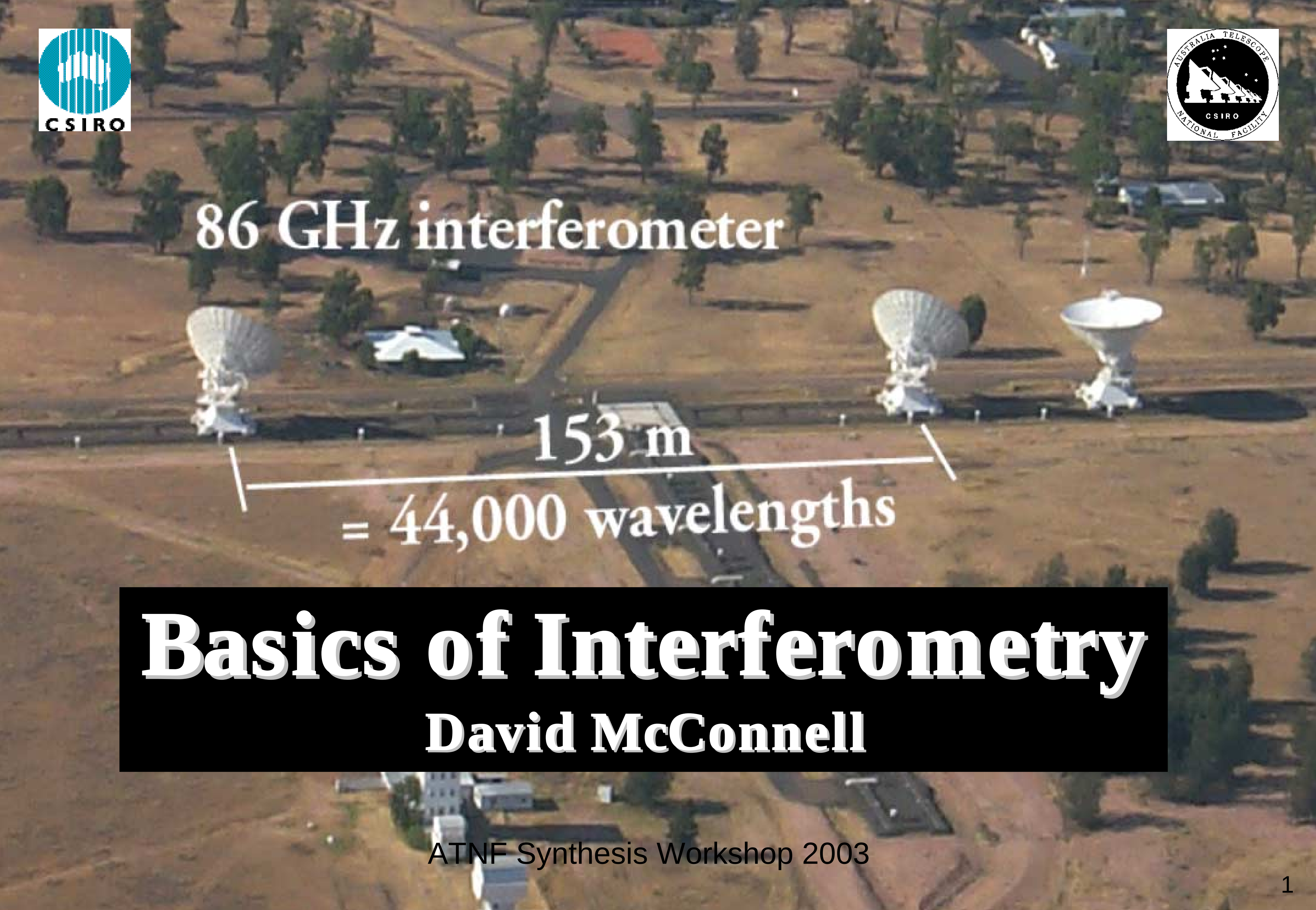


Basics of Interferometry

David McConnell

ATNF Synthesis Workshop 2003

Basic Interferometry

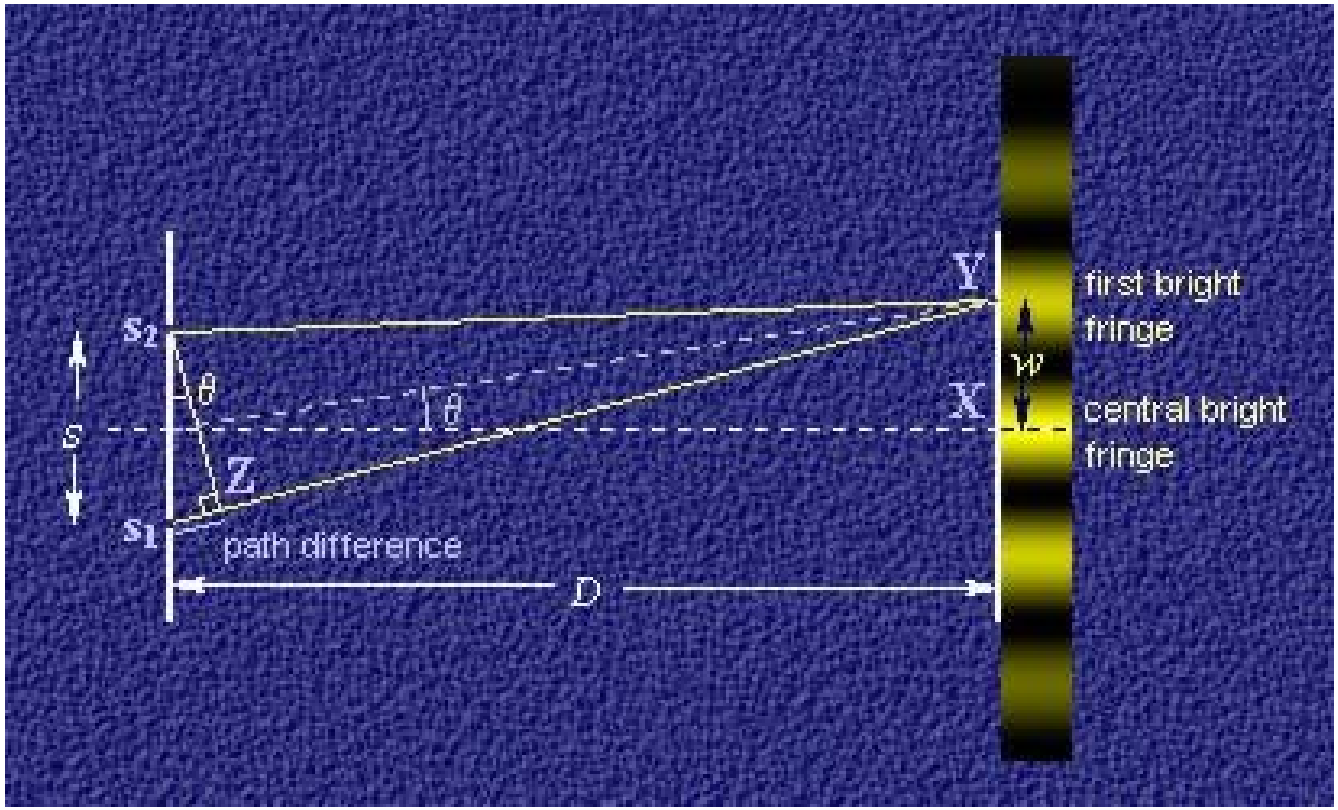
- Coherence in Radio Astronomy
  - » follows closely Chapter 1* by Barry G Clark
- What does it mean?
- Outline of a Practical Interferometer
- Review the Simplifying Assumptions

