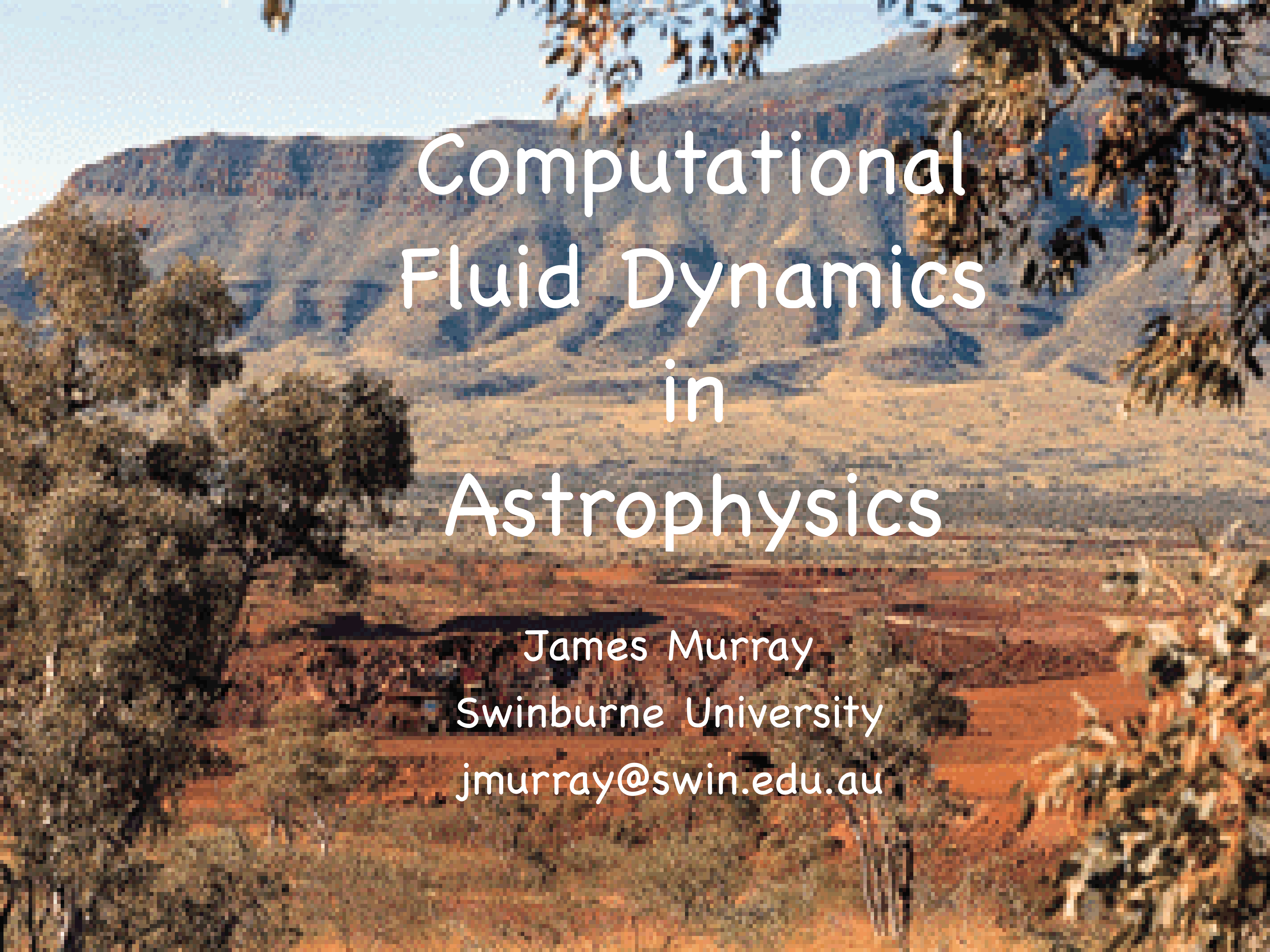
