

# Principles of Interferometry I

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# Acknowledgements

- Material in this presentation has been borrowed and adapted from talks by Ron Ekers, Dave McConnell, Mark Wieringa, Michael Rupen, Ray Norris and others.

“Starting from modest beginnings and by small steps radio astronomers took the separate fields of antennas, receivers and information theory and welded them into image forming systems that have improved by seven orders of magnitude in resolution, surpassing the optical telescope, and inspired developments in fields as diverse as optics, acoustics, seismic probing and X-ray tomography”

*(Bracewell, in Sullivan [1984])*

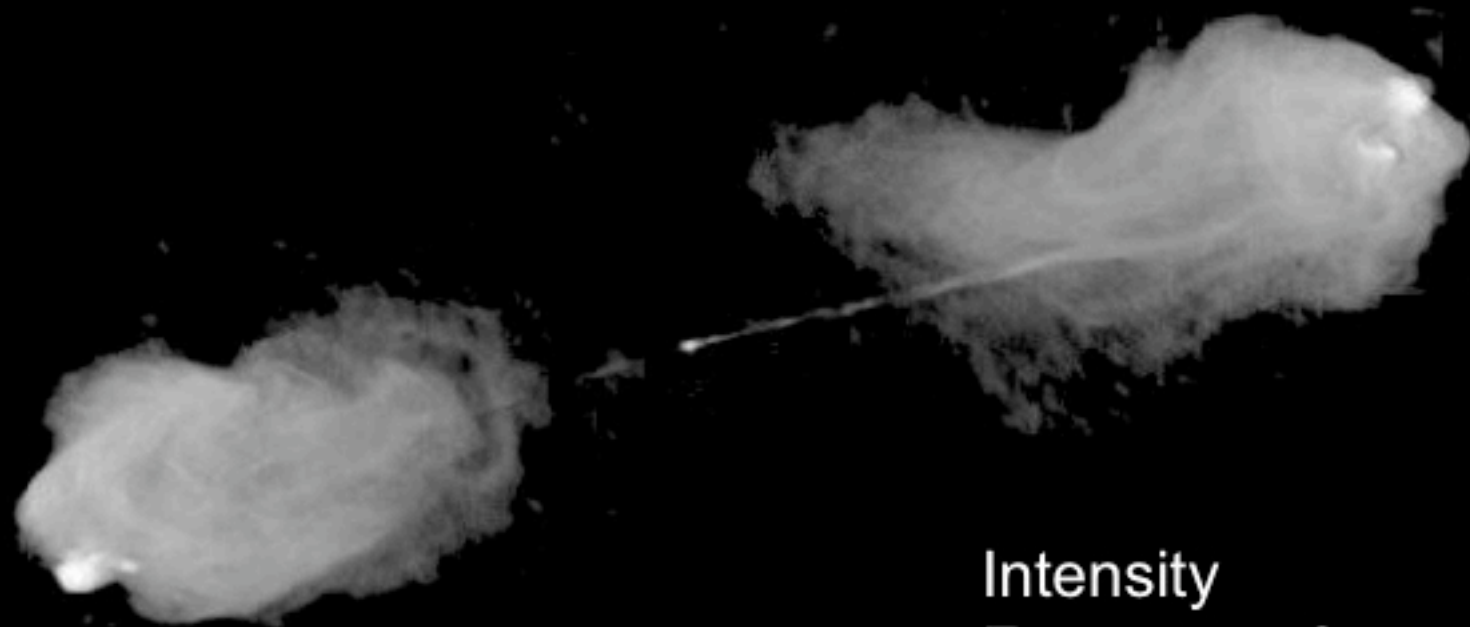
# Introduction to Interferometry

- Synthesis Imaging in Radio Astronomy
  - I (1989) ASP Conf. Series vol. 6
  - II (1999) ASP Conf. Series vol. 180
- Interferometry and Synthesis in Radio Astronomy
  - Thompson, Moran & Swenson (2001)
- Kellermann & Moran, 2001, ARA&A
- [www.atnf.csiro.au/whats\\_on/workshops/synthesis2008/](http://www.atnf.csiro.au/whats_on/workshops/synthesis2008/)

# Astronomy is an observational science

- We cannot interact as the experimental sciences can.
- We have a 2D view of 3D universe.
- Information about the universe comes almost exclusively from electromagnetic radiation.
- High demand on our ability to form images and understand the measurement process.

# Astronomical images



Intensity  
Frequency Spectrum  
Polarization properties

# Diffraction limits

$$\Delta\theta \approx \frac{1.22\lambda}{D}$$

$$\Delta\theta = 1'' \quad \lambda \quad D$$

Optical	500 nm	125 mm
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Radio	20 cm	50 km
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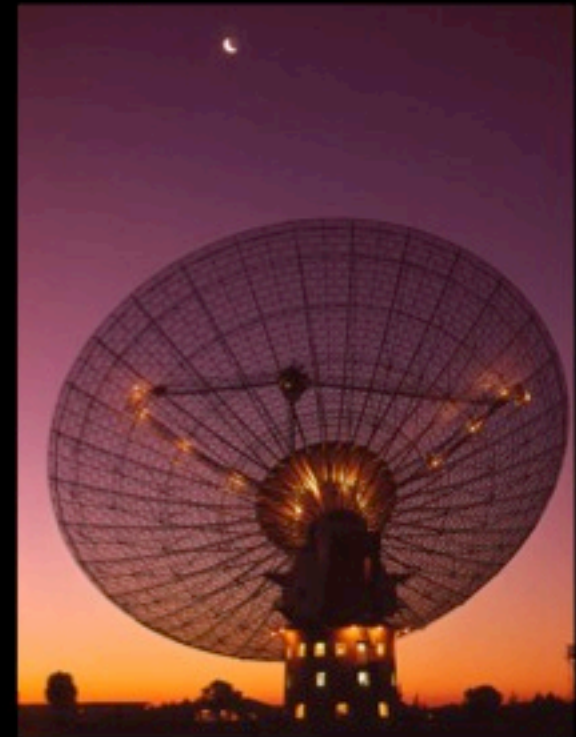
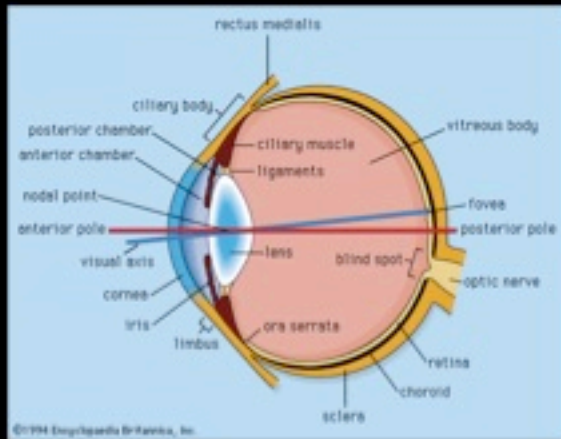
# Direct imaging onto a focal plane

$\Delta\theta$

1'

2''

10'



# Synthesis telescopes



Indirect imaging

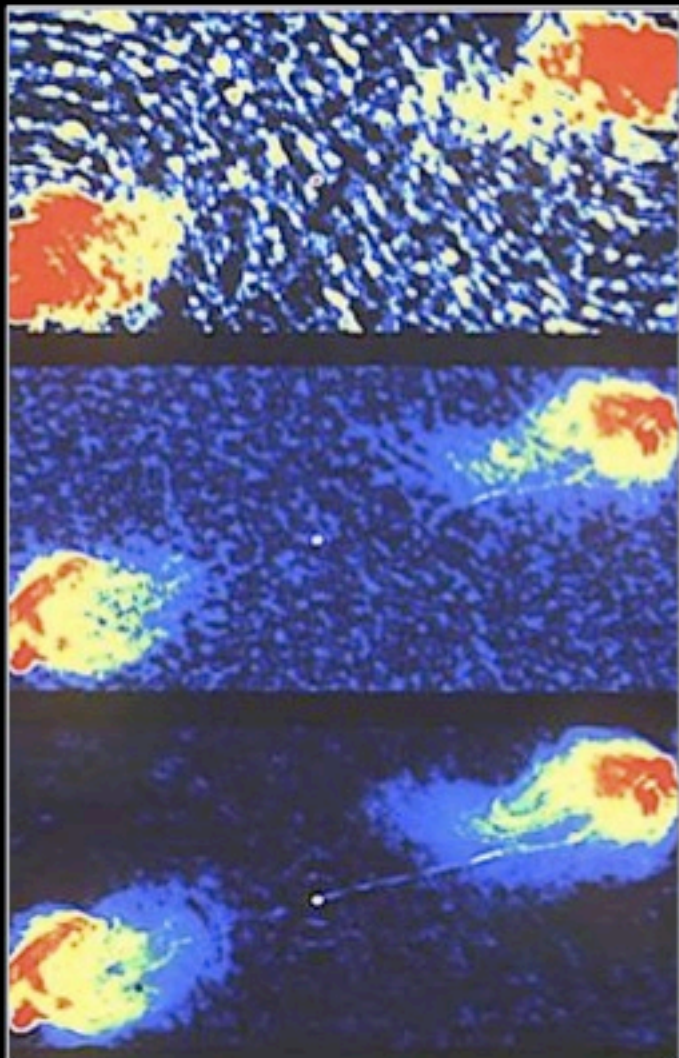
# Indirect imaging

- Used where we cannot form a direct map of the object on the focal plane.
- We infer the properties of the object from certain characteristics of the received electromagnetic field.
- Not intuitive !

# Indirect Imaging Applications

- Interferometry
  - radio, optical, IR, space...
- Aperture synthesis
  - Earth rotation, SAR, X-ray crystallography
- Axial tomography (CAT)
  - NMR, Ultrasound, PET
- Seismology
- Fourier filtering, pattern recognition
- Adaptive optics, speckle

# Cygnus A



← Raw data

VLA continuum

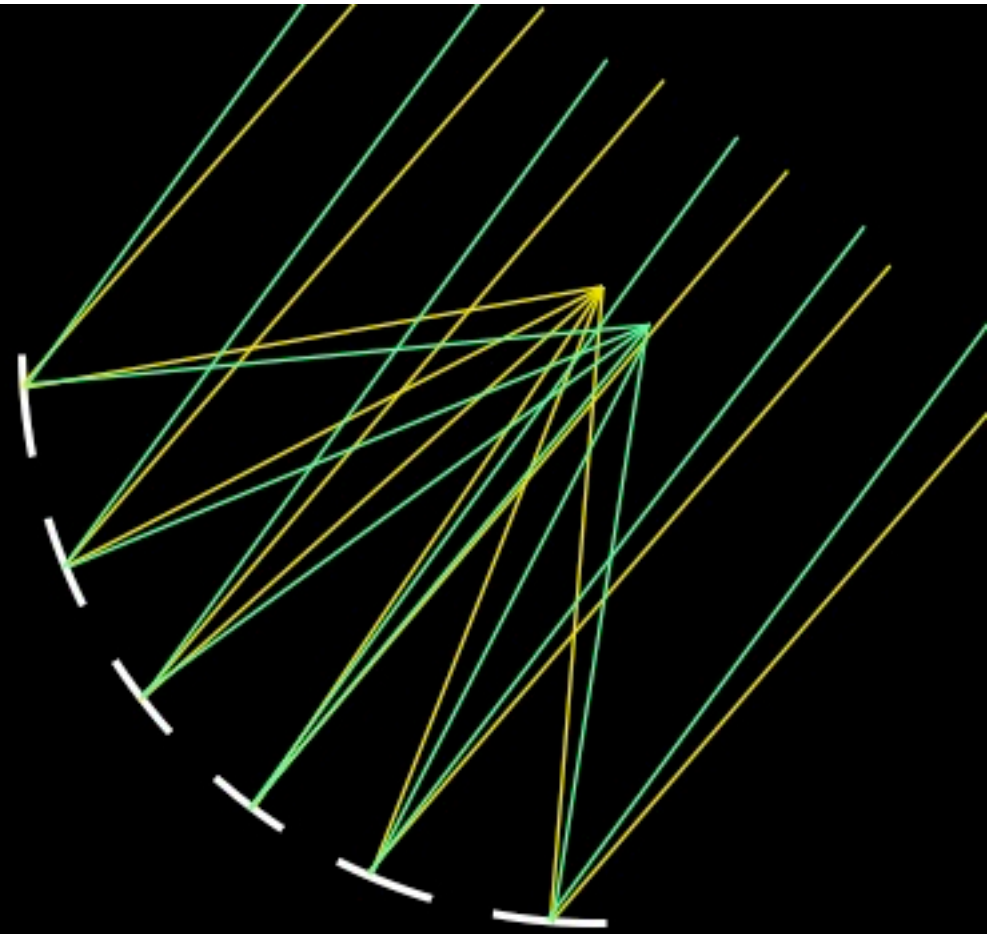
← Deconvolution

correcting for gaps  
between telescopes

← Self Calibration

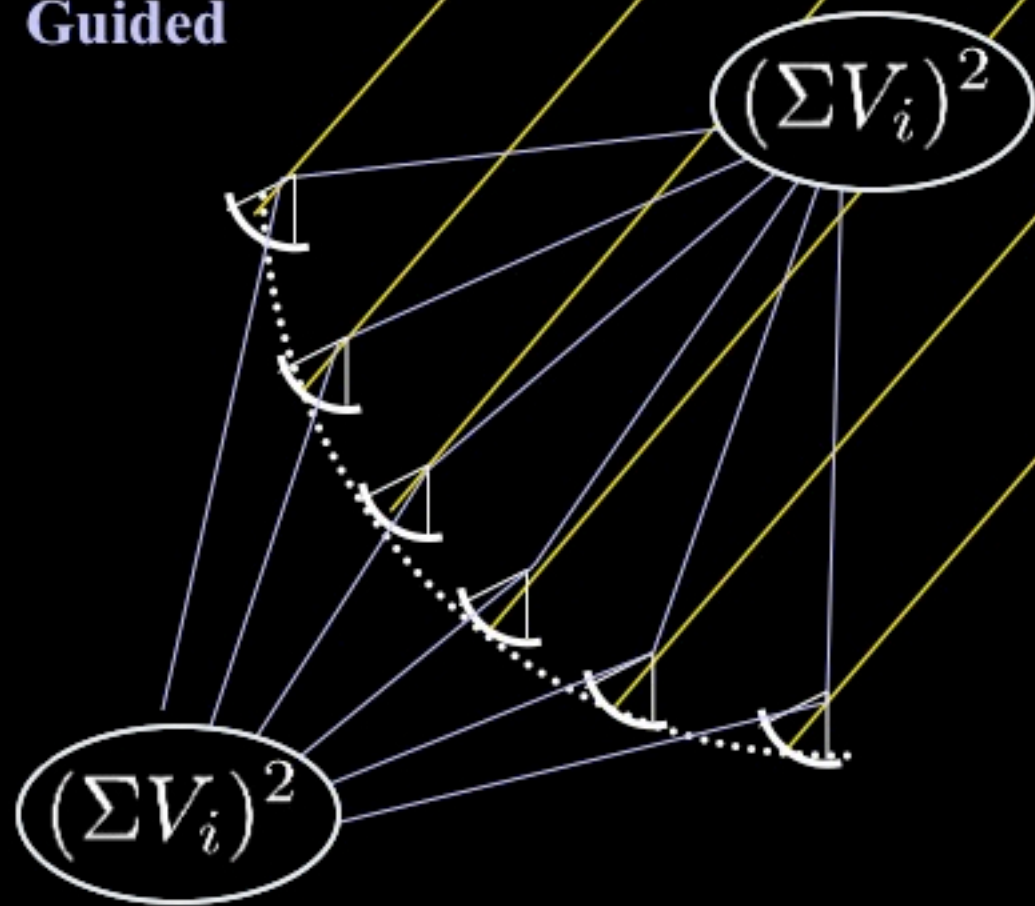
adaptive optics

Single  
dish ...