* Synthesis Imaging in Radio Astronomy II

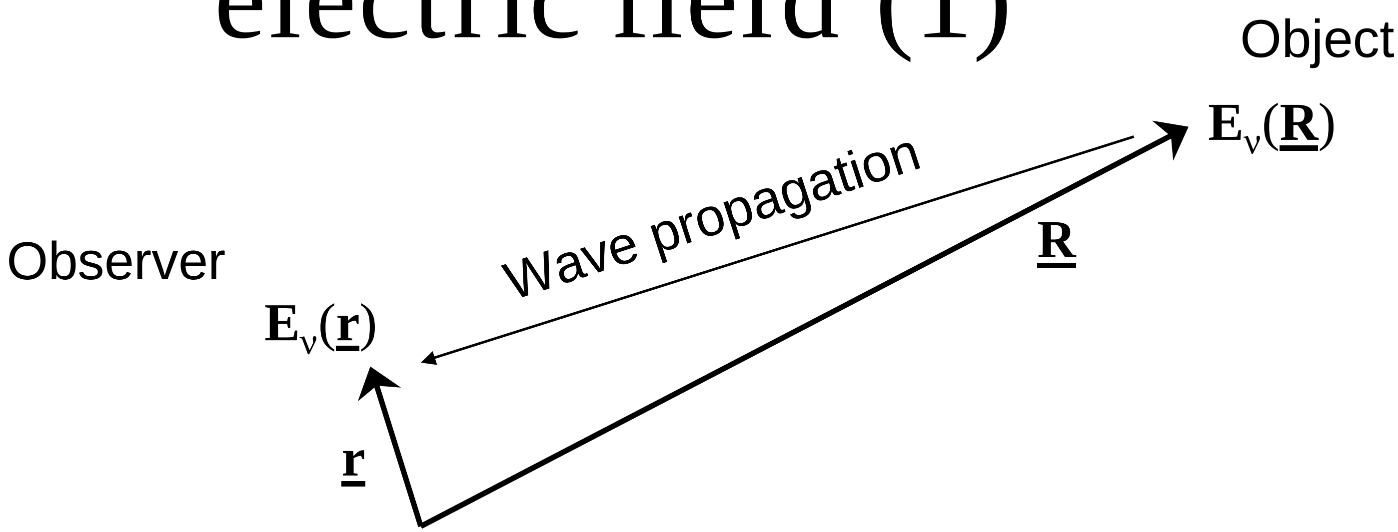
Edited by G.B.Taylor, C.L.Carilli, and R.A Perley

Interference

Young's double slit experiment (1801)



Form of the observed electric field (1)



$$\mathbf{E}_\nu(\mathbf{r}) = \iiint P_\nu(\mathbf{R},\mathbf{r})\mathbf{E}_\nu(\mathbf{R})\,dxdydz$$

Superposition allowed by linearity of Maxwell's equations

The *propagator*

Form of the observed electric field (2)

$$\mathbf{E}_\nu(\mathbf{r}) = \iiint P_\nu(\mathbf{R},\mathbf{r})\mathbf{E}_\nu(\mathbf{R})\,dxdydz$$

Assumption 1: Treat the electric field as a scalar - ignore polarisation

Assumption 2: Immense distance to source; ignore depth dimension; measure “surface brightness”: $E_\nu(\mathbf{R})$ is electric field distribution on celestial sphere

Assumption 3: Space is empty; simple propagator

$$E_\nu(\mathbf{r}) = \int E_\nu(\mathbf{R}) \frac{e^{2\pi i \nu|\mathbf{R}-\mathbf{r}|/c}}{|\mathbf{R}-\mathbf{r}|} dS$$

Spatial coherence function of the field (1)

Define the **correlation** of the field at points $\mathbf{r_1}$ and $\mathbf{r_2}$ as:

$$\mathrm{V}_\nu(\mathbf{r}_1,\mathbf{r}_2) = \left\langle E_\nu(\mathbf{r}_1) E_\nu^*(\mathbf{r}_2) \right\rangle$$

where

$$E_\nu(\mathbf{r}) = \int \mathsf{E}_\nu(\mathbf{R}) \frac{e^{2\pi i \nu |\mathbf{R}-\mathbf{r}|/c}}{|\mathbf{R}-\mathbf{r}|} dS$$

so

$$\mathrm{V}_\nu(\mathbf{r}_1,\mathbf{r}_2) = \left\langle \iint \mathsf{E}_\nu(\mathbf{R}_1) \mathsf{E}_\nu^*(\mathbf{R}_2) \frac{e^{2\pi i \nu |\mathbf{R}_1-\mathbf{r}_1|/c}}{|\mathbf{R}_1-\mathbf{r}_1|} \frac{e^{-2\pi i \nu |\mathbf{R}_2-\mathbf{r}_2|/c}}{|\mathbf{R}_2-\mathbf{r}_2|} dS_1 dS_2 \right\rangle$$

Spatial coherence function of the field (2)

