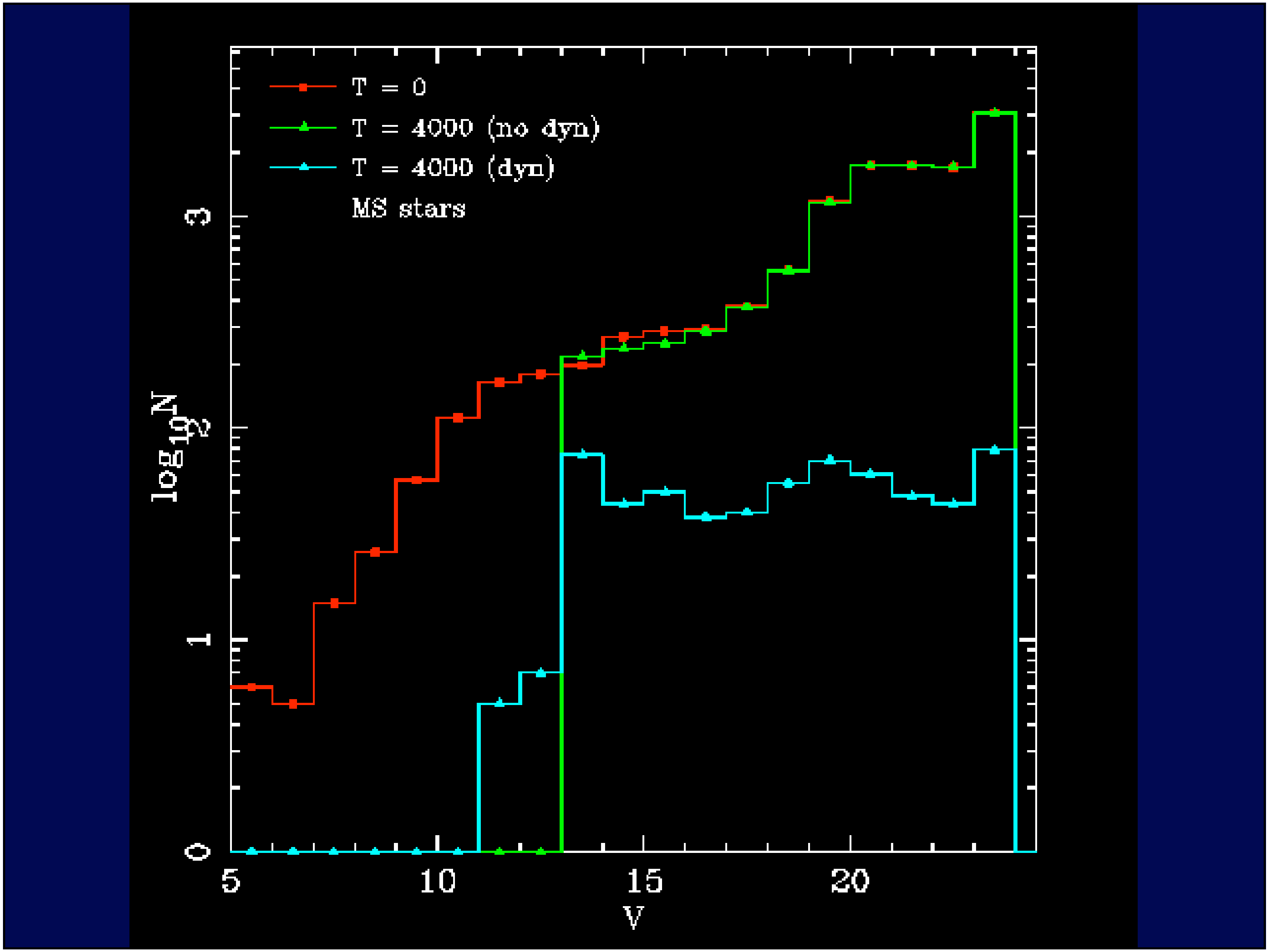
- perturbed: 6000d, e=0.94
- CE + CE -> DWD (0.35d)
- GR -> merger after 10 Gyr
  M = 1.6 Msun

Comparison with Data: Simulation of M67

- ★ 12,000 single stars (KTG1993 IMF: 0.1 - 50 M_{sun})
- ★ 12,000 binaries (q: uniform, e: thermal, a: flat-log, max 50 au)
- ★ Z = 0.02
- ★ Circular orbit at R_{gc} = 8 kpc
- ★ Plummer Sphere in virial equilibrium
  - M ~ 18700 M_{sun}
  - R_t = 32 pc
  - T_{rh} ~ 200 Myr
  - □ ~ 3 km/s
  - n_c ~ 200 stars/pc^3
  - 6-7 Gyr lifetime
  - 4-5 weeks of GRAPE-6 cpu

- show CMD movie (animated gif)

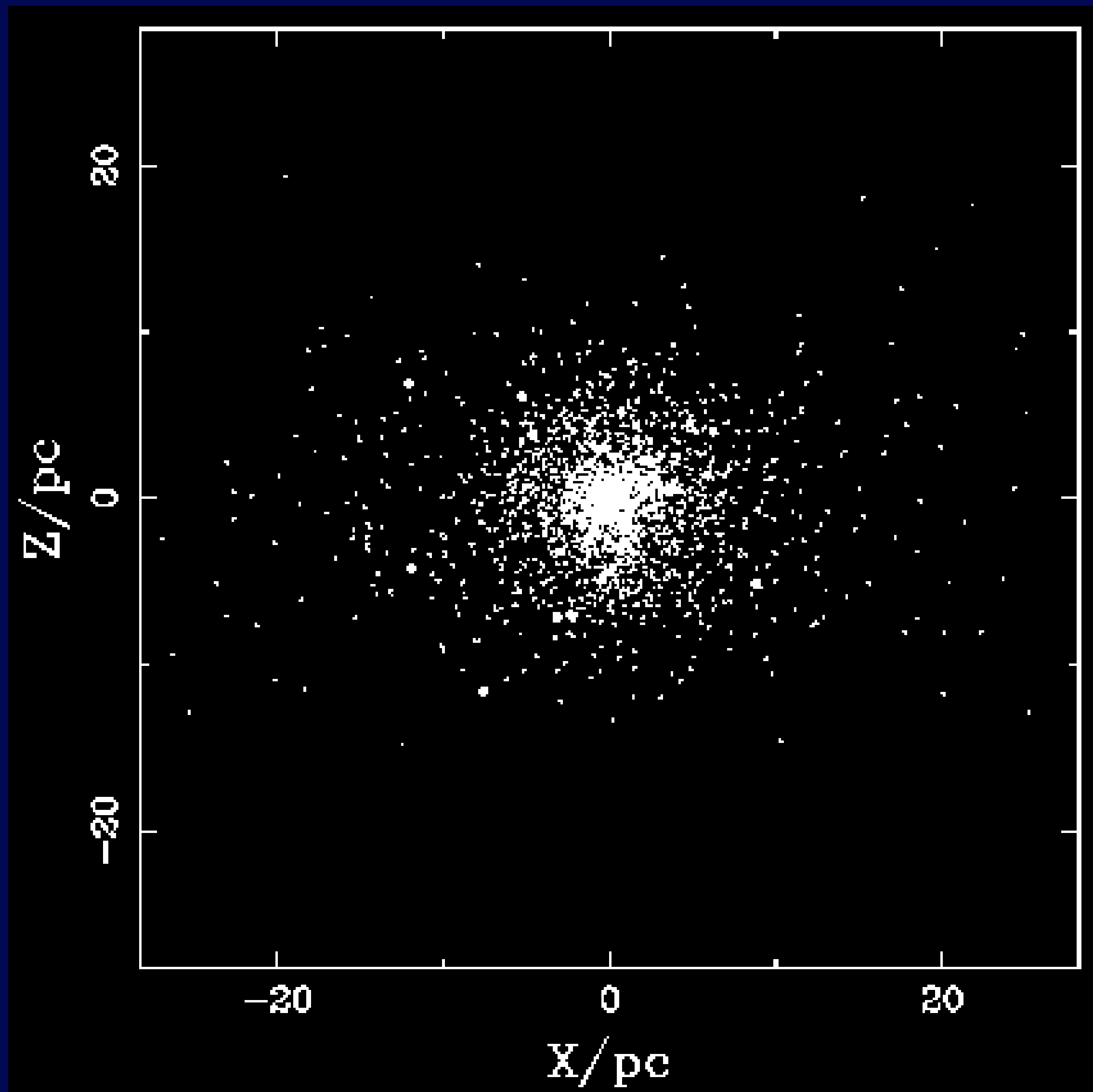

Colour-Magnitude Diagram Legend:
single main-sequence (MS) star, MS-MS binary
single white dwarf (WD)
WD-WD binary
MS-WD binary
[active CV]
MS star in binary (non-MS or WD companion)
Blue Straggler (BS)
sub-giant, giant, or supergiant star
naked Helium star
WD in binary (non-MS or WD companion)
Neutron star or Black Hole (only shown if in binary)
e.g.
BS-WD binary
Upper-Right Panel:
Cumulative radial profiles of selected sub-populations (at current time):
single MS stars
MS-MS binaries
single giants
single WDs
Lower-Right Panel:
Evolution of selected cluster properties to the current time:
number density of stars in the core
cluster mass as fraction of initial cluster mass (scales from 1 to 0)