Free space

Guided



# Phased array

Free space

Guided

Split signal  
no S/N loss

Delay

Phased array

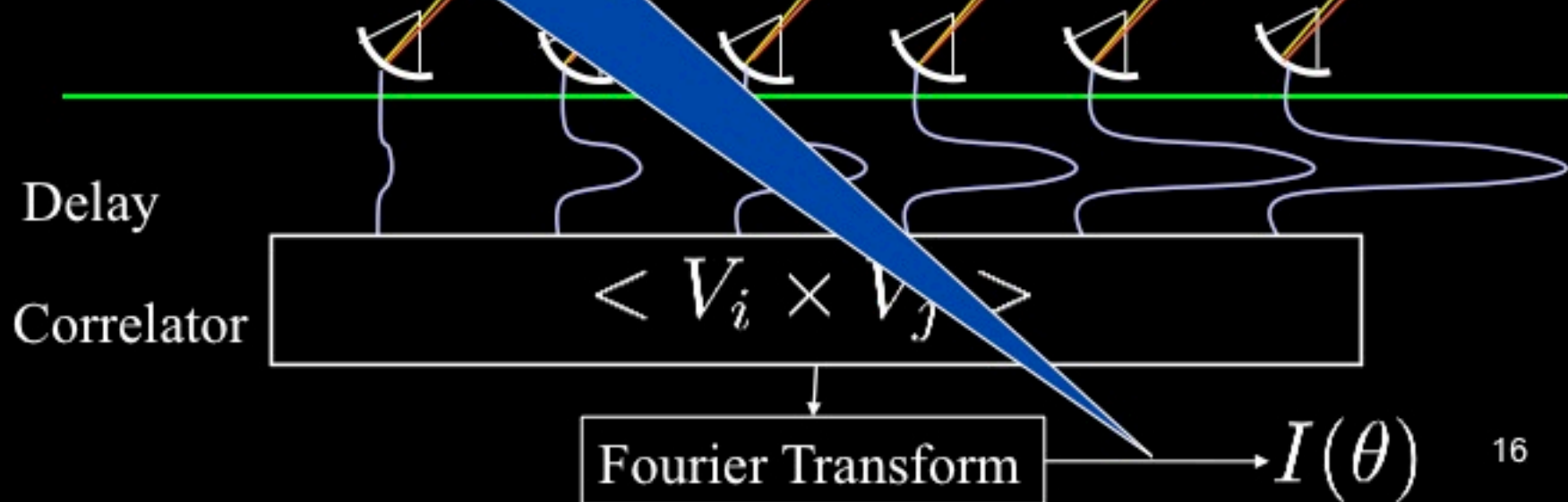
$$(\sum V_i)^2$$

$$(\sum V_i)^2 \quad (\sum V_i)^2$$

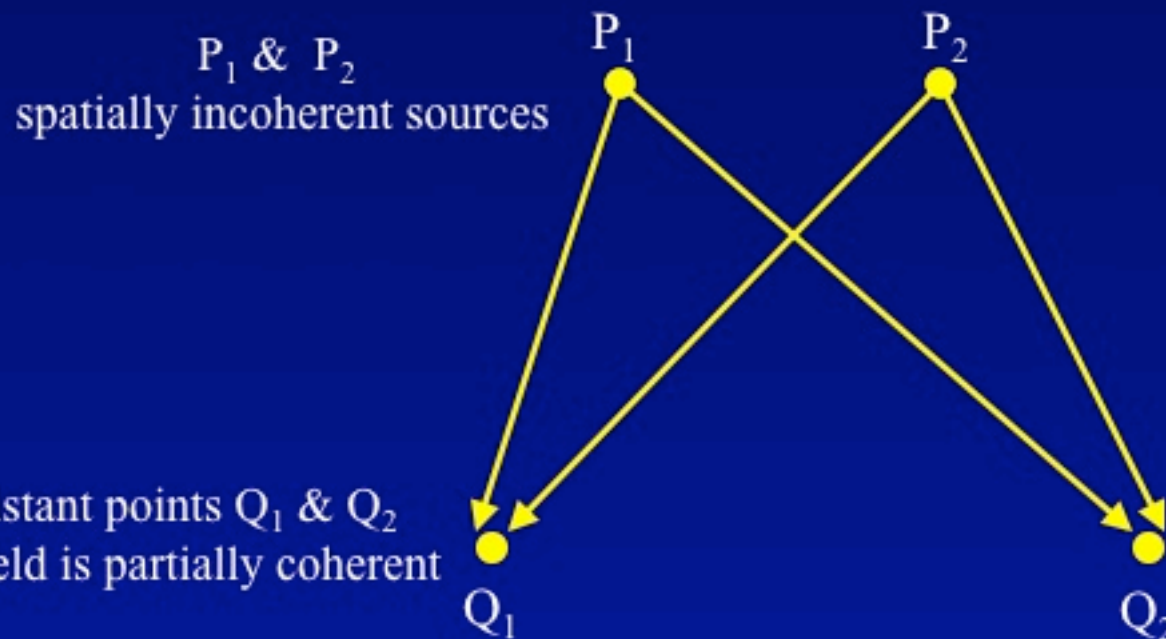
$$I(\theta)$$

# Synthesis imaging

van Cittert-Zernike theorem



# Spatial Coherence



van Cittert-Zernike theorem

The spatial coherence function is the Fourier Transform  
of the brightness distribution



# Physics: propagation of coherence

- Radio source emits independent noise from each element
  - Electrons spiraling around magnetic fields
  - Thermal emission from dust, *etc.*
- As electromagnetic radiation propagates away from source, it remains coherent
- By measuring the correlation in the EM radiation, we can work backwards to determine the properties of the source
- Van Cittert-Zernicke theorem states that the
  - *Sky brightness and Coherence function are a Fourier pair*



# Physics: propagation of coherence

- Mathematically:

$$V(u, v) = \int I(l, m) \cdot e^{j \cdot 2\pi \cdot (ul + vm)} dl \cdot dm$$

- where
  - $l, m$  are the orthogonal direction cosines
  - $u, v$  are the projected baseline coordinates (in  $\lambda$ )
  - $V(u, v)$  is the complex visibility function – what is observed
  - $I(l, m)$  is the intensity distribution of the source
    - this is what we want to know!
  - $j$  (sometimes  $i$  is used) is  $\sqrt{-1}$
- Since  $I(l, m)$  is a real function,  $V(u, v)$  is hermitian
  - So  $V(-u, -v) = V(u, v)$

# Van Cittert-Zernike theorem

- The van Cittert-Zernike theorem states that the distribution of intensity of the incoming radiation (which we want to know) is the Fourier transform of the spatial coherence function (which we measure with an interferometer).
- Named for Pieter van Cittert and Frederik (“Frits”) Zernike

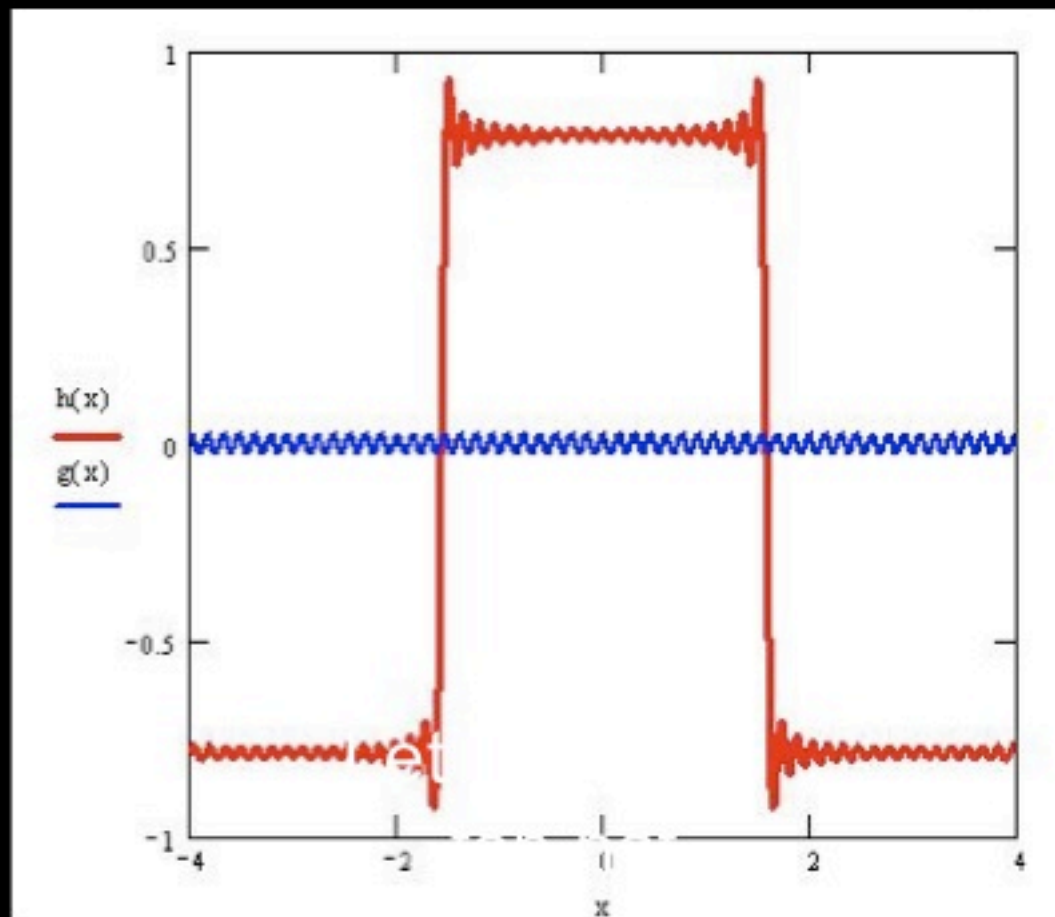
# Fourier decomposition

- Decompose the 2-dimensional intensity distribution into its Fourier components.
- An interferometer (pair of antennas) can measure a Fourier component.
- One dimensional FT
  - $F(u) = \int_{-\infty}^{\infty} f(x) e^{-ixu} dx$
- *Inverse FT*
  - $f(x) = \int_{-\infty}^{\infty} F(u) e^{ixu} du$

# Fourier Transforms

- Recall Euler formula
  - $e^{ix} = \cos x + i \sin x$
- Any function can be represented as a sum of even (cos) and odd (sin) functions

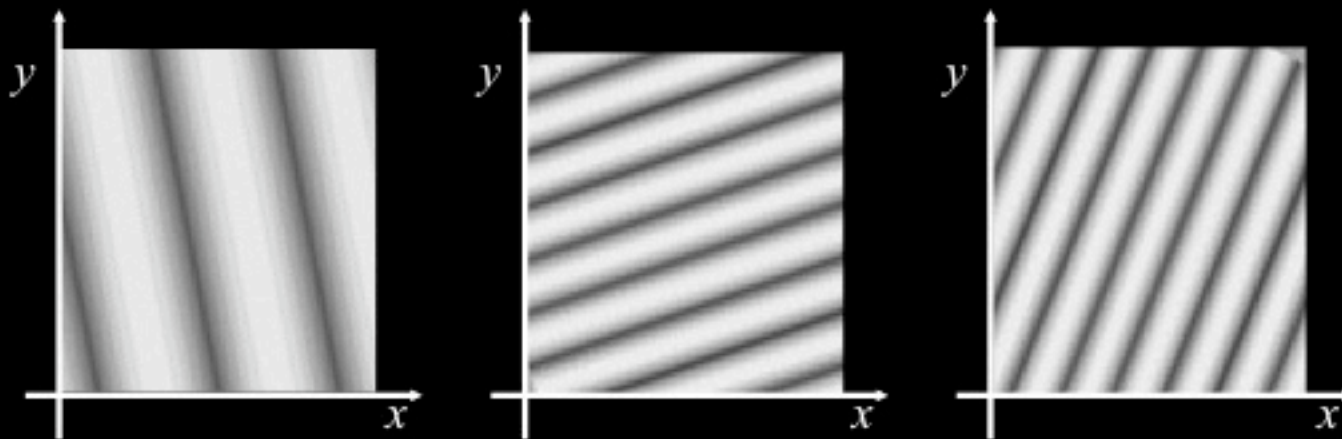
# Function synthesis



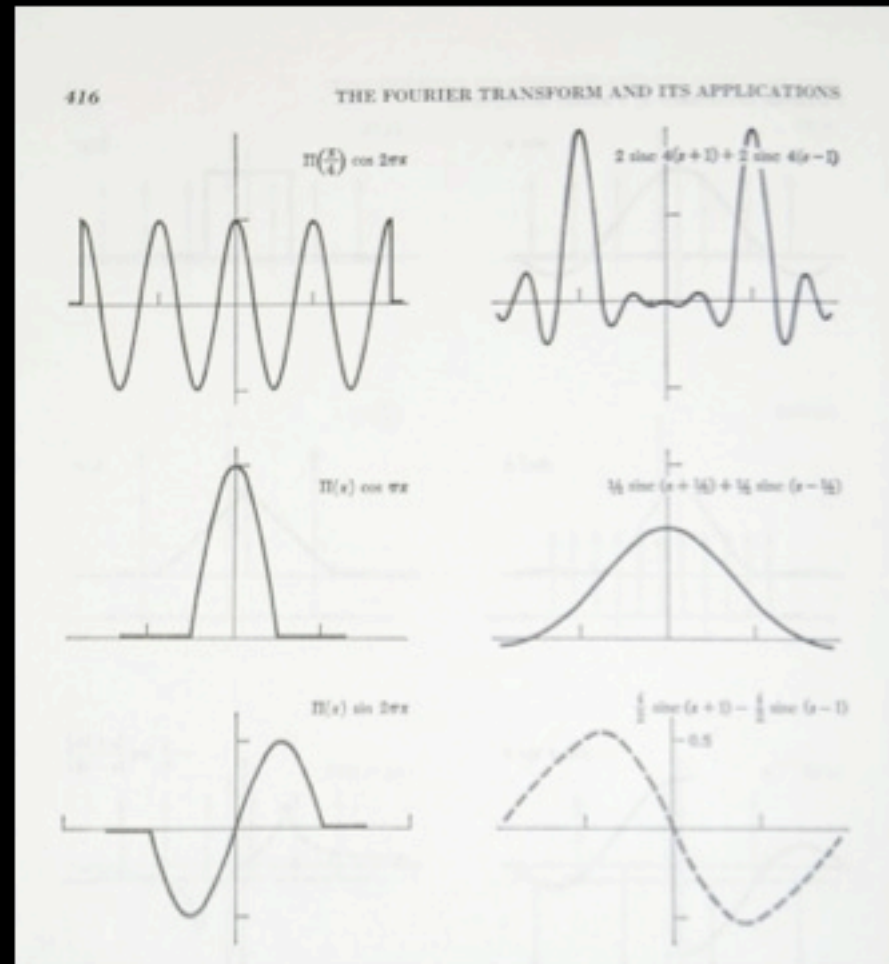
In general, **any** function can be composed – “*synthesised*” – from a number of sines of different periods, amplitudes and phases.

# Image analysis

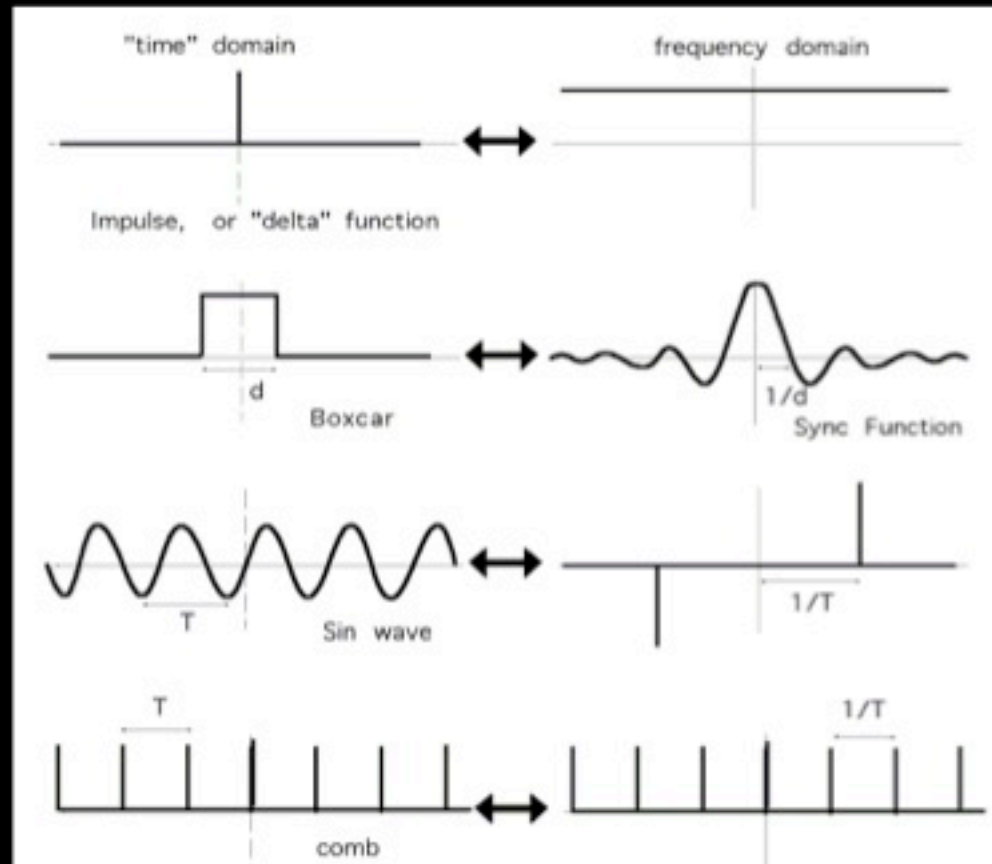
- The image is the 2-dimensional function  $I(x,y)$
- Like 1-dimensional functions, it can be analysed into a number of sinusoidal functions of  $(x,y)$ . These sinusoids are sometimes called “components” of the image.
- These 2-dimensional sinusoids have two periods - with respect to both  $x$  and  $y$ .