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \left\langle \iint E_\nu(\mathbf{R}_1) E_\nu^*(\mathbf{R}_2) \frac{e^{2\pi i\nu|\mathbf{R}_1-\mathbf{r}_1|/c}}{|\mathbf{R}_1-\mathbf{r}_1|} \frac{e^{-2\pi i\nu|\mathbf{R}_2-\mathbf{r}_2|/c}}{|\mathbf{R}_2-\mathbf{r}_2|} dS_1 dS_2 \right\rangle$$

Assumption 4: Radiation from astronomical sources is **not** spatially coherent

$$\left\langle E_\nu(\mathbf{R}_1) E_\nu^*(\mathbf{R}_2) \right\rangle = 0 \qquad \text{for } \mathbf{R}_1 \neq \mathbf{R}_2$$

After exchanging the expectation operator and integrals becomes:

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \int \left\langle \left| E_\nu(\mathbf{R}) \right|^2 \right\rangle \frac{e^{2\pi i\nu|\mathbf{R}-\mathbf{r}_1|/c}}{|\mathbf{R}-\mathbf{r}_1|} \frac{e^{-2\pi i\nu|\mathbf{R}-\mathbf{r}_2|/c}}{|\mathbf{R}-\mathbf{r}_2|} dS$$

Spatial coherence function of the field (3)

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \int \left\langle \left| E_\nu(\mathbf{R}) \right|^2 \right\rangle \frac{e^{2\pi i \nu |\mathbf{R}-\mathbf{r}_1|/c}}{|\mathbf{R}-\mathbf{r}_1|} \frac{e^{-2\pi i \nu |\mathbf{R}-\mathbf{r}_2|/c}}{|\mathbf{R}-\mathbf{r}_2|} dS$$

Write the unit vector as: $\mathbf{s} = \frac{\mathbf{R}}{|\mathbf{R}|}$

Write the observed intensity as: $I_\nu(\mathbf{s}) = \left\langle \left| E_\nu(\mathbf{s}) \right|^2 \right\rangle$

Replace the surface element : $dS = |\mathbf{R}|^2 d\Omega$

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) \approx \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s}\cdot(\mathbf{r}_1-\mathbf{r}_2)/c} d\Omega$$

Spatial coherence function of the field (4)

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s}\cdot(\mathbf{r}_1-\mathbf{r}_2)/c} d\Omega$$

"An interferometer is a device for measuring this spatial coherence function."

$V_\nu(\mathbf{r}_1,\mathbf{r}_2)$ is known as the complex visibility.

Inversion of the Coherence Function (1)

The Coherence Function is invertible after taking one of two further simplifying assumptions:

•Assumption 5(a): vectors ($\mathbf{r}_1 – \mathbf{r}_2$) lie in a plane

•Assumption 5(b): endpoints of vectors **s** lie in a plane

Choose coordinates (u,v,w) for the ($\mathbf{r}_1 – \mathbf{r}_2$) vector space

Write the components of **s** as $\left(l, m, \sqrt{1-l^2-m^2}\right)$

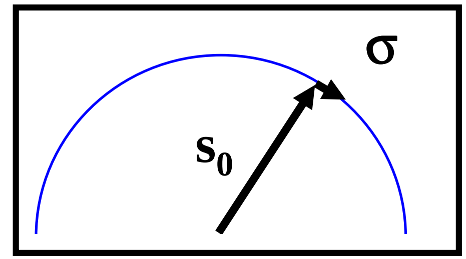
Then with 5(a):

$$V_\nu(u,v,w\equiv 0) = \iint I_\nu(l,m) \frac{e^{-2\pi i(ul+vm)}}{\sqrt{1-l^2-m^2}} dldm$$

Inversion of the Coherence Function (2)

Or, taking 5(b) assuming all radiation comes from a small portion of the sky, write the vector **s** as $\mathbf{s} = \mathbf{s_0} + \boldsymbol{\sigma}$ with $\mathbf{s_0}$ and $\boldsymbol{\sigma}$ perpendicular

Choose coordinates s.t. $\mathbf{s_0} = (0,0,1)$ then



$$\mathrm{V}_\nu(\mathbf{r_1},\mathbf{r_2}) \approx \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s}\cdot(\mathbf{r}_1-\mathbf{r}_2)/c} d\Omega$$

becomes

$$V_\nu'(u,v,w) = e^{-2\pi i w} \iint I_\nu(l,m) e^{-2\pi i(ul+vm)} dl dm$$

$$V_\nu(u,v,w) = e^{2\pi i w} V_\nu'(u,v,w)$$ is independent of *w*

$$V_\nu(u,v) = \iint I_\nu(l,m) e^{-2\pi i(ul+vm)} dl dm$$

Inversion of the Coherence Function (3)

$$V_\nu(u,v) = \iint I_\nu(l,m) e^{-2\pi i(ul+vm)} dl dm$$

⇕

$$I_\nu(l,m) = \iint V_\nu(u,v) e^{2\pi i(ul+vm)} du dv$$

Image analysis / synthesis (1)