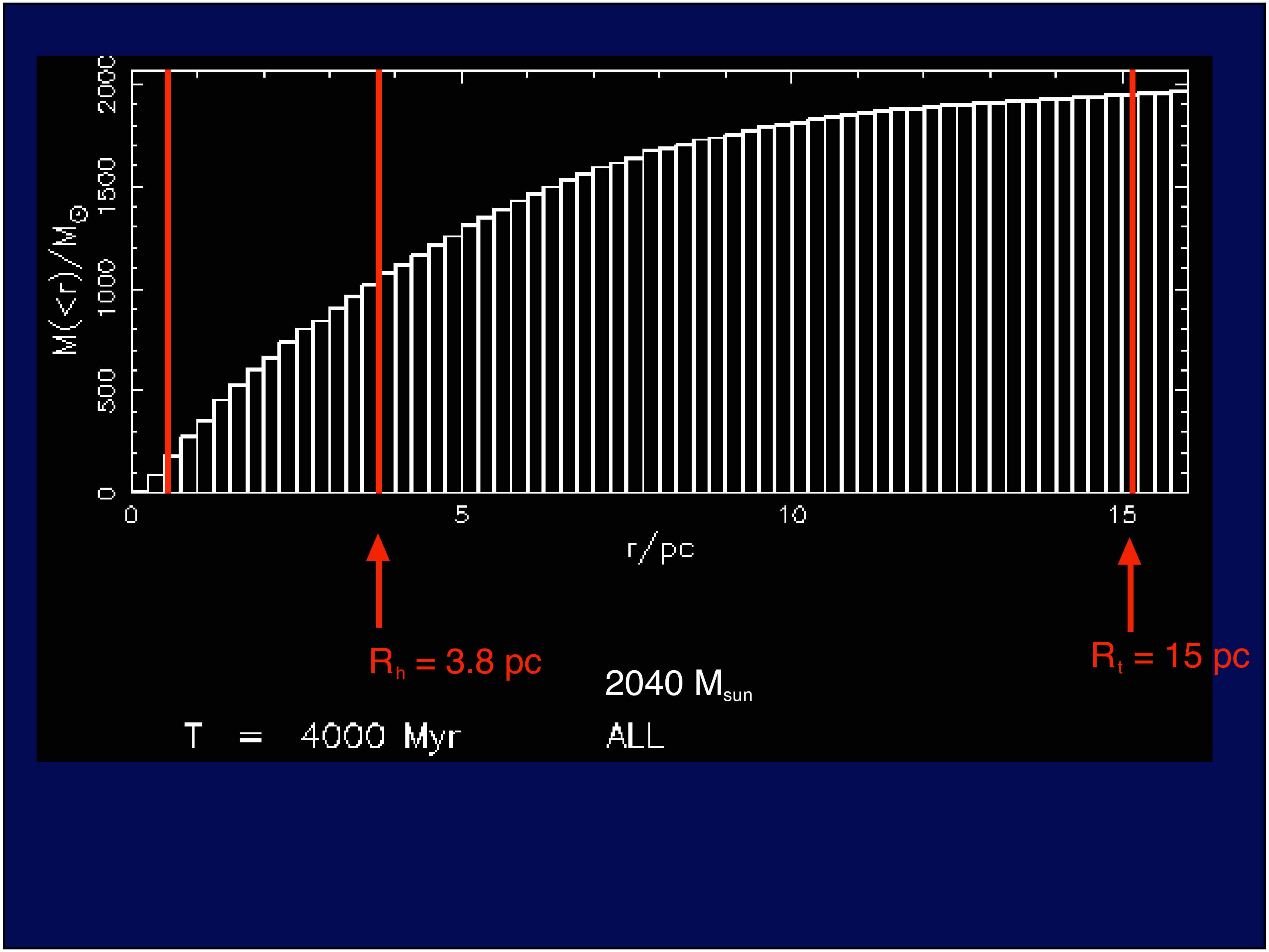
T = 0
T = 4000 (no dyn)
T = 4000 (dyn)
MS stars
$\log_{10}N$
V
0
1
2
3
5
10
15
20

model at 4 Gyr = M67?

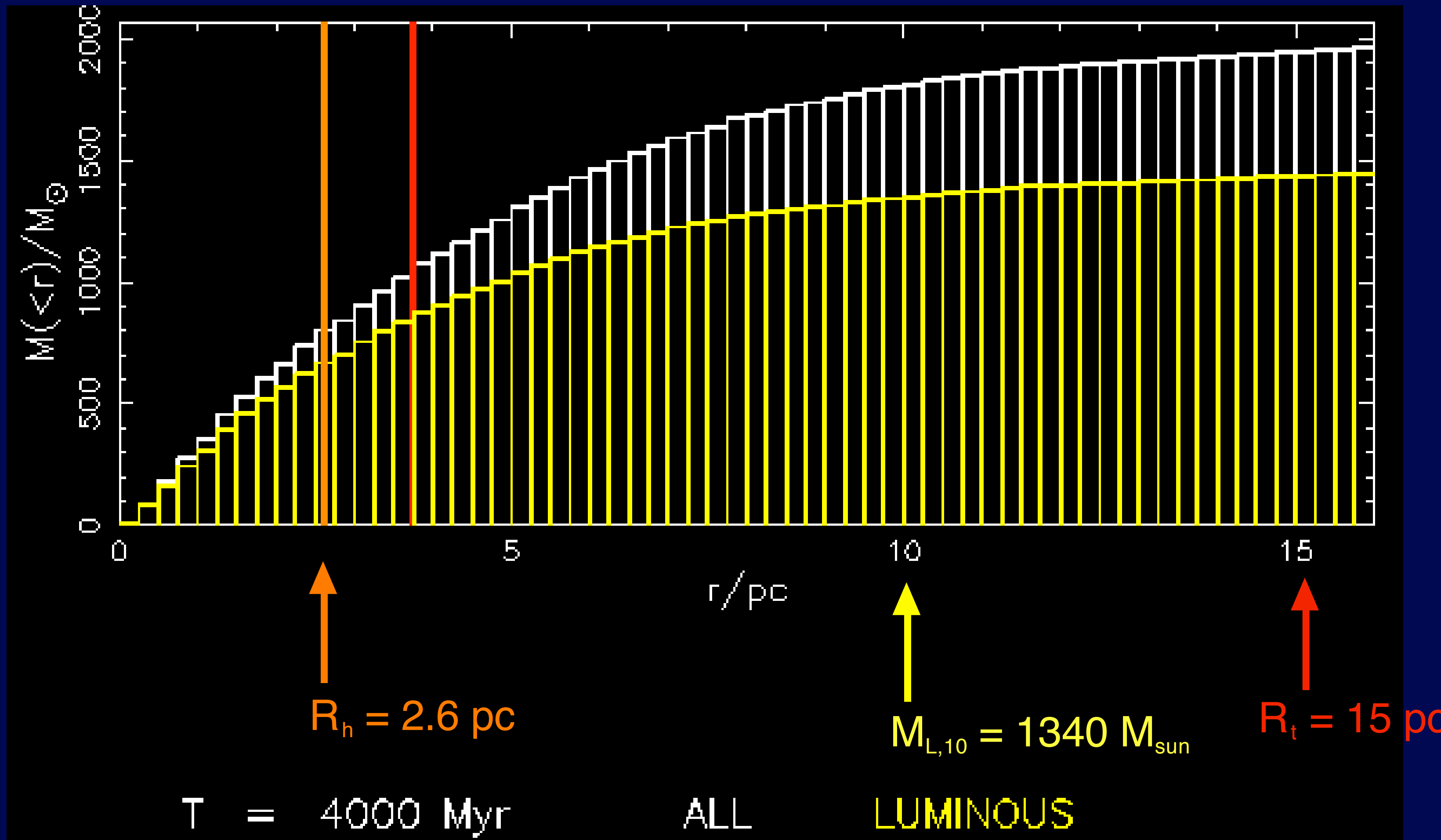
- Age = 4 Gyr (Vandenberg & Stetson 2004) ✔
- Metallicity ~ Solar (OCD: Mermilliod 1996) ✔
- Binary fraction ~ 50% (Fan et al. 1996) ✔
- Mass ~ 1300 M_{sun} in luminous stars within 10 pc (Fan et al. 1996) ✔
- Tidal radius ~ 15 pc (Bonatto & Bica 2005) ✔
- Half-mass radius ~ 2.5 pc (Fan et al. 1996) ✔



