Interferometer sensitivity

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Abstract

The effect of various parameters on the sensitivity of a radio interferometer is introduced, with a few examples.

1. **Temperature as a measure of power**

One can treat the power received from an astronomical source as being roughly equivalent to that of a black body. Formally, a black body is represented by the Planck law, where k is the Boltzmann constant and h is the Planck constant.

$$\mathbf{P\ d\nu\ =\ \frac{h\nu}{e^{h\nu/kT}-1}\ d\nu} \tag{1}$$

However, for low frequencies (and temperatures that are not too low, thus for $h\nu \ll kT$, the "Rayleigh-Jeans law"), this can be approximated as

$$\mathbf{P\ d\nu\ =\ k\ T\ d\nu} \tag{2}$$

This implies that the power received is directly proportional to a temperature. Now, WHICH power are we talking about?

2. Definition of the antenna temperature

Let us assume now that our telescope observes over a limited frequency range, $\Delta\nu$ (the so-called "passband"), with constant response over that range. When pointing at a blank portion of the sky, there are only contributions from what one would call "background" or "noise". The power of this component can be expressed as

$$P_N = k \, T_{sys} \, \Delta\nu \, G \tag{3}$$

Similarly, the power of an astronomical source can be expressed as

$$P_a = k \, T_a \, \Delta\nu \, G \tag{4}$$

where

$$T_a = \frac{\eta_a \, A \, S}{2k} = K \, S \tag{5}$$

T_a is the so-called "antenna temperature", which is a measure of the temperature equivalent of the power received from an astronomical source.

3. Antenna efficiency

K, the so-called "antenna efficiency", is a measure of what antenna temperature one gets out of a fiducial received astronomical source flux density. For the ATCA, K is of order 10-15 K/Jy, depending on the observing frequency.

$$\mathbf{K} = \frac{\eta_\mathbf{a}\mathbf{A}}{\mathbf{2k}} = \frac{\mathbf{A_e}}{\mathbf{2k}}\ [\mathbf{K/Jy}] \tag{6}$$

This implies that we must optimise the conversion rate of incoming signal strength to outgoing power to the correlator. $\mathbf{A}_\mathrm{e}$ is called the antenna's "effective area".

What we need to find out now is which minimal astronomical source power we can detect for a given noise power. If we want to detect an astronomical source, its power must be stronger than the noise power of our equipment (plus sky background).

$$\mathbf{P_a} > \mathbf{P_N}$$

But first: What does the "noise" consist of?

4. Definition of the system temperature

Various sources contribute to the incoming power, a measure of which is the so-called "system temperature".

$$T_{sys} = T_{bg} + T_{sky} + T_{spill} + T_{loss} + T_{cal} + T_{rx} \quad (7)$$

where the different noise contributions originate from:

- T_{bg} = **3 K microwave and galactic background**
- T_{sky} = **Atmospheric emission**
- T_{spill} = **Ground radiation scattering into the feed**
- T_{loss} = **Losses in the feed and input waveguide**
- T_{cal} = **Injected calibration signal**
- T_{rx} = **Thermal receiver/electronics noise**