$$I_\nu(l,m) = \iint V_\nu(u,v) e^{2\pi i(ul+vm)} du dv$$

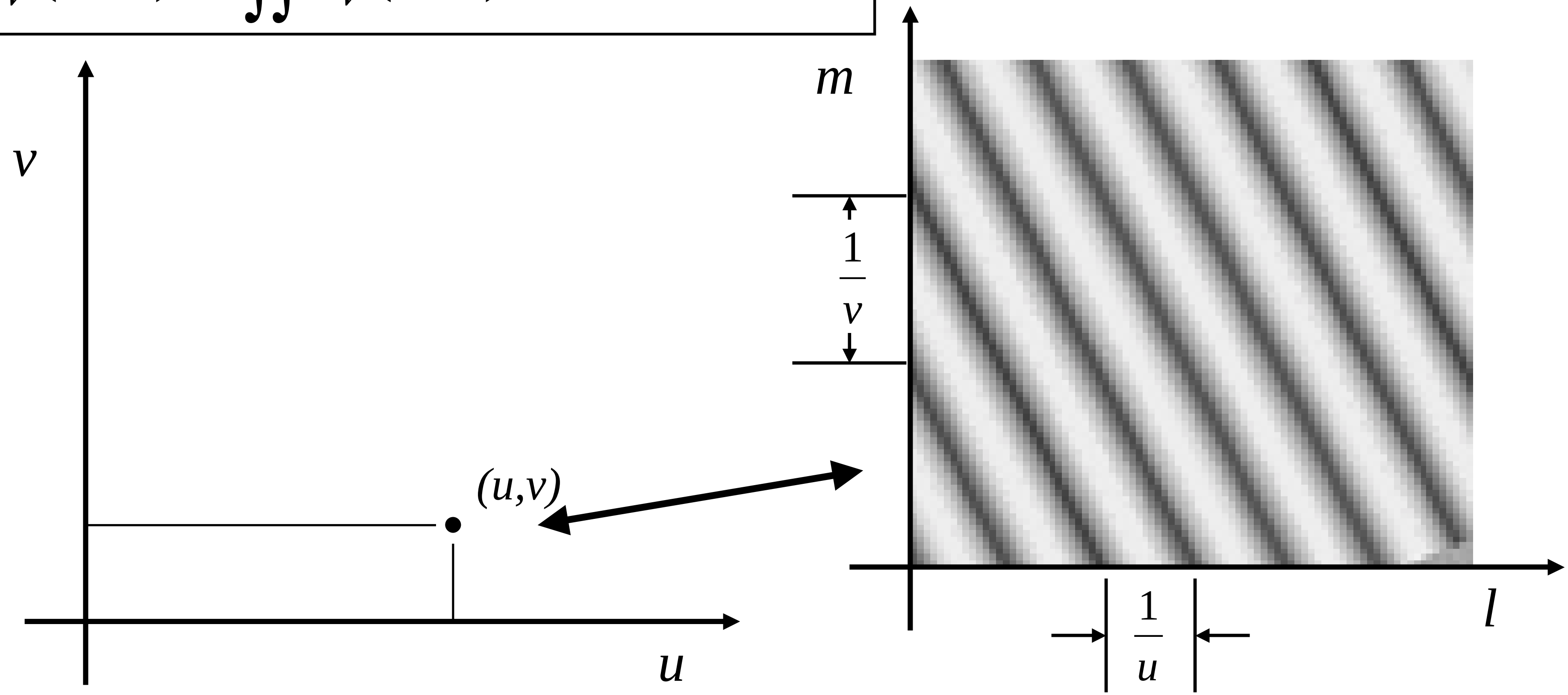
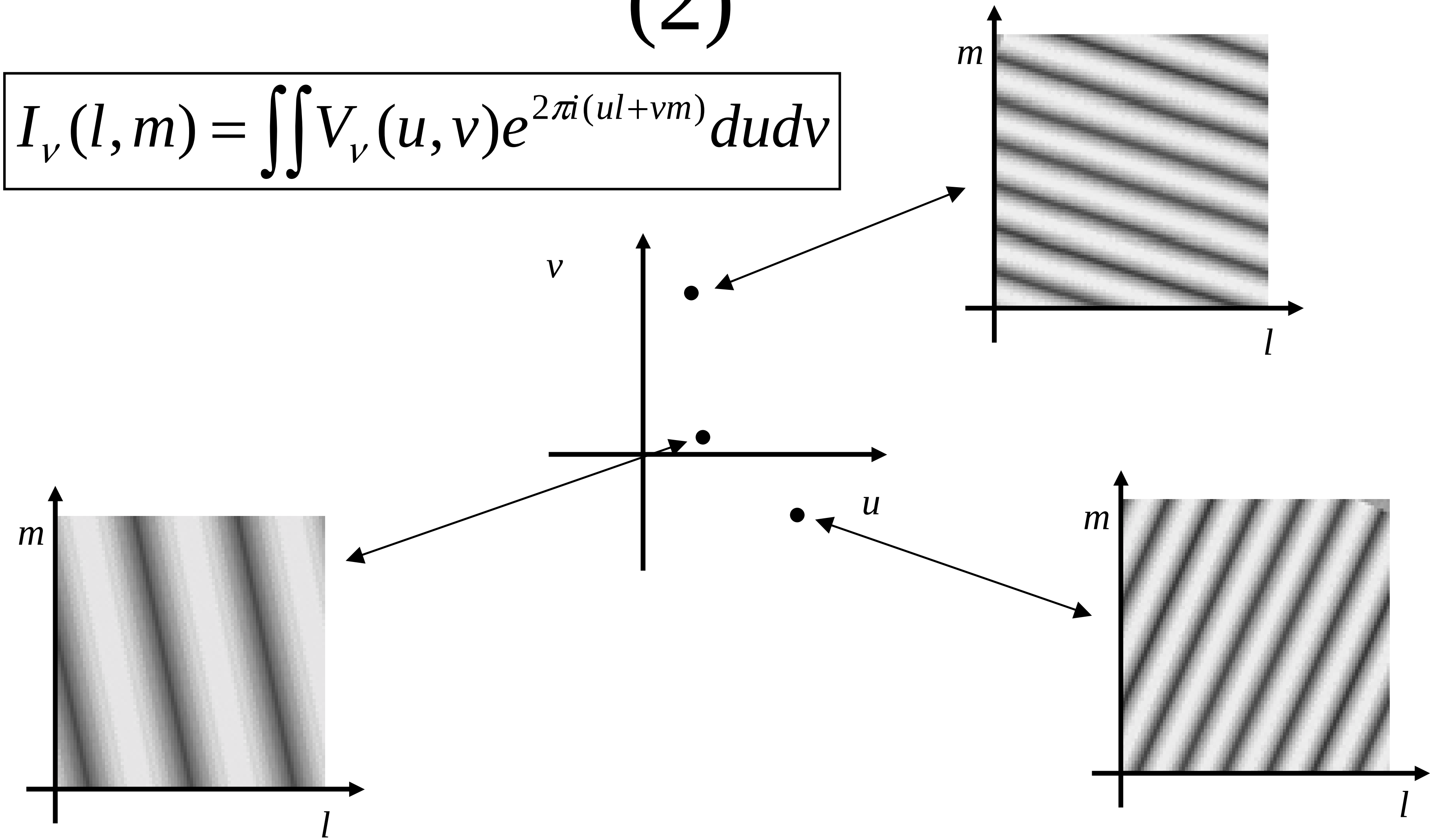


Image analysis / synthesis (2)

$$I_\nu(l,m) = \iint V_\nu(u,v) e^{2\pi i(ul+vm)} du dv$$



Incomplete Sampling (1)

$$I_\nu(l,m) = \iint V_\nu(u,v) e^{2\pi i(ul+vm)} du dv$$

Usually it is not practical to measure $V_\nu(u,v)$ for all (u,v) - our sampling of the (u,v) plane is incomplete.