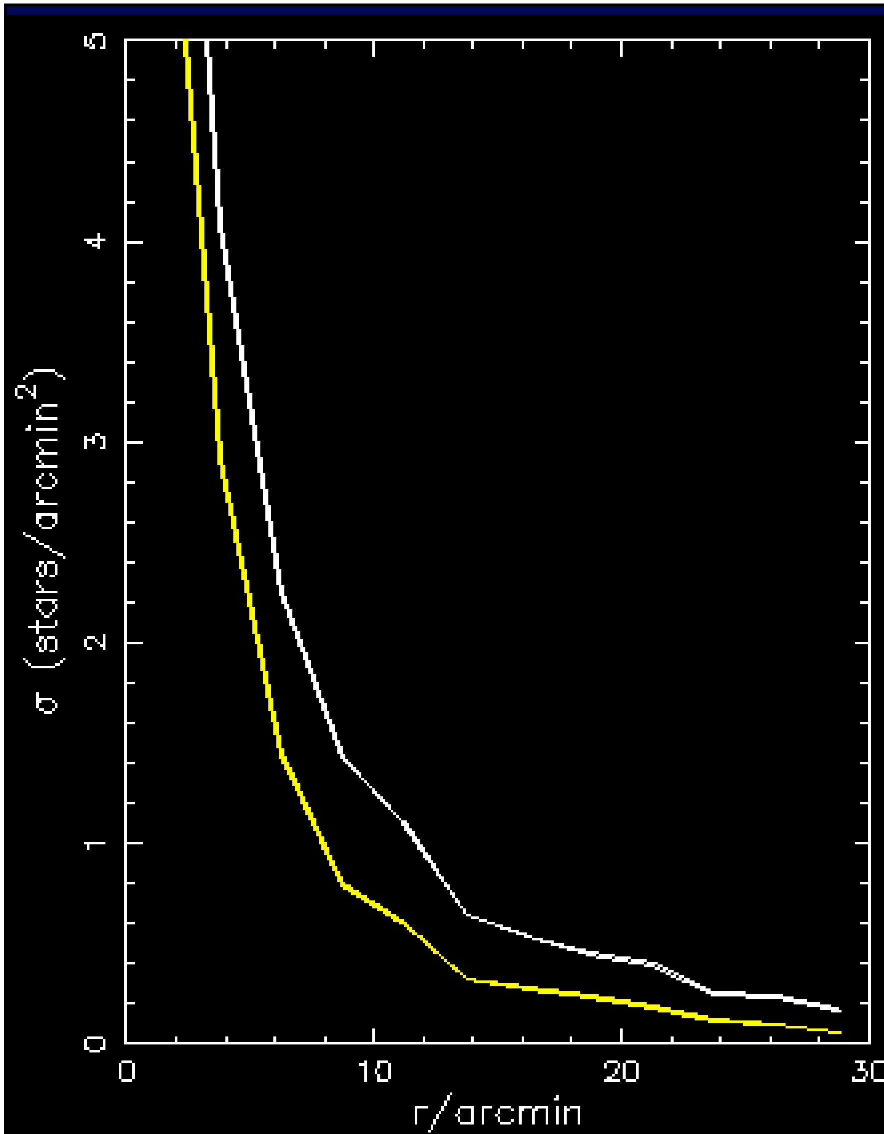
20
0
−20
Z/pc
−20
0
20
X/pc



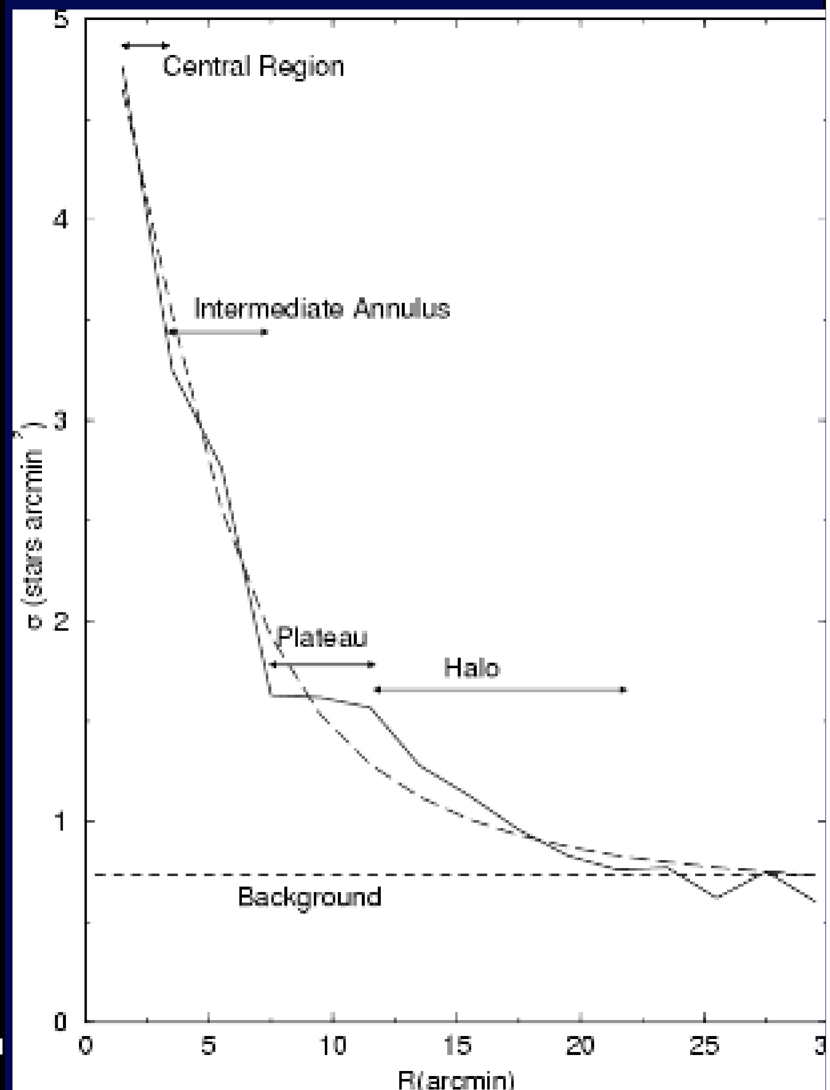
2000
1500
1000
500
0
M(<r)/M⊙
0
5
10
15
r/pc
Rh = 3.8 pc
Rt = 15 pc
2040 Msun
T = 4000 Myr
ALL



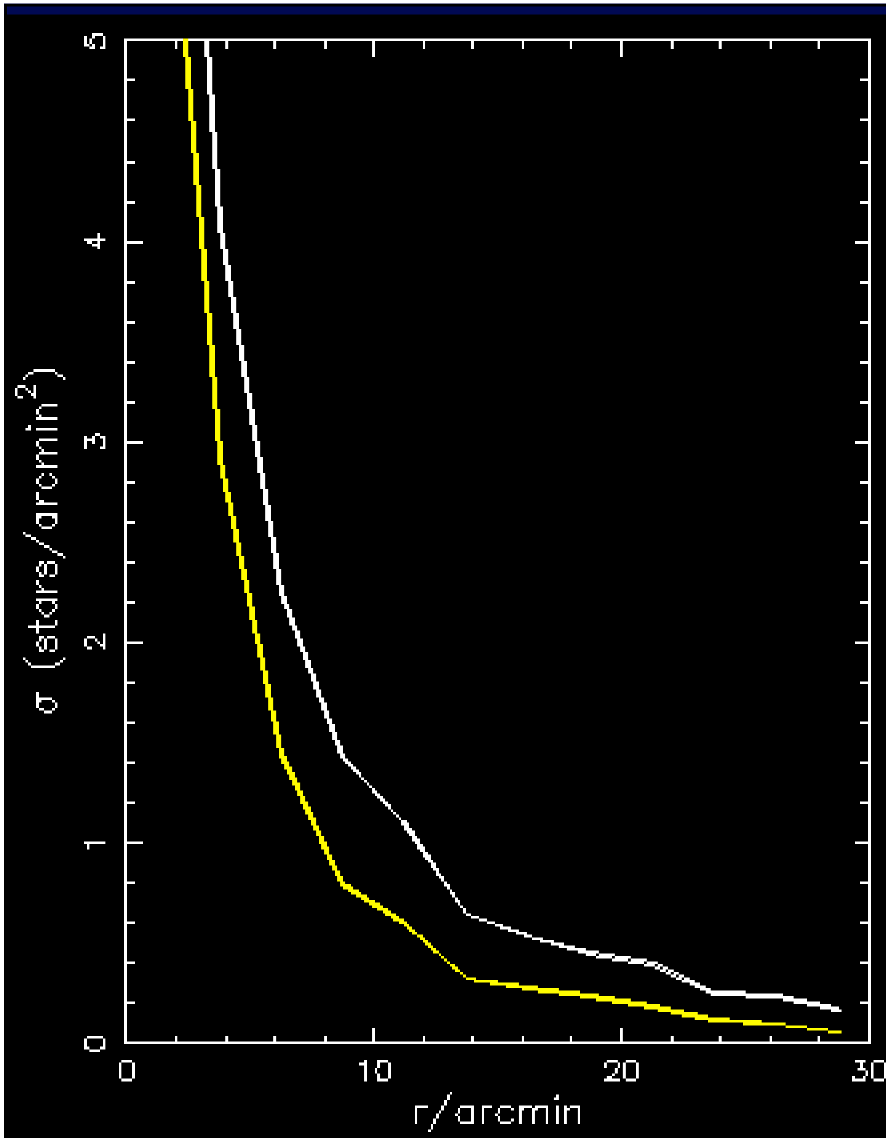
M(<r)/M$_\odot$
0
500
1000
1500
2000
0
5
10
15
r/pc
R_h = 2.6 pc
$M_{L,10}$ = 1340 M_{sun}
R_t = 15 pc
T = 4000 Myr
ALL
LUMINOUS



