- Radio source Luminosity

L_ν $\quad$ $W \; sr^{-1} \; Hz^{-1}$

towards the antenna

at freq ν_e

- Distance to source

$\equiv$ Luminosity distance

D $\quad$ m

- Flux density of radiation at the antenna :

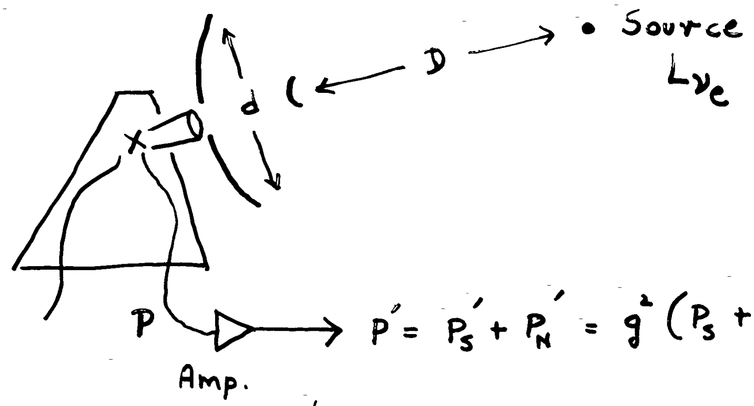
$$S_{\nu_0} = \frac{4\pi \, L_{\nu_e}}{4\pi \, D^2 (1+z)} \quad W \, m^{-2} \, Hz^{-1}$$

at freq ν_0

$\nu_0 \longleftrightarrow \nu_e$

relationship.

- Doppler Effect
- Cosmological redshift
- Gravitational redshift.



$$P' = P_s' + P_N' = g^2 (P_s + P_N)$$

gain $= g$; b.w =

- Signal Power from the source

$$P_s = (f_p \, S_{\nu_0}) \times (A_e = \eta_a A = \eta_a \pi d^2/4) \times \Delta\nu \quad W$$

a single probe picks up one pol.
$f_p = \frac{1}{2}$ if source is randomly polarized

- Post amplification

$$P_s' = g^2 \left(\frac{1}{2} S_{\nu_0} A_e \Delta\nu \right)$$

- Additive noise present post amplification

$$P_N' = P_{bg} + P_{atm} + P_g + P_r$$

P_{bg}: b.g. sources + uniform b.g

P_{atm}: Atmosp. emission

P_g: ground emission

P_r: due to losses before Amp + Noise added amp. in forward

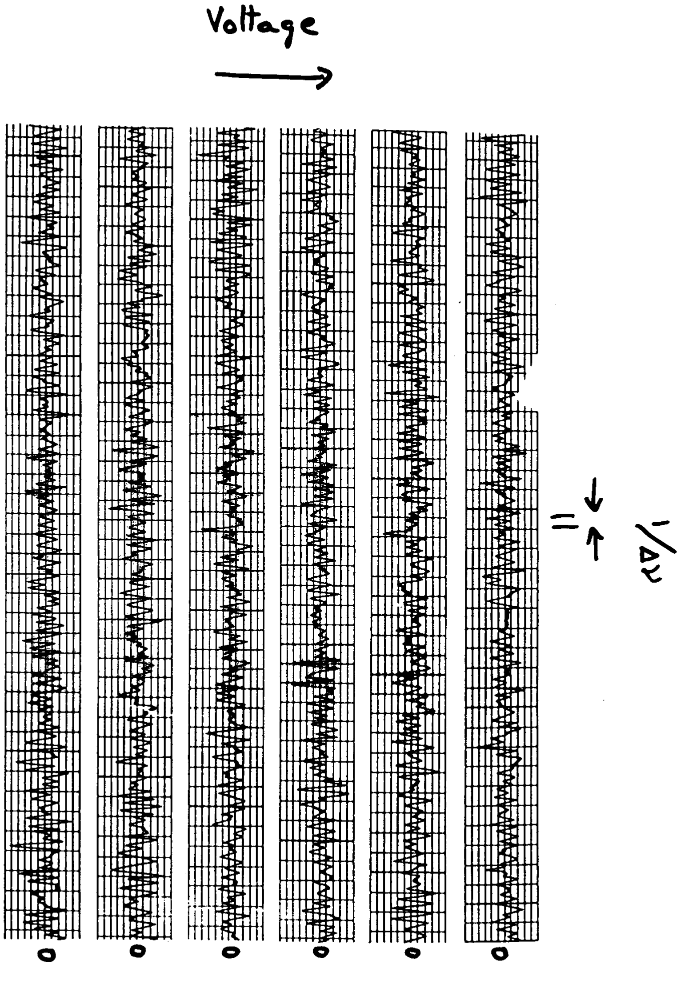
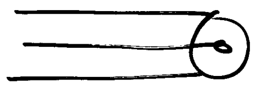


FIG. 1-1 Oscillogram of wide-band thermal noise, 200 to 3,000 cycles.