Define a sampling function: $S(u,v) = 1$ where we have measurements, otherwise $S(u,v) = 0$.

$$I_\nu^D(l,m) = \iint S(u,v) V_\nu(u,v) e^{2\pi i(ul+vm)} du dv$$

$I_\nu^D(l,m)$ is called the "dirty image"

Incomplete Sampling (2)

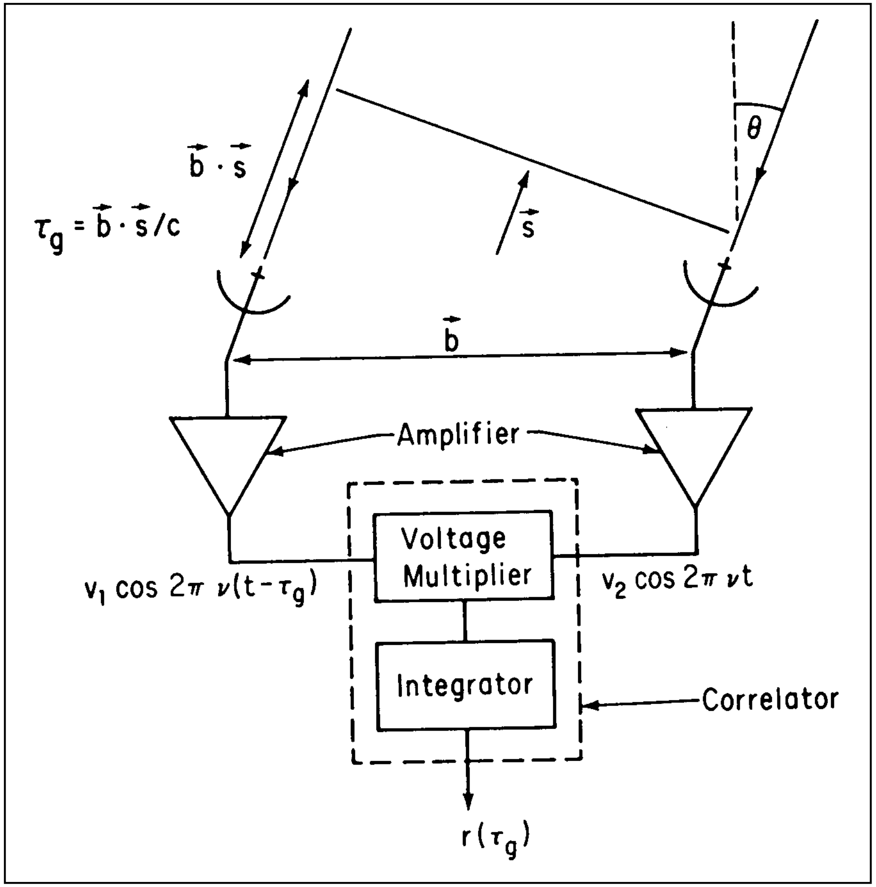
The convolution theorem for Fourier transforms says that the transform of the product of functions is the convolution of their transforms. So we can write:

$$I_\nu^D = I_\nu * B$$

The image formed by transforming our incomplete measurements of $V_\nu(u,v)$ is the true intensity distribution I_ν convolved with $B\,(l,m)$, the “synthesized beam” or “point spread function”.

$$B(l,m) = \iint S(u,v) e^{2\pi i(ul+vm)} du dv$$

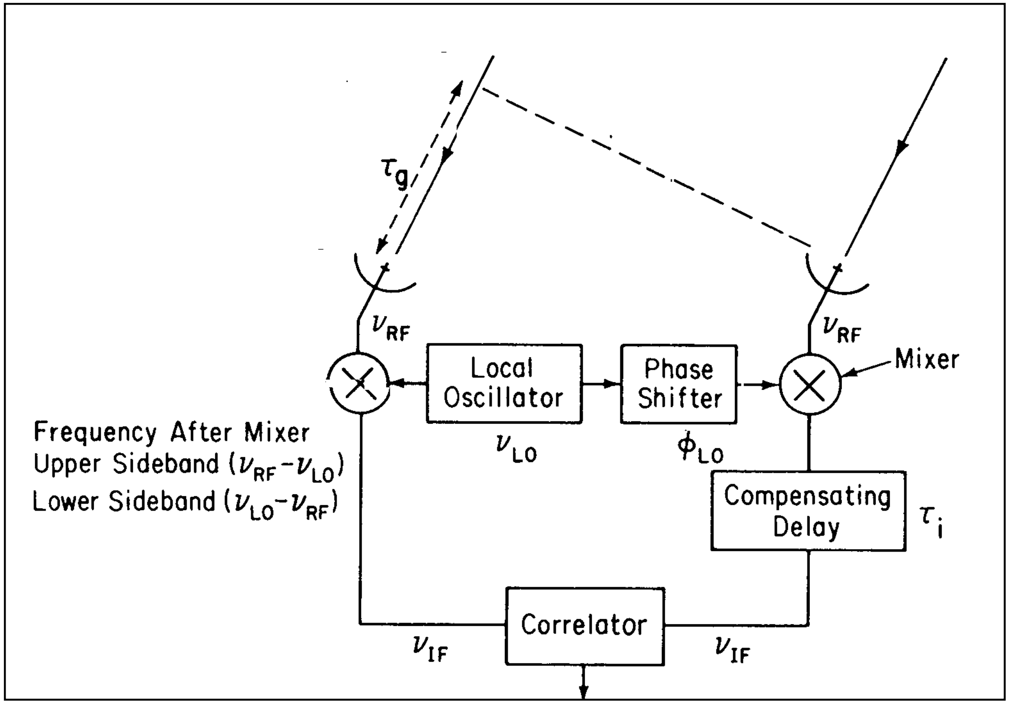
Interferometry Practice (1)



θ and therefore τ_g change as the Earth rotates. This produces rapid changes in $r(\tau_g)$ the correlator output.

This variation can be interpreted as the "source moving through the fringe pattern".

Interferometry Practice (2)



We could introduce a variable phase reference and delay compensation to move the “fringe pattern” across the sky with the source (“fringe stopping”).