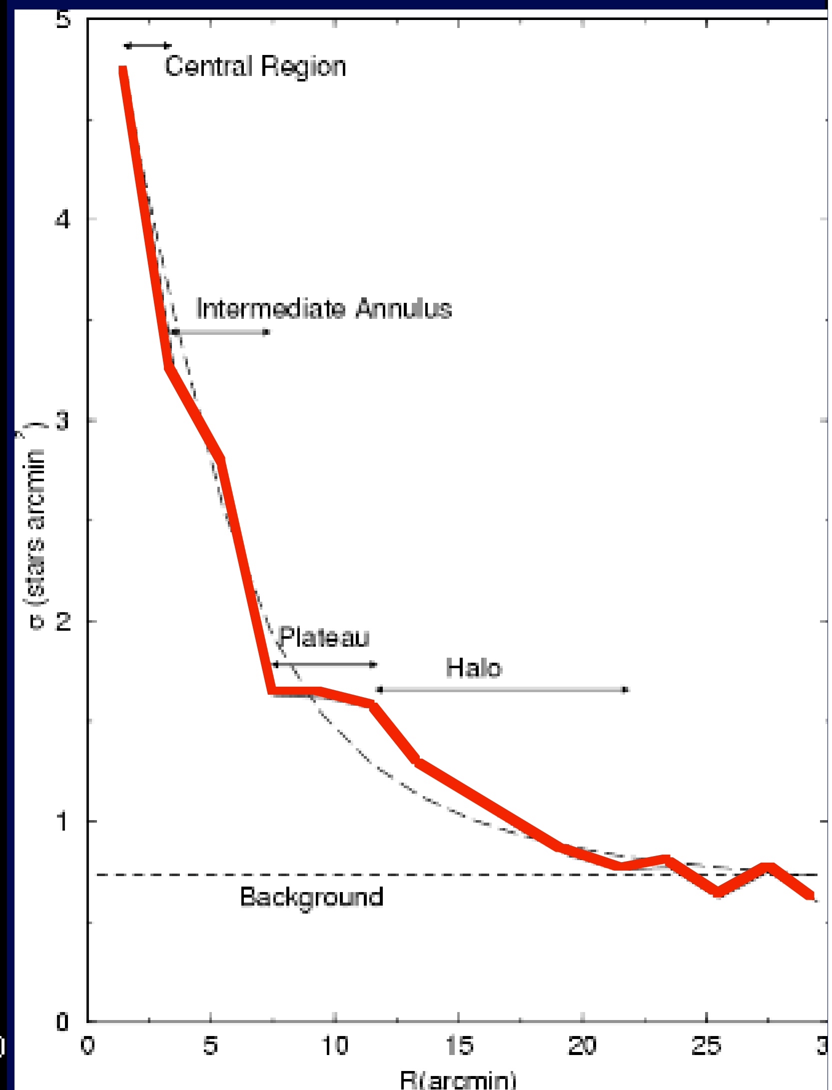
M67 model



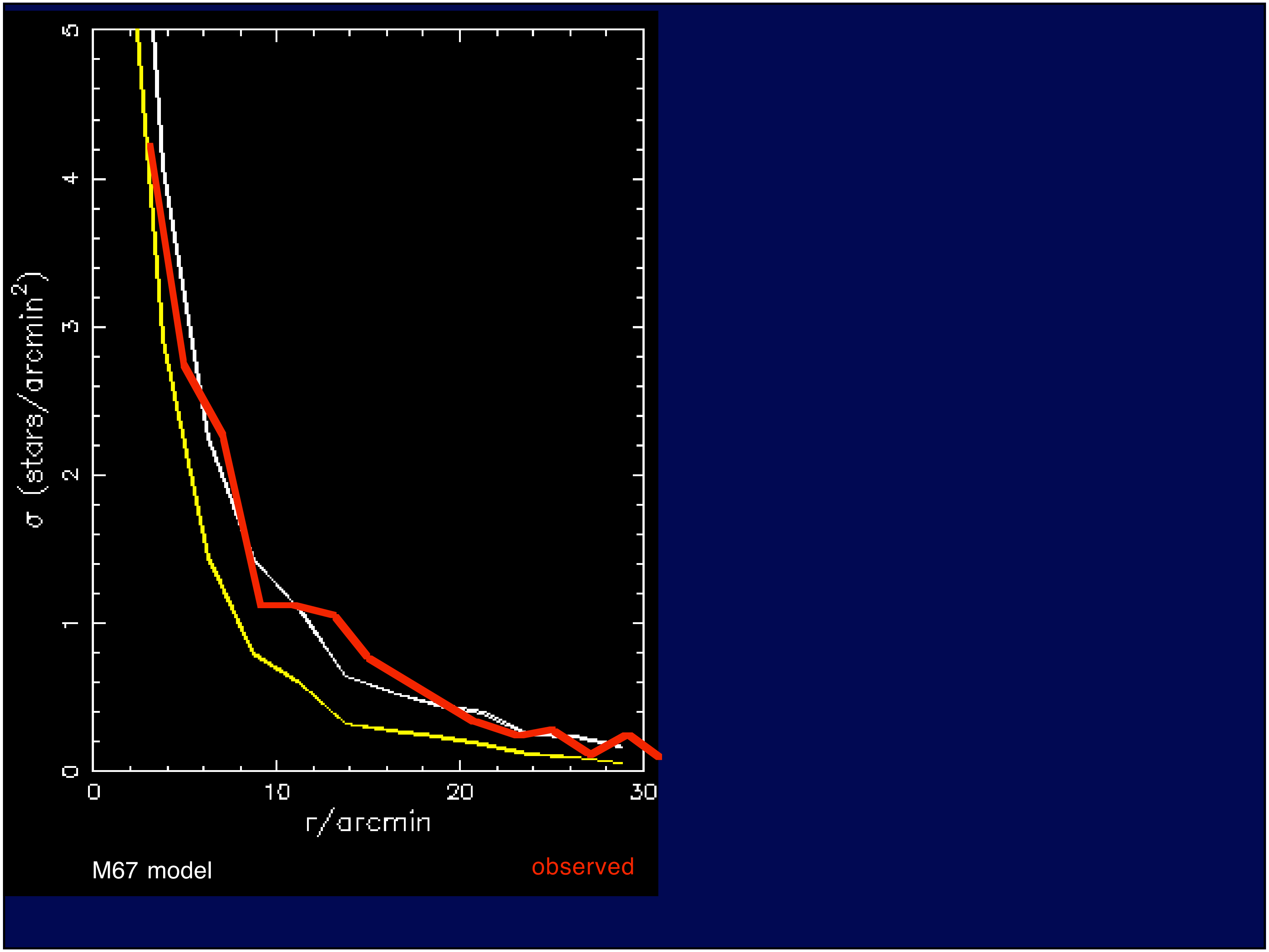
Bonatto & Bica (2003) - M67 observed

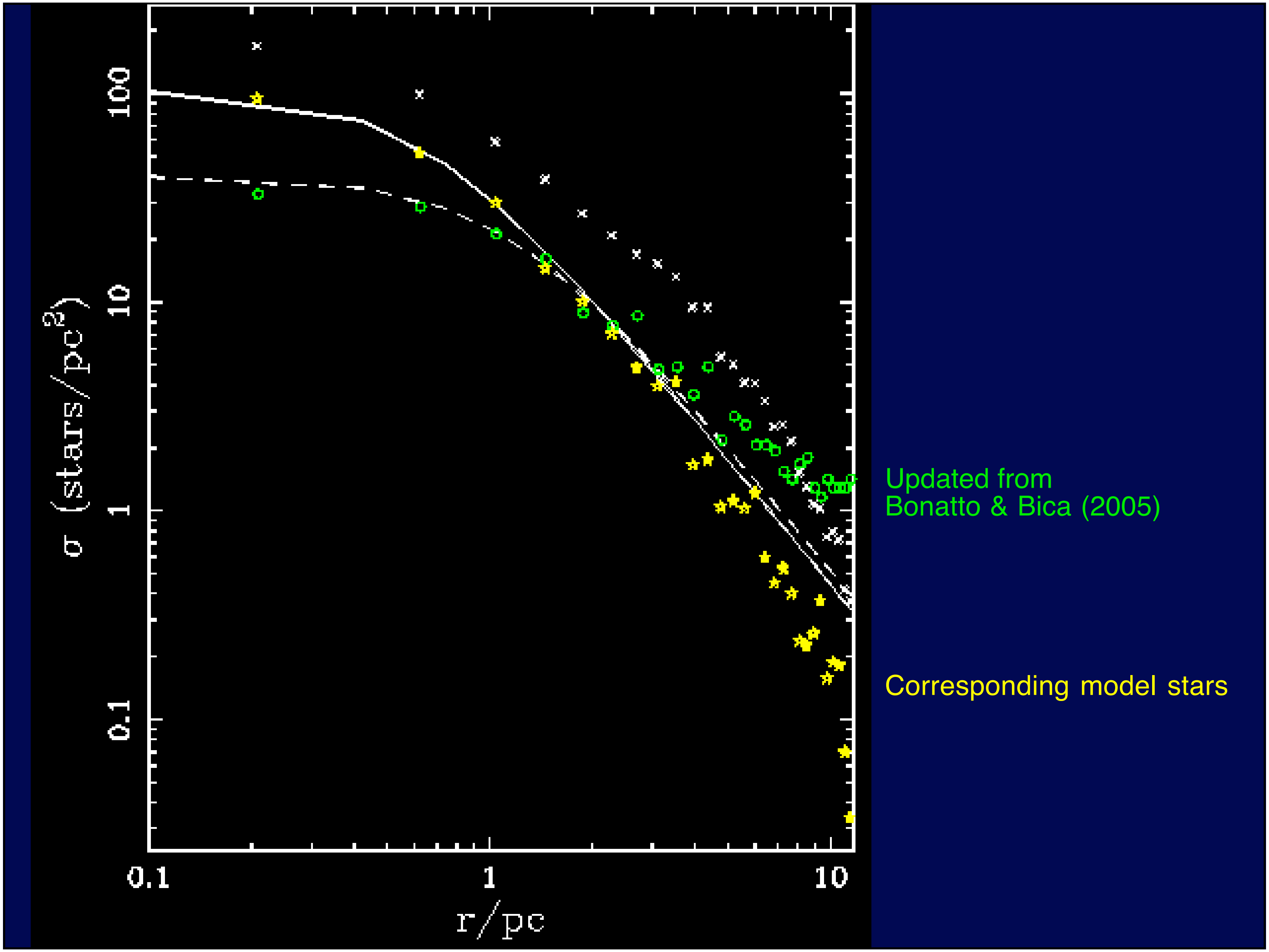


M67 model

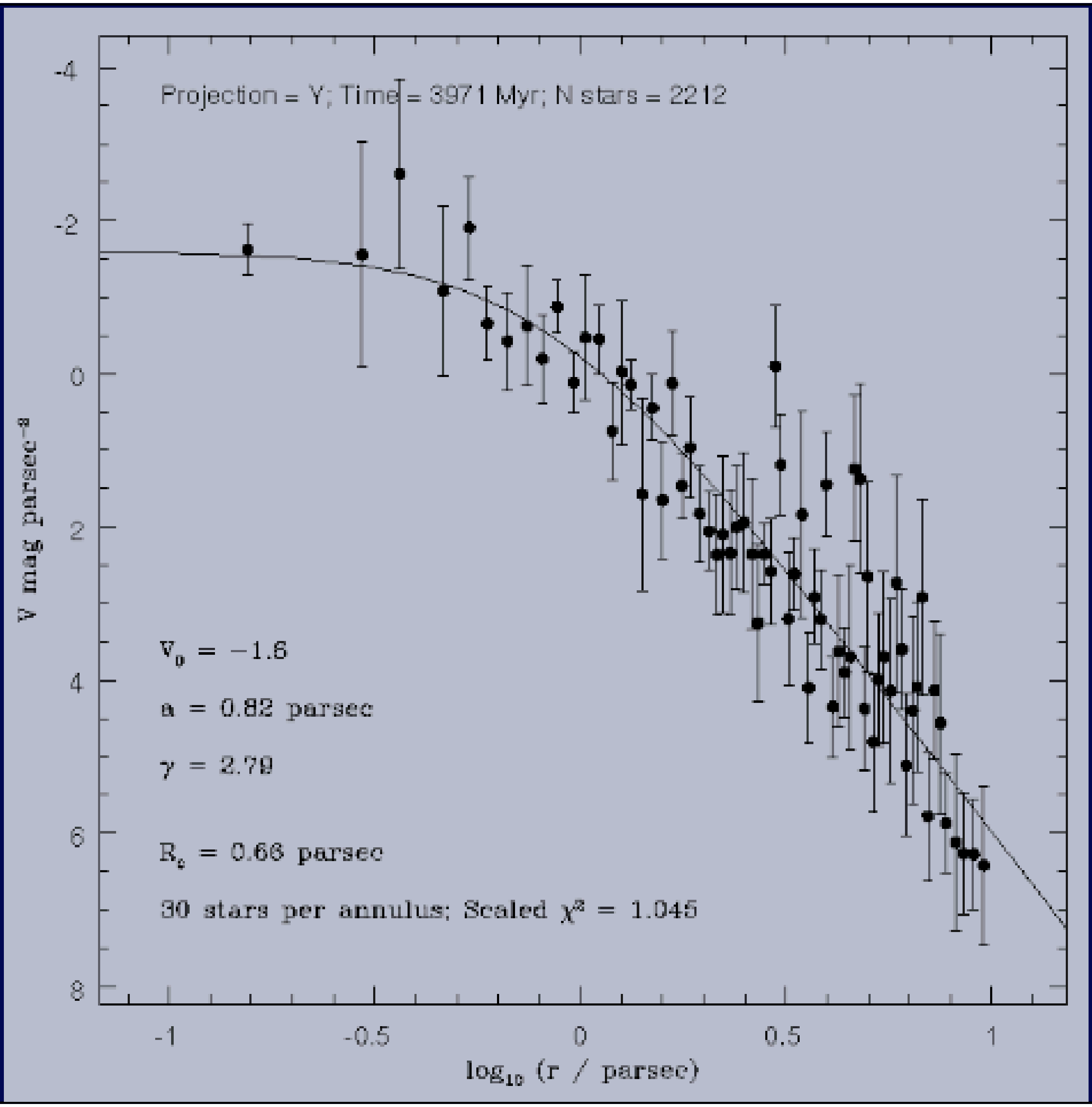


Bonatto & Bica (2003) - M67 observed

