P○lar↕metry

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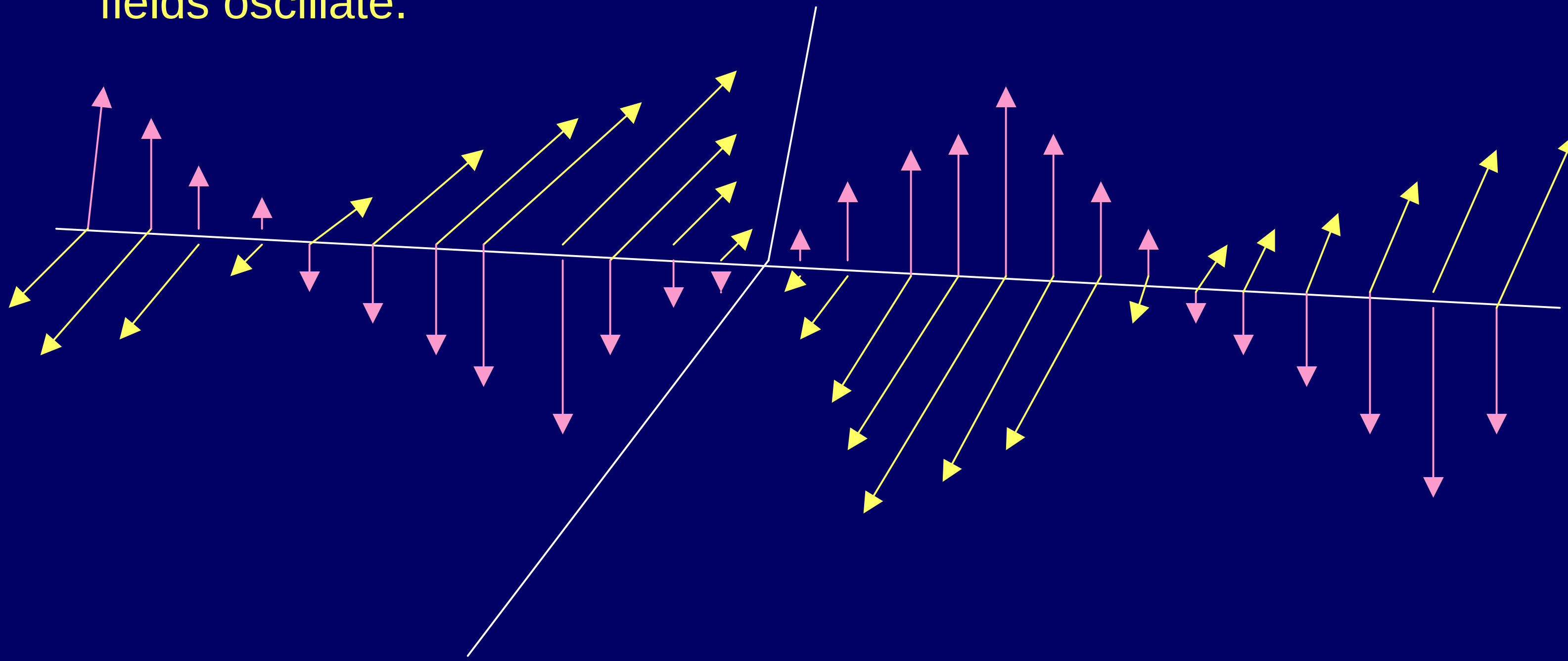
Overview

- What is Polarisation?
- Milestones of polarimetry
- How is it described?
- How is it measured?
- What is its role in Astronomy

What is Polarisation?

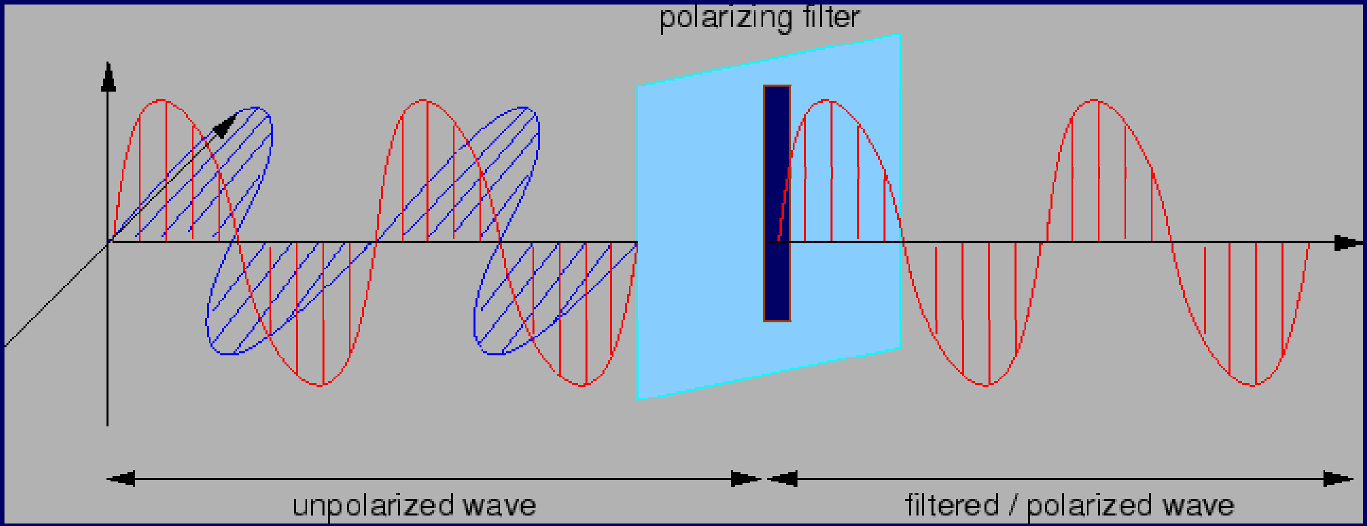
- Electromagnetic waves are vectors: have an Intensity AND a direction of propagation associated with them.

- Transverse to this plane is the plane in which the electric and magnetic fields oscillate.

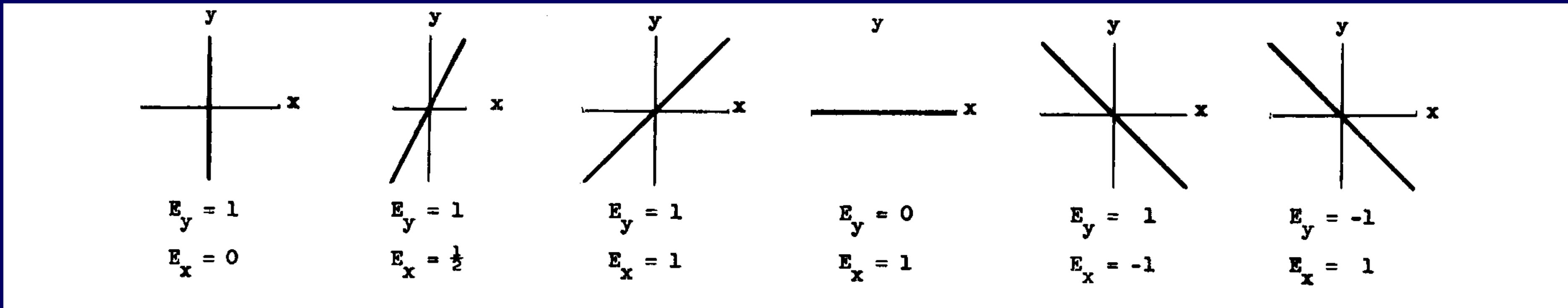


- Imagine one monochromatic (infinite) harmonic wave propagation in some direction.

- If the direction of the E field is unchanged (it is always perpendicular to the direction of propagation, but let's say it always points "up" as well) the wave is linearly polarised.