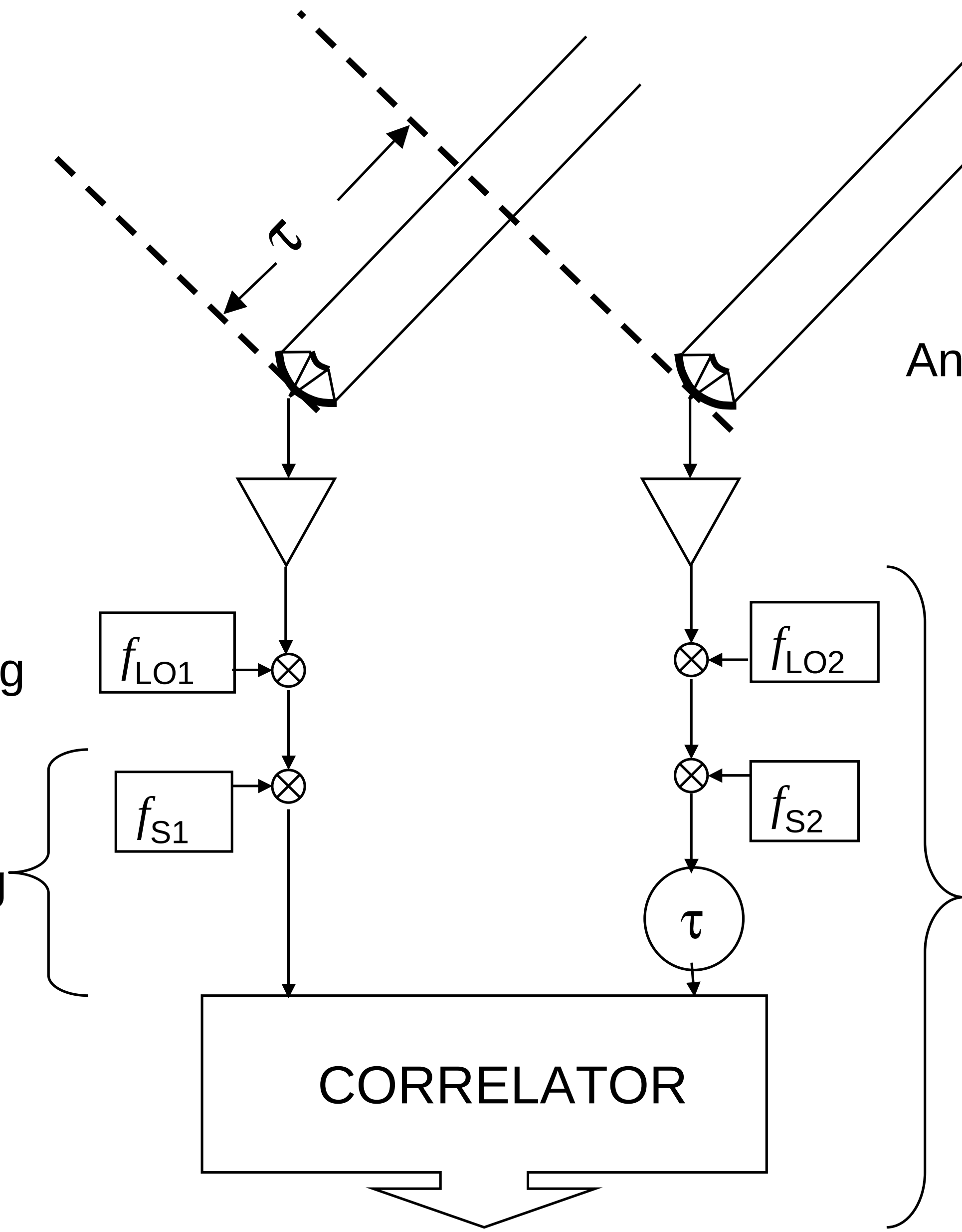
Antennas (Ravi Subrahmanyan)

amplification

Receivers (Graeme Carrad)

fringe stopping

delay tracking



Delay compensation and correlator (Warwick Wilson)

Simplifying Assumptions (1)

Assumption 1: Treat the electric field as a scalar - ignore polarisation

Polarisation is important in radioastronomy and will be addressed in a later lecture (see also Chapter 6).

Assumption 2: Immense distance to source, so ignore depth dimension and measure “surface brightness”: $E_\nu(\mathbf{R})$ is electric field distribution on celestial sphere

In radioastronomy this is usually a safe assumption. Exceptions may occur in the imaging of nearby objects such as planets with very long baselines.

Simplifying Assumptions (2)

Assumption 3: Space is empty; simple propagator

Not quite empty! The propagation medium contains magnetic fields, charged particles and atomic/molecular matter which makes it wavelength dependent. This leads to dispersion, Faraday rotation, spectral absorption, etc.

Assumption 4: Radiation from astronomical sources is **not** spatially coherent

Usually true for the sources themselves; however multi-path phenomena in the propagation medium can lead to the position dependence of the spatial coherence function.

$$V_\nu(\mathbf{r_1},\mathbf{r_2}) \approx \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s}\cdot(\mathbf{r}_1-\mathbf{r}_2)/c} d\Omega$$

Simplifying Assumptions (3)

The Coherence Function is invertible after taking one of two further simplifying assumptions:

•Assumption 5(a): vectors ($\mathbf{r}_1 - \mathbf{r}_2$) lie in a plane

•Assumption 5(b): endpoints of vectors $\mathbf{s}$ lie in a plane

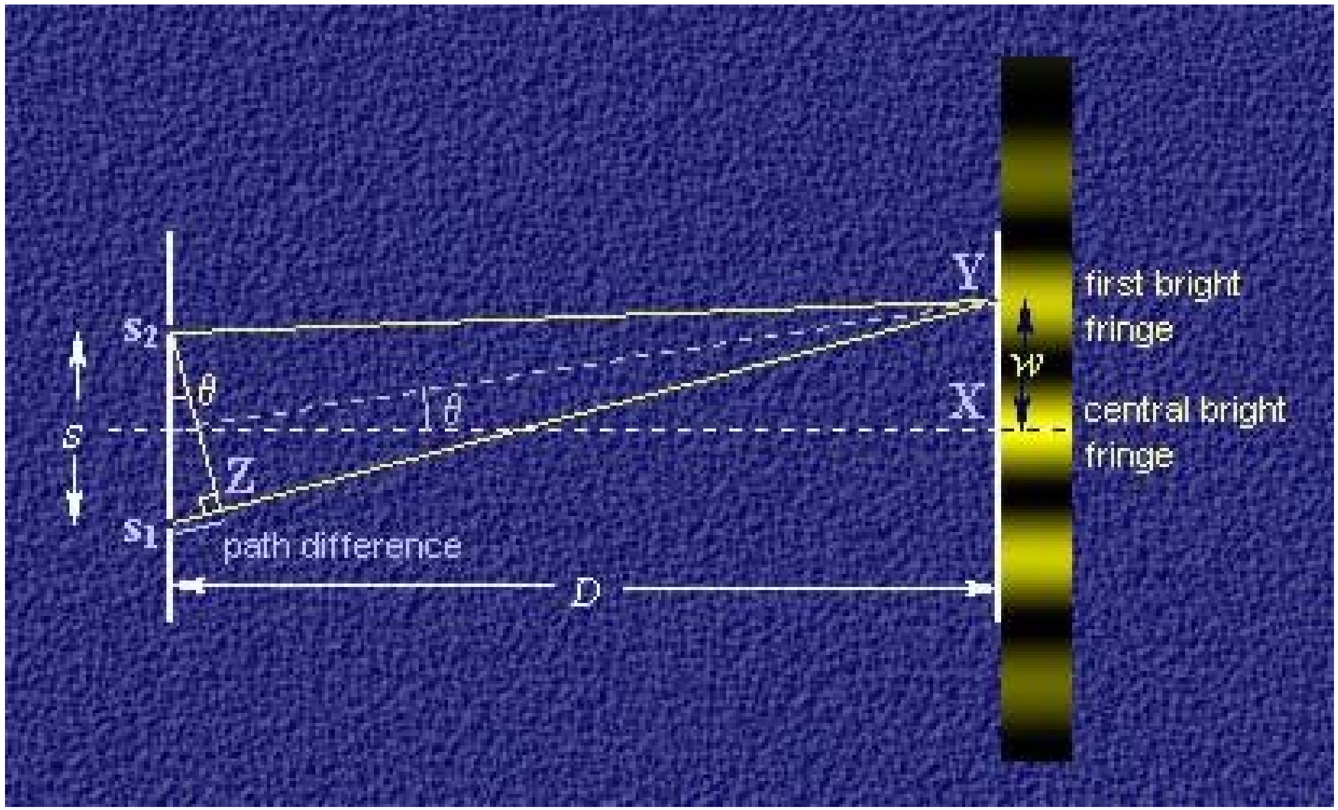
5(a) violated for all but East-West arrays