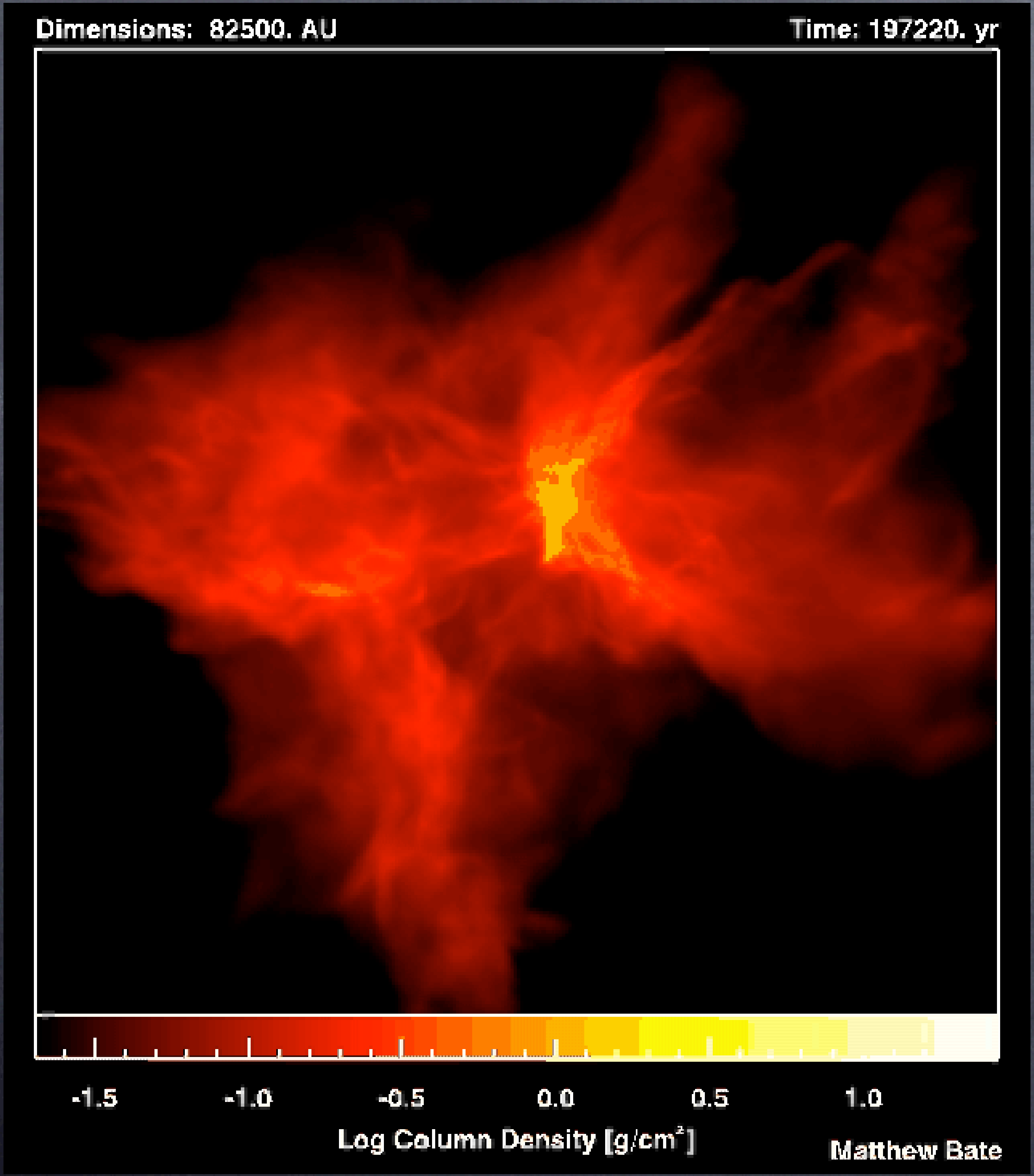