# Fourier and his Transform



# Fourier Transform pairs

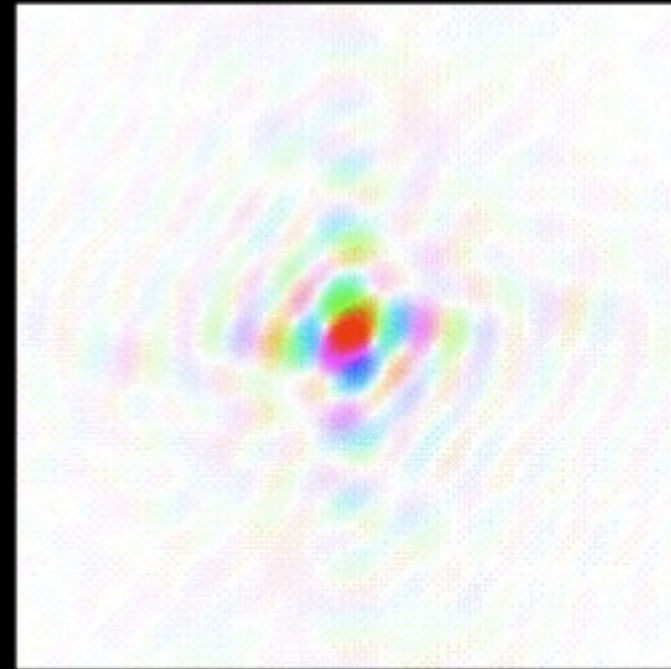
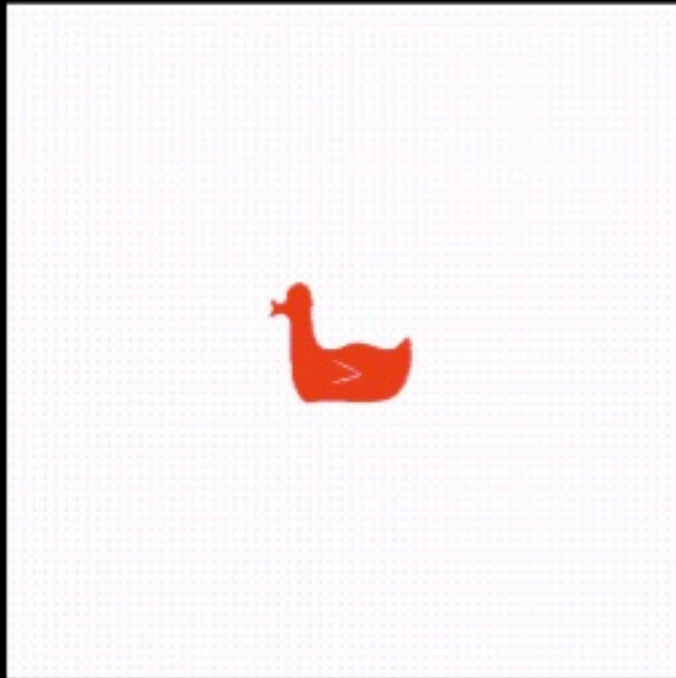


# Fourier Transforms

- Note that  $f(x,y)$  is the image and is *real*, but  $F(u,v)$  is the FT and is, in general, *complex*.
- Generally,  $F(u,v)$  is represented by its *magnitude* and *phase* rather than its real and imaginary parts.
- The *magnitude* tells "how much" of a certain frequency component is present and the *phase* tells "where" the frequency component is in the image.

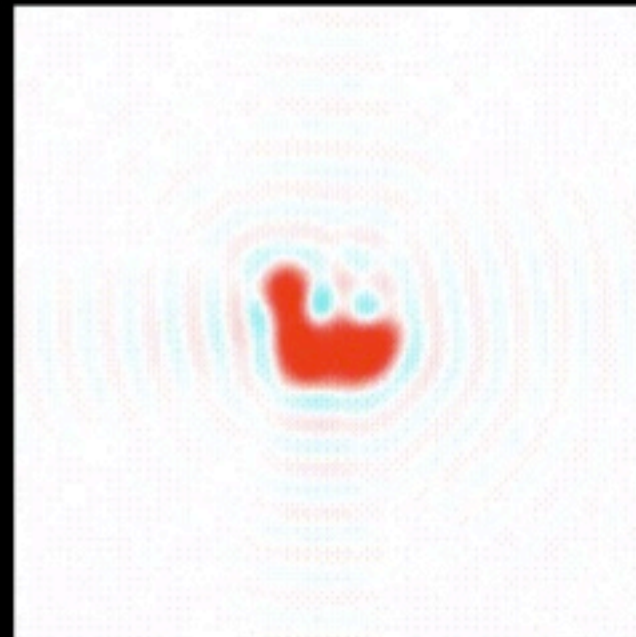
<http://www.cs.unm.edu/~brayer/vision/fourier.html>

# Fourier Transform examples



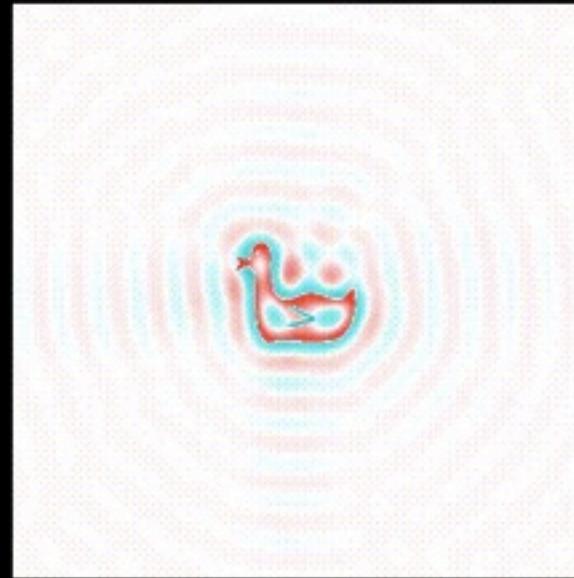
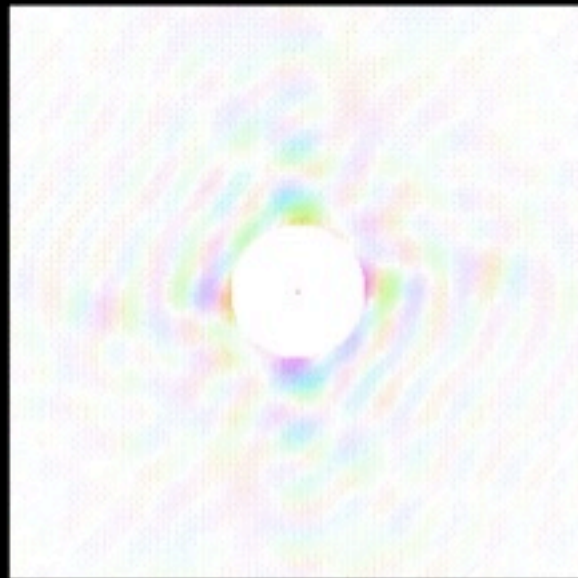
- A duck and its Fourier Transform
- <http://www.general.uwa.edu.au/u/vpatrick/fourier/fourier.html>

# Fourier Transform examples



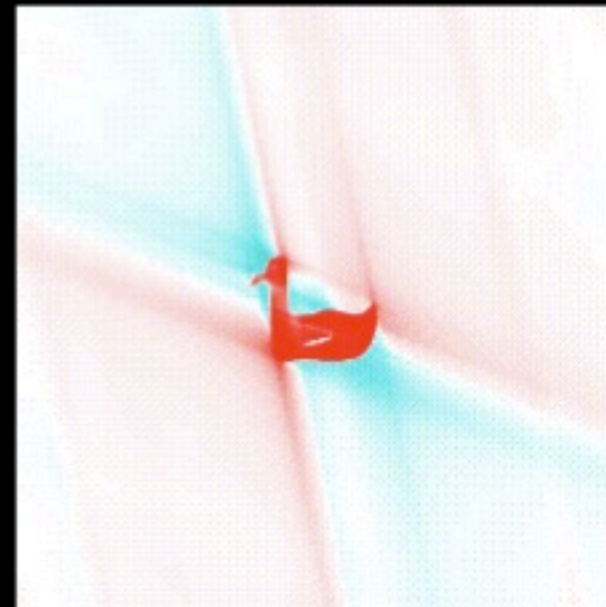
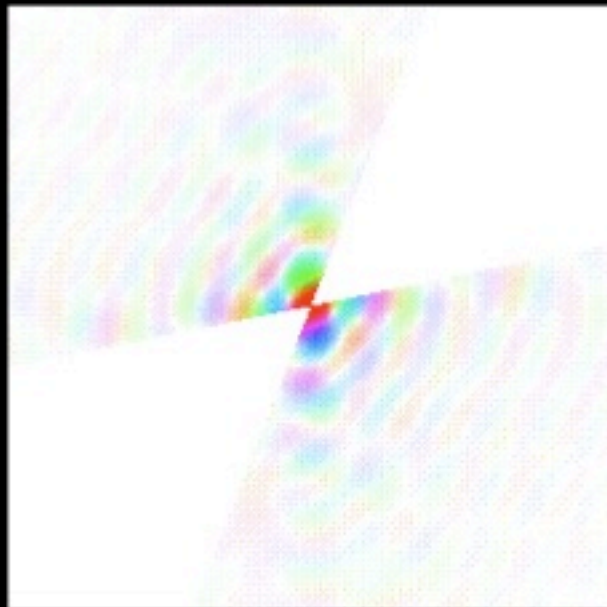
- If we only have the low resolution terms of the diffraction pattern, we only get a low resolution duck
- There is considerable loss of detail. Note the ripples around the duck.

# Fourier Transform examples



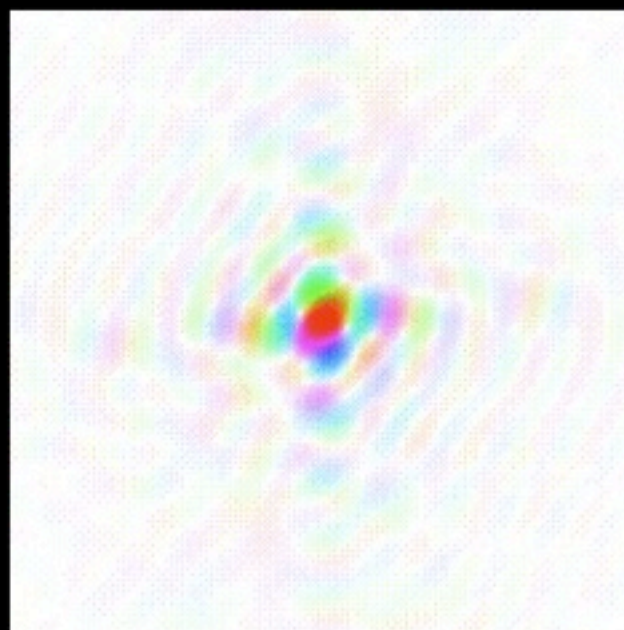
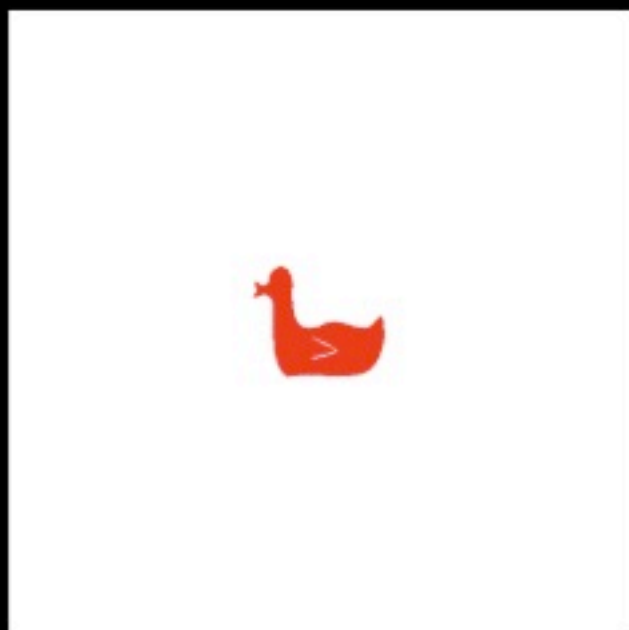
- If we only have the high resolution terms of the diffraction pattern, we see only the edges of the duck
- Do not omit your low resolution data. Collect it and use it!

# Fourier Transform examples

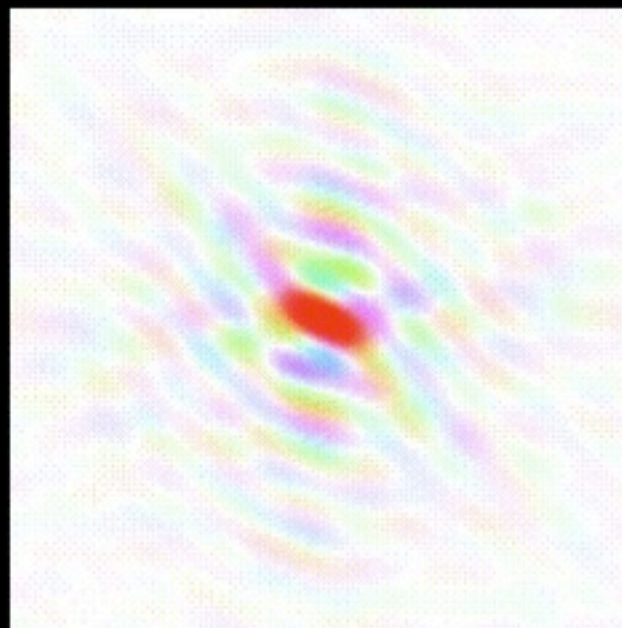
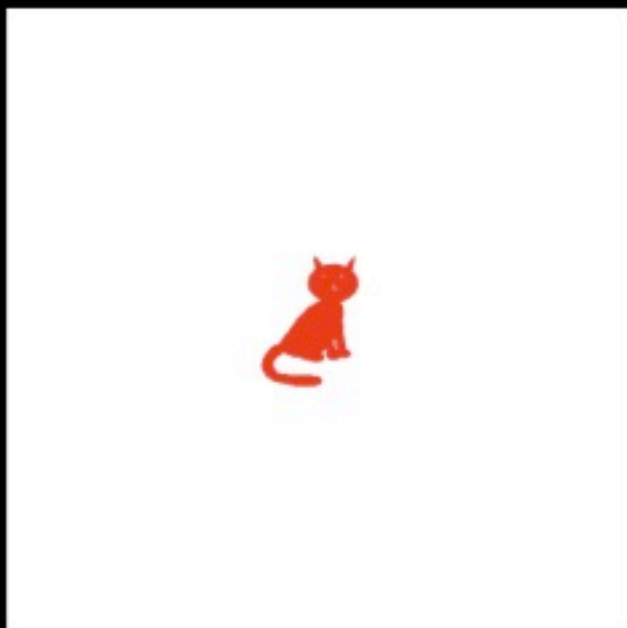


- If a segment of data is missing, features perpendicular to that segment will be blurred.

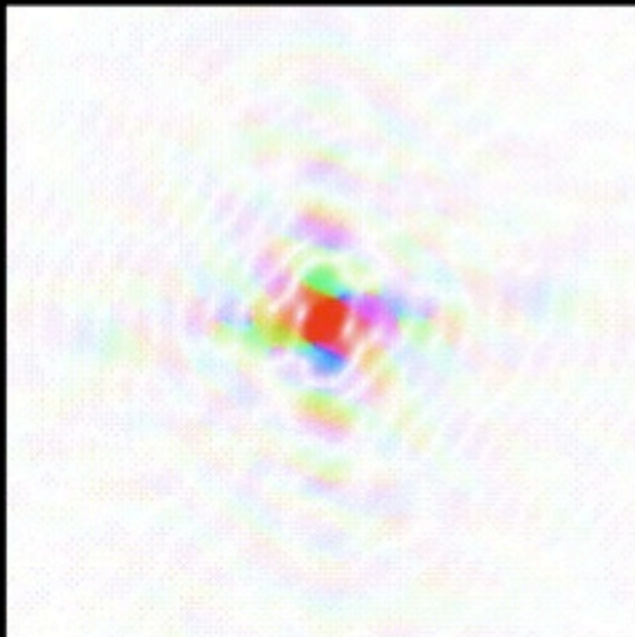
# The duck and its FT



# A cat and its FT



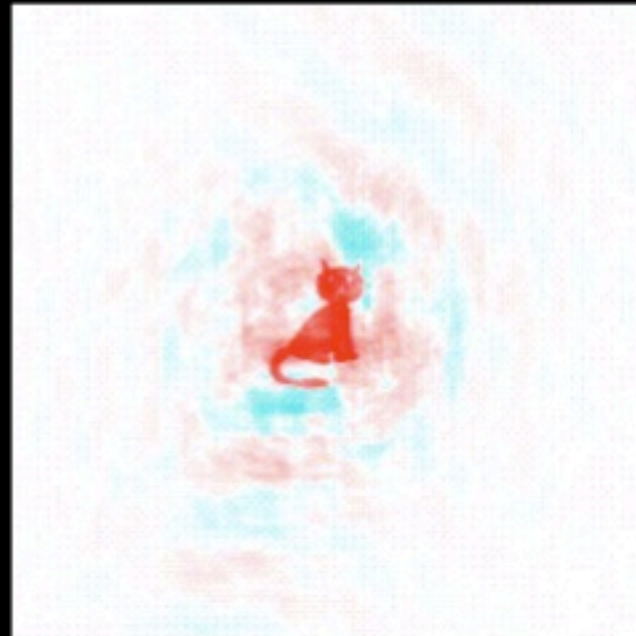
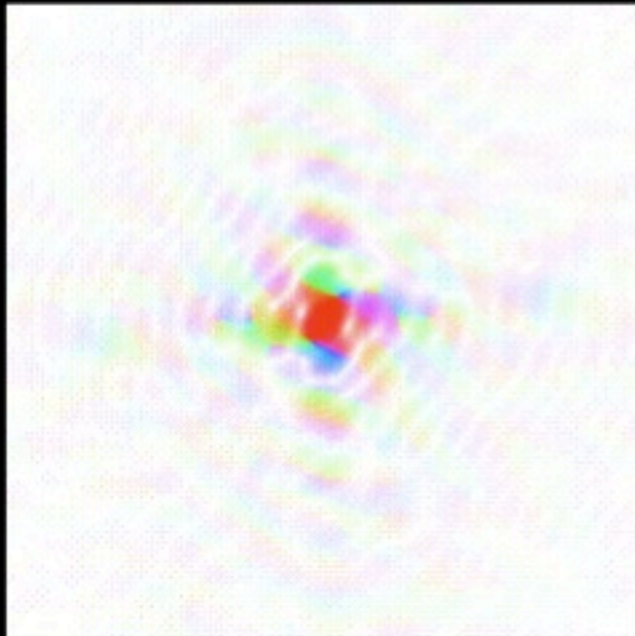
# Fourier Transform examples



FT = ?

