Non-imaging Analysis and Self-calibration

Steven Tingay
Swinburne University of Technology

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Self-calibration (self-cal)

The first half of this talk borrows heavily from chapter 10 of the NRAO synthesis imaging book by Cornwall and Fomalont (1999).

Following the calibration of interferometer data by tracking instrumental effects and observing an astronomical calibration source, self-calibration can allow further calibration using the data for the target source itself, under certain conditions.

Why do we need self-cal?

The complex visibility output of a well-designed interferometer can be closely approximated by:

$$V'_{ij}(t) = g_i(t) g_j^*(t) V_{ij}(t) + \epsilon_{ij}(t)$$

- After ordinary calibration, residual errors remain in the gains.
- Temporal and spatial variations in the atmosphere distort the incoming wavefront (ATCA - mm observing!!);
  - Varies with elevation, frequency, weather etc;
- Weak or resolved calibration source;
- Errors in the geometric model;

How does self-cal work?

Self-cal can be used to estimate these residual errors by *treating the complex gains as free parameters*. In many cases, the target source data contain enough information to solve for the source structure as well as the complex gains for each antenna.

For an array of N elements, there are N unknown complex gains corrupting the ½N(N-1) visibility measurements. Therefore at least ½N(N-1)-N "good" complex numbers remain in the data; these can be used to constrain the intensity distribution of the target source.