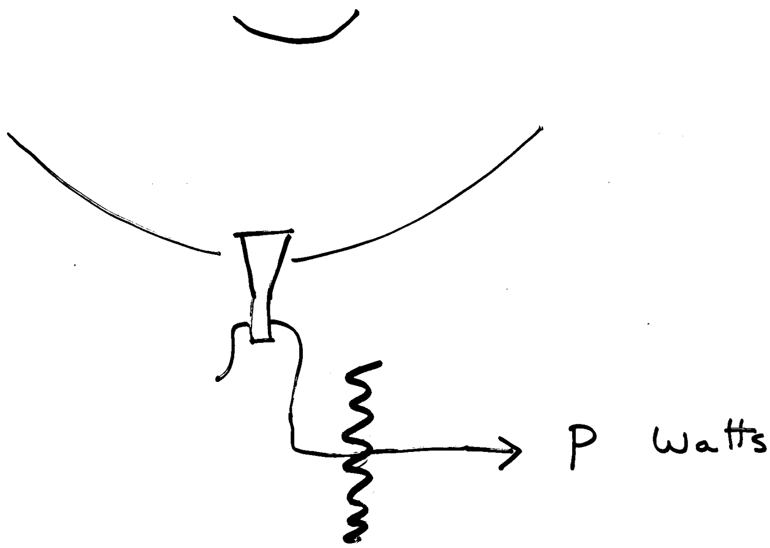
Power in a Gaussian Random Voltage Wave form



$$P = \frac{\langle v^2(t) \rangle}{R}$$

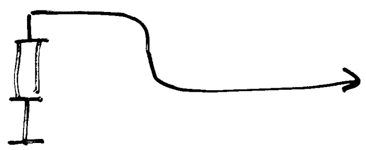
Cable characteristic impedance

$50\,\Omega$?



Replace antenna by resistor R at Temp. T

R, T



$$P = \frac{h\nu\,\Delta\nu}{e^{h\nu/k_B T} - 1} \quad \text{Watts}$$

$$\approx k_B T \,\Delta\nu \qquad h\nu \ll k_B$$

when T is adjusted to get same power P as from the antenna

$$\boxed{T = \frac{P/\Delta\nu}{k_B}.}$$

Total Power $\longrightarrow$ T_{sys} System Temperature

Signal power $\longrightarrow$ T_s Antenna Temp.

Noise

b.g $\longrightarrow$ T_{bg}

gnd $\longrightarrow$ T_g

atm $\longrightarrow$ T_a

Rec.

$g^2 P_r$ post. amp. $\longrightarrow$ T_r Receiver Temp.

ATCA

	20cm (L)	13cm (S)	6cm (C)	3cm (X)
Apert. Eff η_a	0.68	0.52	0.69	0.65
$\frac{1}{K}$ Jy / K	10.7	14.0	10.5	11.2
b.g. + Atm + gr.	11	11	12	14
losses	10	10	9	13
T_r (LNA)	9	13	12	13
T_N (K)	30	34	33	40
K/T_N	0.0031	0.0021	0.0029	0.0022

- Similar contrib. from $\left\{ \begin{array}{l} \text{bg. + Atm + gr.} \\ \text{losses} \\ T_r \end{array} \right.$

- $T_S \ll T_N$: At all stages, including before LNA, $\langle v_S^2 \rangle^{1/2} \ll \langle v_N^2 \rangle^{1/2}$

  Signal is buried in noise!