- Let us combine the magnitudes from the Duck transform with the phases from the Cat transform.

# Fourier Transform examples



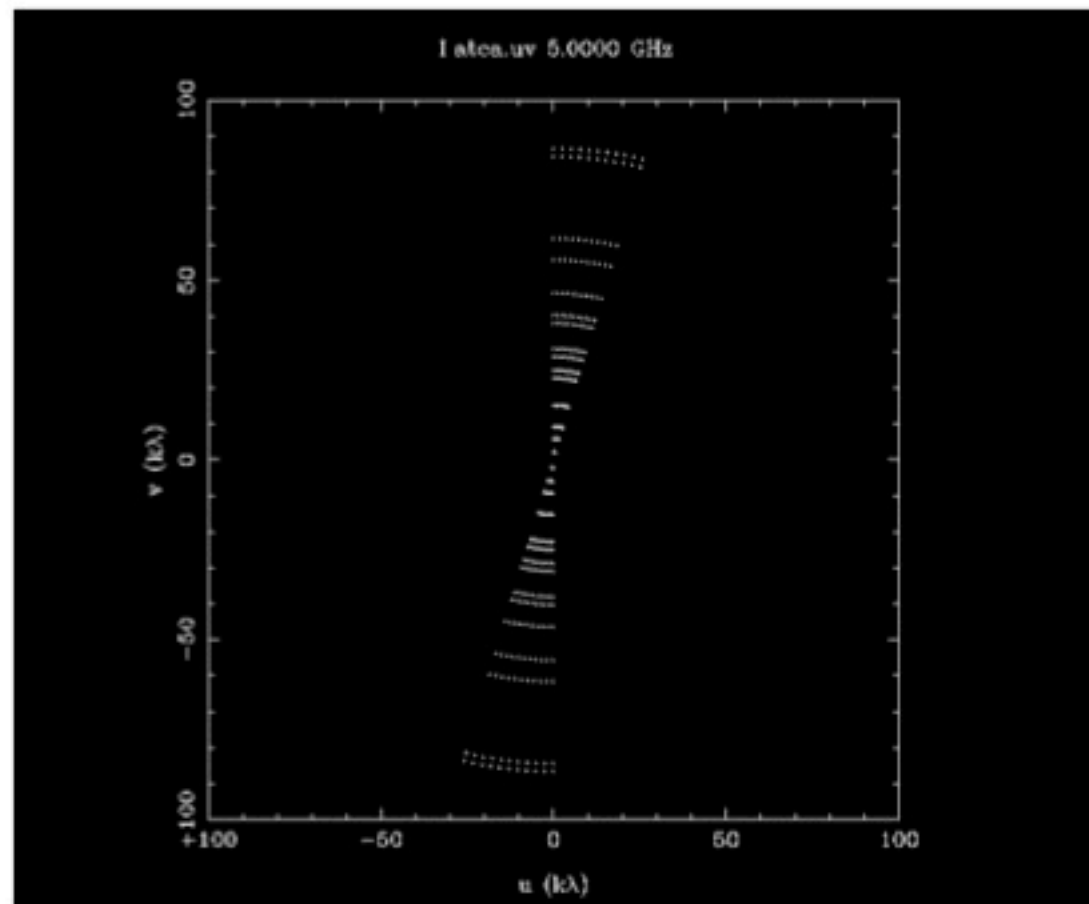
- Let us combine the magnitudes from the Duck FT with the phases from the Cat FT.
- The image which contributed the phases is still visible, whereas the image which contributed the magnitudes has gone!

# “Anthropomorphism” in Astronomy

- Snake
  - ApJ 443, 638 (1995); ApJ, 462, 768 (1996)
- Rabbit
  - ApJ 515, 712 (1999)
- Kookaburra
  - ApJ 561, L187 (2001)
- Duck
  - ApJ 573, L111 (2002); ApJ 652, 1523 (2006)
- CaT
  - MNRAS 393, 846 (2009)

# The (u,v)-plane

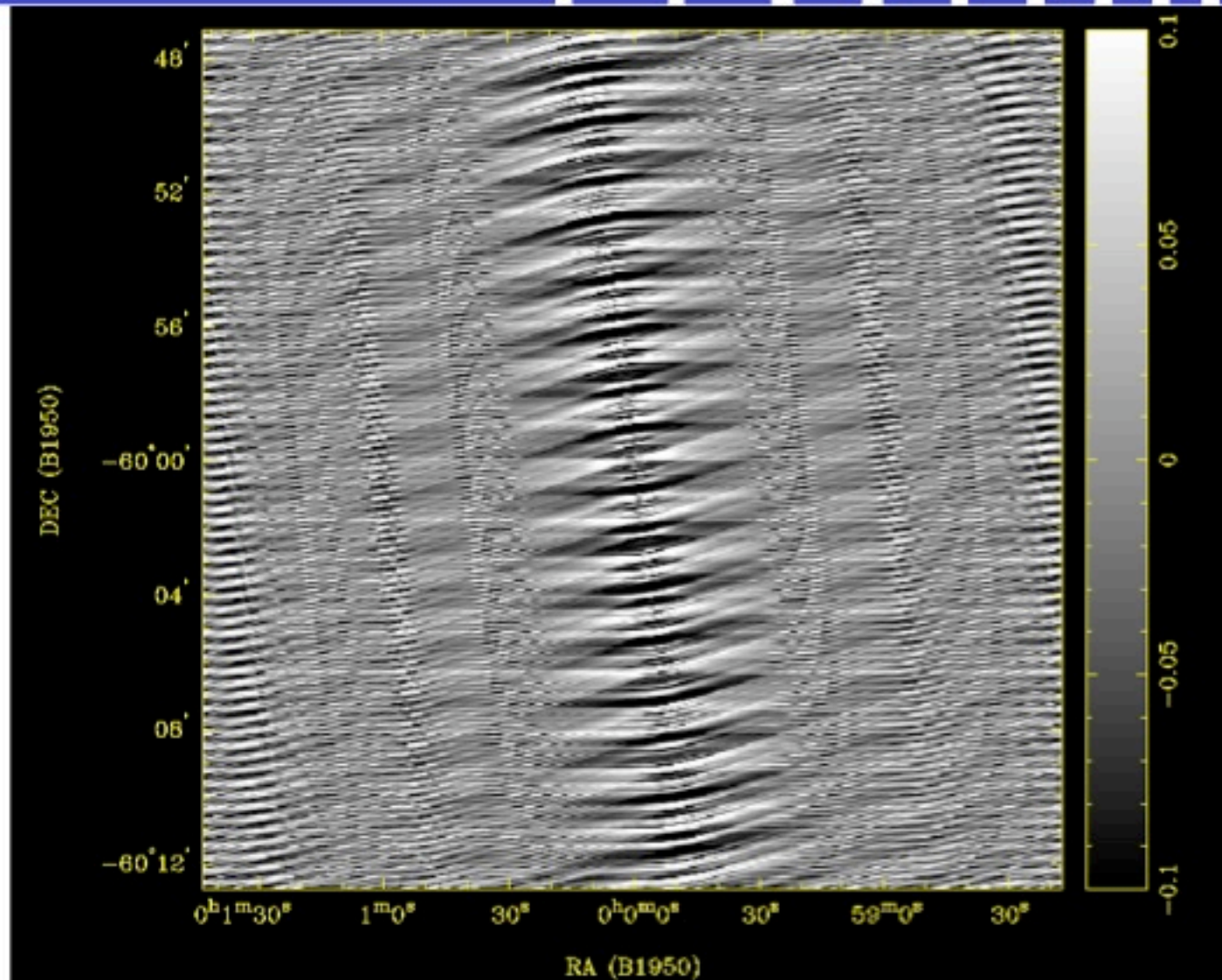
- Visibilities have coordinates:  $u, v$
- Baselines trace out arcs in  $(u, v)$  plane
- Earth rotation fills the plane
- As  $V(-u, -v) = V(u, v)$  only need a 12 hr synthesis



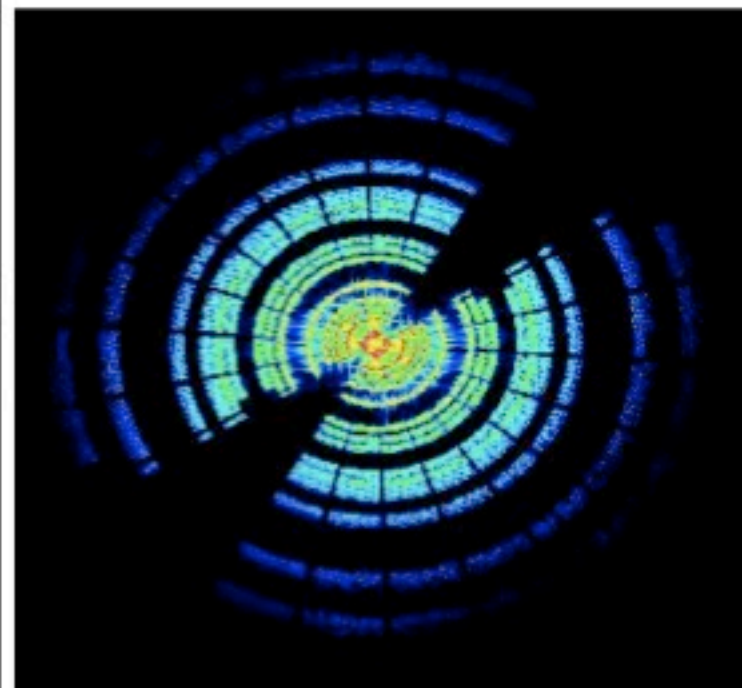
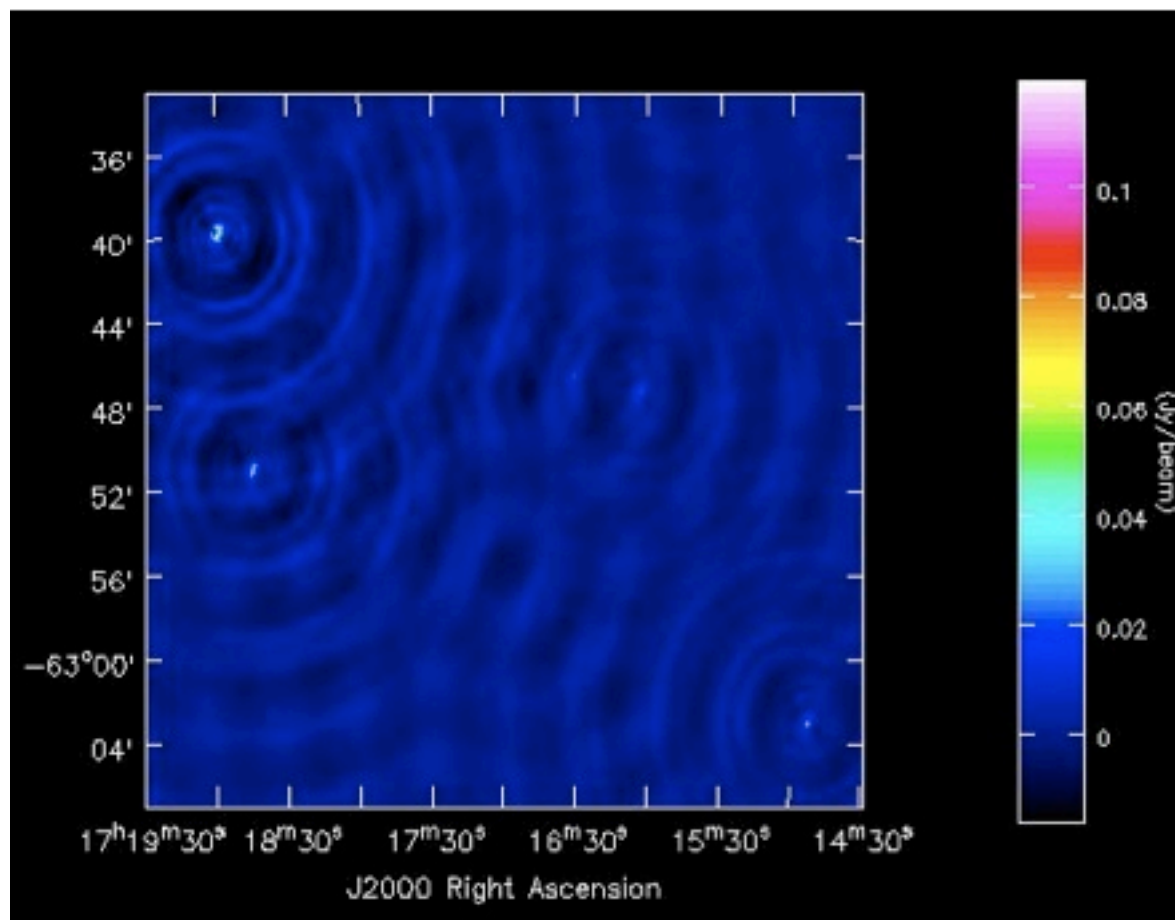
E-W array

# The dirty beam

Dirty  
beam =  
PSF =  
Response  
to unit  
point  
source at  
field  
center =  
FT of  
(u,v)  
coverage