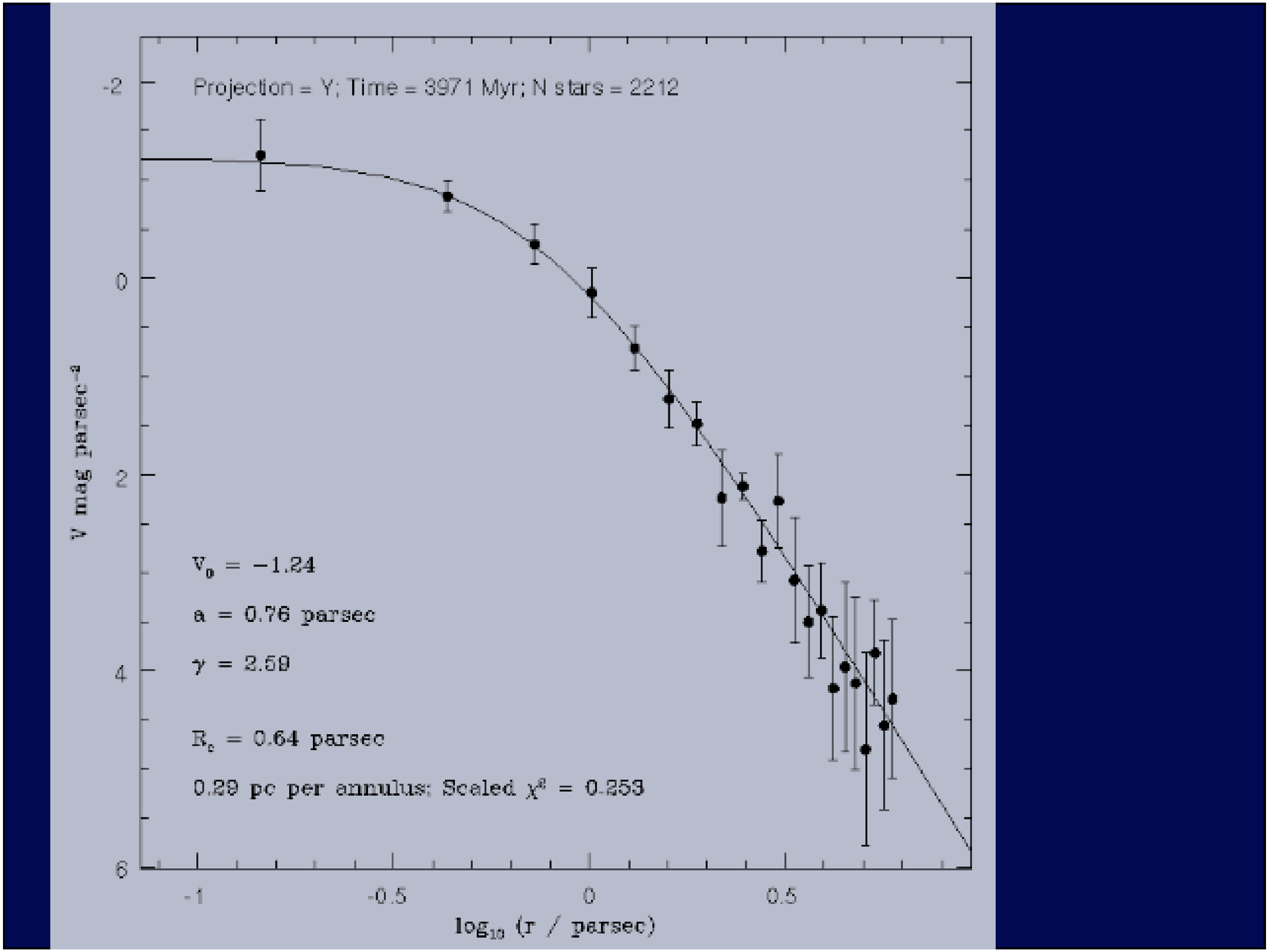

σ (stars/arcmin²)
r/arcmin
0
10
20
30
1
2
3
4
5
M67 model
observed



σ (stars/pc²)
100
10
1
0.1
0.1
1
10
r/pc
Updated from
Bonatto & Bica (2005)
Corresponding model stars

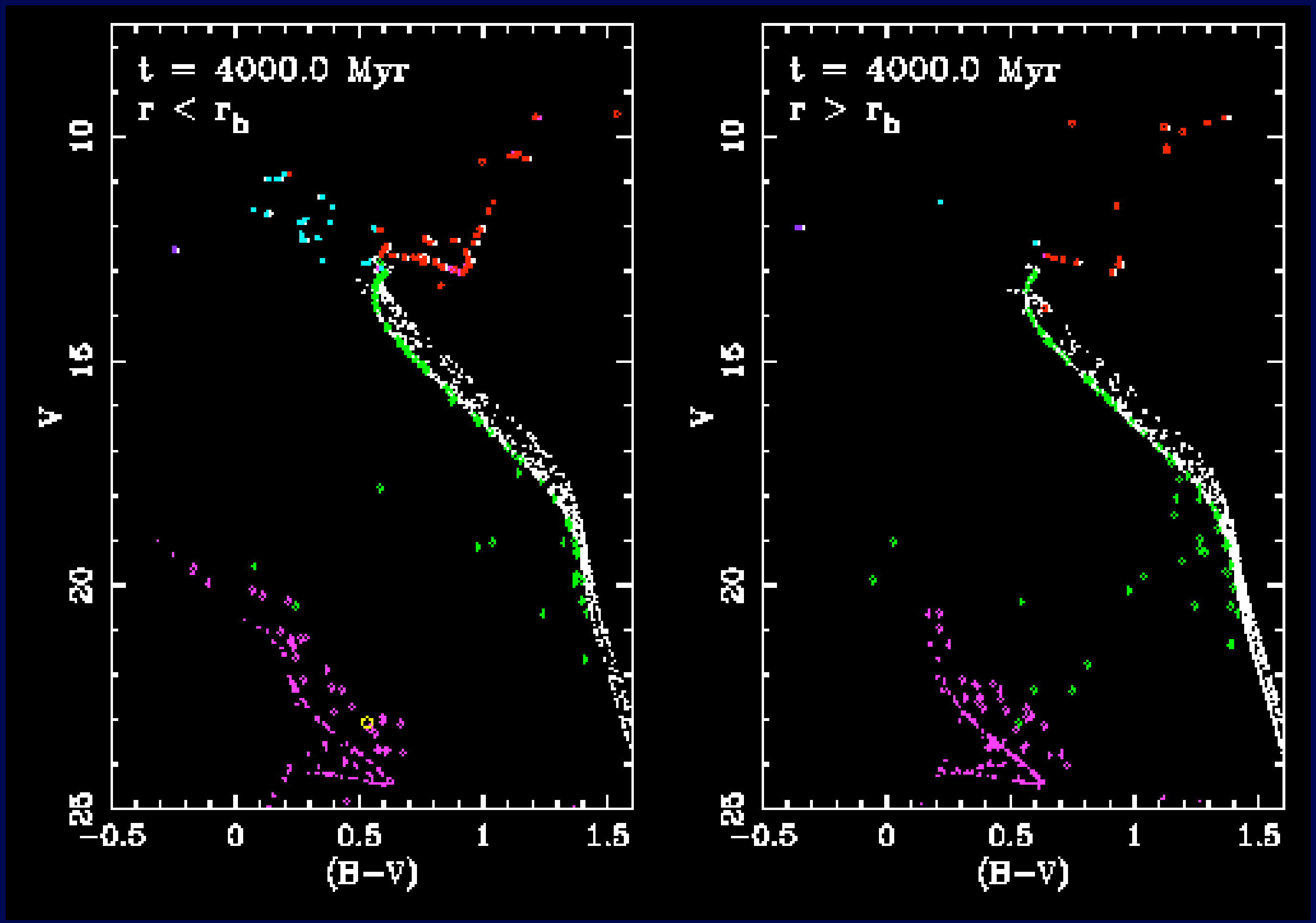