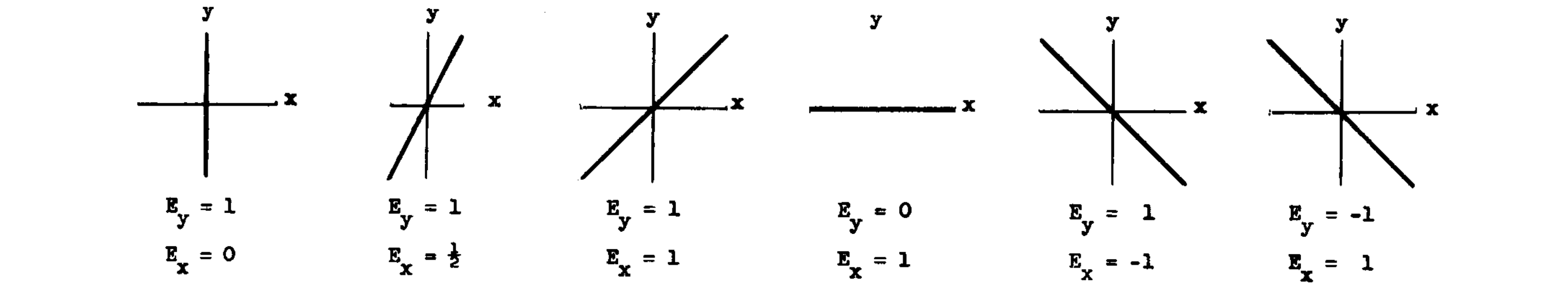
- Now imagine two such harmonic waves of the same freq propagation through the same region of space in the same direction.
- Let us choose the direction of the E vectors (I.e. the plane of polarisation) to be orthogonal between the two waves.
- Further, there is some phase difference, between these waves.
- The resultant wave is just the vector superposition of the two orthogonal components.



Superposition of x-vibrations and y-vibrations in phase

(From Feynman Lectures in Physics)

- If $\varepsilon = n \times 2\pi$, the resultant wave is also linearly polarised, and the plane of polarisation is at 45 degrees (for equal amplitudes).
- Note that we can resolve any linearly polarised wave into two orthogonal components.
- If $\varepsilon = n(\text{odd}) \times \pi$, the resultant wave is also linearly polarised, but the plane of polarisation is rotated 90 degrees from the previous example



Superposition of x-vibrations and y-vibrations in phase

(From Feynman Lectures in Physics)

- If the amplitudes of the orthogonal components are equal and ε +- π /2+2nπ what comes out is a wave where the amplitude of the E vector is constant but rotating in a circle (if viewed at one location in space).

- The tip of the electric vector rotates clockwise or anticlockwise depending on the sense of the phase shift. These are circularly polarised waves.

- We can also combine two oppositely circularly polarised waves of equal amplitude. What comes out is a linearly polarised wave.

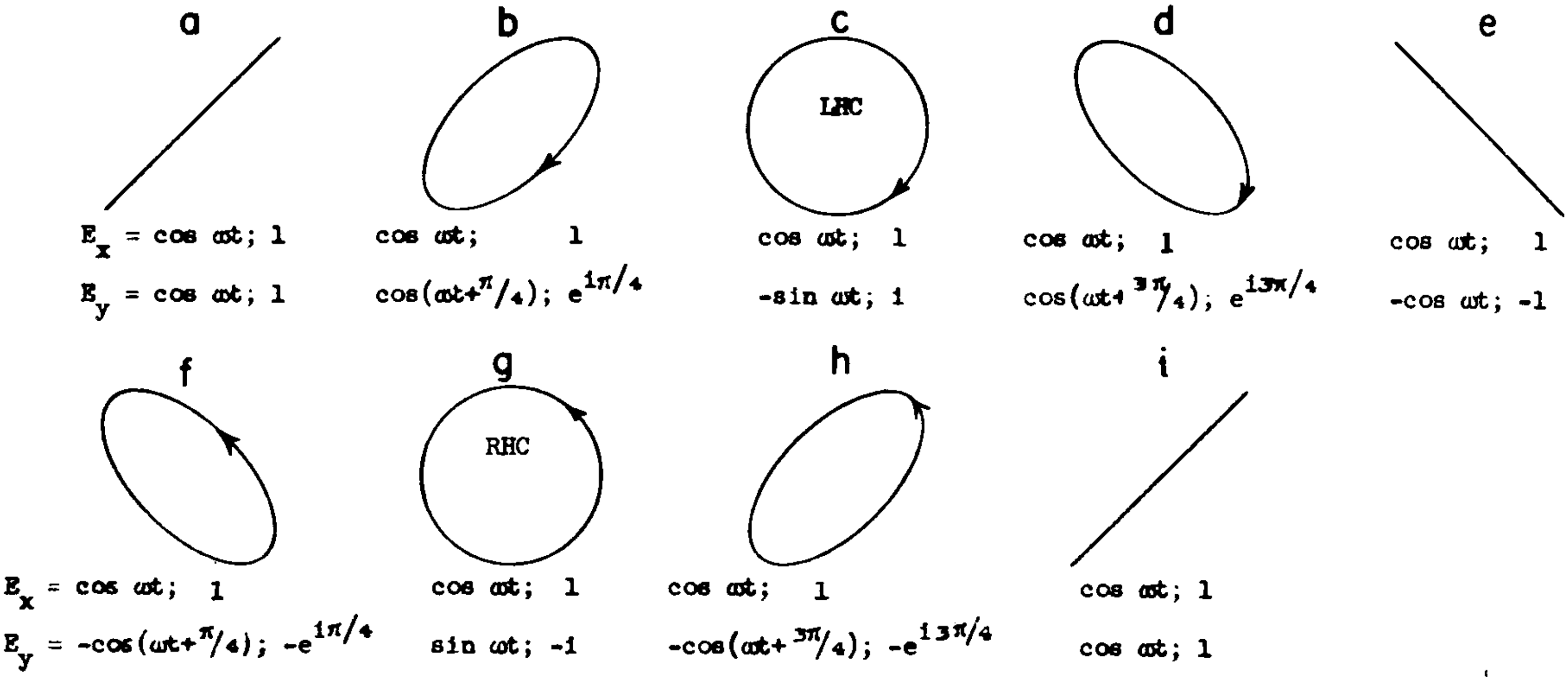
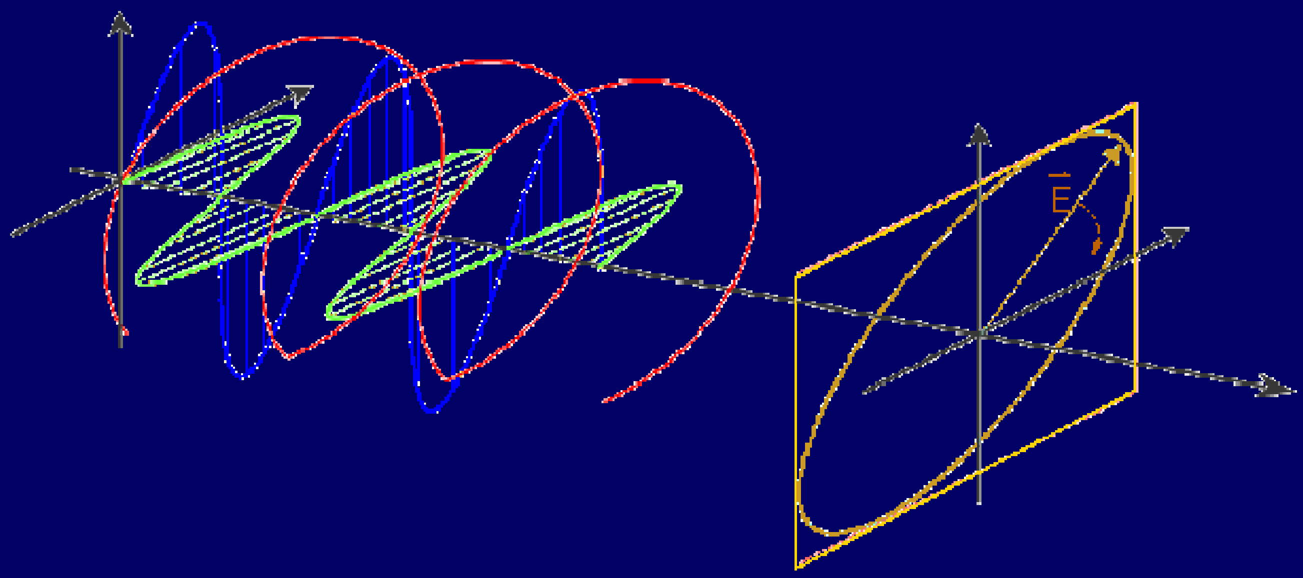


Fig. 33–2. Superposition of *x*-vibrations and *y*-vibrations with equal amplitudes but various relative phases. The components E_x and E_y are expressed in both real and complex notations.

Superposition of x-vibrations and y-vibrations with equal amplitudes but various relative phases. The components E_x and E_y are expressed in both real and complex notations.