ATCA (6 ant): 15 visibilities, 6 gains

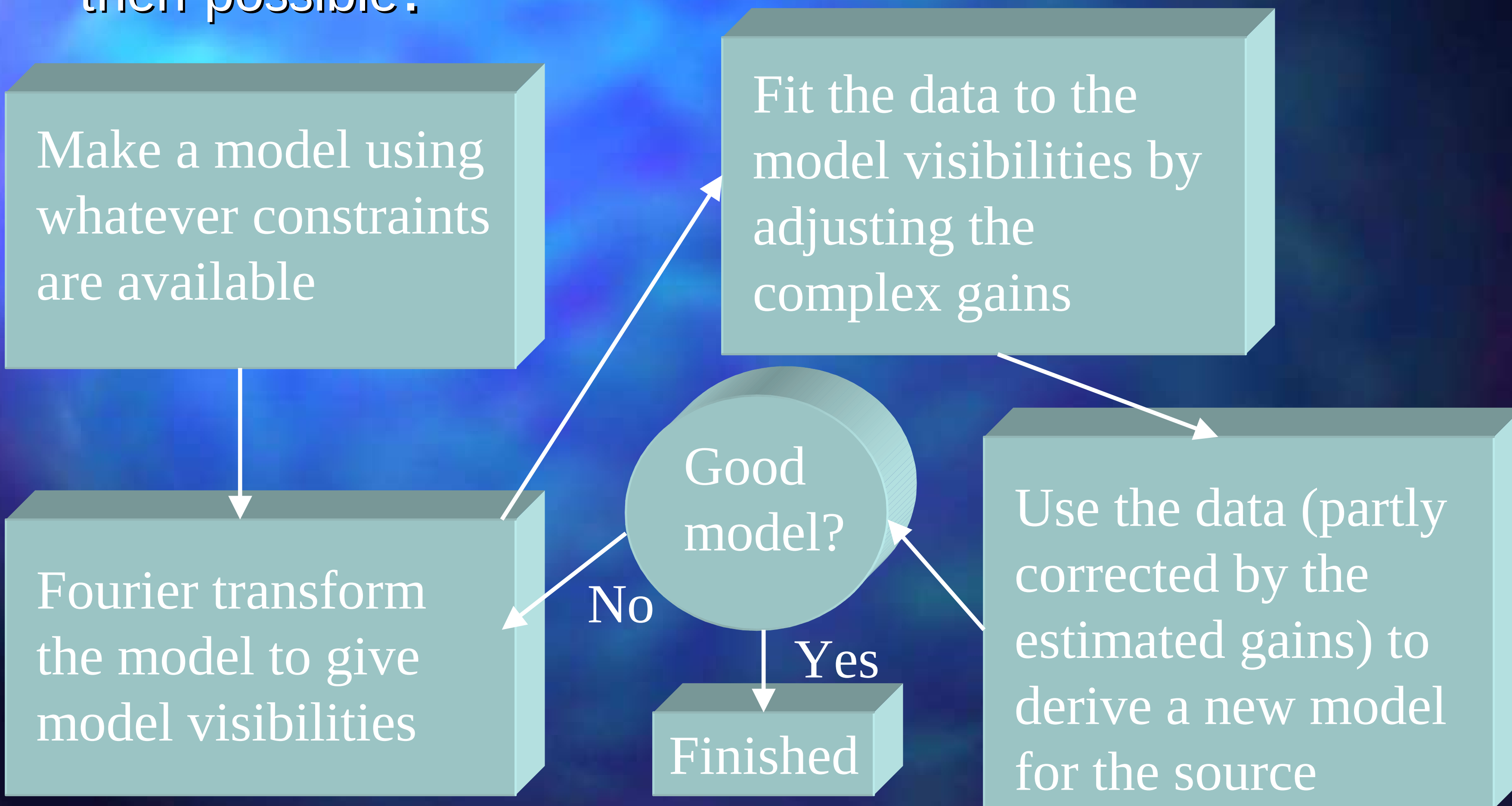
ATCA (5 ant): 10 visibilities, 5 gains

In allowing the gains to be free parameters, something is lost:

- The absolute position of the source;
- Absolute source strength information;
- The ability to distinguish between various types of source structures;
- But as N increases, the ratio of constraints to the number of unknown gains increases without bound, so for large N little is lost.

The degrees of freedom introduced due to the gains are balanced in self-cal by *a priori* knowledge of the source. For example, the corrupted data may still be used to produce an adequate model for the structure of the source, to be used as a starting point in an iterative self-calibration of the data.

The model of the source derived from the corrupted data (or by other means) can be used to partially correct the data (similar to calibration using an astronomical source). An iterative approach to estimating the unknown gains is then possible:



Why does it work?

That this iterative procedure should converge has never been rigorously proven. However:

- Self-cal is most successful in arrays with large N, when the number of constraints is far greater than the degrees of freedom due to the gains;
- Most sources are simple relative to the uv plane coverage of an observation and are effectively oversampled, allowing the addition of a small number of degrees of freedom bearable.

Caveats

- Self-cal fails in low signal to noise regimes – quantitative estimate is possible - ~100 mJy with the ATCA, 100 MHz bandwidth (cm wavelengths);
- Self-cal can also fail when the source is too complex relative to the model.