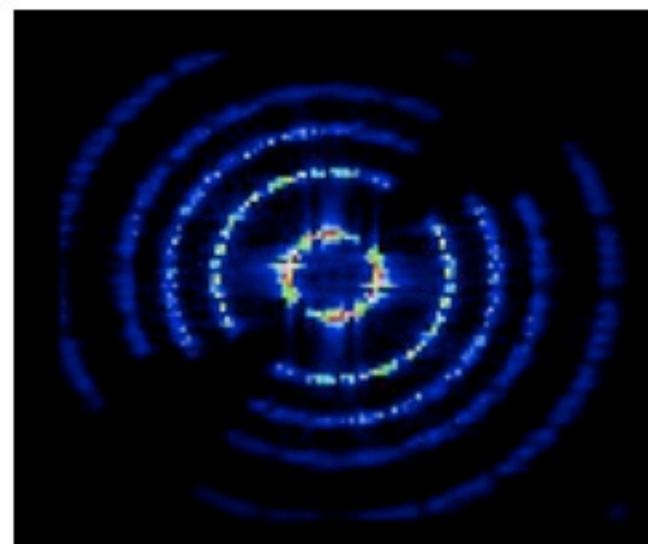


# Image Plane and (u,v) plane



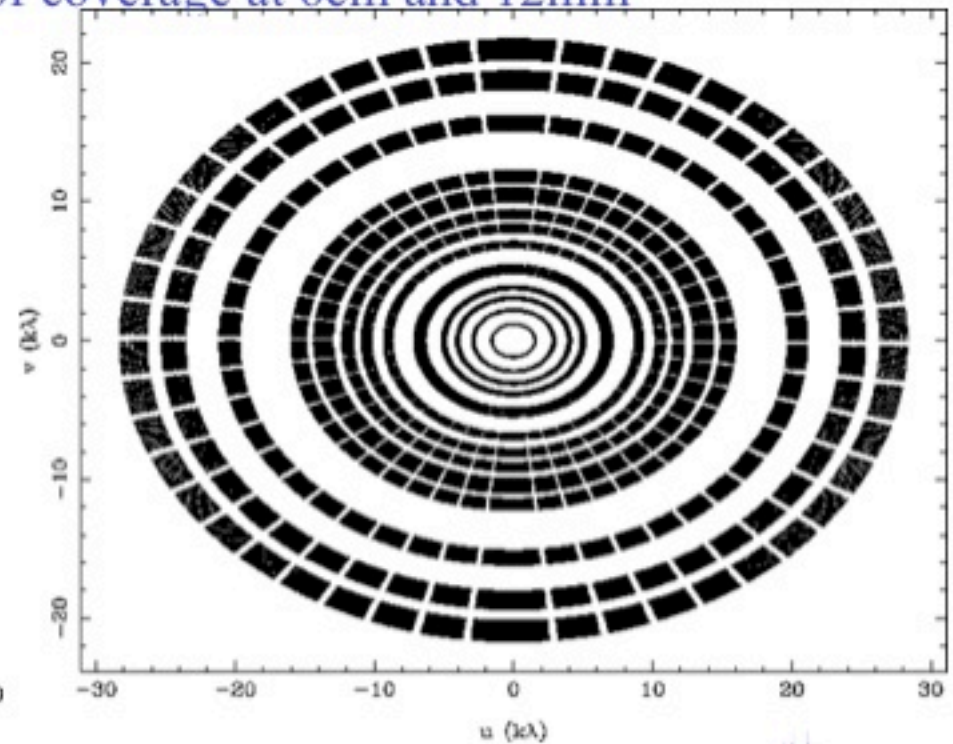
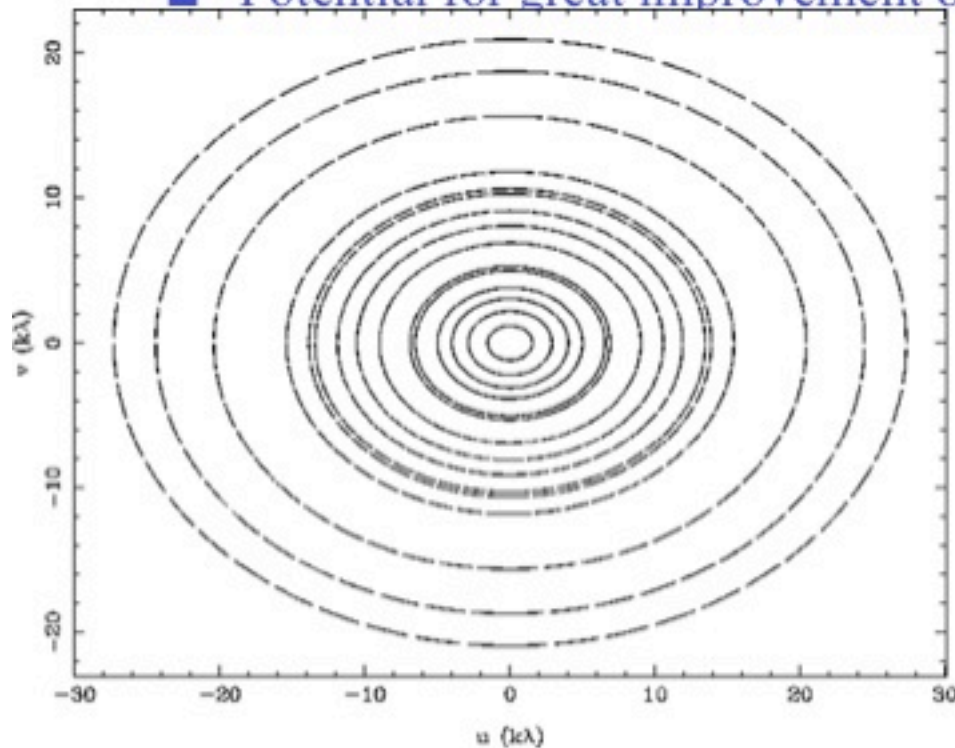
# (u,v) plane features

- Dense rings
  - Baselines – tracks in  $(u,v)$  plane
- Low level in between rings
  - Gaps in coverage, missing information
- Hole in center
  - No information on low 'spatial frequencies', i.e., no info on large scale structure
- Outer boundary
  - No info on small scale structure – resolution limit
- Ways to fill in more of the  $(u,v)$  Plane
  - Add single dish data
  - Multiple configurations (needs more time)
  - Multi-frequency synthesis



# Multi-frequency synthesis

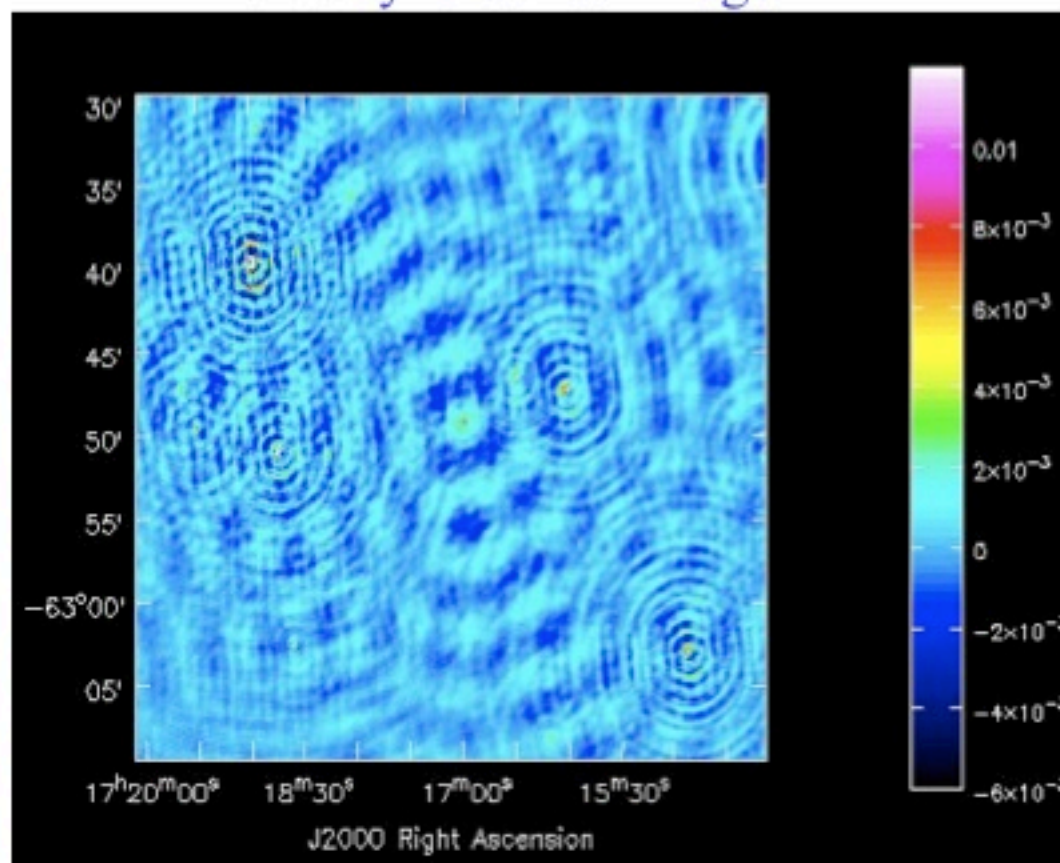
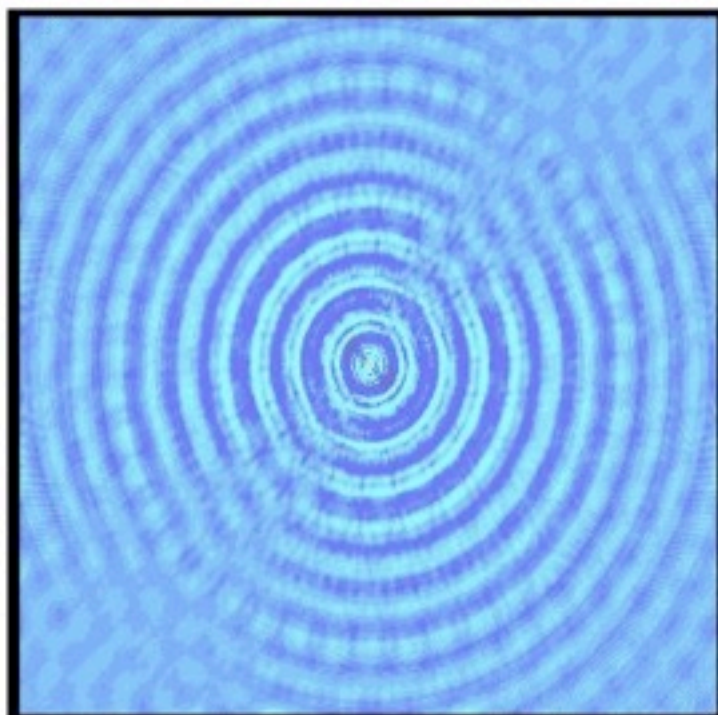
- As  $(u,v)$  coordinates are measured in wavelengths, another way of filling the Fourier plane is to observe at multiple wavelengths simultaneously.
- Standard for ATCA continuum (128MHz/32Chn)
- Potential for great improvement of coverage at 6cm and 12mm



# Image plane

Partially Cleaned Image

PSF or 'dirty beam'



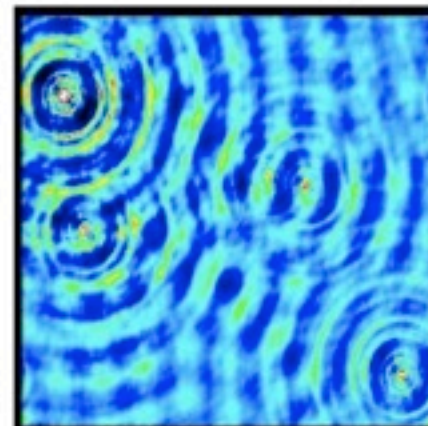
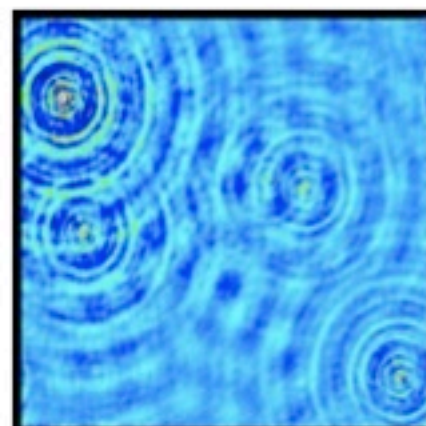
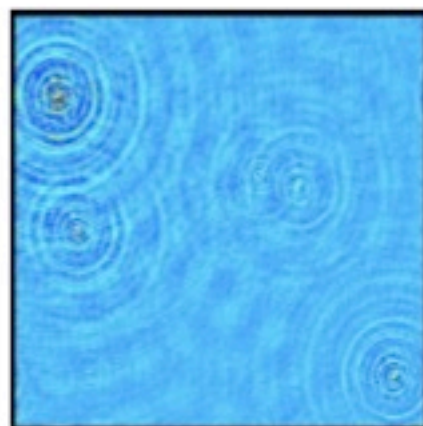
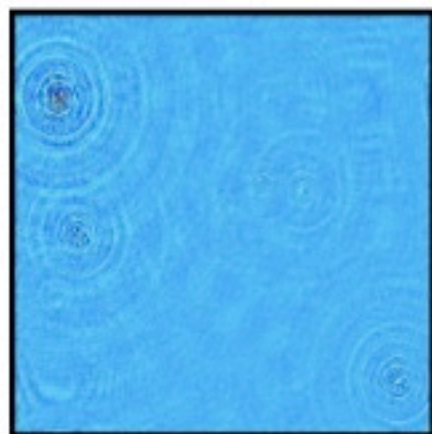
Note extended source and sources outside field

# Imaging Decisions

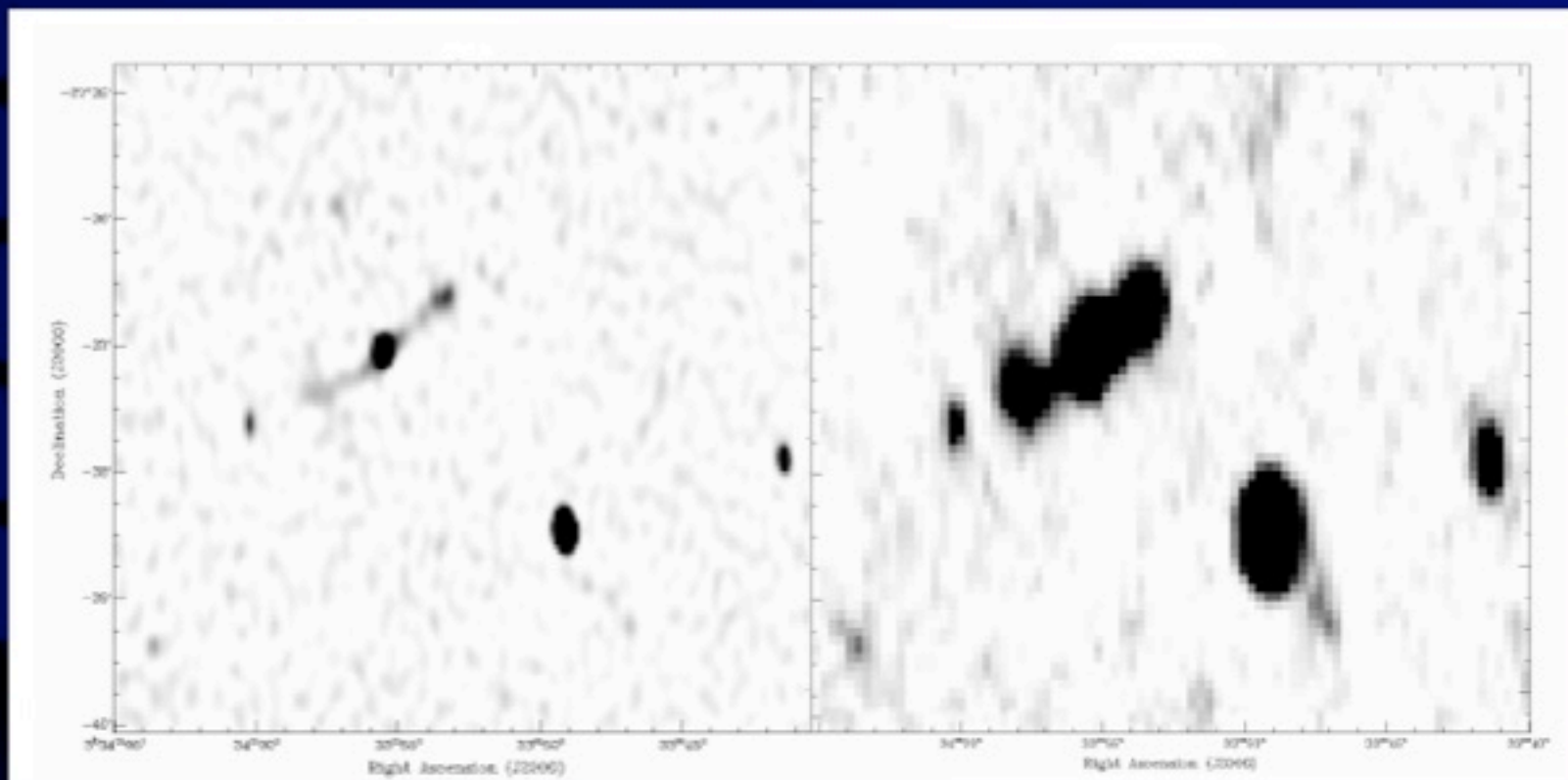
## ■ Weighting scheme

- Uniform (minimum sidelobe level)
- Natural (minimum noise level)
- Robust (optimal combination of above two)

Uniform	Robust=0.5	Robust=1.0	Natural
Beam: 7x5"	8x6.5"	9.6x7.5"	12x8"
Sens.: 1.45	1.16	1.06	1.0



# The effect of weighting



Uniform weighting  
beam-size = 11 x 5 arcsec  
 $\sigma = 25 \mu\text{Jy}$

Natural weighting  
beam-size = 28 x 16 arcsec  
 $\sigma = 17 \mu\text{Jy}$

# Deconvolution



- A good (non-linear) deconvolution algorithm is one that picks plausible ('invisible') distributions to fill the unmeasured parts of the Fourier plane.
- Need to make assumptions to get a realistic estimate
  - Main assumption: Real sky does NOT look like typical dirty beam
    - » Rings, spokes, negative regions, infinite extent: all very unlikely
- Different algorithms make different assumptions:
  - CLEAN (pixel based), Point Source Fitting
    - » Sky is mostly empty, with occasional peaks
    - » Works well for field with point sources, poor for extended emission
  - MEM (Maximum Entropy Method) (pixel based)
    - » Sky is uniform (& positive)
    - » Works well for very extended sources, poor for point sources
  - Scale-sensitive algorithms: multi-scale clean, asp clean, Source fitting
    - » Sky consists of bounded, overlapping, regions of emission