(from Feynman Lectures in Physics)

- Linearly and circularly polarised radiation are specific cases of elliptically polarised radiation. In this case, the tip of the electric vector traces out an ellipse (viewed at one place in space). The general case of an arbitrary phase, *e* , between our two orthogonal waves, and arbitrary amplitudes, gives us elliptically polarised radiation.



- The locus of the tip of the electric vector is referred to as the polarisation ellipse.
- So we now know how to refer to the polarisation state of the monochromatic wave and that the general case of elliptically polarised radiation can be decomposed into two orthogonal, unequal-amplitude, linear (or circular) states.

What is a Randomly Polarised (Unpolarised) Wave ?

- Monochromatic waves are 100 percent polarised: at every instant the wave is in some specific and invariant polarisation state.
- However, the monochromatic wave is an idealisation, as it is of infinite extent
- Consider a light source with a large number of randomly oriented atomic emitters. Depending exactly upon its motions, each excited atom emits a fully polarised wave train for a very short time (the coherence time), delta t).
- If we look in a direction over a time that is short compared with the average coherence time, the electric field from all of the individual atomic emissions will be roughly constant in amplitude and phase (i.e. in some polarisation state).

- Thus if we were to look for an instant in some direction, we would “see” a coherent superposition of states; the resultant wave would be in some particular elliptically polarised state. That state would last for a time less than the coherence time before it changed randomly (as the emitters are incoherent) to some other state.

- As each wave train has a beginning and an end, it is not infinite and therefore not monochromatic; it has a range of frequency components, the bandwidth (delta nu ~ 1/ delta t) about some dominant frequency. If the bandwidth is large, the coherence time is short, and any polarisation state is short lived. Polarisation and coherence are intimately related.

- A randomly polarised (often called unpolarised, an inaccurate description) is one which does not prefer any polarisation state over its orthogonal state over the period of time you are looking at it.

- It has become a statistical issue: on average, what state is the radiation in ?
- If the wave is said to have no linear polarisation, the it actually has equal amounts of orthogonal linearly polarised states (which could be zero) on short time scales
- A wave that prefers one state over its orthogonal one is said to be partially polarised.
- A wave which spends all of its time in one state over the time you look at it is completely polarised

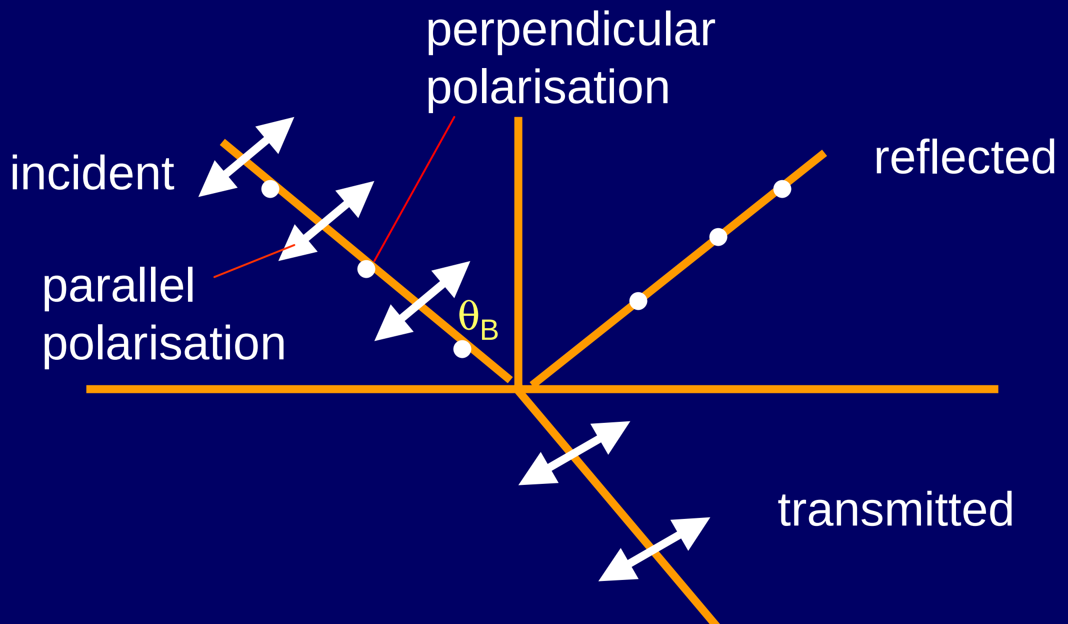
Polarisation by Reflection

Fraction of light reflected at different angles of incidence depends on its linear polarisation

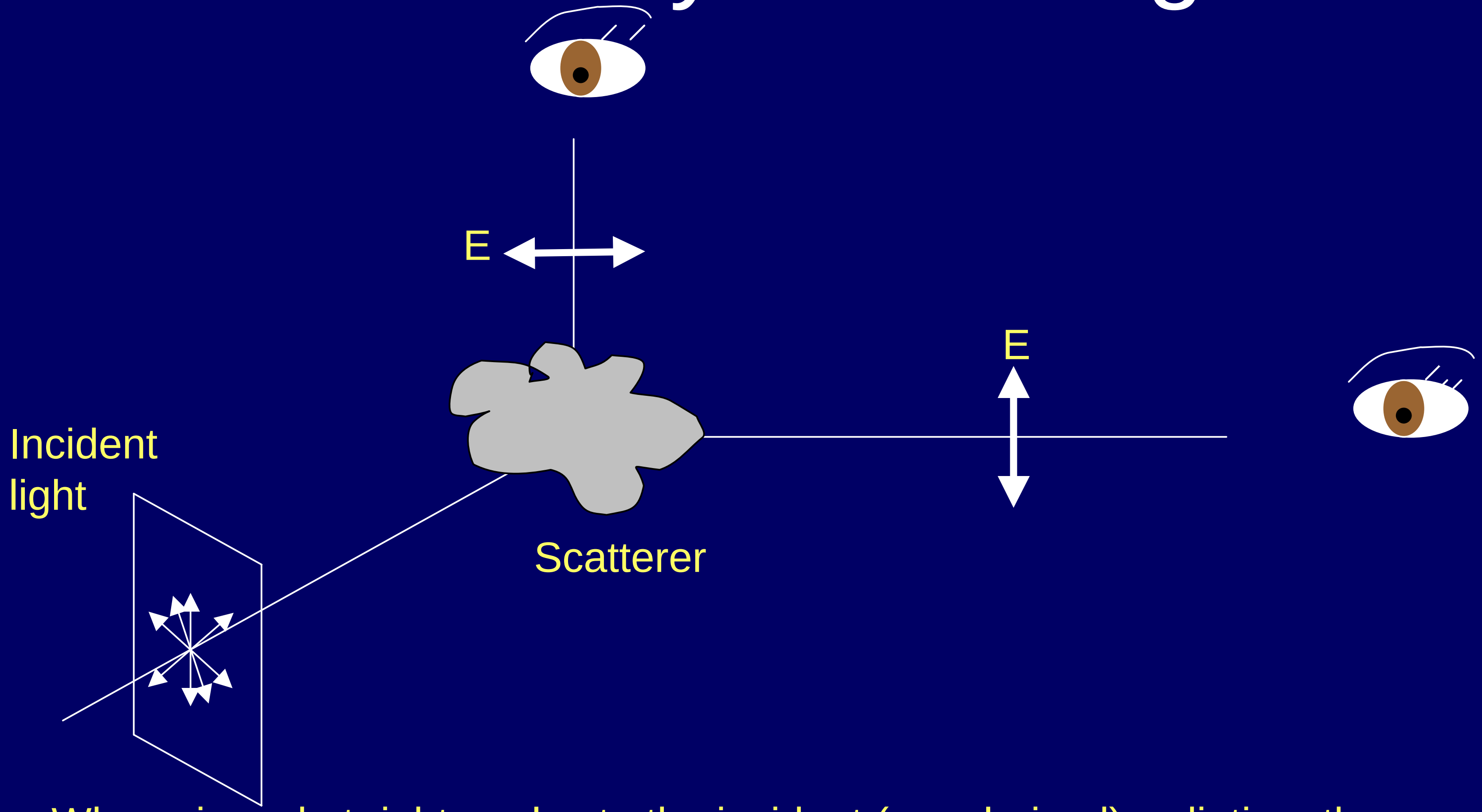
Brewster angle, θ_B , is the angle at which the reflected light is fully polarised, perpendicular to the plane of incidence