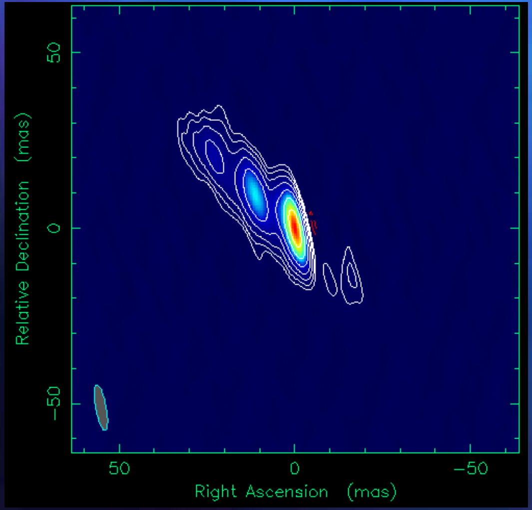
Miscellany

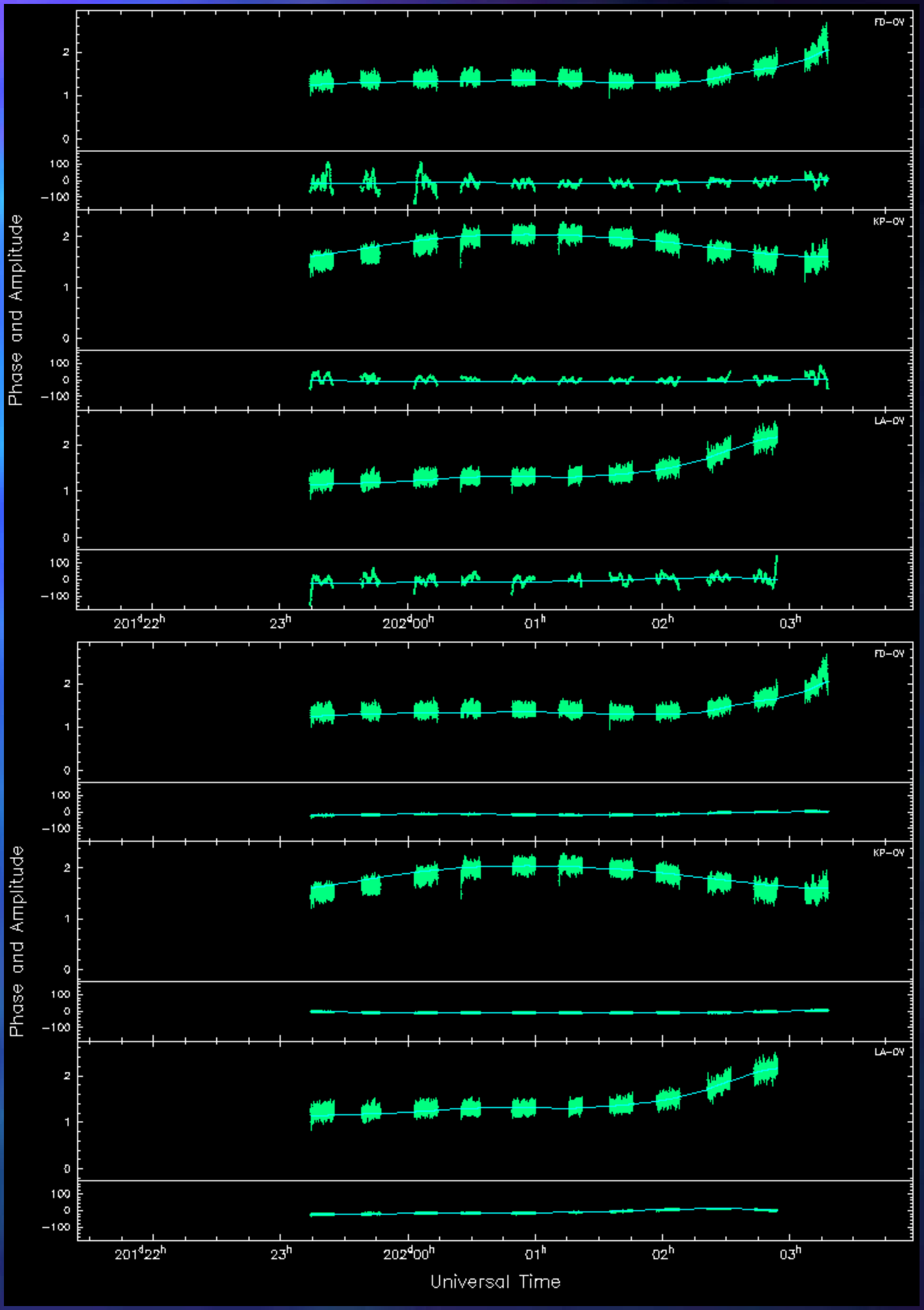
- Amplitude/phase self-calibration;
- Different weighting schemes;
- Averaging times;
- Spectral line self-cal;
- Image errors;

An example

VLBA data for Centaurus A:

- 8.4 GHz;
- 7 antennas;
- Elevation ~ 20 deg;
- Clean/phase self-cal





Non-imaging Analysis

Again I borrow heavily from a chapter in the synthesis imaging book, chapter 16 by Tim Pearson (1999).