Computational Fluid Dynamics in Astrophysics

James Murray
Swinburne University
jmurray@swin.edu.au

- Computational Fluid Dynamics: how boring is that?
- What is CFD?
- What are the various approaches?
- If I were starting my PhD now

Matthew Bate, University of Exeter, UKAFF



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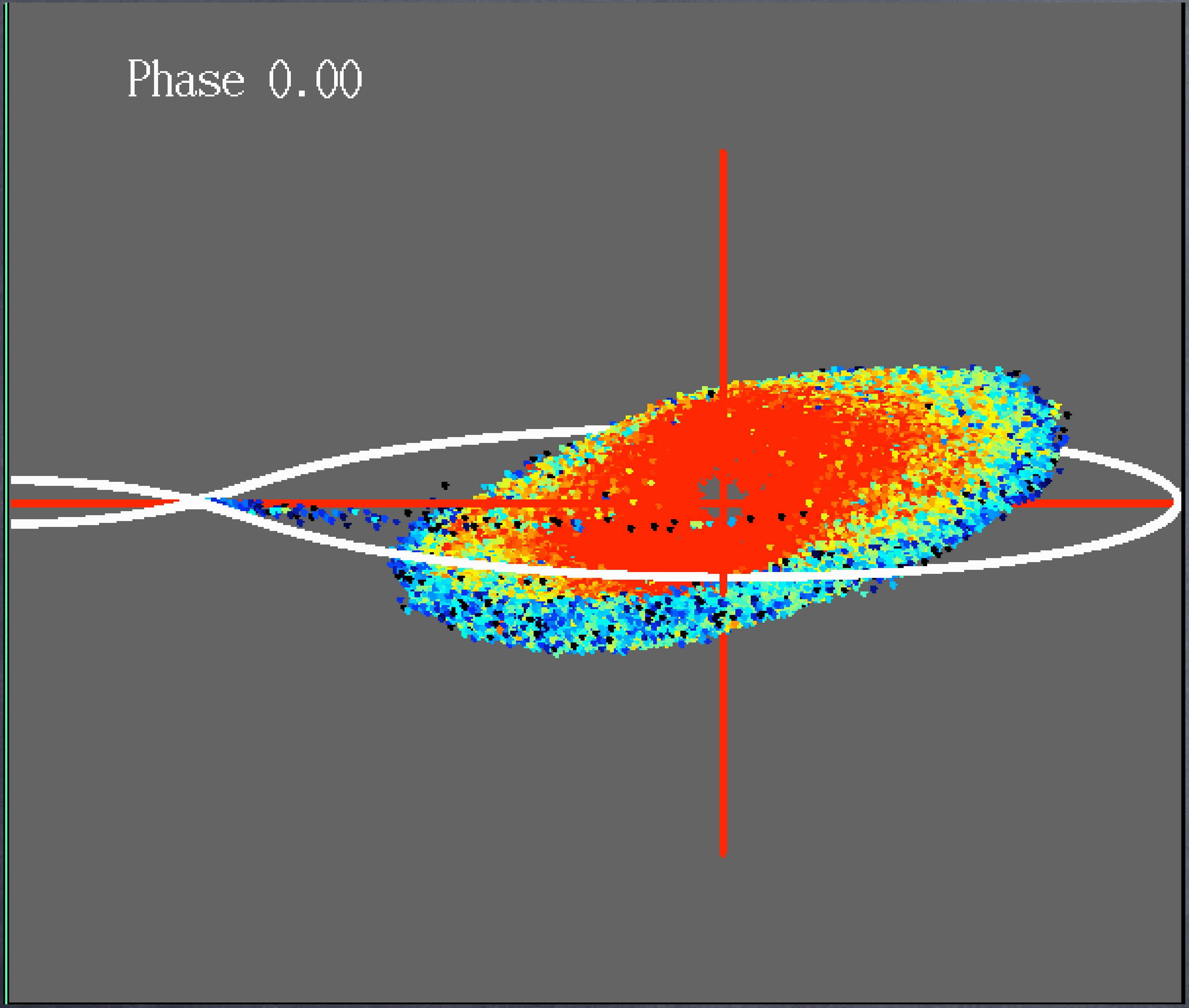
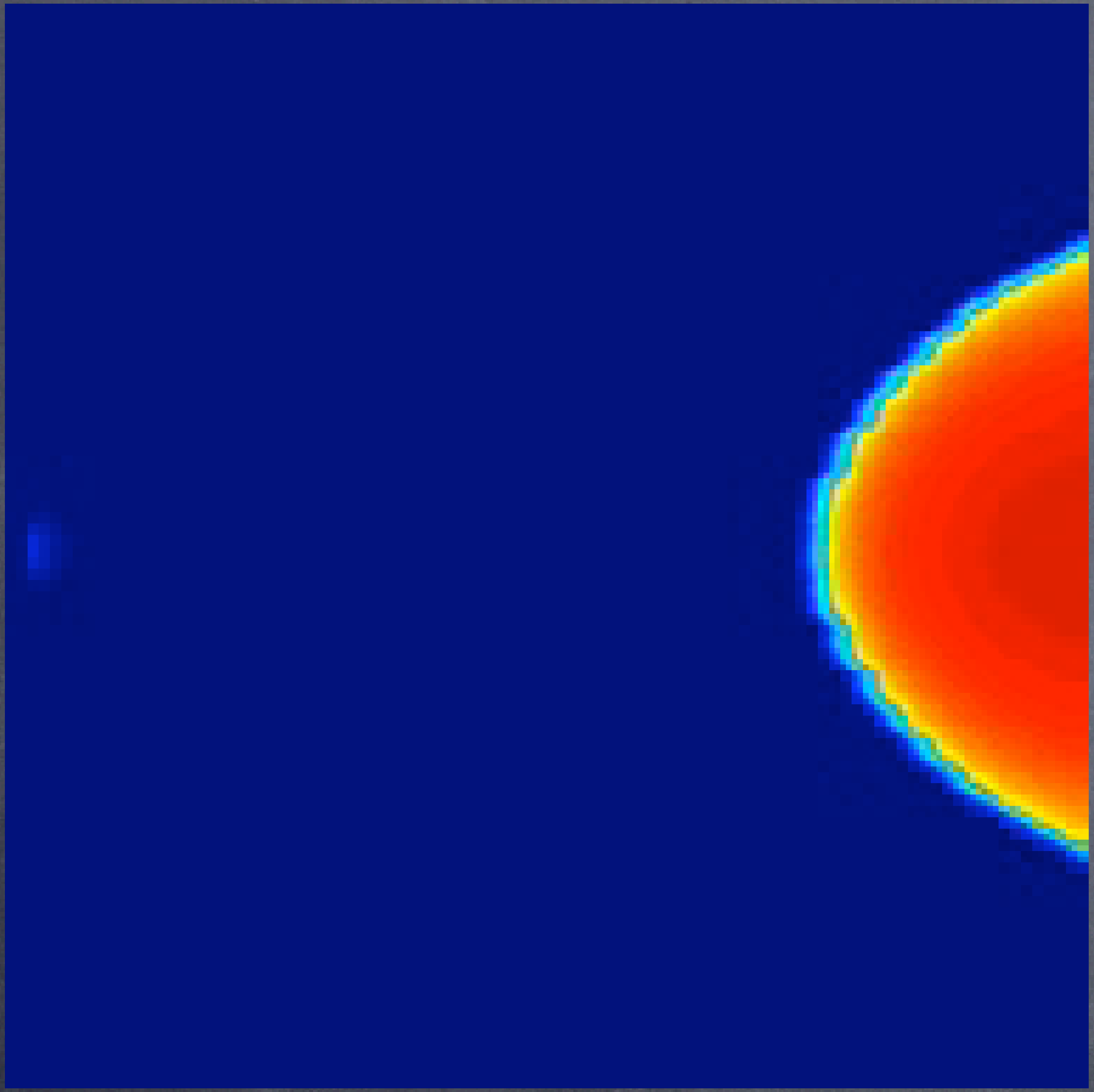