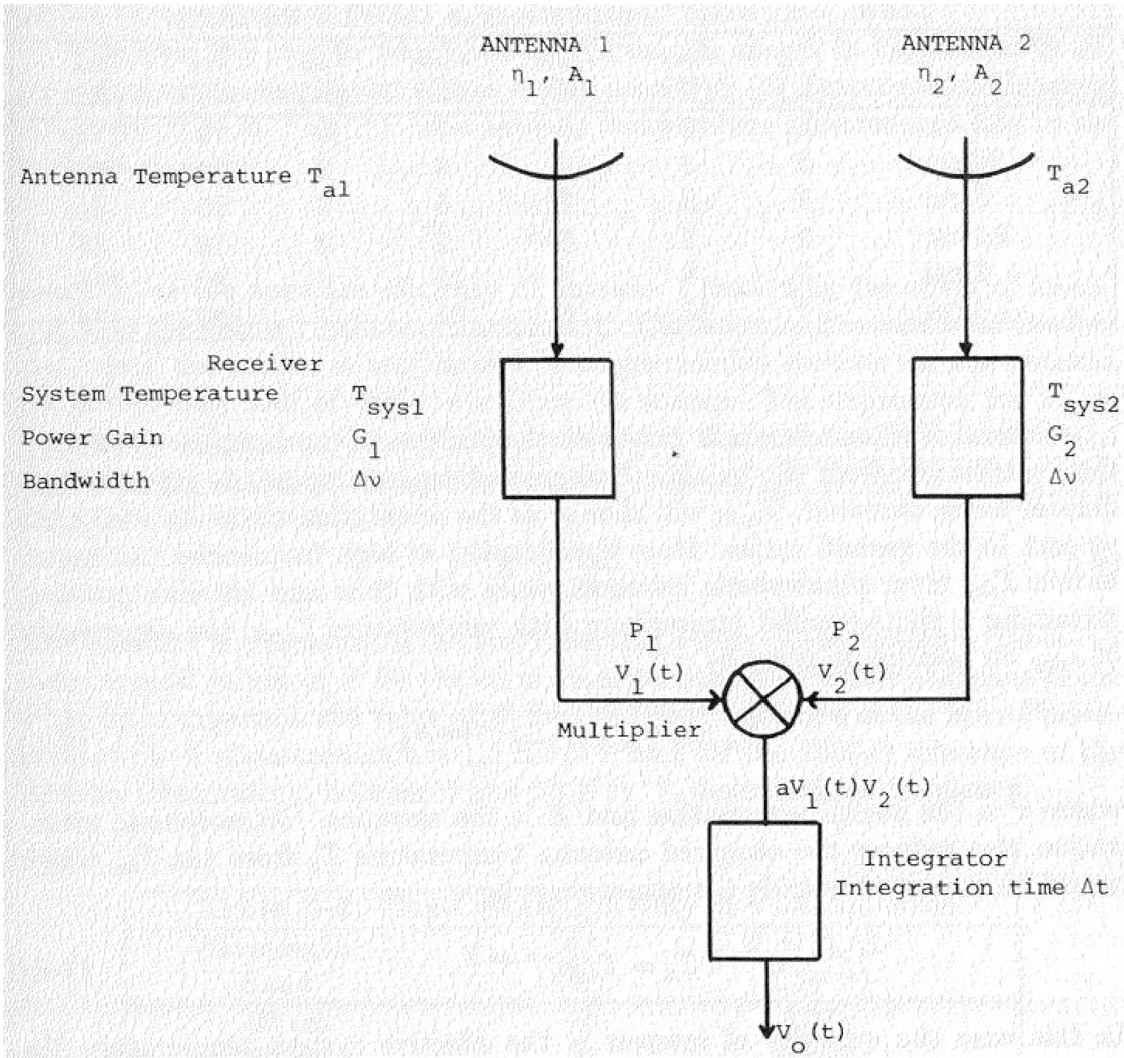
5. Effects not considered here

The following effects which can also contribute to the "noise" are not taken into consideration here:

- Errors in the calibration of the complex gains,
- atmospheric amplitude and phase instabilities,
- sidelobe effects,
- source confusion,
- radio frequency interference,
- bandwidth distortions,
- correlator DC offsets.

The term noise is put in parentheses, because these effects are in part systematic, not random, and thus influence the data quality in a different way than real noise. They must also be treated differently in the process of data reduction, if detected.