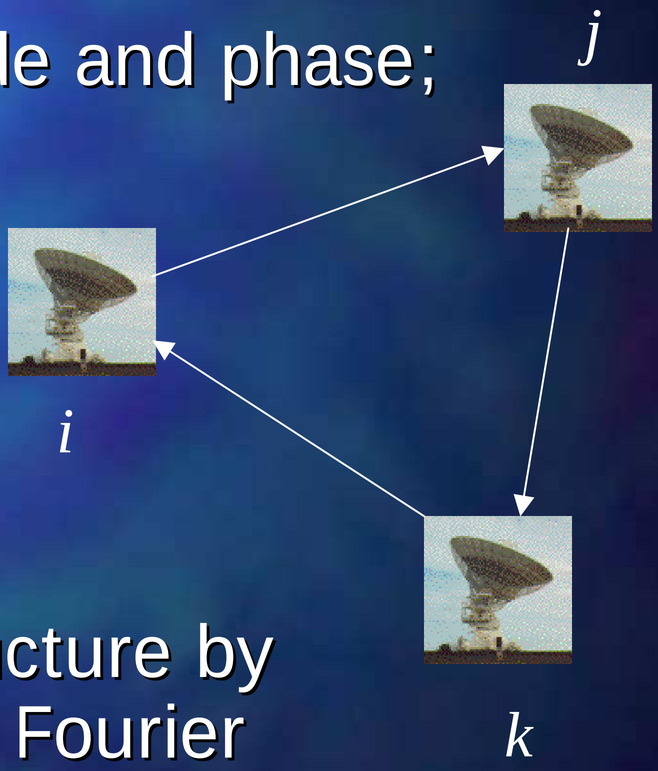
Interferometer data are measured in the uv plane. Thus the most direct analysis of the data occurs in this plane. Errors are also often easier to recognise in the uv plane than in the image plane and are generally better understood in the uv plane. Sometimes datasets are too sparse to image and analysis in the uv plane is the most sensible option.

Inspection of visibility data

- Plots of amplitude and phase against time and distance (along various position angles) in the uv plane;
- Closure quantities – amplitude and phase;

$$\phi'_{ij}(t) = \phi_{ij}(t) + \theta_i(t) - \theta_j(t) + noise$$

$$C'_{ijk}(t) = \phi'_{ij}(t) + \phi'_{jk}(t) + \phi'_{ki}(t)$$
$$C'_{ijk}(t) = \phi_{ij}(t) + \phi_{jk}(t) + \phi_{ki}(t) + noise$$
$$C'_{ijk}(t) = C_{ijk}(t) + noise$$



- Get a feel for the source structure by comparing the uv data to the Fourier transforms of basic intensity distributions;

Model-fitting

This is what is generally meant by non-imaging data analysis – a method of building a detailed model for the source structure that does not involve Fourier transforms of the uv plane data, or de-convolution.

- Model is generated in the uv plane by operations on the uv plane data, as opposed to operations in the image plane when using clean;
- Certain similarity to self-calibration.

Three steps to model-fitting success:

- The user defines (guesses) a model for the source, parameterised by a known number of free (adjustable) parameters.
- The *model* is then Fourier transformed into the uv plane to produce model visibilties
- Compare the model visibilities to the uv plane data and adjust the free parameters of the *model* so as to fit the model visibilities to the data (this is the similarity to self-cal).

Model-fitting success:

1. Best-fit values for the free parameters of the model;
2. A measure of the goodness-of-fit for the best-fit model (relative to the measurement errors);
3. Estimates of the uncertainty in the best-fit parameters.

Limitations to this approach include:

- May be difficult to define a starting model parameteristion;
- Solutions are not unique;
- Slower than conventional imaging (Fourier invert/clean);
- Least-squares method probably not strictly appropriate;
  - assumes that the errors are Gaussian, uncorrelated, and no calibration errors;
  - degrees of freedom introduced during self-cal should be taken into account.
- Uncertainties on the model parameters can be difficult to quantify.