# CLEAN



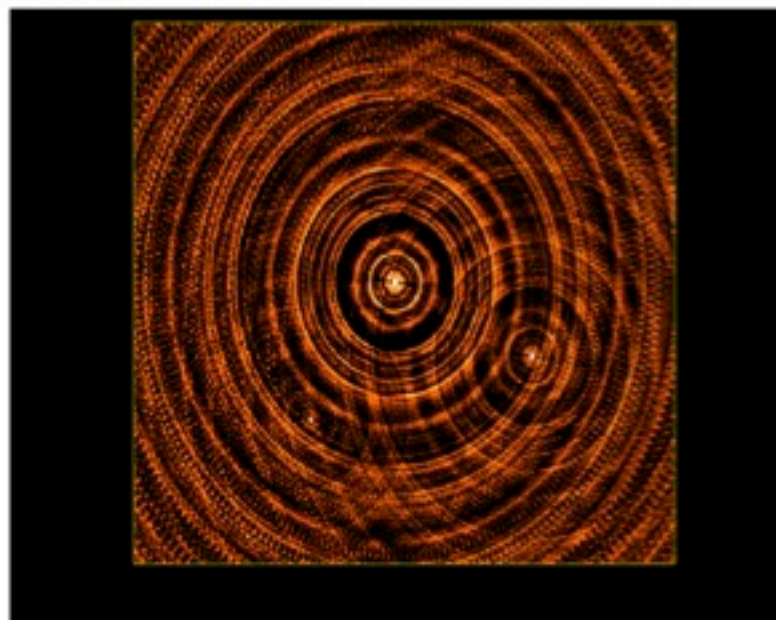
- Original version by Högbom (1974)
  - Purely image based
  - Later versions (Clark, Cotton-Schwab, SDI) add FFT speedups, model visibility subtraction and try to cope with extended emission
  
- Algorithm:
  1. Find position of highest peak in image – assume this is a point source
  2. Subtract a fraction of this peak ('gain factor') using a scaled dirty beam at this position
  3. Add model component to list
  4. Go to 1, unless prescribed flux limit or iteration limit reached

# CLEAN example 1

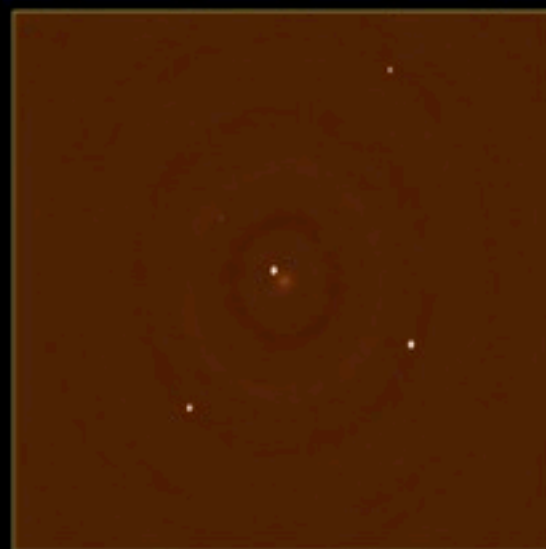
Model: 5 point sources + 1 gaussian

Point: 1,0.5,0.25,0.1,0.01 Jy

Gaussian: 0.1Jy, 10''x10''

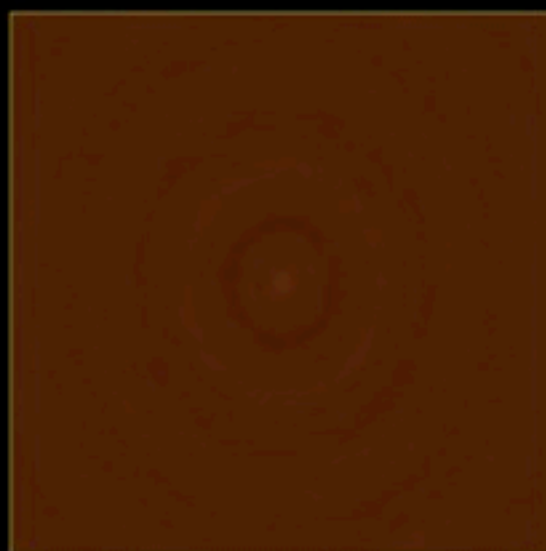


Dirty image



Restored

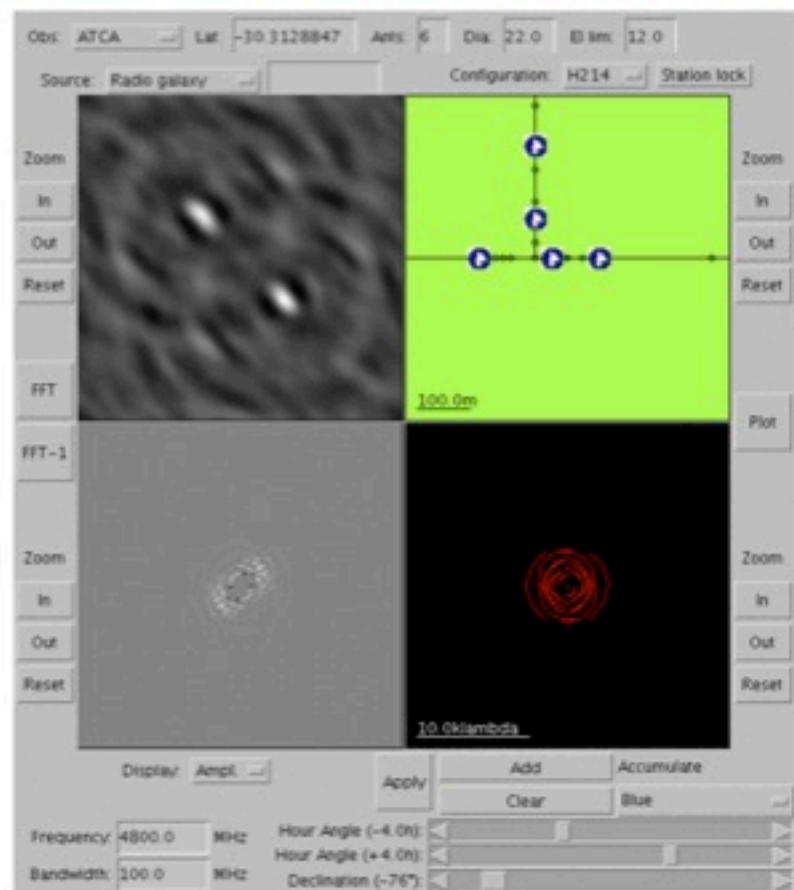
1,5,10,20,  
50,100,  
200,500,  
1000 CC's



Residual

# VRI

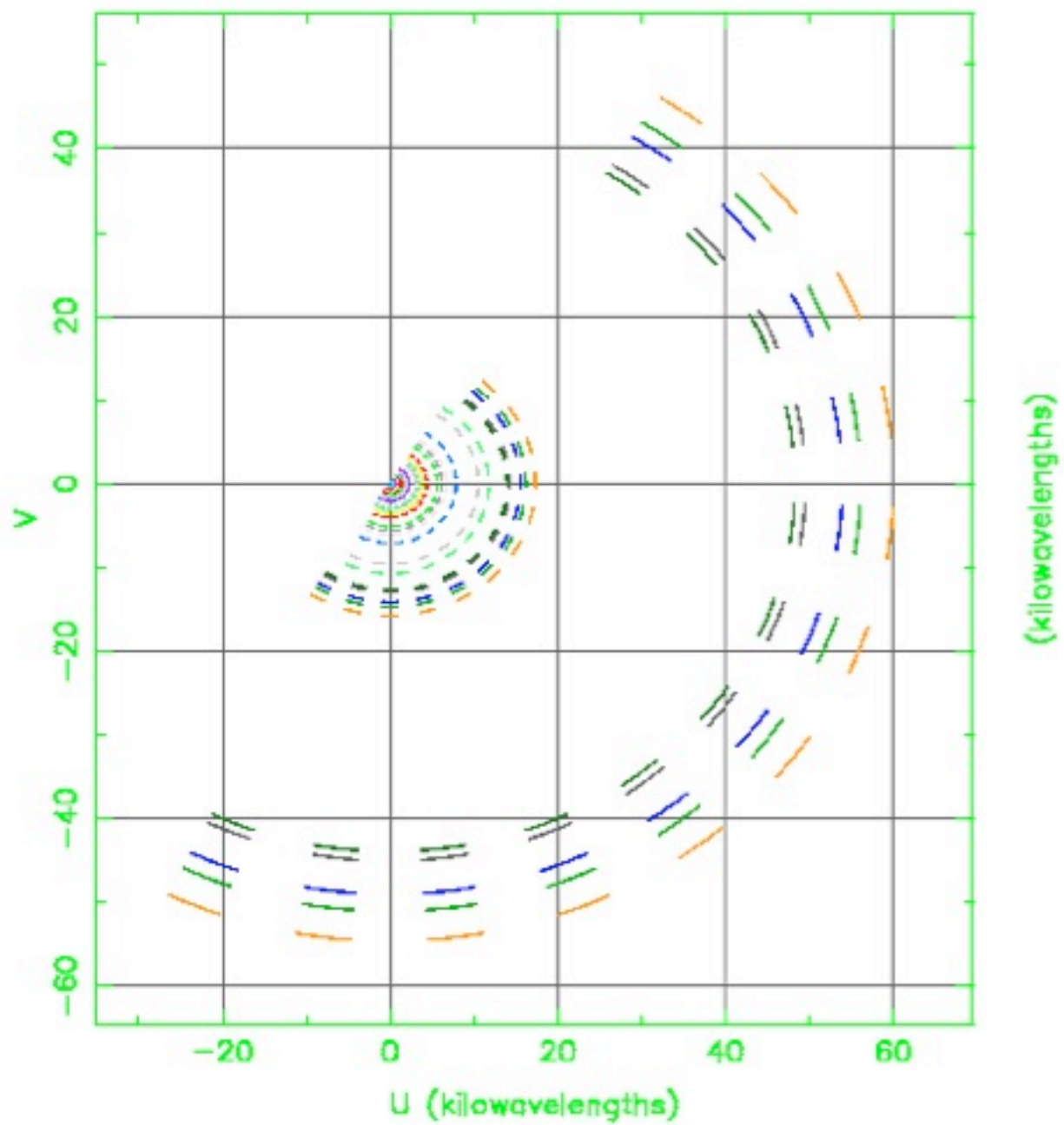
- VRI, the virtual radio interferometer
  - Type vri in searchbox on ATNF website
  - Lets you experiment with Fourier transforms and ATCA configurations



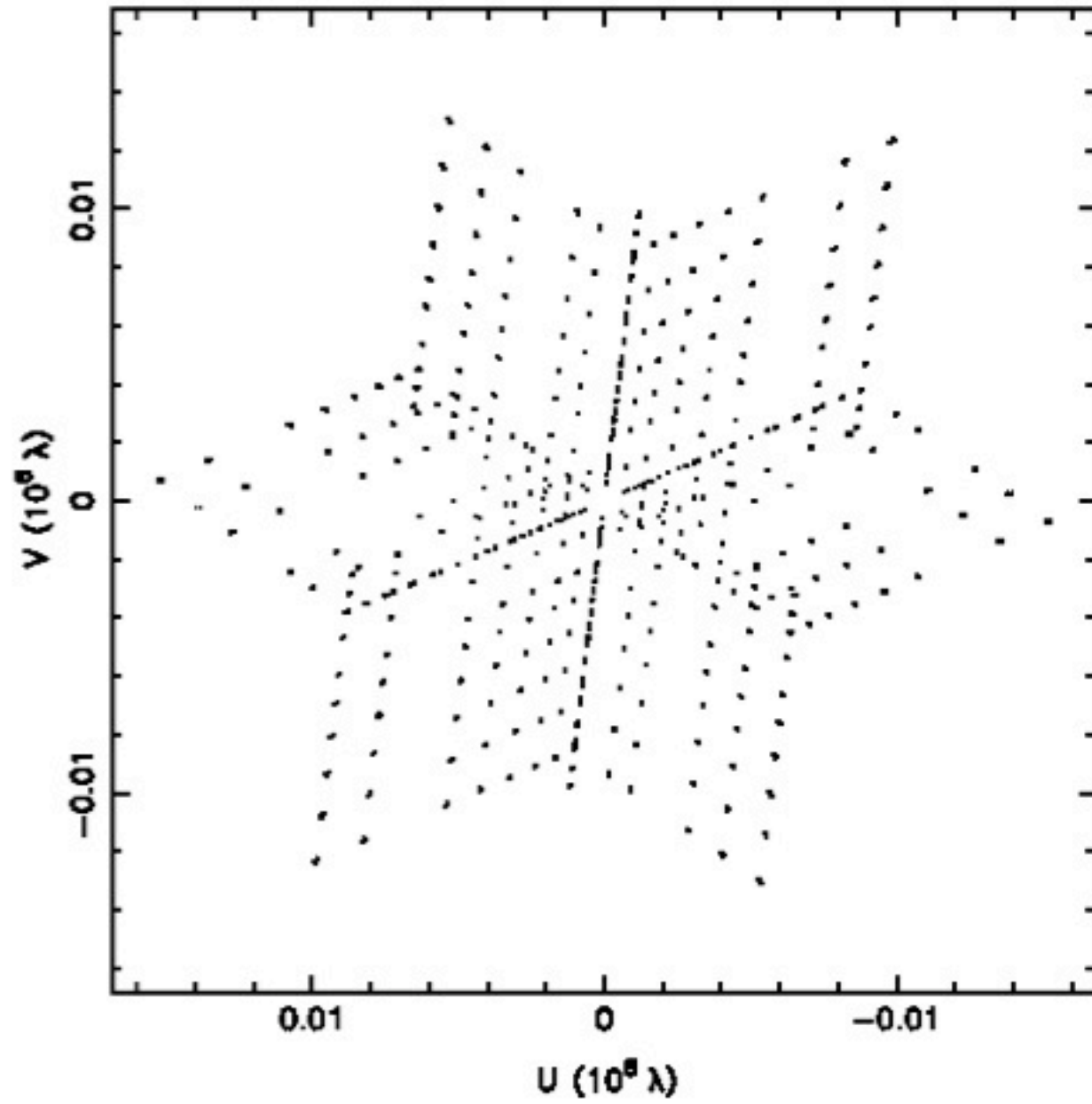
<http://www.narrabri.atnf.csiro.au/astronomy/vri.html>

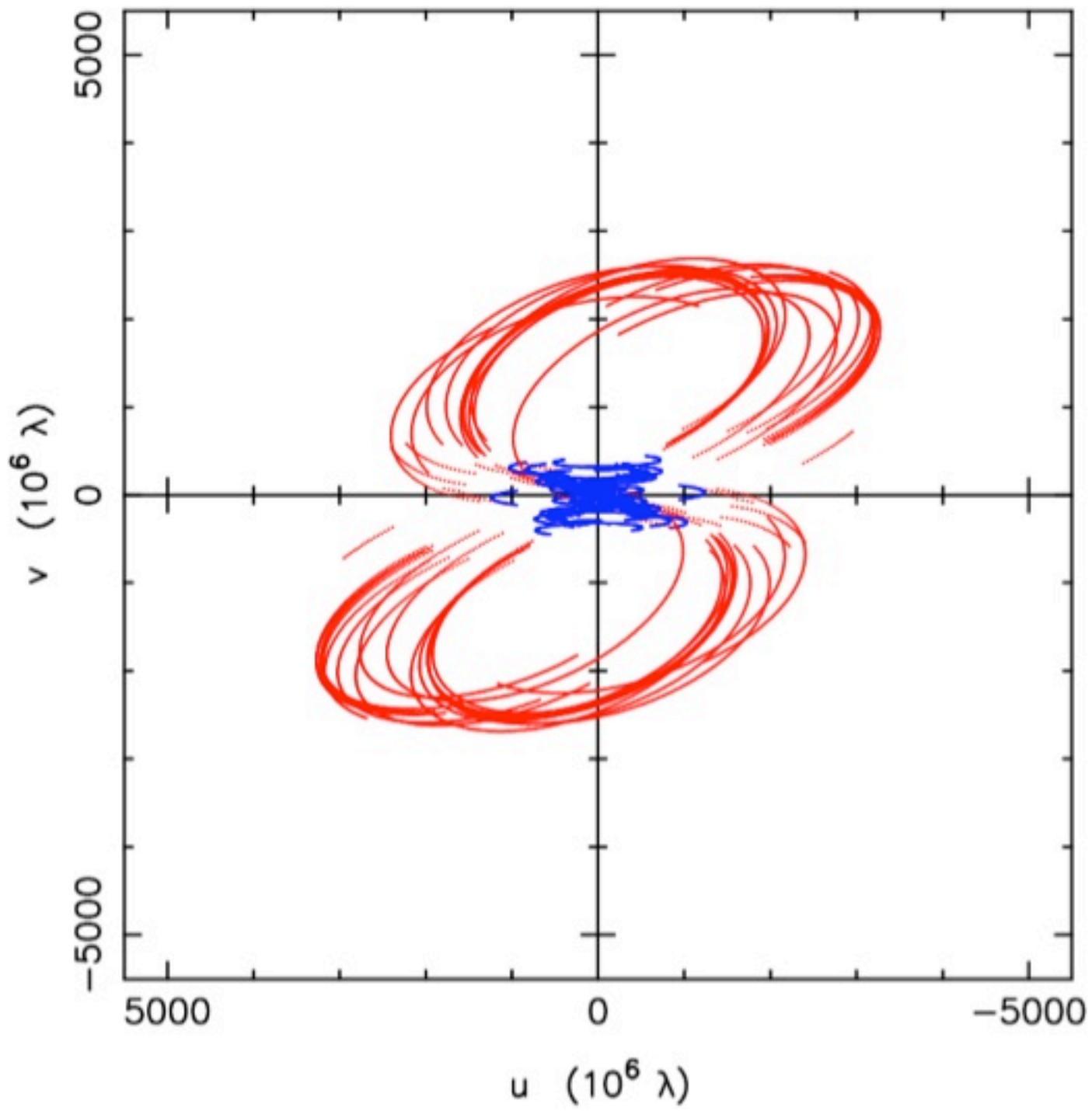
# Arrays

- An array of  $n$  antennas contains  
$$n(n-1)/2$$
  
pairs of two-element interferometers, each with its own projected baseline length and orientation
- Each baseline samples a different Fourier component of the source
- Array configurations carefully chosen to avoid redundancy and fill  $(u,v)$  plane optimally
- Let's now look at some examples!