6. Two-antenna single-multiplier correlation interferometer



7. Signal correlation in an interferometer

The output of a multiplier is the amplified power of the input signals from a pair of antennae.

$$\mathbf{P_{12} = G_1 G_2 (s_1 + n_1)(s_2 + n_2)} \tag{8}$$

Integrating over a short time interval, i.e. creating an average measurement, this transforms into

$$\begin{aligned} < \mathbf{P_{12}} > &= < \mathbf{G_1 G_2 (s_1 + n_1)(s_2 + n_2)} > \\ &= \mathbf{G_1 G_2} < \mathbf{s_1 s_2 + s_1 n_2 + s_2 n_1 + n_1 n_2} > \\ &= \mathbf{G_1 G_2} < \mathbf{s_1(t) s_2(t)} > \end{aligned} \tag{9}$$

The power from each antenna, for a source of flux density S, is

$$\mathbf{P_i \;=\; G_i^2 \frac{A_e \Delta\nu}{2} \, S = \; G_i^2 \, K_i \; k \; \Delta\nu \; S} \tag{10}$$

The voltage waveform from each antenna is

$$\mathbf{S_i(t) \;=\; G_i \sqrt{K_i \; k \; \Delta\nu} \; \tilde{S}_i(t)} \tag{11}$$

Inserting these, we obtain for $< \mathbf{P_{12}} >$

$$\begin{aligned} < \mathbf{P_{12}} > &= \mathbf{G_1 G_2 \sqrt{K_1 K_2} k \Delta\nu} \;\; < \mathbf{\tilde{S}_1(t) \tilde{S}_2(t)} > \\ &= \mathbf{G_1 G_2 \sqrt{K_1 K_2} k \Delta\nu \; S_C} \end{aligned} \tag{12}$$

The quantity that really interests us here is the uncertainty of $< \mathbf{P_{12}} >$, $\sigma(< \mathbf{P_{12}} >)$. The derivation of this quantity was described by Ravi Subrahmanyan during the last workshop (notes are available online).

8. Total system noise

For measurements on a blank area of the sky, $\sigma(< \mathbf{P_{12}} >)$ corresponds to the total rms noise of our system, ΔS.

For the sake of simplicity, let us assume that the system temperatures, efficiencies and gains of all antennae are equal. Then, for weak sources

$$\mathbf{\Delta S} = \frac{\mathbf{T_{sys}}}{\mathbf{K}} \frac{\mathbf{1}}{\sqrt{\mathbf{2\Delta t\Delta}\nu}} = \frac{\sqrt{\mathbf{2}}\mathbf{kT_{sys}}}{\eta_{\mathbf{a}}\mathbf{A}\sqrt{\mathbf{\Delta t\Delta}\nu}} \tag{13}$$

Taking into account that also a correlator causes losses, another factor is added - the correlator efficiency

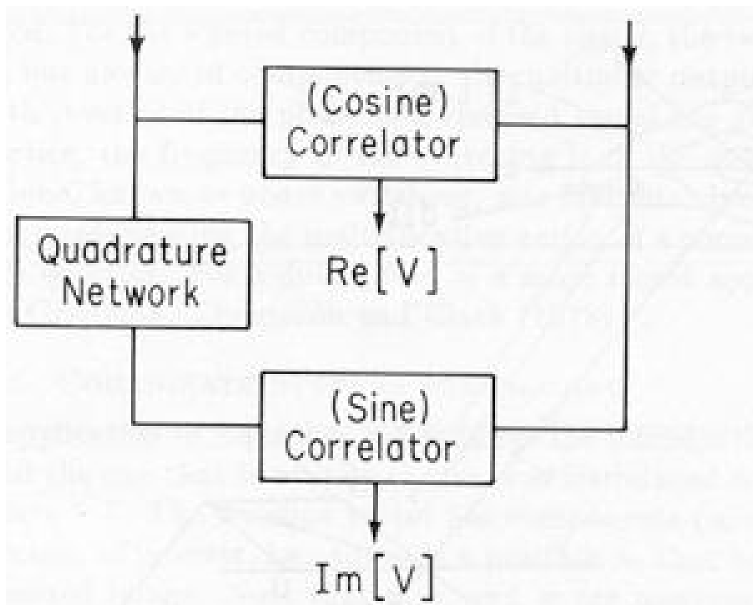
$$\mathbf{\Delta S} = \frac{\sqrt{\mathbf{2}}\mathbf{kT_{sys}}}{\eta_{\mathbf{c}}\eta_{\mathbf{a}}\mathbf{A}\sqrt{\mathbf{\Delta t\Delta}\nu}} \tag{14}$$

9. Two-antenna complex correlation interferometer

A visibility is a complex quantity.

$$\mathbf{V(u,v)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{A(l,m)I(l,m)e^{-2\pi i(ul+vm)}\,dl\,dm} \qquad (15)$$

A single correlator (=multiplier+integrator) samples only the real part of a visibility, i.e. the cos term. Introducing a 90° phase shifter and using a second, independent correlator to register this signal, one samples both the real and imaginary part.