Reflected and transmitted rays are mutually perpendicular

Brewster's law: $\tan\theta_B = n$



Polarisation by scattering



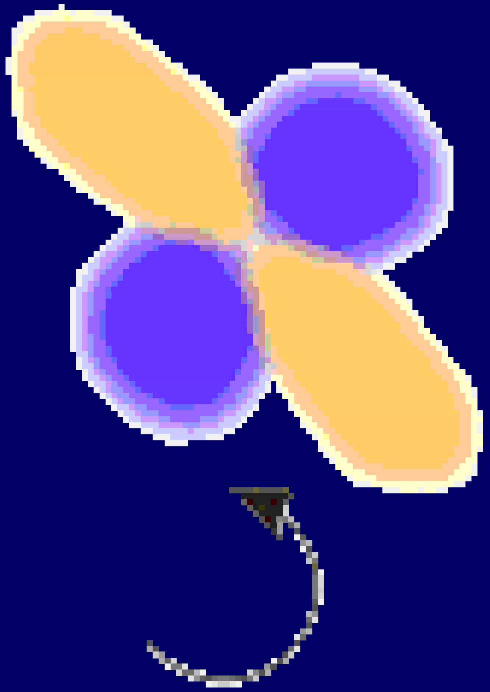
When viewed at right angles to the incident (unpolarised) radiation, the scattered fraction will be fully linearly polarised with the E vector perpendicular to the two directions. Thus, light scattered through 90 degrees is strongly polarized e.g. blue sky 90 degrees from the sun

Bees, Beetles, Happily ever after

- Bees see polarisation pattern of sky (Aristotle, von Frisch)
- Beetles are left wing (rose chaffer, cock chaffer, summer chaffer, garden chaffer)
- Rainbows are tangentially polarized
  - Primary: (42 degrees), 96 % by internal reflection
  - Secondary: (51 degrees), 90%polarized (by the two internal reflections)
- Polarisation clock – direction and strength of skylight polarisation depends on the relative position of the sun and the patch of sky doing the scattering
- Viking fairy tale

- Wilhelm K. Von Haidinger discovered an extra ‘sense’ in 1846, we can detect linear polarisation. Also circular (William Shurcliff, 1954)

- Diffuse elongated yellowish pattern, pinched at center. Bluish leaves (usually shorter) cross it at 90 degrees



- Yellow pattern points perpendicular to vibration plane for linearly polarized light

- Circularly polarized light generates inclined brush wrt line bisecting face, going up to the right and down to the left for RCP (tell by inclining your head)

- Brush is small, about 3 to 5 degrees
- Effect is weak, need at least 60 % polarisation to see it
- Happens only towards blue side of spectum (bees detect polarization in uv)
- Skylight at 90 degrees from sun is highly polarised, uniform, and blue.
- Probably caused by dichroic, long chain pigment Lutein which absorbs more light polarized parallel to molecular axis than perp. These molecules are partially aligned as concentric circles around fovea (or effect would average out)

Milestones of Polarimetry

- 1699 Bartholinus (re)discovers double refraction in calcite
- c. 1670 Huygens interprets this in terms of a spherical wavefront and discovers extinction by crossed polarisers
- 1672 Newton considers the light and the crystal to have “attractive virtue lodged in certain sides” and refers to the poles of a magnet as an analogy; this eventually leads to the term “polarisation”.
- 1808 Malus looks at the reflection of sunlight off a window through a crystal of calcite. He notices that the intensity of the two images in the reflection varied as he rotated the crystal. The reflection process has linearly polarised the light.

- 1812 Brewster relates the degree of polarisation with the angle of reflection and the refractive index.

- 1817 Fresnel and Young suggest the transverse nature of light and give a theoretical explanation of Malus' observation.

- 1845 Faraday links light with electromagnetism using polarisation. He showed that a piece of isotropic glass became birefringent when threaded with a mag field (circular modes, Faraday Rotation of linear polarisation). Faraday's insights were fully developed by Maxwell.

- 1852 Stokes studies the incoherent superposition of polarised light beams and introduces four parameters to describe the (partial) polarisation of noise-like signals.

- 1880's Hertz produces radio waves in the lab (m to dm range). He shows they can be reflected, refracted and diffracted, just like optical light. Also did polarisation experiments; previously polarisation was only associated with light.

- 1890's Bose makes wave guides, horn antennas, lens antennas, polarised mirrors. Made microwave polarimetry a science. Demonstrated wireless transmissions (to the Royal Institution) in 1896, a year before Marconi.

- 1923 Polarimetry of sunlight scattered by Venus by Lyot. Regarded as the start of polarimetry as an astronomical technique.

- 1930's Birth of radio astronomy with Jansky and later (1940's) Reber. Clear that Galactic radiation had a non-thermal component.

- 1942 polarisation concepts and sign conventions defined by the Institute of Radio Engineers (IRE, nowadays IEEE); adopted by radio astronomers.

- 1946 Chandrasekhar introduces the Stokes parameters into astronomy and predicts linear polarisation of electron-scattered starlight, to be detected in eclipsing binaries.

- 1949 Hiltner and Hall actually find interstellar polarisation. Bolton first identifies a discrete radio source (Taurus A) with the Crab nebula. Shklovskii suggests the featureless optical spectrum is a continuation of the radio spectrum and that both were synchrotron radiation.

- 1950 Alfven and Herlofson also suggested the diffuse radiation was from the synchrotron mechanism. People realized that synchrotron radiation should be linearly polarised (E perp to B) but nobody could detect a (confirmable, Razin) polarised component.

- 1954 Optical polarisation detected in Crab Nebula by Dombrovsky and Vashakidze. And later by Oort and Walraven. The first map of mag field inside an astrophysical object had been made.

- Soon, extragalactic objects were identified with discrete radio sources e.g. Virgo A(M 87)

- 1956 Optical polarisation in the jet of Virgo A detected by Oort, Walraven and Baade. Detection of polarisation was crucial evidence in support of the synchrotron hypothesis.
- 1957 First detection of polarised radio waves by Mayer et al. From Crab at 3cm they found 8% polarisation.
- Next 5 years Hundreds of discrete radio sources (local and extragalactic) found, many with spectra suggesting synchrotron radiation. But NO reliable polarisation detections.
- 1961 Radhakrishnan et al find Crab 2% polarised at 20 cm. The other three brightest non-thermal sources (Cas A, Cen A, Cyg A) were only a few tenths of a percent polarised (theoretical maximum is 72%). Big mystery!
- 1962 Mayer found Cyg A and Cen A polarised at 3% at 3cm. Westerhout detected polarised Galactic emission at 75 cm.

- 1972 First detection of polarised X-ray emission (Crab Nebula) by Columbia Uni group.
- 1973 the IAU (commissions 25 and 40) endorses IEEE definitions for elliptical polarisation.
- 1974 the first source book of astronomical polarimetry is published ed. Gehrels
- 1990's polarimeters become easy to use in many wavelengths and their use is spurred by theoretical developments.

Lessons from History

- Early attempts failed because
  - Large spurious instrumental effects (telescopes not designed for polarimetry)
  - Changes in direction of mag field in the emitting source within the (poor) telescope resolution averaged down the polarised flux
  - Faraday rotation (internal, beam, band)

The Future

- Future of polarimetry lay (and lies) in high resolution, high frequency and sensitive interferometer observations.

- MORAL: Polarisation lies at the heart of our understanding of light, emission mechanisms and astronomical sources.

How is it described ?

- By a set of four quantities, called the Stokes parameters, which completely specify the nature of incoherent, noise-like radiation from an astronomical source.
- Devised by Sir G. G. Stokes (1852) and adapted for astronomy by S. Chandrsekhar (1949).

- Idea was to write down polarisation state of wave in terms of observables (hard to get hold of varying polarisation ellipse!)

- Observables are intensities averaged over time

- Stokes wrote down his parameters in terms of the intensity passed by some polarizing filters that if illuminated by a randomly polarised wave, transmit half of the incident light.

- Filter 0 passes all states equally, giving intensity I_0

- Filters 1 and 2 pass linearly polarised light at position angles of 0 (horizontal) and 45 degrees, respectively.