A radiatively warped accretion disk in a low mass x-ray binary

Steve Foulkes, Open University

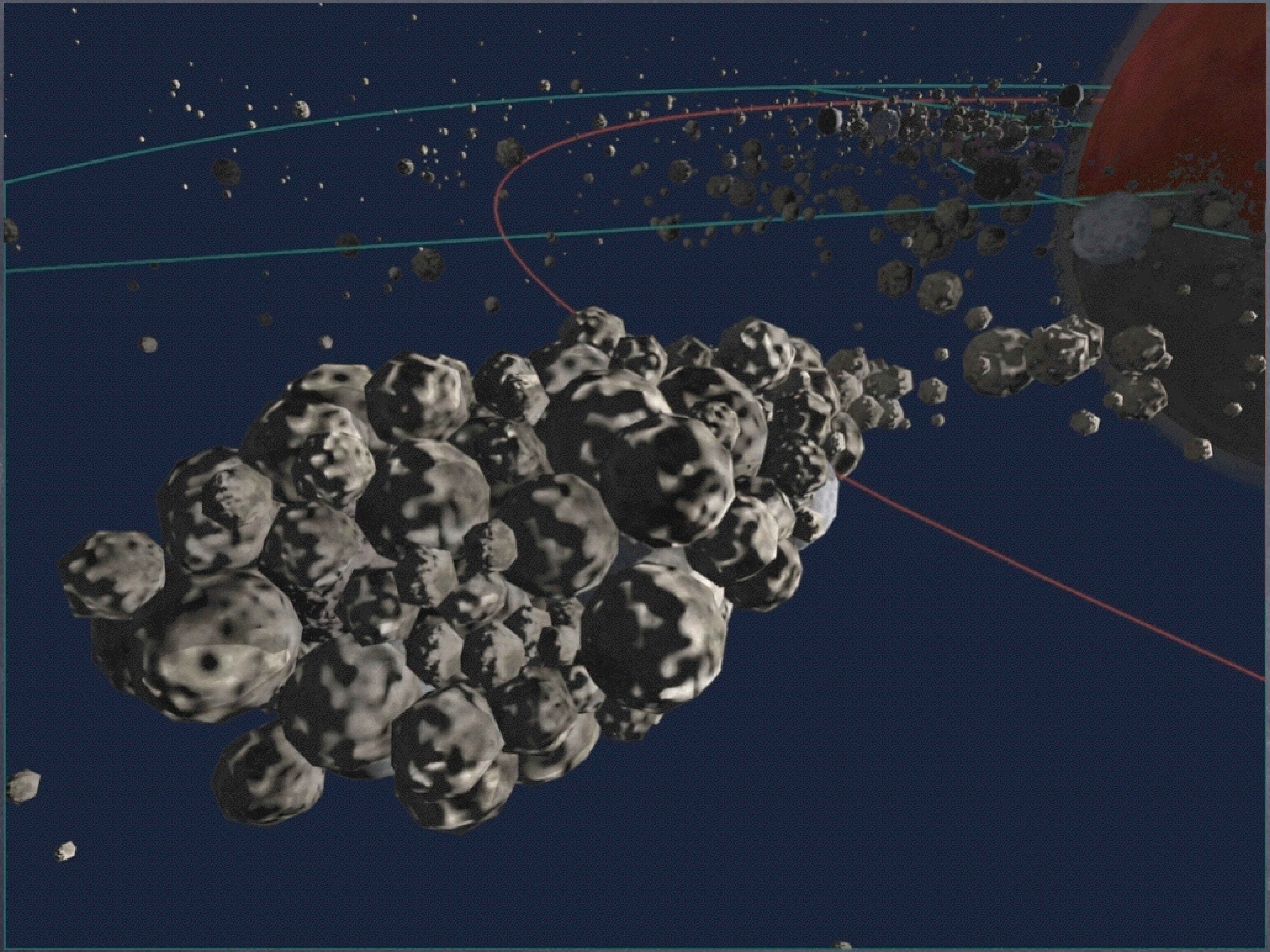


Side view of an adiabatic accretion disk surrounding a neutron star / black hole

Jim Stone et al., Princeton

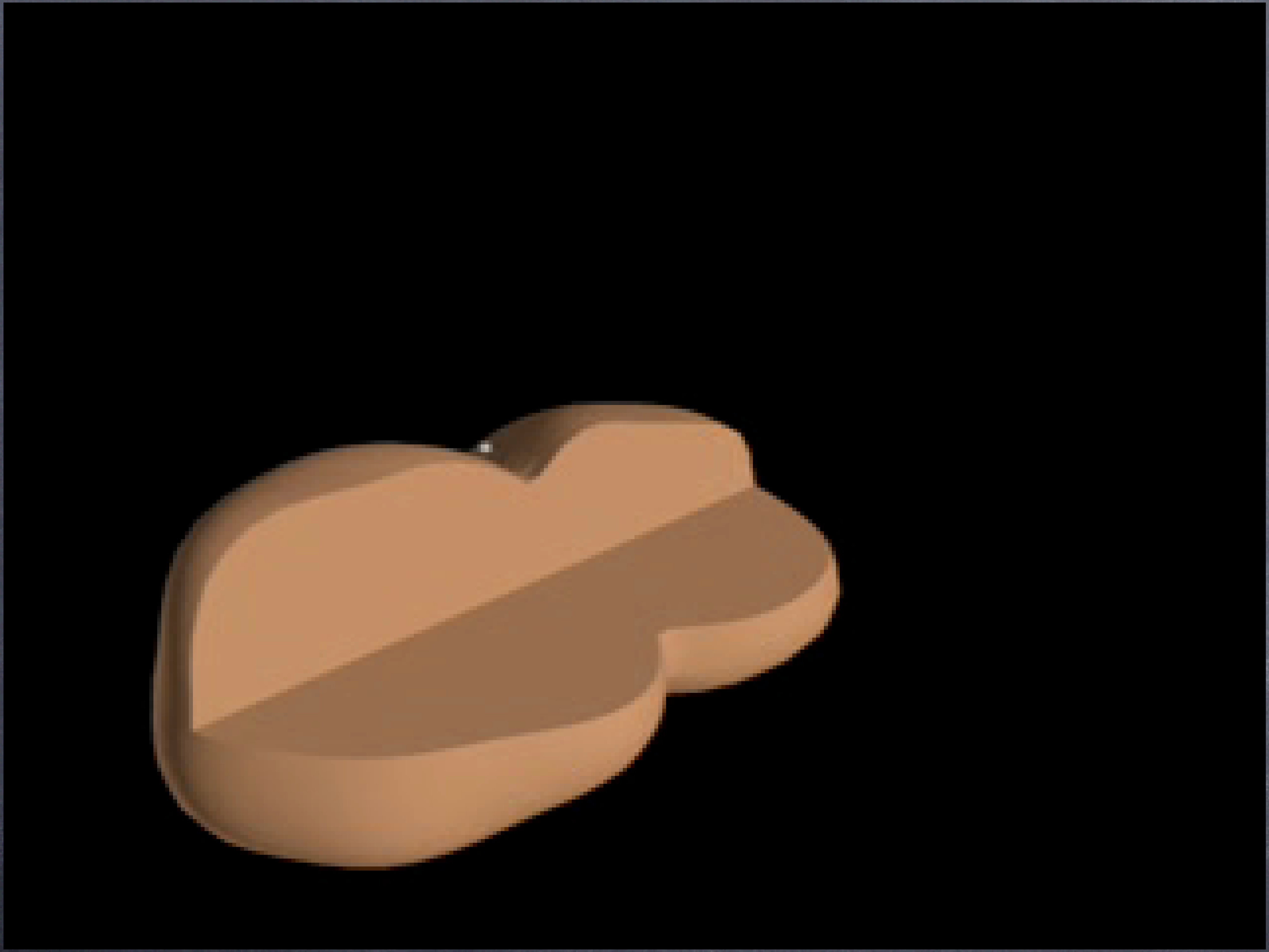
Turbulence in a molecular cloud

Jim Stone, Princeton



formation
of the
Moon
after a
giant
impact

Eiichiro Kokubo, National Astronomical Observatory of Japan, HARP

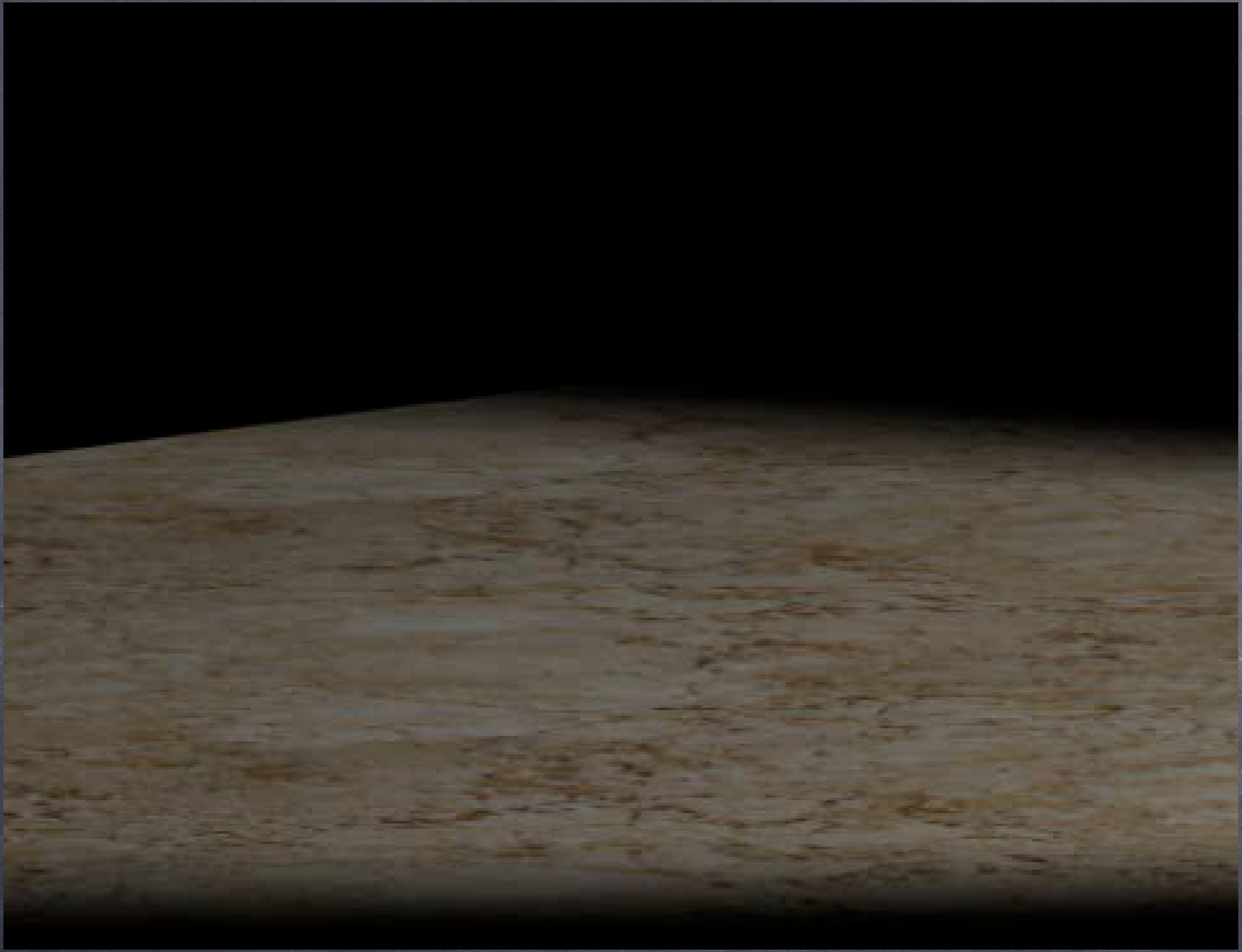


the effect
of a
nuclear
weapon
detonating
on an
asteroid

Erik Asphaug, UCSC

Dam breaking in the Rocky Mountains

Joe Monaghan, Monash & Paul Cleary, CSIRO



Water filling a bath

me

Drops
into a
"square"
vase

What is computational fluid dynamics?

The numerical solution of
the equations of motion and
state of a fluid

What is computational fluid dynamics?

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P$$

$$P = k\rho^{\gamma}$$

What sorts of astrophysical problems can we solve?