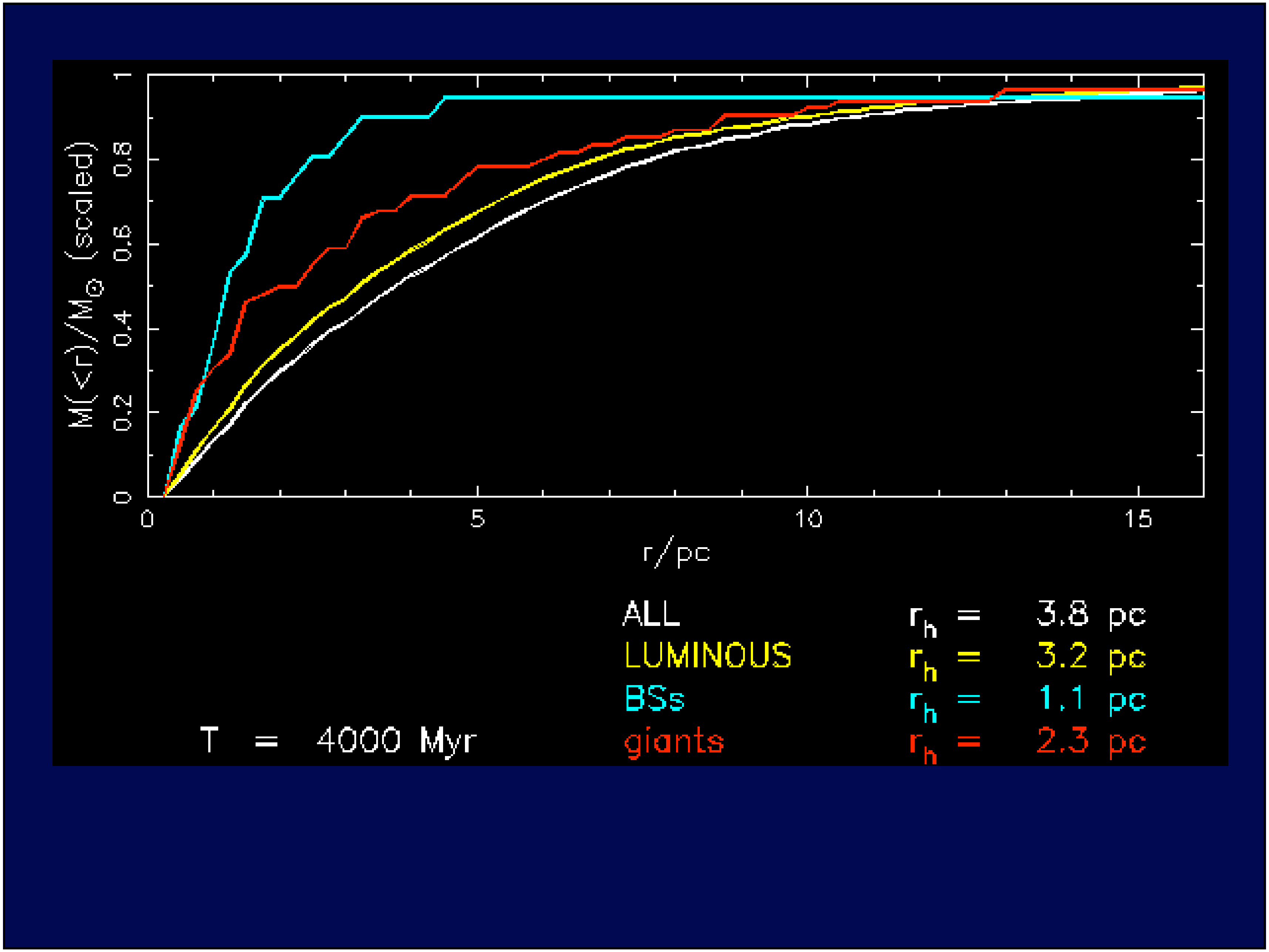
Software:
Mackey &
Gilmore 2003



Projection = Y; Time = 3971 Myr; N stars = 2212
V mag parsec⁻²
log₁₀ (r / parsec)
V₀ = −1.24
a = 0.76 parsec
γ = 2.59
R_c = 0.64 parsec
0.29 pc per annulus; Scaled χ² = 0.253
-2
0
2
4
6
-1
-0.5
0
0.5