- Filter 3 is opaque to left handed circular polarisation

$$I = 2I_0$$

$$Q = 2I_1 - 2I_0$$

$$U = 2I_2 - 2I_0$$

$$V = 2I_3 - 2I_0$$

- I is the total intensity
- Q reflects the tendency for the light to be in a linear state which is horizontal (Q>0), vertical (Q<0) or neither (Q=0)
- U reflects the tendency for the light to be in a linear state at 45 degrees (U>0) or -45 degrees (U<0), or neither (U=0).
- V reflects the tendency for the light to be in a circular state which is right handed (V>0), left handed (V<0) or neither (V=0)

- In general, all four parameters are functions of time and wavelength

- While I >=0, Q, U and V may be negative.

- We can think of a polarised wave as consisting of a completely polarised bit and an unpolarised bit. The latter contributes only to the total power. Thus, $I^2 >= Q^2 + U^2 + V^2$

- Degree of polarisation is defined as the length of the Stokes vector divided by I

- The position angle of the linear polarised radiation is $0.5 \tan^{-1}(U/Q)$. It is its phase measured east from north

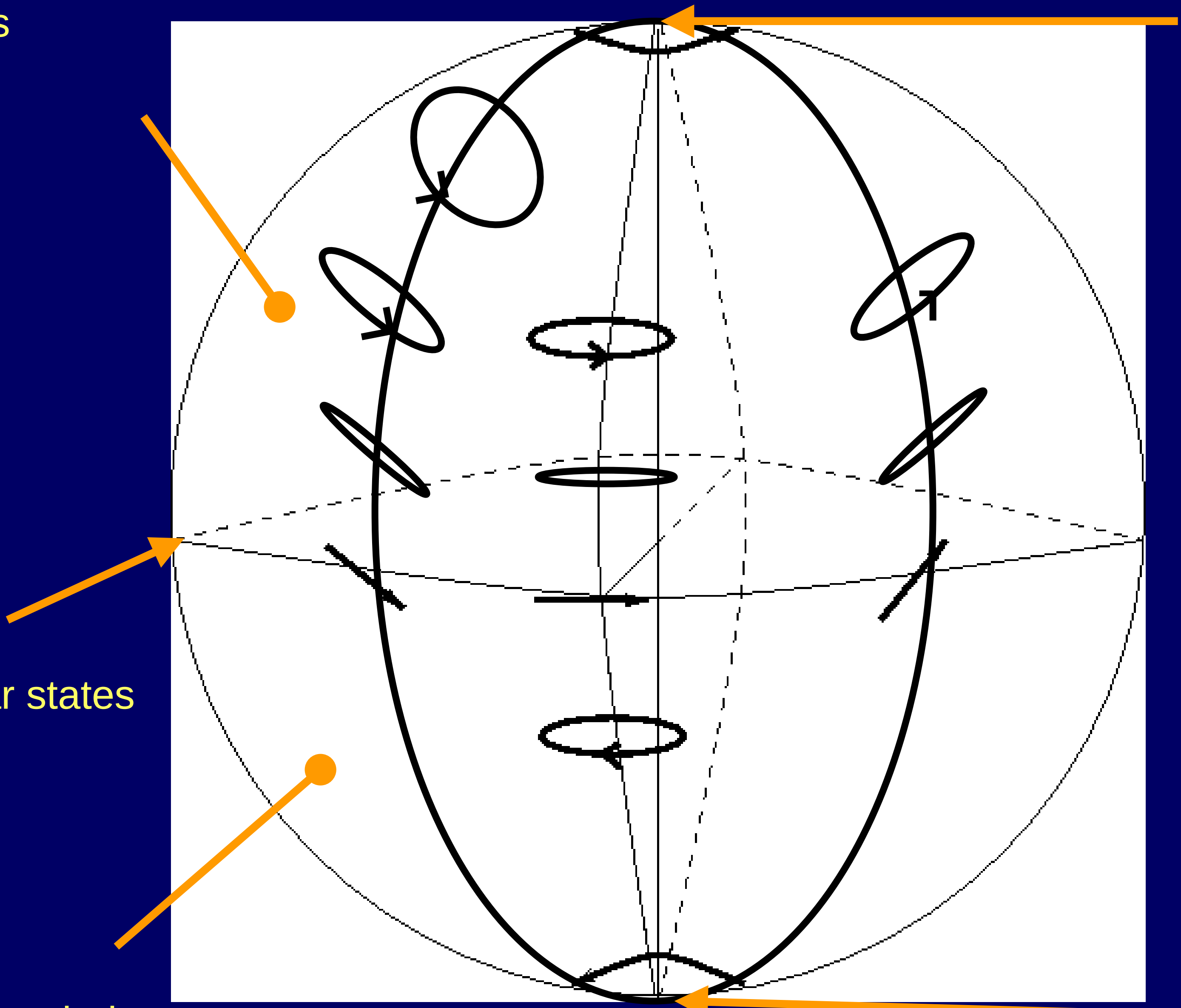
- Stokes parameters are additive for incoherent waves. Thus, in the case of many waves propagating through the same volume of space, the Stokes parameters of the resultant is simply the sum of the individual Stokes parameters

Poincaré Sphere

- Useful representation of all possible polarisation states on surface of sphere
- Poles represent the two circulars
- Equator represents linear
- Rest of surface elliptical
- Longitude represents orientation/tilt angle
- Latitude represents ellipticity/axial ratio
- NOTE: must double angles (dipole nature of electromagnetism)