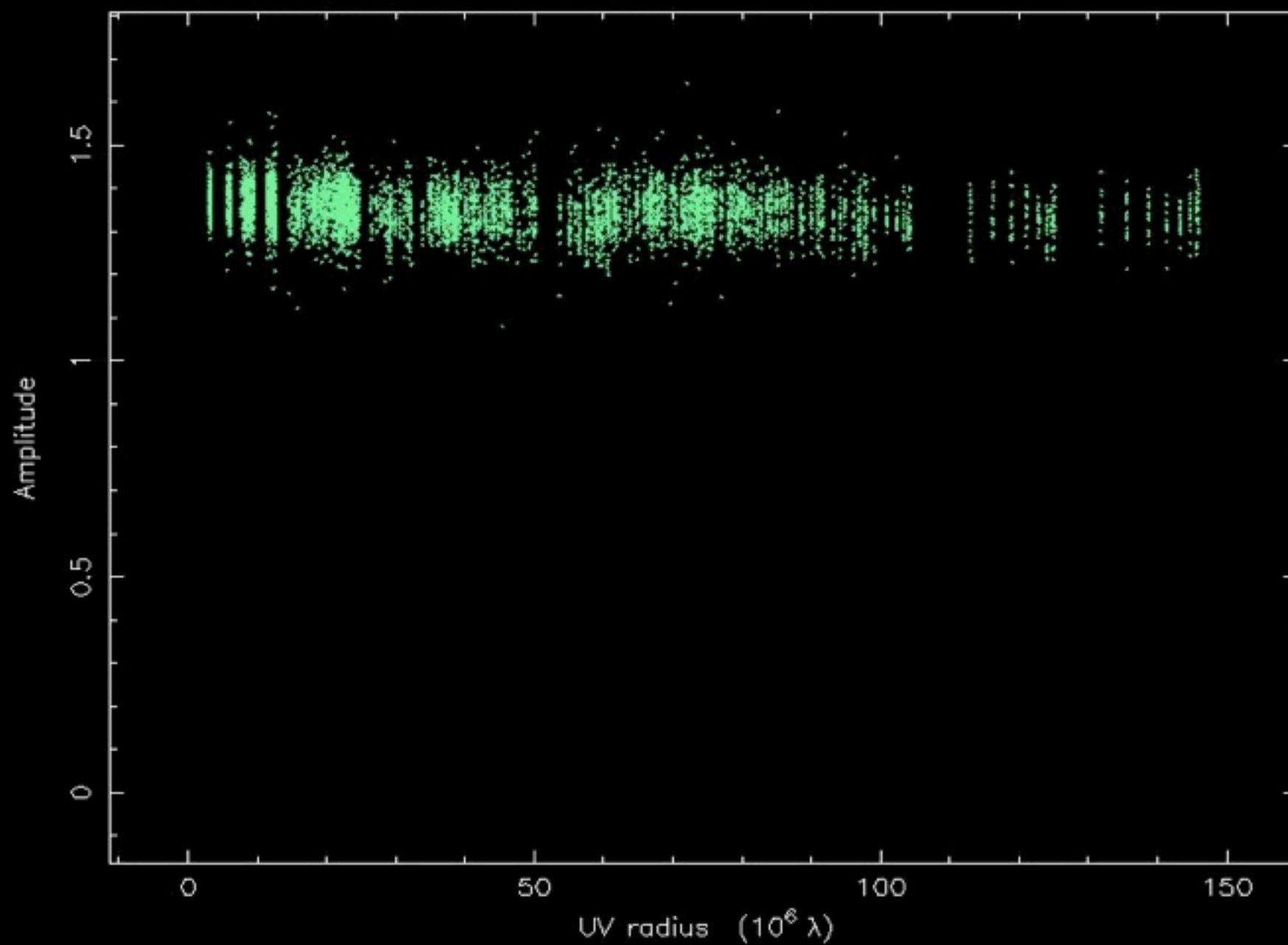


J0746+25 at 4.860 GHz in RR 2005 Nov 03

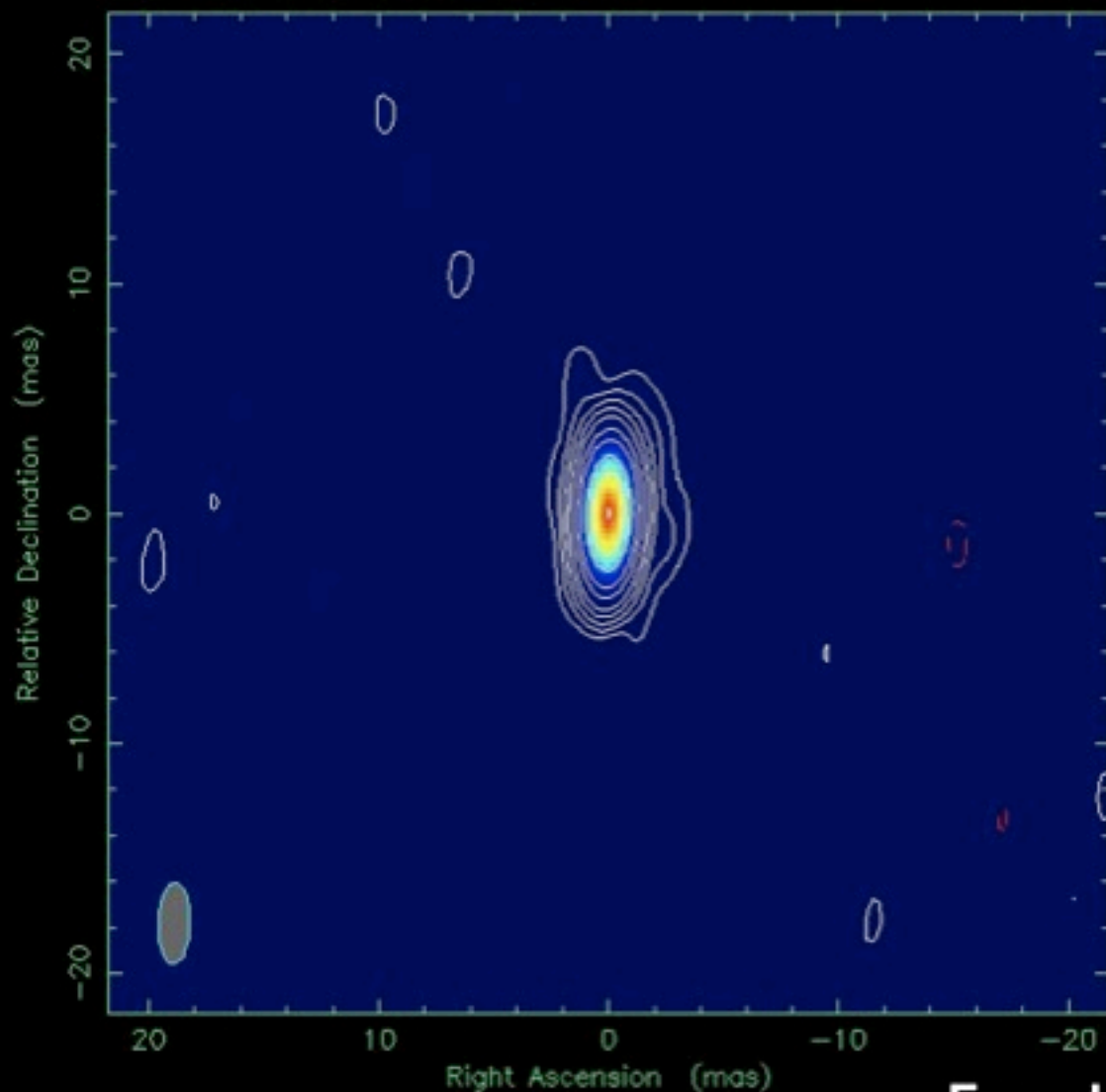




0050-092 at 4.916 GHz 1996 Jun 05



Clean map. Array: BFHKLMNOPS  
0050-092 at 4.916 GHz 1996 Jun 05



Map center: RA: 00 50 41.317, Dec: -09 29 05.209 (2000.0)  
Map peak: 1.35 Jy/beam  
Contours %: -0.1 0.1 0.25 0.5 1 2 5 10 25 50 99  
Beam FWHM: 3.54 x 1.36 (mas) at  $-1.71^\circ$



Map peak .135 Jy/beam

Model-fit:

1.35 Jy

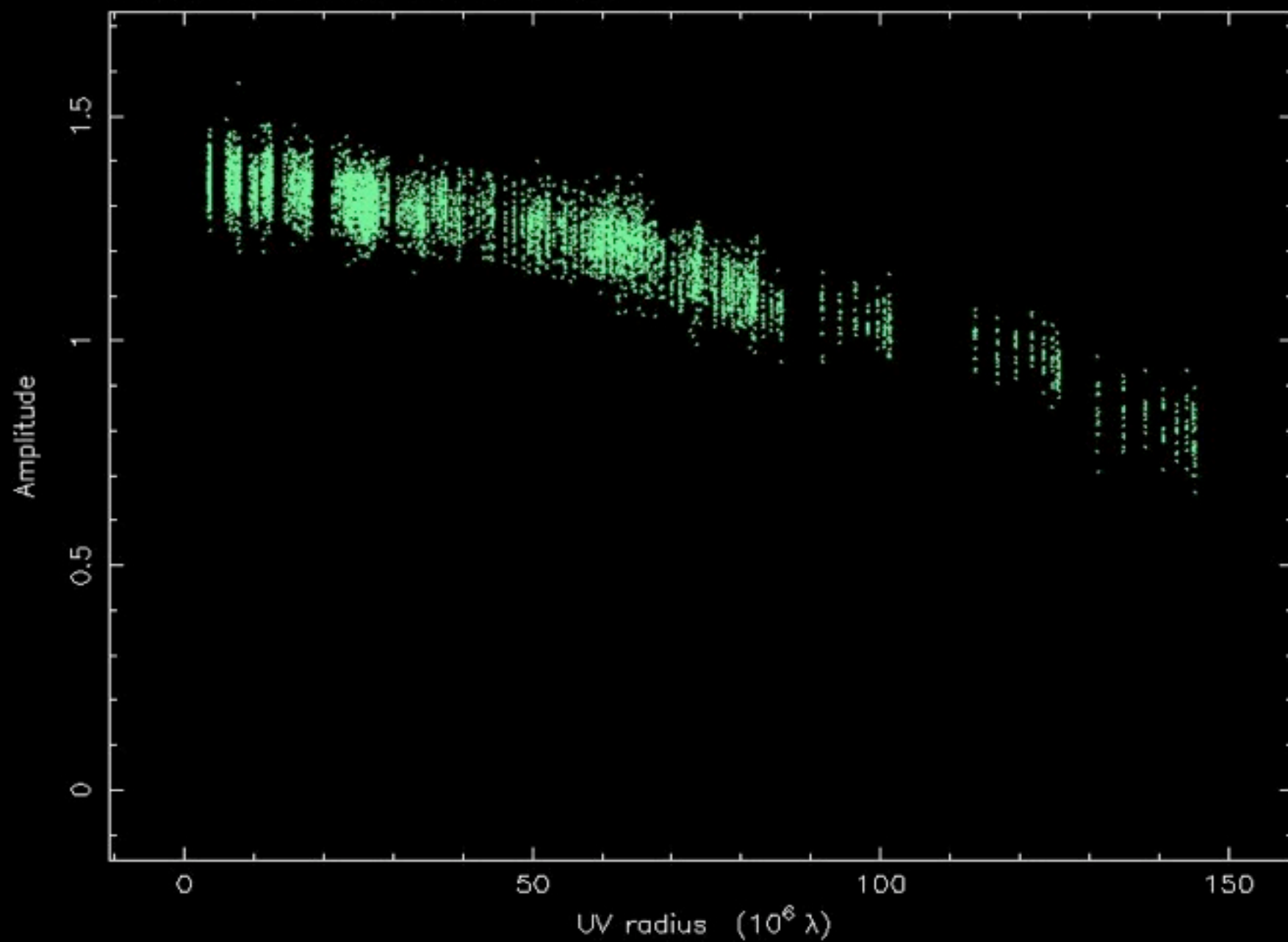
$a < 0.6$  mas

$b < 0.2$  mas

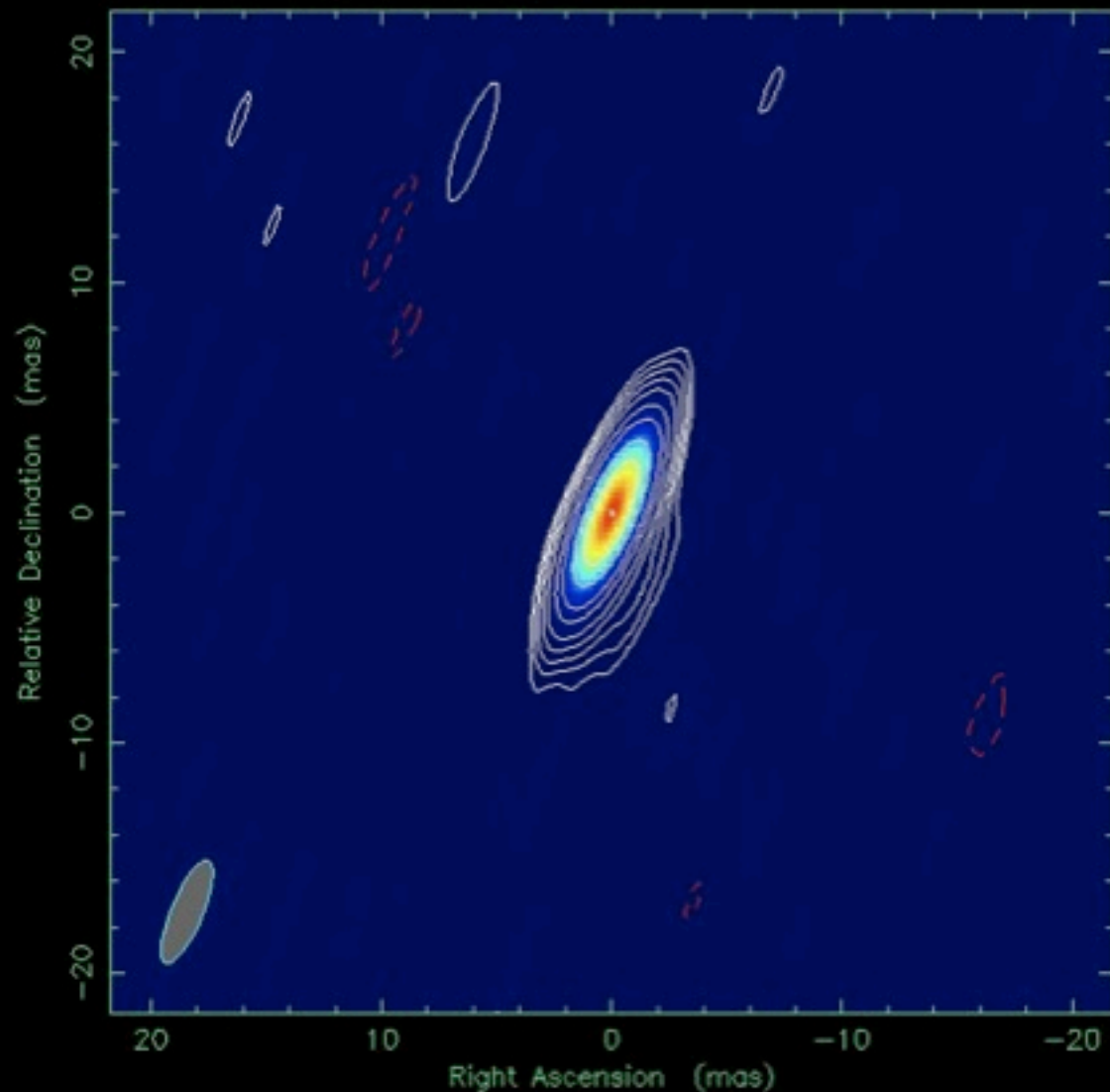
**Fomalont et al. (2000), ApJS, 131, 95**