Stellar Populations





M(<r)/M⊙ (scaled)
r/pc
ALL r_h = 3.8 pc
LUMINOUS r_h = 3.2 pc
BSs r_h = 1.1 pc
giants r_h = 2.3 pc
T = 4000 Myr

Observed CMD

29 blue stragglers

+ 1/2 in binaries

+ $N_{bs}/N_{ms,2to} \sim 0.15$ (high)

+ $R_{h,BS} = 1.6$ pc (cf. 2.5 pc)

Simulation:

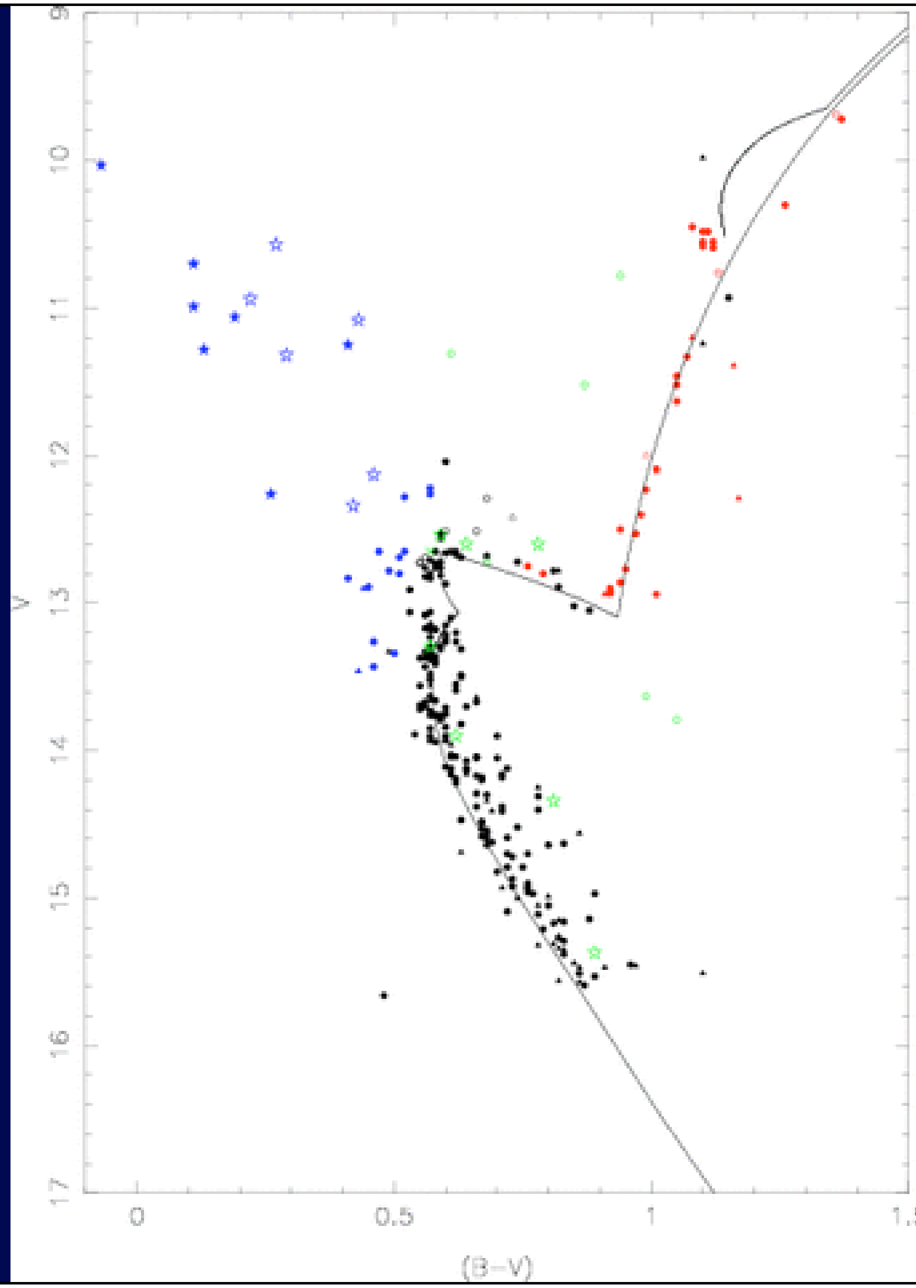
21 blue stragglers

+ 1/2 in binaries

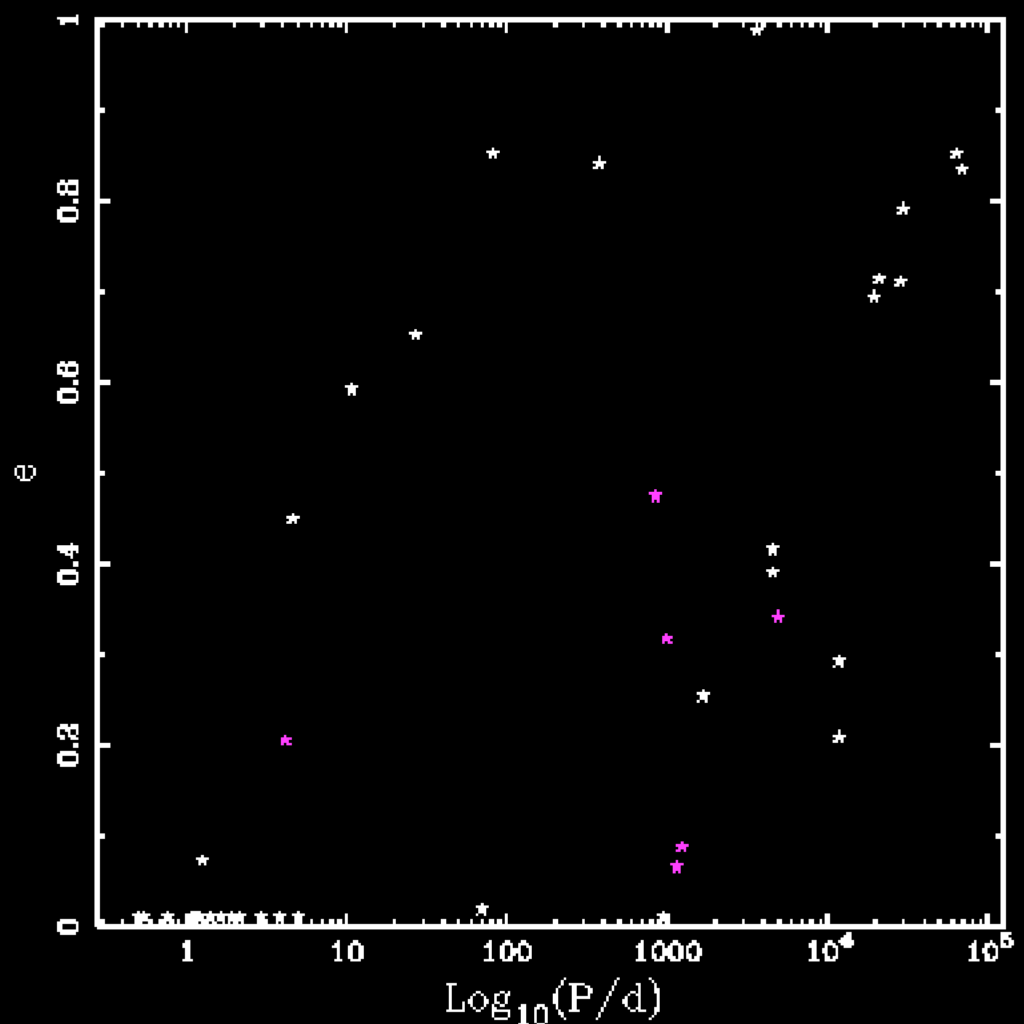
+ $N_{bs}/N_{ms,2to} = 0.18$

+ $R_{h,BS} = 1.1$ pc

→ 50%+ via dynamics



BS binary orbital parameters



investigate other populations

- 6 RS CVn binaries (cf. Belloni, Verbunt & Mathieu 1998)
- white dwarfs

predictions for future observations

- e.g. BY Draconis X-ray binaries

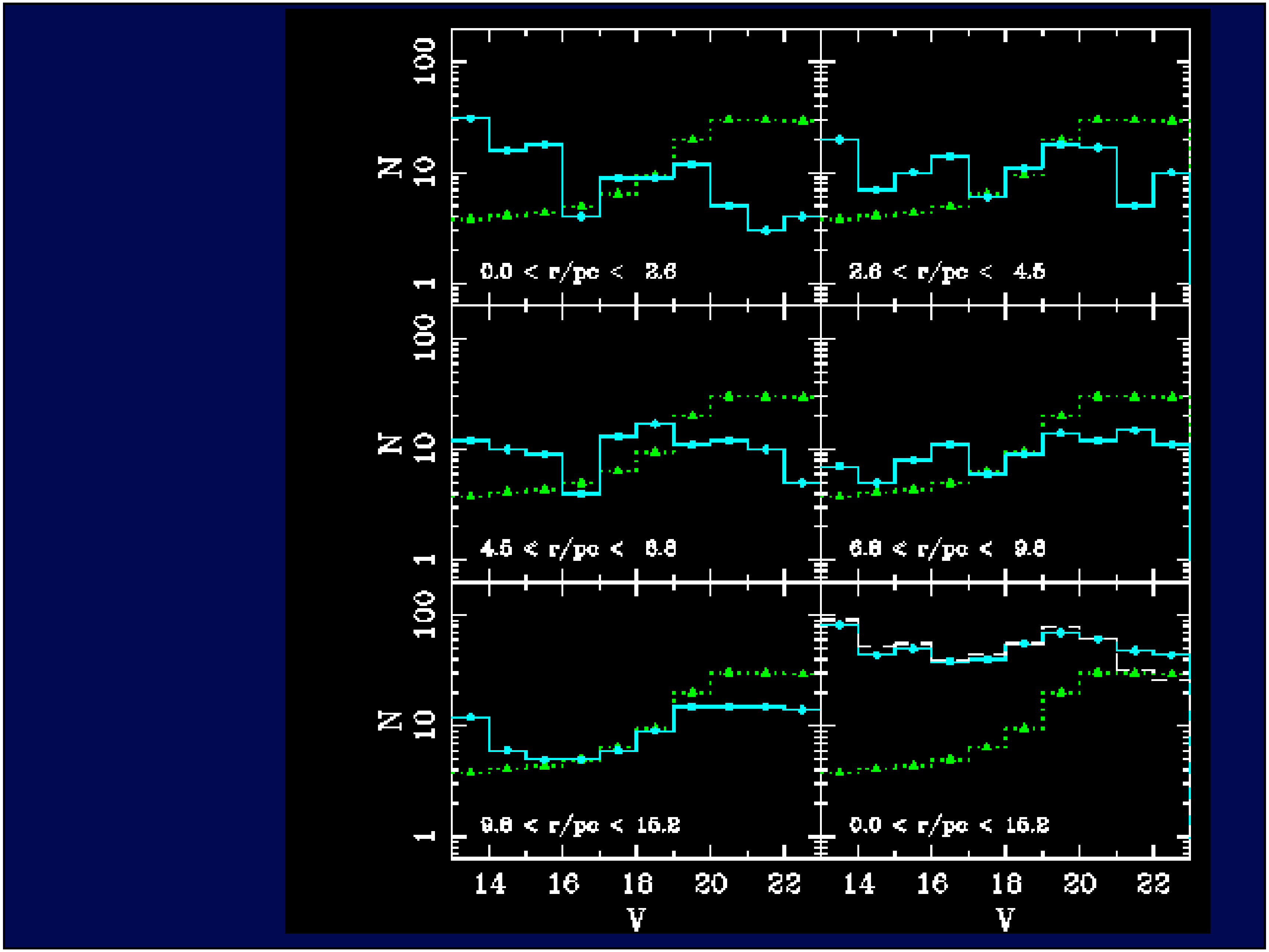
constrain initial conditions

- alternative binary period distributions ruled out by blue straggler analysis

understand cluster evolution

- luminosity functions -> mass segregation
  -> initial mass function?
- core collapse
- binary “burning”
- nucleosynthesis

... globular clusters with GRAPE-8


0.0 < r/pc < 2.6
2.6 < r/pc < 4.5
4.5 < r/pc < 6.8
6.8 < r/pc < 9.8
9.8 < r/pc < 15.2
0.0 < r/pc < 15.2
N
V
1
10
100
14
16
18
20
22

Some papers ...

A Complete N-body Model of the Old Open Cluster M67

Hurley, Pols, Aarseth & Tout, 2005, submitted

White Dwarf Sequences in Dense Star Clusters

Hurley & Shara, 2003, ApJ, 589, 179

Star Clusters as Type Ia Supernova Factories

Shara & Hurley, 2002, ApJ, 571, 830

Collaborators

Sverre Aarseth
Christopher Tout
Onno Pols
Rosemary Mardling
Mike Shara

Hermite Integration:

Predict all j particles to time t:

$$
\begin{aligned}
\Delta t &= t - t_j \\
\mathbf{x}_{p,j} &= \frac{\Delta t^4}{24}\mathbf{a}_{0,j}^{(2)} + \frac{\Delta t^3}{6}\mathbf{a}_{0,j}^{(1)} + \frac{\Delta t^2}{2}\mathbf{a}_{0,j} + \Delta t\,\mathbf{v}_{0,j} + \mathbf{x}_{0,j} \\