Right handed states

RHC



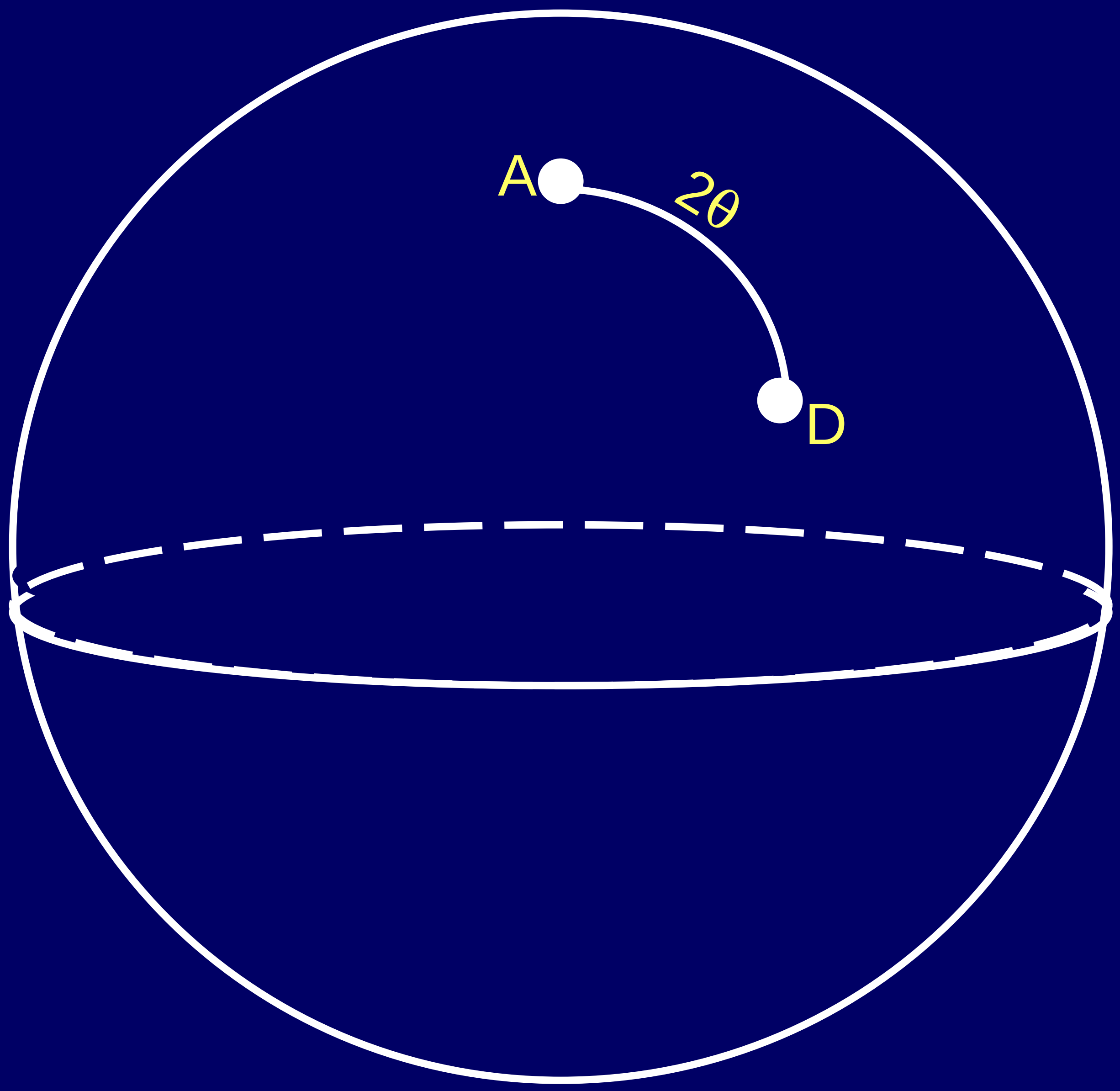
Latitude represents axial ratio

Linear states

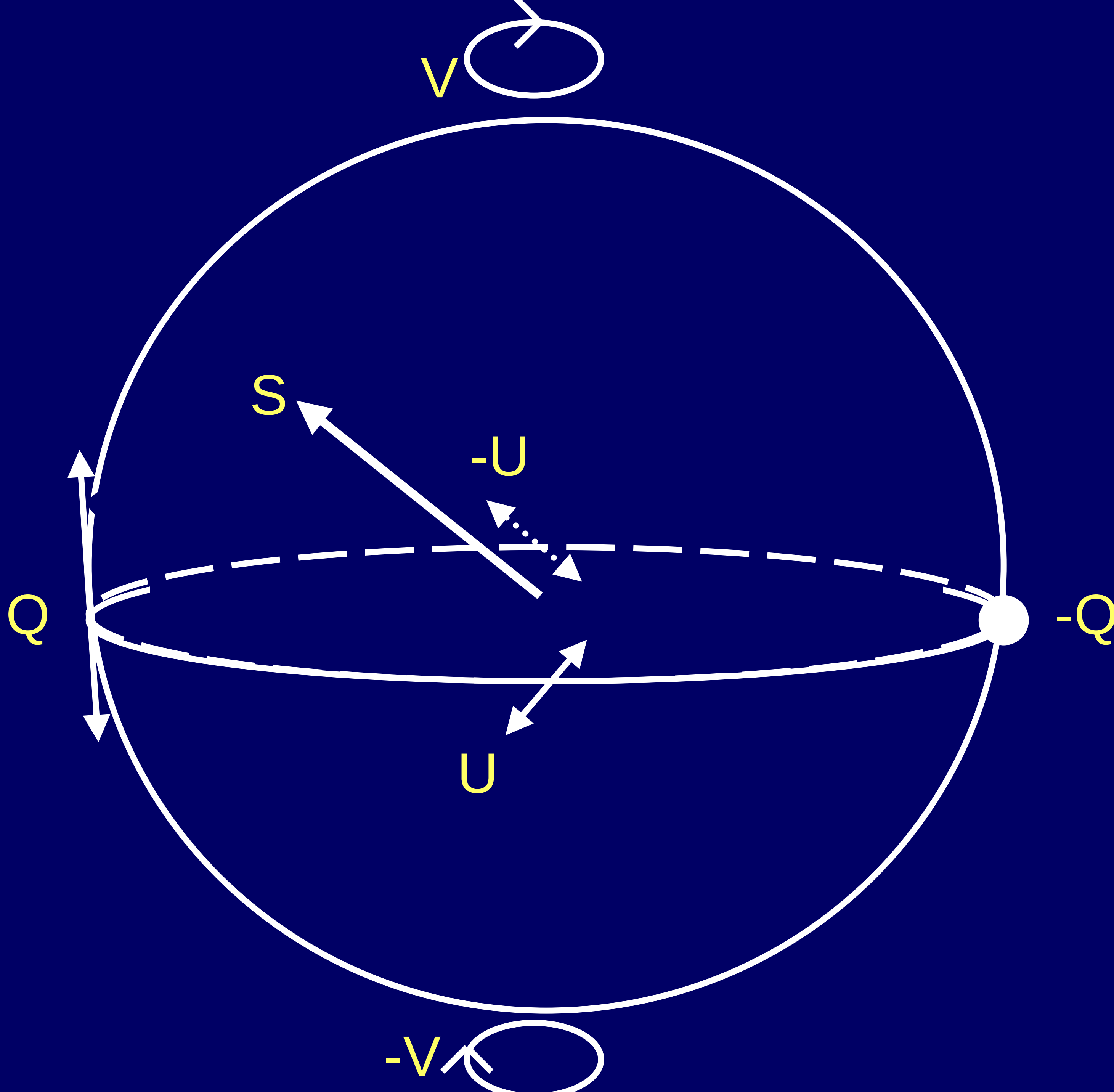
Longitude represents tilt angle

Left handed states

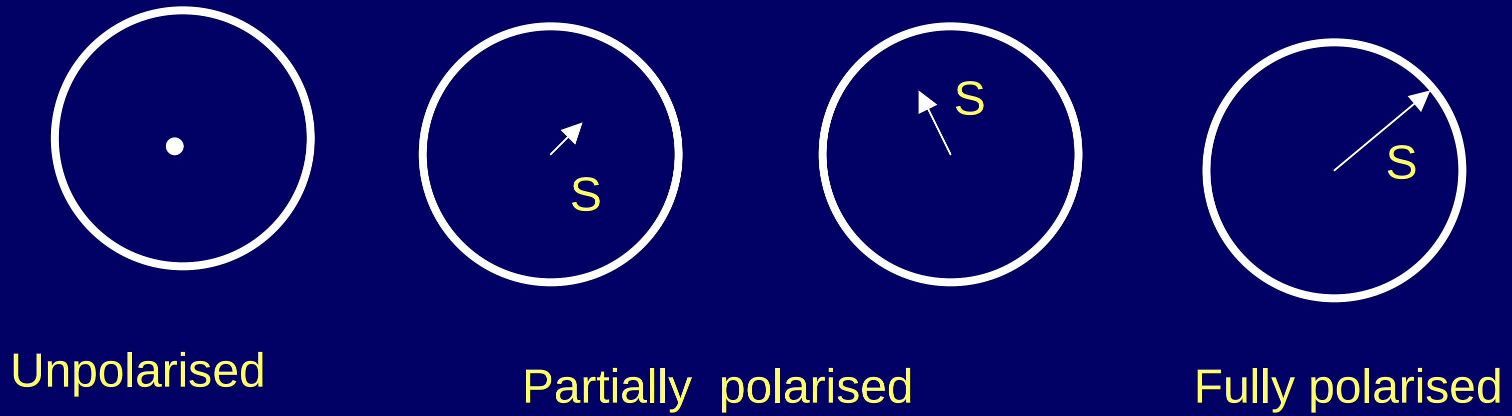
LHC

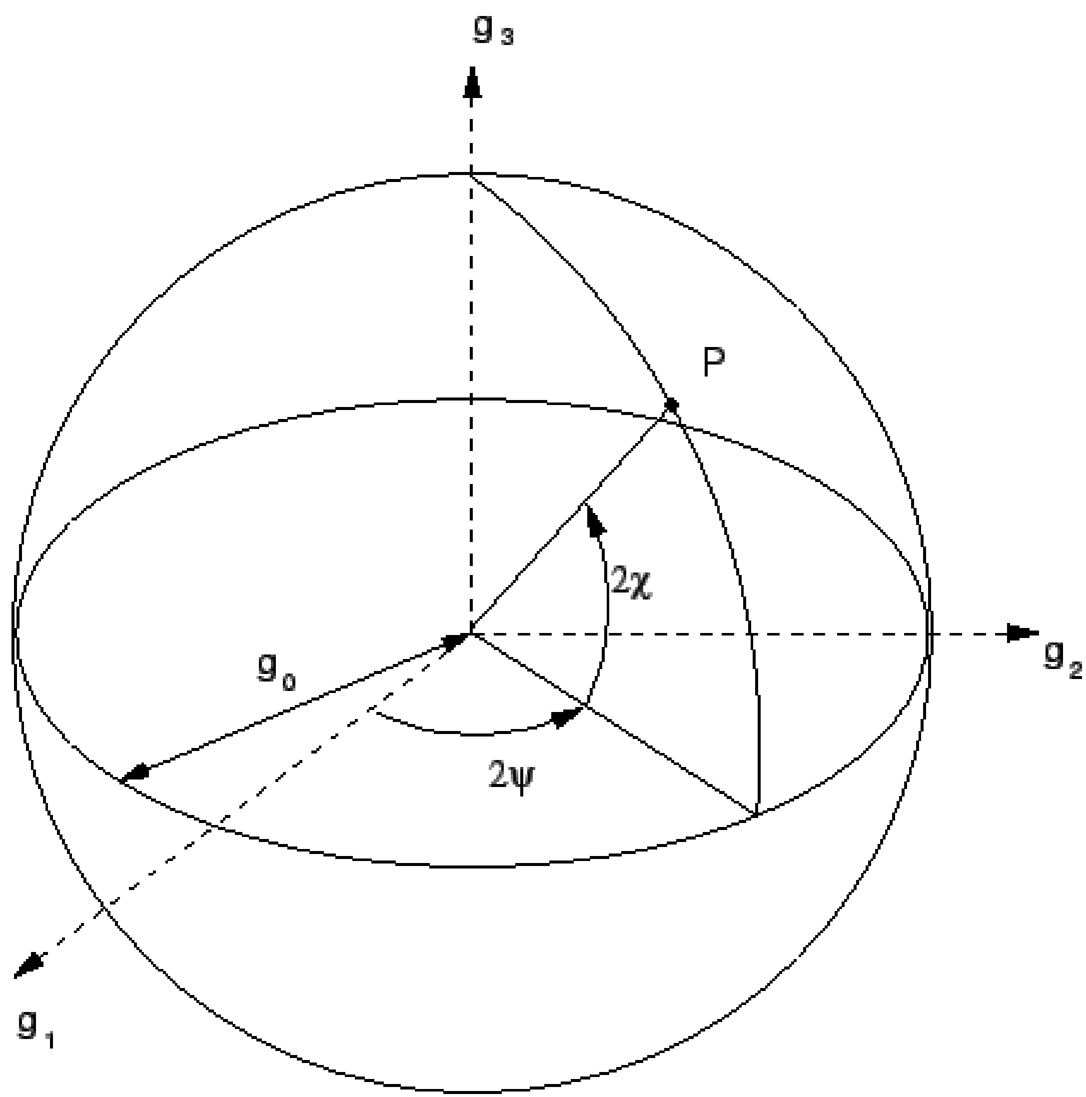


Fraction of energy received is $\cos^2\theta$


V
S
-U
Q
-Q
U
-V

Pancharatnam’s Extension





g_3
P
2χ
g_0
g_2
2ψ
g_1

How is it measured ?

- Our objective is to obtain the sky brightness distribution for each of the 4 Stokes parameters I, Q, U, V.
- Given a pair of antennas, 1 and 2, with feeds sensitive to right and left circularly polarised light, the four complex cross-correlations that can be formed are:

$$R_1 R^*_2 = \tilde{I}_{12} + \tilde{V}_{12}$$

$$L_1 L^*_2 = \tilde{I}_{12} - \tilde{V}_{12}$$

$$R_1 L^*_2 = \tilde{Q}_{12} + i\tilde{U}_{12}$$

$$L_1 R^*_2 = \tilde{Q}_{12} - i\tilde{U}_{12}$$

For ideal feeds and data. * denotes complex conjugation

- In terms of the time averages of the cross correlations of two circularly polarised electric fields, the Stokes parameters are

$$I_{12} = \tfrac{1}{2}\,(E_{1R}E^*_{2R}> + < E_{1L}E^*_{2L}>)$$

where the angle brackets indicate a time average

- Real, non-ideal, feeds pick up some of the component of polarisation, orthogonal to the one to which they are nominally sensitive. The response of such a feed can be approximated by the linear expressions:

$$V_R = G_R\,(E_R e^{-i\phi} + D_R E_L e^{+i\phi})$$

$$V_L = G_L\,(E_L e^{+i\phi} + D_L E_R e^{-i\phi})$$

- Where the G's are complex, multiplicative, time-dependent gains with amplitude g and phase ϕ