Model-fit errors

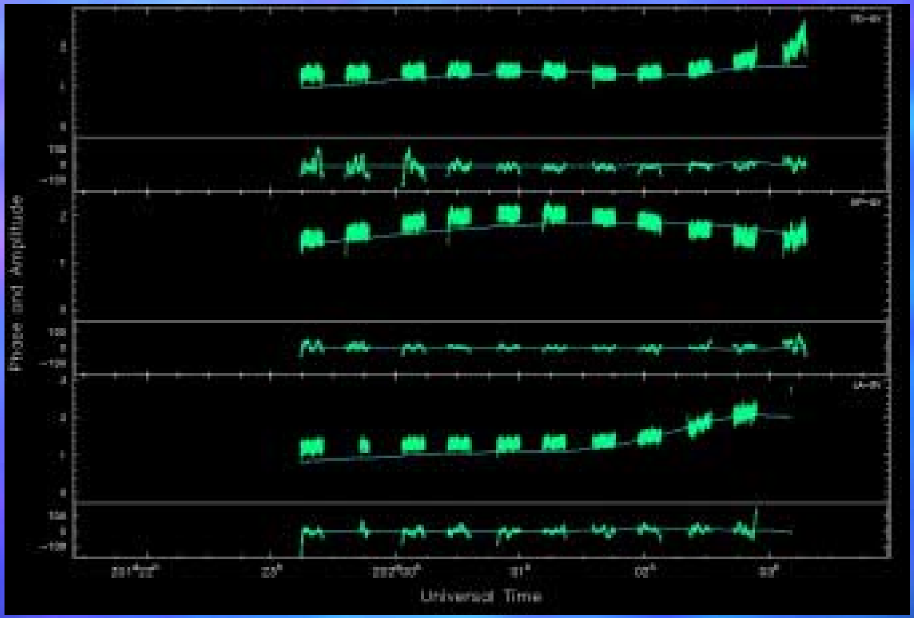
- Covariance matrix to see which parameters are constrained and how they are correlated;
- Contours of constant chi-square for single parameters or sets of parameters;
- Caution must be exercised when using any theoretical measures of confidence since they assume fully independent data for which the visibility errors are fully understood and are distributed appropriately for the statistical tests being used;
- Monte Carlo tests are useful but time-consuming!!!

Software

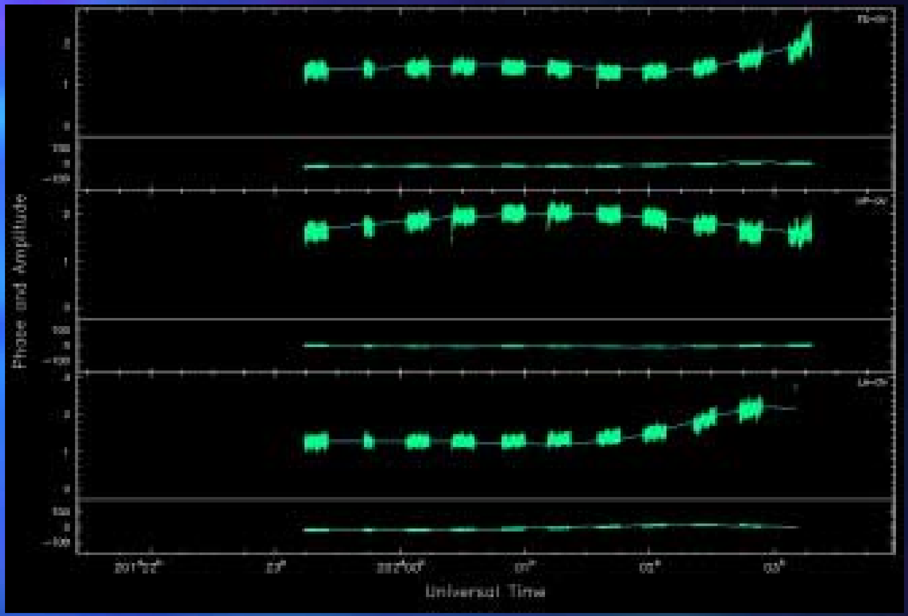
- UVFIT in AIPS does a least-squares fit to the real and imaginary parts of the visibilities;
- SLIME, an AIPS add-on by Chris Flatters combines model-fitting and a graphical interface;
- DIFMAP (Shepherd 1994) contains a very nice model-fitting interface (supplemented by DIFWRAP for error estimation – Jim Lovell);
- MIRIAD has UVFLUX which fits point source models to the real and imaginary parts of the visibilities.