\mathbf{v}_{p,j} &= \frac{\Delta t^3}{6}\mathbf{a}_{0,j}^{(2)} + \frac{\Delta t^2}{2}\mathbf{a}_{0,j}^{(1)} + \Delta t\,\mathbf{a}_{0,j} + \mathbf{v}_{0,j}\,.
\end{aligned}
$$

The force and time-derivative are then calculated

$$
\begin{aligned}
\mathbf{a}_i &= \sum_j G m_j \frac{\mathbf{r}_{i,j}}{\left(r_{i,j}^2 - \epsilon^2\right)^{3/2}} \\
\mathbf{a}_i^{(1)} &= \sum_j G m_j \left[\frac{\mathbf{v}_{i,j}}{\left(r_{i,j}^2 - \epsilon^2\right)^{3/2}} - \frac{3\left(\mathbf{v}_{i,j}.\mathbf{r}_{i,j}\right)\mathbf{r}_{i,j}}{\left(r_{i,j}^2 - \epsilon^2\right)^{5/2}}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{r}_{i,j} &= \mathbf{x}_{p,j} - \mathbf{x}_i\,, \\
\mathbf{v}_{i,j} &= \mathbf{v}_{p,j} - \mathbf{v}_i\,.
\end{aligned}
$$

The corrector terms

$$
\begin{aligned}
\Delta \mathbf{x}_i &= \frac{\Delta t_i^4}{24}\mathbf{a}_{0,i}^{(2)} + \frac{\Delta t_i^5}{120}\mathbf{a}_{0,i}^{(3)} \\
\Delta \mathbf{v}_i &= \frac{\Delta t_i^3}{6}\mathbf{a}_{0,i}^{(2)} + \frac{\Delta t_i^4}{24}\mathbf{a}_{0,i}^{(3)}
\end{aligned}
$$

are then applied.

The formulation requires the force and derivative to be written as Taylor series to third-order about a reference time t:

$$
\begin{aligned}
\mathbf{a} &= \mathbf{a}_0 + \mathbf{a}_0^{(1)} t + \frac{1}{2}\mathbf{a}_0^{(2)} t^2 + \frac{1}{6}\mathbf{a}_0^{(3)} t^3 \\
\mathbf{a}^{(1)} &= \mathbf{a}_0^{(1)} + \mathbf{a}_0^{(2)} t + \frac{1}{2}\mathbf{a}_0^{(3)} t^2\,.
\end{aligned}
$$

Substitution of these expressions into each other leads to expressions for the second and third derivative correctors:

$$
\begin{aligned}
\mathbf{a}_0^{(2)} &= \left(-3\left(\mathbf{a}_0 - \mathbf{a}\right) - \left(2\mathbf{a}_0^{(1)} + \mathbf{a}^{(1)}\right) t\right)\frac{2}{t^2} \\
\mathbf{a}_0^{(3)} &= \left(2\left(\mathbf{a}_0 - \mathbf{a}\right) + \left(\mathbf{a}_0^{(1)} + \mathbf{a}^{(1)}\right) t\right)\frac{6}{t^3}\,.
\end{aligned}
$$