- The D's are the complex fractional responses of each feed to the orthogonally polarised radiation

ϕ is the orientation of the feeds with respect to the source, known as the *parallactic angle*. It is formally defined as the angle between the local vertical and north at the position of the source in the sky.

$$R_1L_2^* = \langle V_{R1}V_{L2}^* \rangle = G_{1R}G_{2L}^*[\tilde{P}_{12}e^{i(-\phi1-\phi2)}$$

$$+ D_{1R}D_{2L}^*\tilde{P}_{21}^*e^{i(+\phi1+\phi2)}$$

$$+D_{1R}(\tilde{I}_{12} - \tilde{V}_{12})e^{i(+\phi1-\phi2)}$$

$$+D_{2L}^*(\tilde{I}_{12} + \tilde{V}_{12})e^{i(-\phi1+\phi2)}]$$

- Calibration is the determination of the G's and D's in the above equations so that the total intensity and polarisation of a source can be recovered.

- Note that the instrumental contribution to the cross polarised response is not affected by parallactic angle, whereas the contribution from the source does

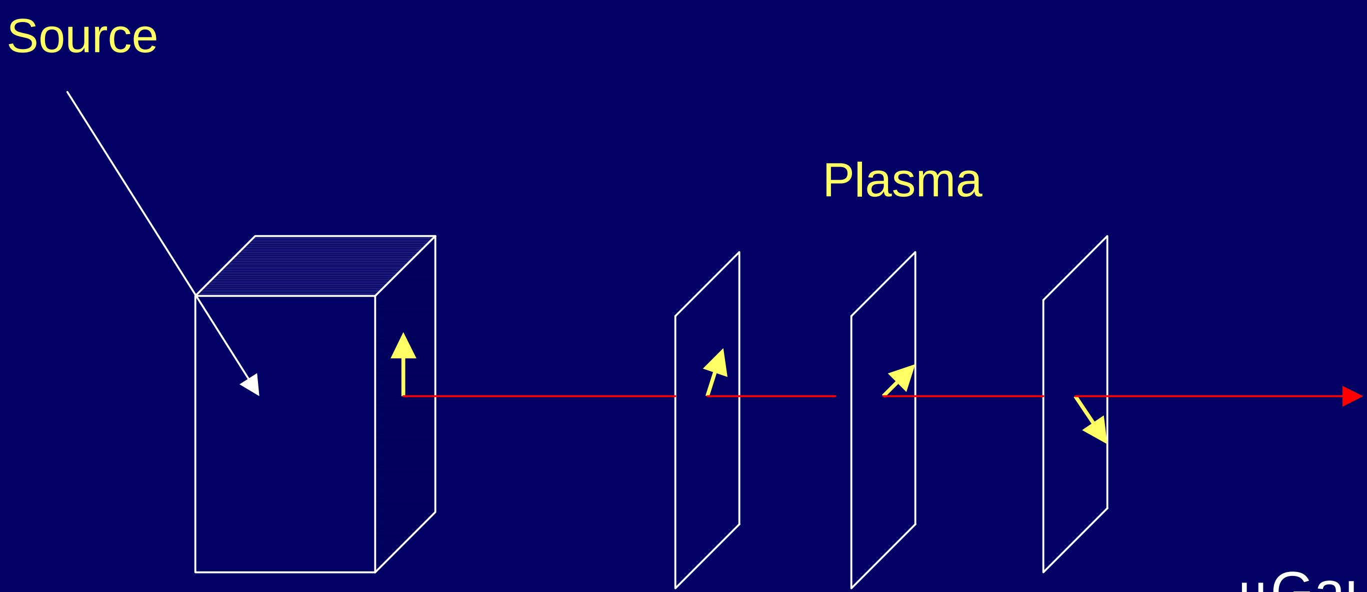
- Hence for interferometers with alt-az mounts, observations of a calibrator over a range of parallactic angles can separate source and instrumental polarisation

- The total intensity distribution is the average of the transform of the parallel hand (RR and LL) correlations

- The linear polarisation information resides in the cross-hands (RL and LR).

- Circular polarisation information is obtained from the difference of the parallel hand correlations

Faraday Rotation



$$\Psi = \Psi_0 + RM\lambda^2$$

μGauss

Kpc

$$RM = 812 \int_0^L n_e B_{11} dl \text{ radians/m}^2$$

cm^{-3}

Why do polarimetry ?