- Shocks
- Explosions
- Collisions
- Formation
- Destruction

Possible forces that need be considered

- Gas pressure
- Gravity
- Viscosity
- Turbulence
- Magnetic fields
- Radiation

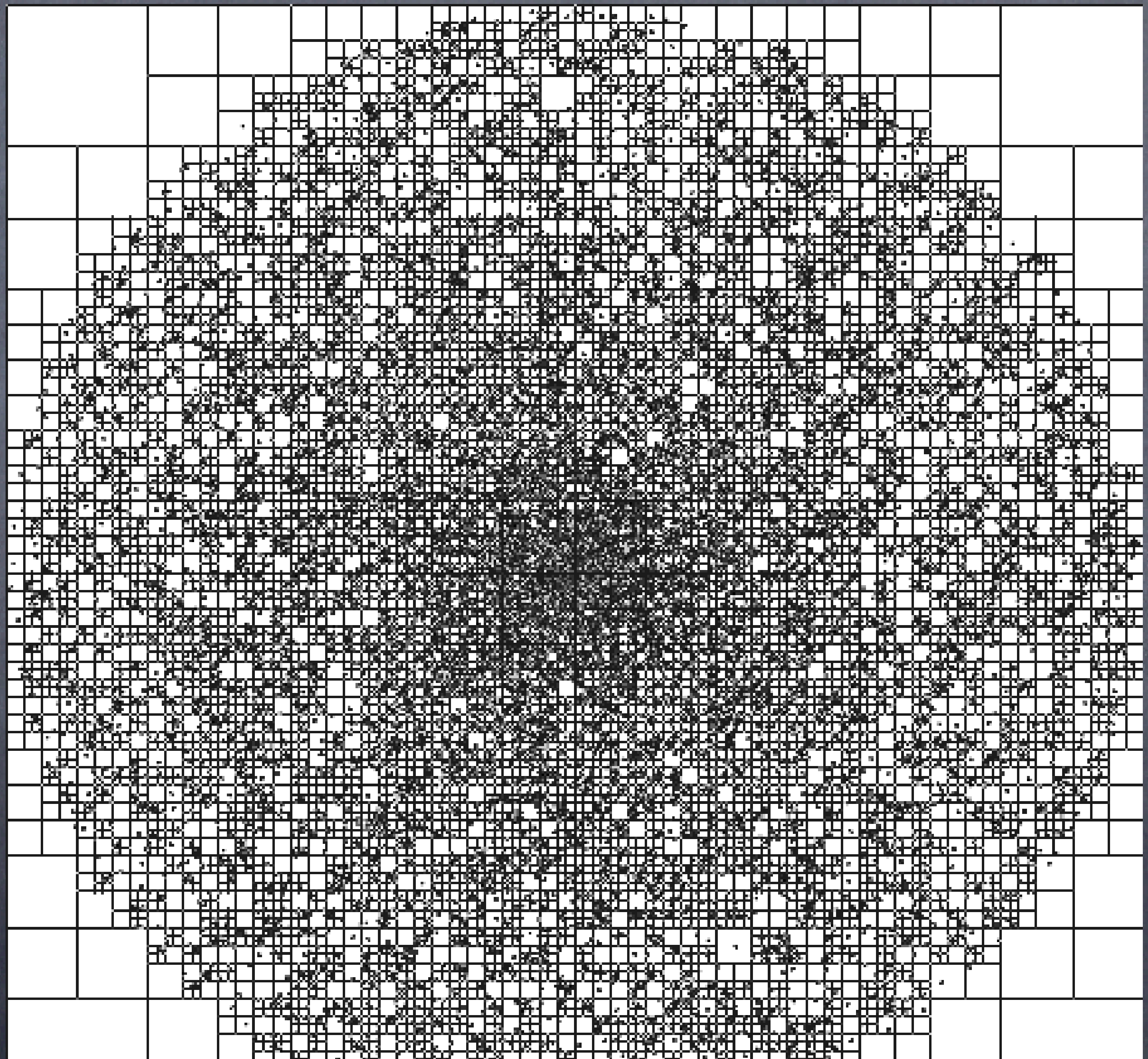
Equations of state

- Isothermal
- Adiabatic
- Polytropic
- Gas with cooling
- Solid!

Discrete equations

- The computer can only store quantities such as velocity, density, pressure and temperature at a finite number of places.
- How do we choose where those places are?
- How do we calculate spatial derivatives?

The two approaches to CFD



Smoothed Particle Hydrodynamics

- Fluid is represented by a collection of particles that move with the flow
- The key to SPH is a method for interpolating fluid quantities from a “disordered” set of points

Kernel interpolation

- For any fluid property A, we can write an interpolant
- W is the interpolating "kernel"

$$A_I(\mathbf{r}_I) = \int_V A(\mathbf{r})\, W(\mathbf{r} - \mathbf{r}_I, h)\, dV$$

Interpolating from particles

$$A_I(\mathbf{r}_I) = \int_V \frac{A(\mathbf{r})}{\rho} W(\mathbf{r}-\mathbf{r}_I,h)\,dm$$

The integral is approximated by a summation

$$A_I(\mathbf{r}_i) \approx \sum_{j=1}^{n} m_j \frac{A_j}{\rho_j} W(\mathbf{r}_i-\mathbf{r}_j,h)$$

Interpolating fluid properties

The interpolant for the density

$$\rho_I(\mathbf{r}_i) \approx \sum_{j=1}^{n} m_j W(\mathbf{r}_i - \mathbf{r}_j, h)$$

Spatial derivatives also become a simple summation

$$\nabla A_I(\mathbf{r}_i) \approx \sum_{j=1}^{n} m_j \frac{A_j}{\rho_j} \nabla W(\mathbf{r}_i - \mathbf{r}_j, h)$$