Source with flux density S_{ν_0} Jy

$$1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$$

gives power

$$P_S = \frac{1}{2} S_{\nu_0} A_e \Delta\nu$$

or an ant. temp.

$$T_S = \frac{P_S / \Delta\nu}{k_B} = \frac{S_{\nu_0} A_e}{2 k_B}.$$

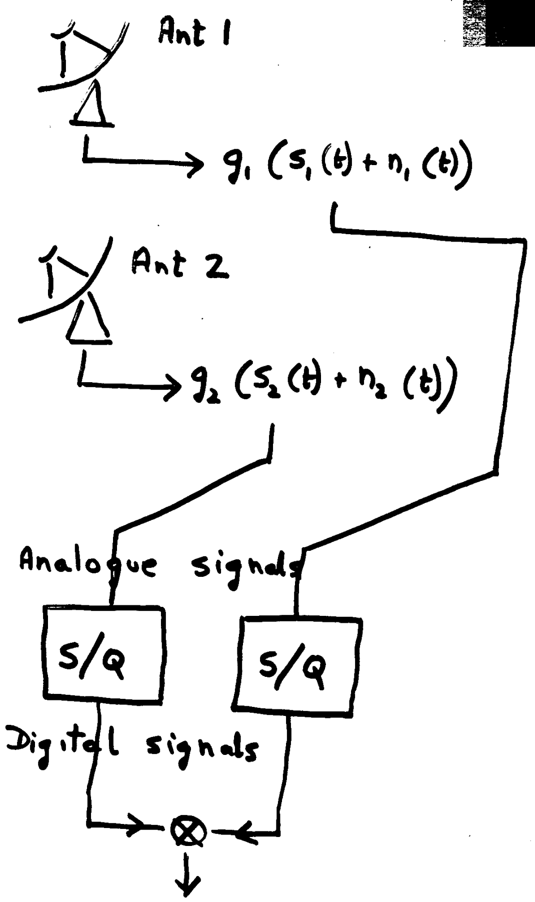
Antenna Sensitivity :

$$\boxed{K = \frac{T_S}{S_{\nu_0}} = \frac{A_e}{2 k_B}}$$

Signal.

$T_S = K S_{\nu_0}$ is the signal

T_{sys} is the noise.



s_1, s_2 : Voltage waveforms due to source.

n_1, n_2 : noise waveforms.

n_1, n_2 are indep. Gaussian random var

$\langle n_1(t)\, n_2(t)\rangle \equiv 0$

noise comp. are not correlated.

$$P_{12} = g_1 g_2 (s_1+n_1)(s_2+n_2)$$

$$\langle P_{12}\rangle = \langle g_1 g_2 (s_1+n_1)(s_2+n_2)\rangle$$

$$= g_1 g_2 \langle s_1 s_2 + \underbrace{s_1 n_2 + s_2 n_1 + n_1 n_2}_{\to 0}\rangle$$

$$= g_1 g_2 \langle s_1(t)\, s_2(t)\rangle$$

ERROR: undefined
OFFENDING COMMAND: x
STACK:

- Source of flux density S_T :

Power in cable from ant. i :

$$P_i = g_i^2 \, \frac{1}{2} S_T \, A_e \, \Delta\nu$$

$$= g_i^2 \, K_i \, k_B \, \Delta\nu \, . \; S_T$$

Voltage wave form from ant i

Source part :

$$s_i(t) = g_i \sqrt{K_i \, k_B \, \Delta\nu} \;\; \tilde{s}_i(t)$$

Voltage

Vector average field across ant. aperture weighted by "illuminati

units : [flux density]$^{1/2}$

- $$\langle P_{12} \rangle = g_1 g_2 \sqrt{K_1 K_2} \; k_B \, \Delta\nu \; \langle \tilde{s}_1(t) \, \tilde{s}_2(t) \rangle$$

$$= g_1 g_2 \sqrt{K_1 K_2} \; k_B \, \Delta\nu \; S_c$$

correlation between the E fields at the two ant. with electronics delays.

- $\tilde{s}_i(t)$ fields are due to S_T

$$|S_c| \leq S_T$$

▷ What is the uncertainty in $\langle P_{12} \rangle$?