The big advantage is that, because the noise of the two correlators is independent, one gains a factor of $\sqrt{2}$.

10. Interferometers vs. single dishes

The formula for ΔS from above is valid for a 2-element interferometer.

$$\Delta S = \frac{\sqrt{2} k T_{sys}}{\eta_c \eta_a A \sqrt{\Delta t \Delta \nu}} \tag{16}$$

A single dish with the same effective area as the pair of interferometer antenna, but otherwise exactly the same instrumental setup, would perform better by a factor of $\sqrt{2}$.

However, interferometers can get close to compensating for this disadvantage.

The full formula for the imaging sensitivity of an interferometer is

$$\Delta I_m = \frac{\sqrt{2} k T_{sys}}{\eta_c \eta_a A \sqrt{N_b N_{IF} \Delta t \Delta \nu}} \tag{17}$$

N_b is the number of baselines in the interferometer. If at any time the input signal of each antenna is correlated with that of each of the others, the total number of baselines is

$$N_b = \frac{N(N-1)}{2} \tag{18}$$

For large numbers of N, this term approaches $N^2/2$, which leads to the formula valid for single dishes.

11. Extended sources

All formulae derived above are valid for point sources at the phase centre of the interferometer. The behaviour of extended sources is slightly different.

For a point source, the quantity ΔI_m, which can be expressed in units Jansky per beamsize, is *independent* of the beam size. This means that also the signal-to-noise ratio will be independent of the beam size.

The same is not true for extended sources! For an extended source (with constant surface brightness) that is larger than the synthesised beam, the flux density per beam varies as a function of beam size as

$$\frac{I\Omega_s}{\Delta I_m}$$

where $I\Omega_s$ is the flux density per beam area and ΔI_m is the imaging sensitivity of the interferometer.

RESOLUTION KILLS!

12. Influence of other important factors

- **Use of more than one IF. When making use of different passbands, with independent receivers and amplifiers, one gains sensitivity.**
- **Primary beam attenuation reduces the sensitivity for imaging in the outer regions of the field of view.**
- **Polarisation measurements - important in terms of sensitivity, because measuring two orthogonal wavefronts yields a gain of a factor of $\sqrt{2}$ for unpolarised sources. They also provide NEW information on magnetic fields.**
- **Convolution and gridding in the post-observation data processing can also lead to losses.**

13. Special considerations for millimetre astronomy

- The optical depth of the atmosphere, τ, becomes important.
- Up to now, no low-noise amplifiers were used in front of the mixers. The ATCA is the first interferometer to make use of this new technology.
- This is because T_{rx} is important in the mm regime.
- Consideration of field-of-view vs. collecting surface area: Maximising the antenna surface area A, which makes the telescope sensitive, decreases at the same time the field of view of an interferometer. Especially in the millimetre regime, but in general for imaging of extended sources on scales of the primary beam or larger, one ideally should have as many as possible moderate-size antennae.

14. Follow-up reading

This lecture was developed using parts of several articles from the available literature. I recommend for further reading the books from which these were taken.

- "Synthesis Imaging in Radio astronomy", Crane P. C. and Napier P. J., ASP Conf. Ser. 6, 1989, Chapter 7
- "Synthesis Imaging in Radio astronomy II", Wrobel J. M. and Walker R. C., ASP Conf. Ser. 180, 1999, Chapter 9
- "Tools of Radio Astronomy", Rohlfs K., 1986, Springer
- "Interferometer Sensitivity", Subrahmanyan R., ATNF Synthesis Workshop 2001