In summary, despite these limitations, working with the data in the uv plane still offers some advantages over imaging, especially for simple sources and/or sparse datasets – imaging and model-fitting are in some sense complementary:

- Errors in the data are most easily recognised and are better understood in the uv plane;
- Quantitative estimates of source structure can be made for reasonably simple sources with high signal to noise, well-calibrated data;
- When the data are poor, sometimes there is little choice.



Phase and Amplitude
Universal Time



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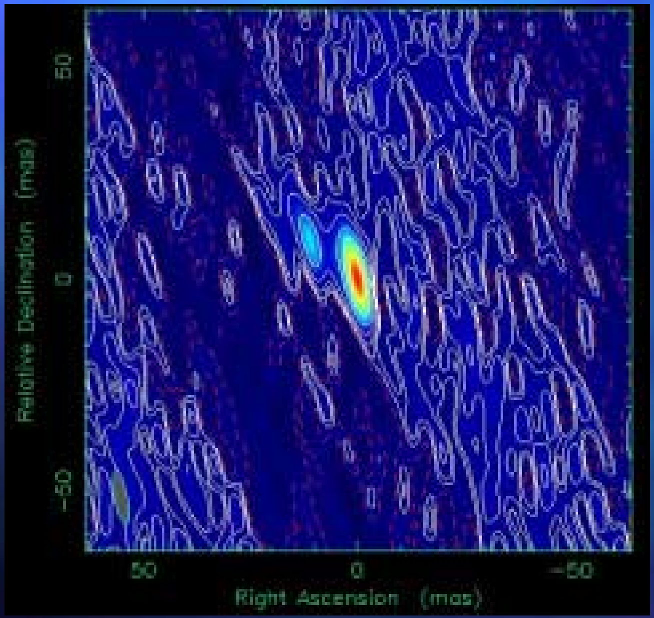
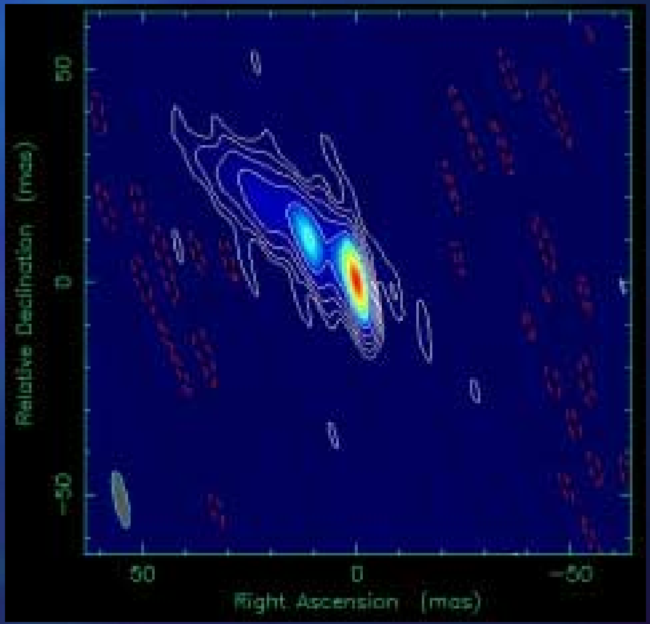
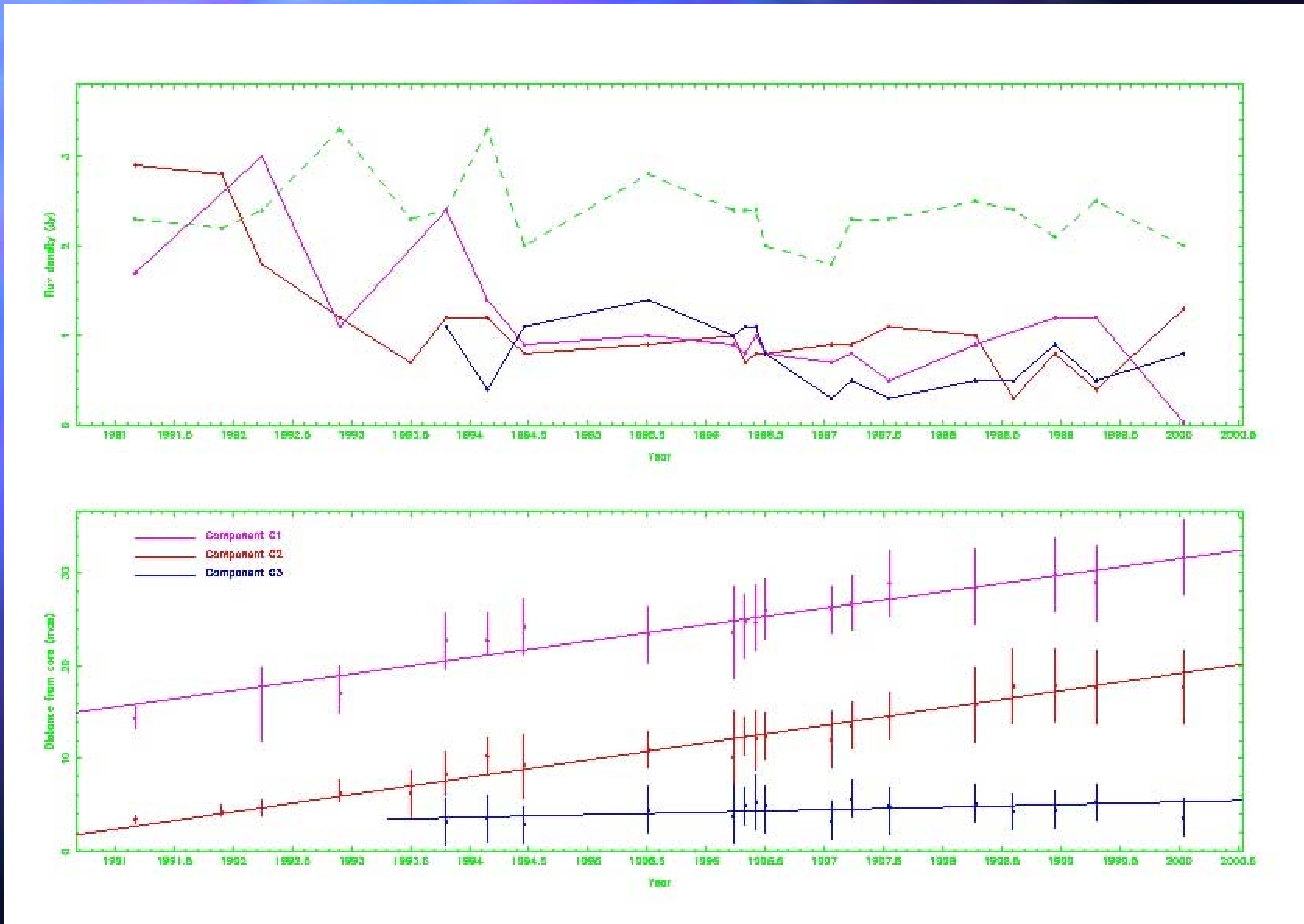


Relative Declination (mas)
Right Ascension (mas)



Relative Declination (mas)
Right Ascension (mas)


Flux density (Jy)
Year
Component C1
Component C2
Component C3
Distance from core (mas)
Year