$$\sigma^2(P_{12}) = g_1^2 g_2^2 \langle [(s_1+n_1)(s_2+n_2)]^2 \rangle - g_1^2 g_2^2 \langle (s_1+n_1)(s_2+n_2) \rangle^2$$

$$g_1^2 g_2^2 k_B^2 (\Delta\nu)^2 K_1 K_2 S_c^2$$

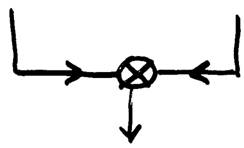
$$g_1^2 g_2^2 \Big[2 \underbrace{\langle (s_1+n_1)(s_2+n_2) \rangle^2}_{k_B^2 \Delta\nu^2 K_1 K_2 S_c^2} + \underbrace{\langle (s_1+n_1)^2 \rangle}_{k_B \Delta\nu (K_1 S_T + T_{N1})} \underbrace{\langle (s_2+n_2)^2 \rangle}_{k_B \Delta\nu (K_2 S_T + T_{N2})} \Big]$$

$$\sigma^2(P_{12}) = k_B^2 (\Delta\nu)^2 g_1^2 g_2^2 \times \left\{ K_1 K_2 S_c^2 + K_1 K_2 S_T^2 + K_1 S_T T_{N2} + K_2 S_T T_{N1} + T_{N1} T_{N2} \right\}$$

$$\boxed{\sigma(P_{12}) = g_1 g_2 k_B \Delta\nu \sqrt{K_1 K_2} \left\{ S_c^2 + S_T^2 + S_T \left(\frac{T_{N2}}{K_2} + \frac{T_{N1}}{K_1} \right) + \frac{T_{N1} T_{N2}}{K_1 K_2} \right\}^{1/2}}$$

[redacted]

▷ <u>Integration</u> of the products :



$$P_{12} = g_1 g_2 \,(s_1 + n_1)(s_2 + n_2)$$

$$\boxed{\text{INTEGRATOR}} \leftarrow$$

Accumulates the products P_{12} over time τ s. Normalizes to give $\langle P_{12} \rangle$

$$\downarrow$$

$$\langle P_{12} \rangle$$

• Uncertainty in the integrator output :

$$\sigma(\langle P_{12} \rangle) = \frac{\sigma(P_{12})}{\sqrt{\#\text{ of prod. av.}}}$$

$$= \frac{\sigma(P_{12})}{\sqrt{\tau \, . \, 2 \, . \, \Delta\nu}}$$

▷ Calibration :

The integrator output is divided by

$$g_1 g_2 k_B \Delta\nu \sqrt{K_1 K_2}$$

to convert $\langle P_{12} \rangle$ to Jy units.

$$\langle P_{12} \rangle \Big|_{cal} = S_c$$

$$\sigma(\langle P_{12} \rangle)\Big|_{cal} = \frac{\left\{ S_c^2 + S_T^2 + S_T \left(\frac{T_{N2}}{K_2} + \frac{T_{N1}}{K_1} \right) + \frac{T_{N1} T_{N2}}{K_1 K_2} \right\}^{\frac{1}{2}}}{\eta_c \sqrt{2 . 2 . \Delta\nu}}$$

This is the rms noise expected in the real and imaginary parts of complex visibility separately.

- No distintion between

  Receiver Noise
  Sky background
  Uncorrelated source flux density.

Special Case:

- Usually $\langle s^2(t)\rangle \ll \langle n^2(t)\rangle$:

$$S_c,\ S_T \ll \frac{T_{N1}}{K_1},\ \frac{T_{N2}}{K_2}.$$

$$\sigma(\langle P_{12}\rangle)\big|_{cal} \approx \frac{1}{\eta_c}\sqrt{\frac{T_{N1}\,T_{N2}}{2\,K_1 K_2\,\tau\,\Delta\nu}}$$

The fluctuations / uncertainty is uncorrelated on different baselines

For antennae with $T_{N1} = T_{N2} = T_N$
$K_1 = K_2 = K$

$$\sigma(\langle P_{12}\rangle)\big|_{cal} \approx \frac{T_N}{\eta_c K\sqrt{2\,\tau\,\Delta\nu}}$$

<u>ATCA</u>

3 cm band

$\Delta\nu = 100$ MHz

$\tau = 10$ s

$\sigma(\langle P_{12}\rangle)\big|_{cal} \approx 11$ mJy.

§ Complex Vis $(S_R, S_I) \longrightarrow$ Image Pixels (I)

Linear transformation:

$$I(l,m) = \frac{1}{2\sum_{1}^{L} T_k \omega_k} \sum_{k=1}^{2L} T_k \, \omega_k \, \underbrace{(S_R + i S_I)}_{L \text{ complex measurements.}} e^{i2\pi(ul + vm)}$$

taper $f(u,v)$ weights $f(\sigma)$

- $\sigma_{k_R} = \sigma_{k_I} = \sigma_k$
- Noise is same in all image pixels.