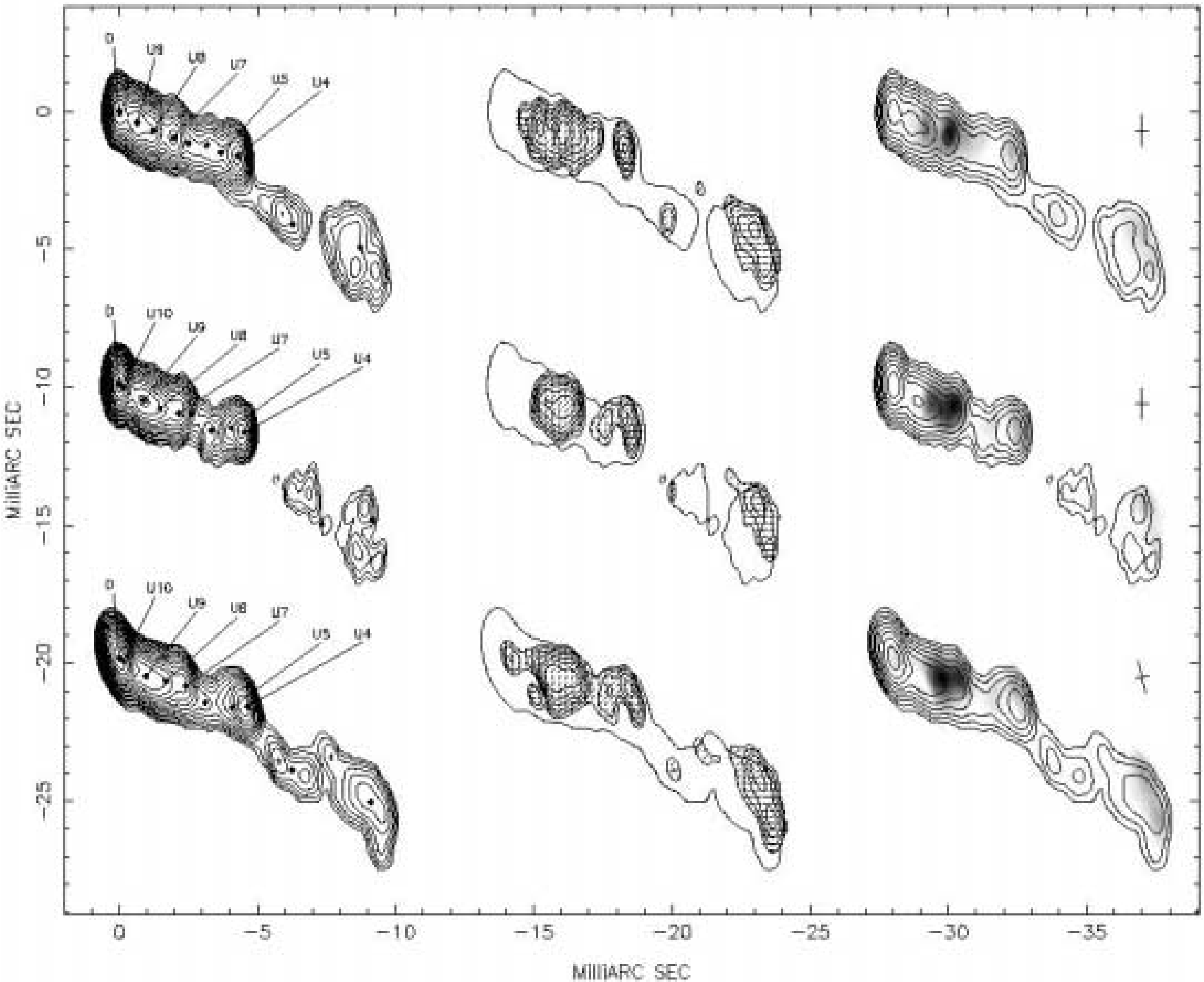
- Polarimetry yields information on the physical state and geometry of the source and the intervening material, that cannot be obtained by other observations. A non-exhaustive list would include:

- Can determine the orientation and order of magnetic fields (through the direction of E vectors and degree of polarization)

- Decide the nature of emission mechanism (e.g. synchrotron, thermal) which in turn casts light on nature of the source

- See the effects of fluid dynamical structures such as shocks (through their effect on the magnetic field)

- Polarisation observations are sensitive to the bulk motion of the radiating plasma (through relativistic aberration)



D
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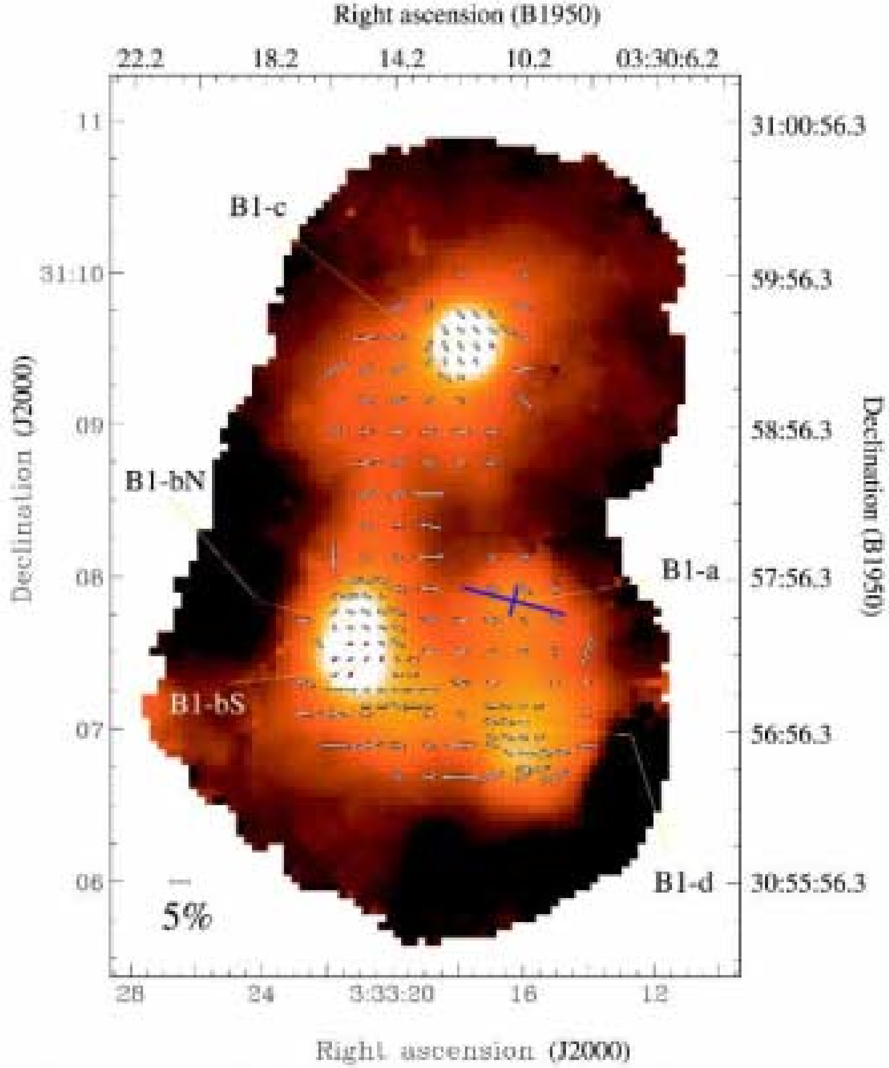

FIG. 1.—850 μm polarization pattern toward the "main core" of Barnard 1 overlaid on the Stokes I map. The gray-scale range is −1 to 3 σ. Polarization data were sampled at 3" and have been binned to 6" (approximately half the JCMT beamwidth of 14") on the bright cores where signal-to-noise ratio (σ_p) is high and 12" in the fainter regions. All vectors are associated with Stokes I values greater than 20% of the B1-d peak flux, $\sigma_p > 3$, and an uncertainty in polarization percentage, dp, less than 1%. Red vectors have polarization percentage, p, less than 1% and were not included in any analysis of the polarization pattern. The vectors are accurate in position angle to better than 10°. The peaks B1-a, B1-bN, and B1-bS have been labeled according to the designations of Hirano et al. (1999). We have designated the other two peaks B1-c and B1-d. The position of IRAS 03301+3057 is marked in blue where the lines denote the extent of the uncertainty in the *IRAS* position. The mean polarization percentage of the plotted vectors is 3.6% with a standard deviation of 2.2%. Coordinates in J2000.0 and B1950.0 are shown.

- They are also sensitive to the thermal particle environment, both mixed into and surrounding the radiating material (by the Faraday effect)

- WELCOME TO THE THIRD DIMENSION!