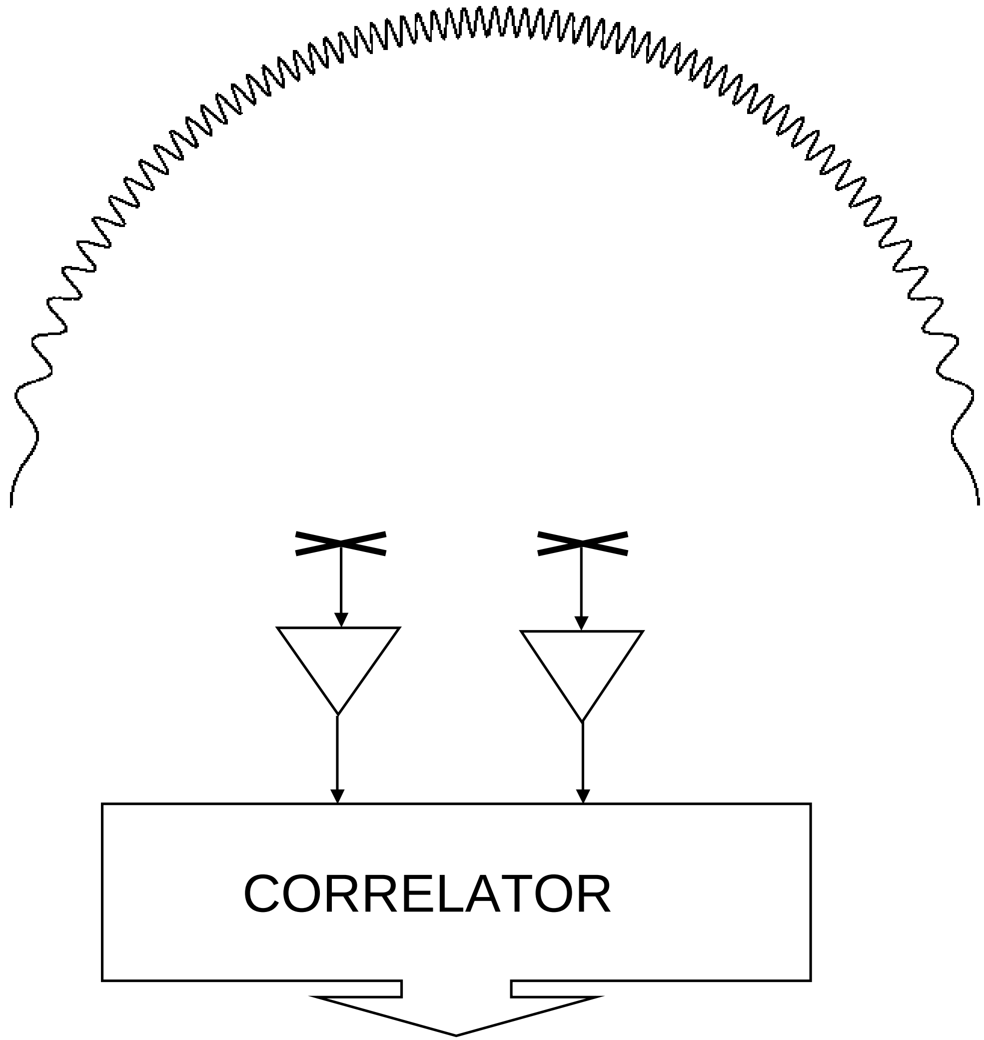
5(b) violated for wide field of view

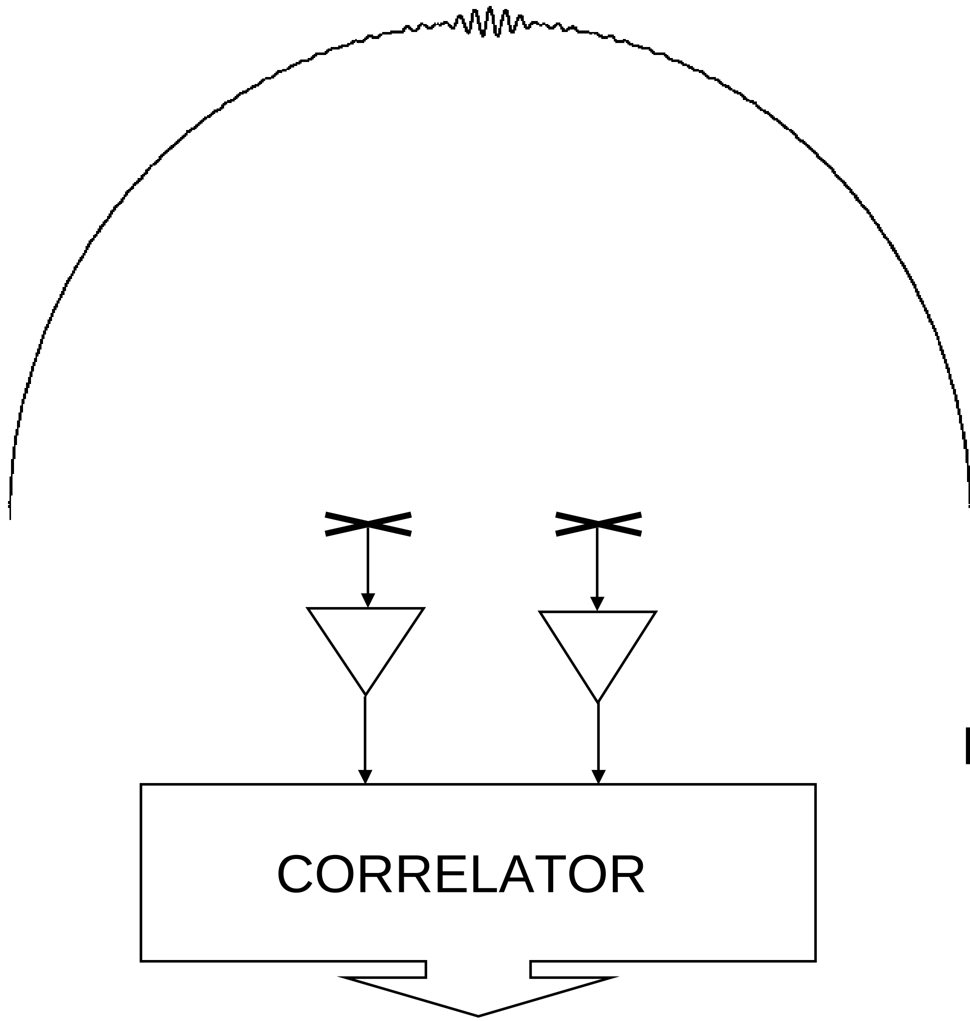
The problem is still tractable, but the inversion relation is no longer simply a 2-dimensional Fourier Transform (chapter 19).

Interference

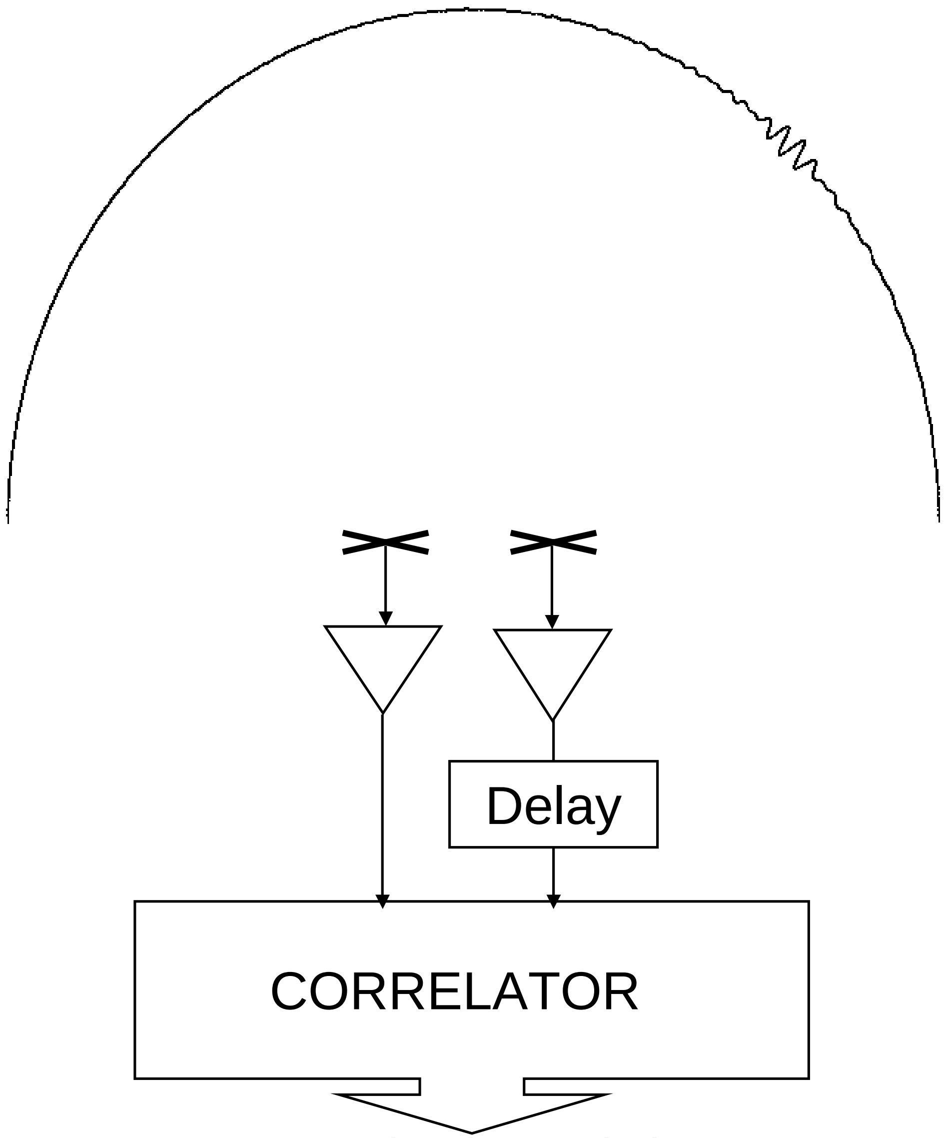
Young's double slit experiment (1801)




CORRELATOR



Finite bandwidth



Finite bandwidth

Delay tracking