▷ At pixel $l=0$, $m=0$:

$$I(0,0) = \sum_{1}^{2L} T_k \omega_k S_R \Big/ 2 \sum_{1}^{L} T_k \omega_k \, .$$

$$\sigma_I^2 = \sum_{1}^{L} T_k^2 \omega_k^2 \sigma_k^2 \Big/ \left(\sum_{1}^{L} T_k \omega_k \right)^2 .$$

assuming that fluctuations in all L measurements are uncorrelated.

L = # of Vis. measurements.

$$= \frac{N(N-1)}{2} \cdot \frac{T}{\tau} \cdot N_b \cdot N_p$$

bands # of pol. XX, YY

▷ The σ_k may vary if

- Averaging times τ are different
- T_N varies with Antenna elevation or weather.

w_k may be used to down weight measurements with high σ_k.

▷ If σ_k are same : $w_k \equiv 1$.

No taper : $T_k \equiv 1$.

"Natural wt."

$$\sigma_I^2 = \frac{\sigma^2}{L}$$

$$\sigma_I = \frac{2 k_B T_N}{\eta_c \eta_a A \sqrt{2 \Delta\nu \, N(N-1) \, (T/\tau) \, N_p N_b}}$$

Point source <u>Sensitivity</u>.

This image has lowest σ_I

Best for detection of 'point srcs'

Sources unresolved on longest baselines

Equal vis. amp. on all bsl.

$$\sigma_Q = \sigma_U = \sigma_V = \sigma_I$$

▷ • Extended sources :

sources resolved on longest baselines
vis. ampl. smaller on longer bsl.

Source size $\Omega_s \approx \frac{\pi \theta_s^2}{4} > \Omega_b$: beam size.

$\Rightarrow$ Vis ampl. low for u,v-dist $> \frac{1}{2\theta_s}$

"Optimal taper" : T_k used to down weight vis. beyond $\frac{1}{2\theta_s}$: $\Omega_b \rightarrow \Omega_s$

• Noise :

$$\sigma_I \Big|_{\substack{\text{with} \\ \text{taper}}} \approx \sigma_I \Big|_{T_k = 1} \sqrt{\frac{\Omega_s}{\Omega_b}}$$

• Signal :
(source flux density / beam)

$$S \Big|_{T_k \equiv 1} \lesssim S_T \frac{\Omega_b}{\Omega_s} = S \Big|_{\substack{\text{with} \\ \text{taper}}} \frac{\Omega_b}{\Omega_s}$$

• Signal / Noise :

$$\frac{S_T}{\sigma_I} \left(\frac{\Omega_b}{\Omega_s} \right) \xrightarrow{\text{taper}} \frac{S_T}{\sigma_I} \left(\frac{\Omega_b}{\Omega_s} \right)^{1/2}$$

Detection sensitivity improves for extended sources : factor $\approx \left(\frac{\Omega_s}{\Omega_b} \right)^{1/2}$

▷ σ_I derived are uncertainties in image pixel values : ΔI

▷ σ_I is rms pixel intensities in
- dirty images - before deconvolution
- source free images.

▷ If all sources are deconvolved :
σ_I will be rms pixel intensity in source - free regions.

▷ Deconvolution / Restoration of source free regions (noise) :
rms pixel intensity $\neq \sigma_I$
(effectively T_k is changed)

▷ Convolution - Gridding - FFT - Grid. conv. co
$\Rightarrow \sigma_I$ higher at edges of image.

▷ σ_I uniform within and outside primary beam region.

▷ Correction for p.b. attenuation
$\Rightarrow \sigma_I$ higher away from pointing cen

- For those who wish to think of the incident radiation as <u>photons</u> :

$\nu_0 = 5$ GHz

$\Delta\nu = 100$ MHz

$T_N = 35\,K \Rightarrow 1.5 \times 10^{10}$ photons s^{-1}

$S_{\nu_0} = 1$ Jy $\Rightarrow 4.10^{7}$ photons s^{-1}

$10\,\mu$Jy $\Rightarrow 400$ photons s^{-1}

On timescale $\approx$ inverse of b.w $\times 2$

≈ 5 ns

10^2 noise photons

0.1 signal (1 Jy) photons.

Number of signal photons per sampling time ($\tau \sim \frac{1}{\Delta\nu}$)

$\ll 1$.