Kernel properties

For the magic to work we require

$$\lim_{h \to 0} W(\mathbf{r}, h) = \delta(\mathbf{r})$$

and

$$\int_V W(\mathbf{r}, h)\, dV = 1$$

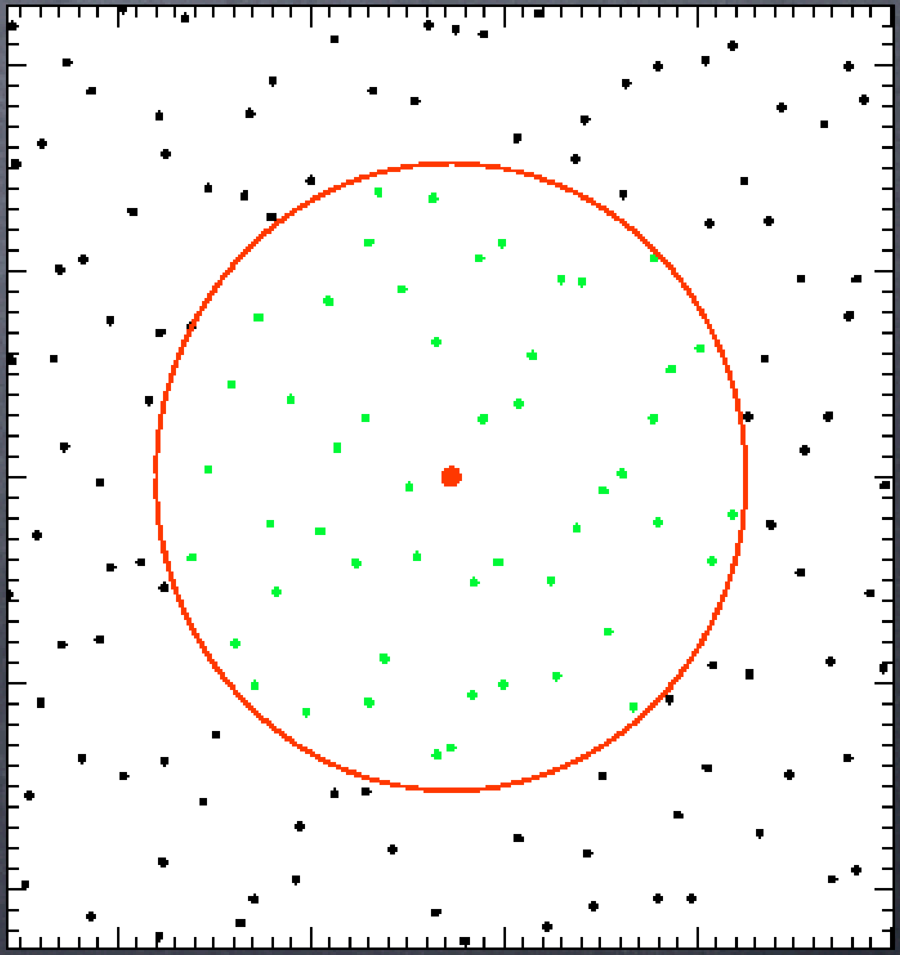
hence toy SPH equations usually use
a gaussian kernel

$$\frac{1}{h\sqrt{\pi}} e^{-\frac{r^2}{h^2}}$$

Computational Issues

$$\rho_I(\mathbf{r}_i) = \frac{1}{h\sqrt{\pi}} \sum_{j=1}^{n} m_j \, e^{-\frac{(\mathbf{r}_i - \mathbf{r}_j)^2}{h^2}}$$

- With a gaussian kernel every particle is neighbour to every other.



The equations of motion

$$\frac{d\mathbf{v}}{dt}_i = -\sum_{j=1}^{n} m_j \left(\frac{P_i}{\rho_i^{2}} + \frac{P_j}{\rho_j^{2}} + \mu_{ij} \right) \nabla W(\mathbf{r}_i - \mathbf{r}_j, h)$$

$$\frac{du}{dt}_i = \sum_{j=1}^{n} m_j \left(\frac{P_i}{\rho_i^{2}} + \frac{P_j}{\rho_j^{2}} + \mu_{ij} \right) \left(\mathbf{v}_i - \mathbf{v}_j \right) \cdot \nabla W(\mathbf{r}_i - \mathbf{r}_j, h)$$

Grids and Particles

- Grids split the domain up into a set of regular boxes or cells. Differential equations become finite difference equations. (Eulerian)
- A second approach is to sample the volume with a set of points that move. This is the particle or Lagrangian approach.

Don’t write your own code

- To write and test your own fluid dynamics code takes months or years…
- Writing a code is “dead time”
- Several public domain codes are available
- These codes have undergone years of testing on a range of problems.
  - Stone & Norman, 1992, ApJS, 80, 753
- One new piece of physics = one PhD thesis

ATHENA

- Finite difference magnetohydrodynamics
- Written by Jim Stone, John Hawley and collaborators (based upon ZEUS)
- The grids are regular and nonadaptive
- Perhaps the easiest code to adapt to your own nefarious porpoises
- http://www.astro.princeton.edu/~jstone/athena.html

ENZO

- Mike Norman, Greg Bryan
- Adaptive Mesh Refinement
- hydrodynamics + N-body
  - http://cosmos.ucsd.edu/enzo/
- Designed for supercomputers (difficult to install)

Smoothed Particle Hydrodynamics

- Monaghan 1992, Annual Reviews of Astronomy and Astrophysics
- Several hydro+self-gravity SPH codes available
- GADGET (Volker Springel)
  - http://www.mpa-garching.mpg.de/gadget/
- HYDRA (Hugh Couchman)
  - http://hydra.mcmaster.ca/hydra/

Which code do I use?

- What physics is important (hydrodynamics, magnetic fields, radiation)?
- What is the geometry of the problem?
- What are the boundary conditions?
- Am I looking for gross features or fine details?

And then?

- Having obtained a code you must learn how to use it
- ATHENA for example comes bundled with a few test problems.
- Expect to spend “some” time learning the code.
- Refer to “Numerical Recipes”

What to look out for

- Accuracy. Do the answers look anything like what is expected?
- Stability. Do the answers converge when you improve resolution?
- Resolution. Particularly in three dimensions extra resolution is costly.

Ancient Proverb

Do not base your PhD upon any simulation that takes more than 5 minutes to run

John
Dubinski,
University
of Toronto
(Blue Gene)

His philosophy was a mixture of three famous schools -- the Cynics, the Stoics and the Epicureans -- and summed up all three of them in his famous phrase, "You can't trust any bugger further than you can throw him, and there's nothing you can do about it, so let's have a drink."
Dydactylos the philosopher (Terry Pratchett, Small Gods)


SHREK
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