<http://www.vsop.isas.jaxa.jp/VLBApls>

0019+732 at 4.916 GHz 1996 Jun 05



Clean map. Array: BFHKLMNOPS  
0019+732 at 4.916 GHz 1996 Jun 05



Map center: RA: 00 19 45.786, Dec: +73 27 30.017 (2000.0)  
Map peak: 1.24 Jy/beam  
Contours %: -0.1 0.1 0.25 0.5 1 2 5 10 25 50 99  
Beam FWHM: 4.81 x 1.37 (mas) at -22.9°



Map peak 1.24 Jy/beam

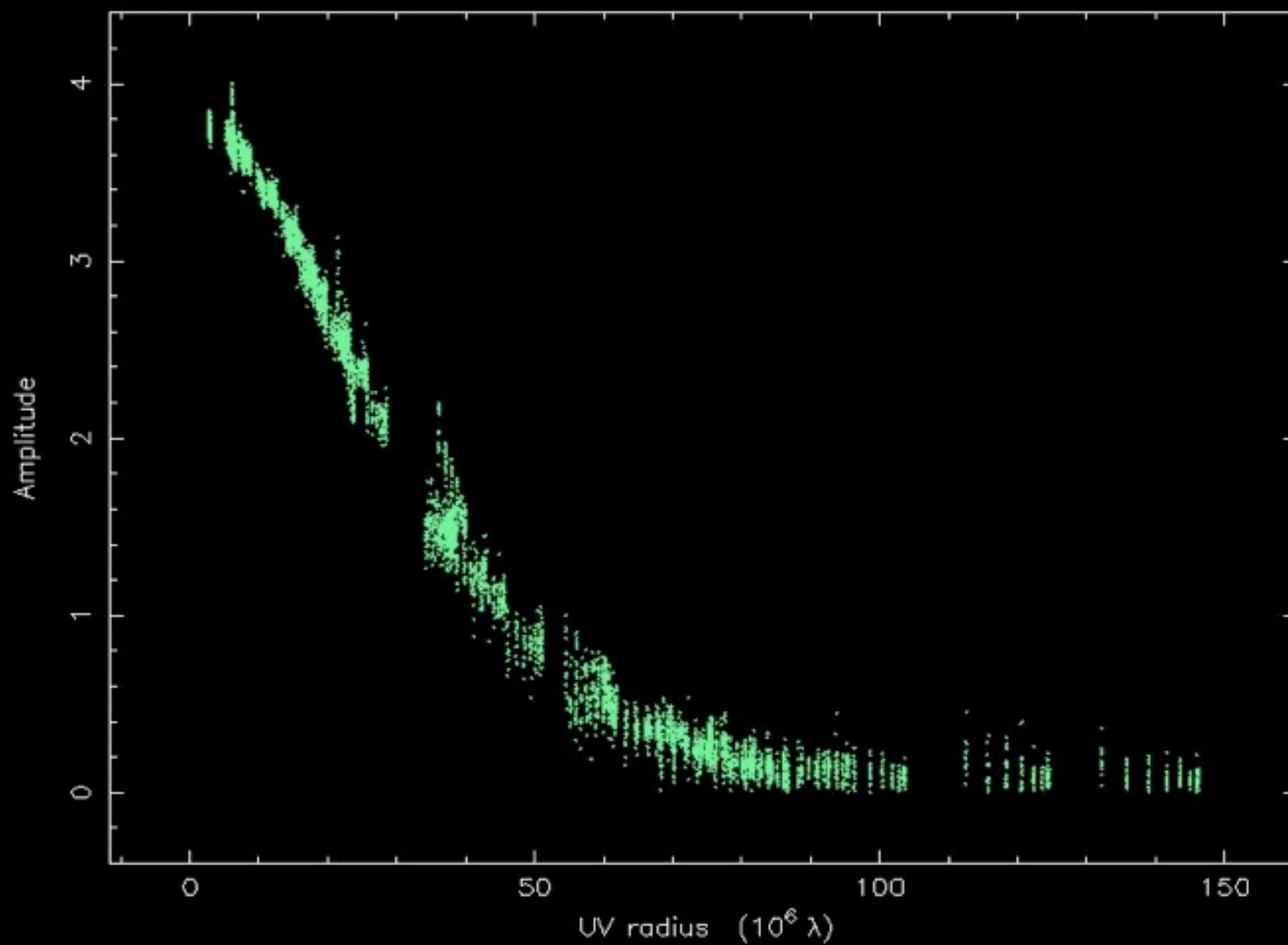
Model:

1.33 Jy

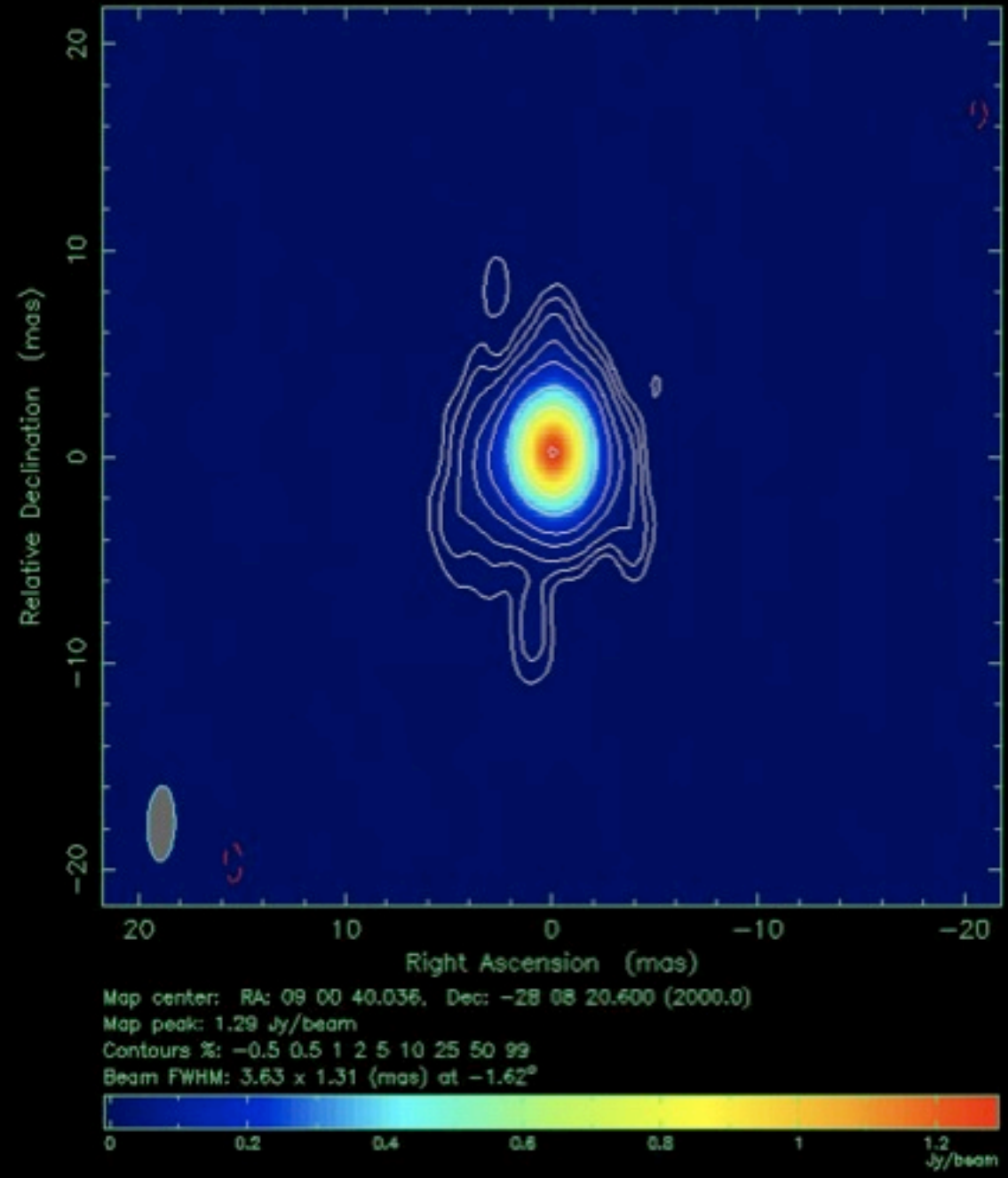
$a = 0.7$  mas

$b < 0.3$  mas

0900-280 at 4.916 GHz 1996 Jun 05



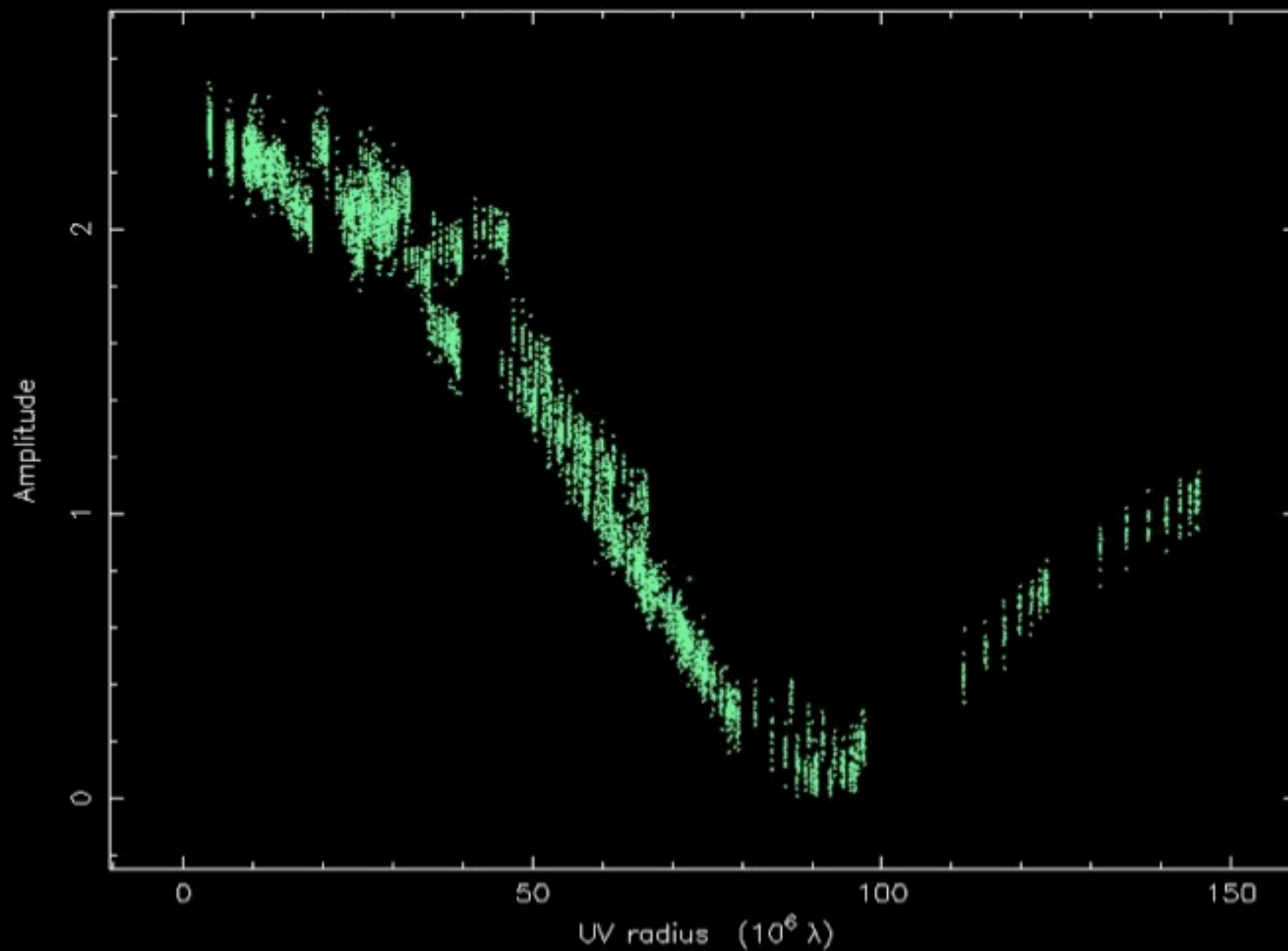
Clean map. Array: BFHKLMNOPS  
0900-280 at 4.916 GHz 1996 Jun 05



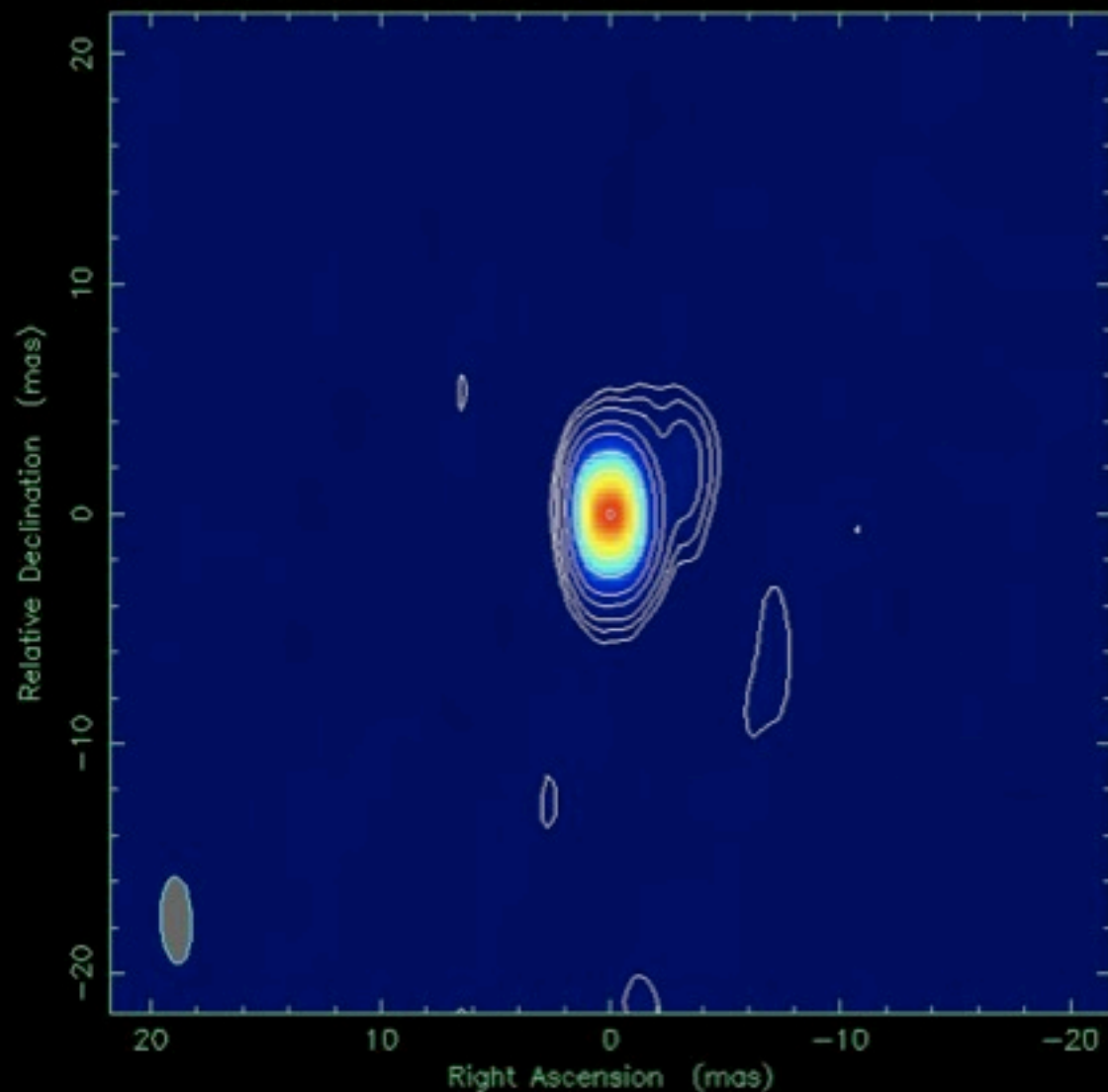
Map peak 1.3Jy/beam

Model:  
3.7 Jy  
a = 2.8 mas  
b = 1.9 mas

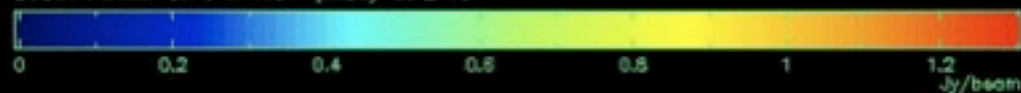
1635+380 at 4.916 GHz 1996 Jun 06



Clean map. Array: BFHKLMNOPS  
1635+380 at 4.916 GHz 1996 Jun 06



Map center: RA: 16 35 15.493, Dec: +38 08 04.500 (2000.0)  
Map peak: 1.31 Jy/beam  
Contours %: 0.5 1 2 5 10 25 50 99  
Beam FWHM: 3.79 x 1.34 (mas) at 2.15°



Map peak 1.3 Jy/beam

Model:

(i) 1.4 Jy

$a = 0.9$  mas

$b < 0.3$  mas

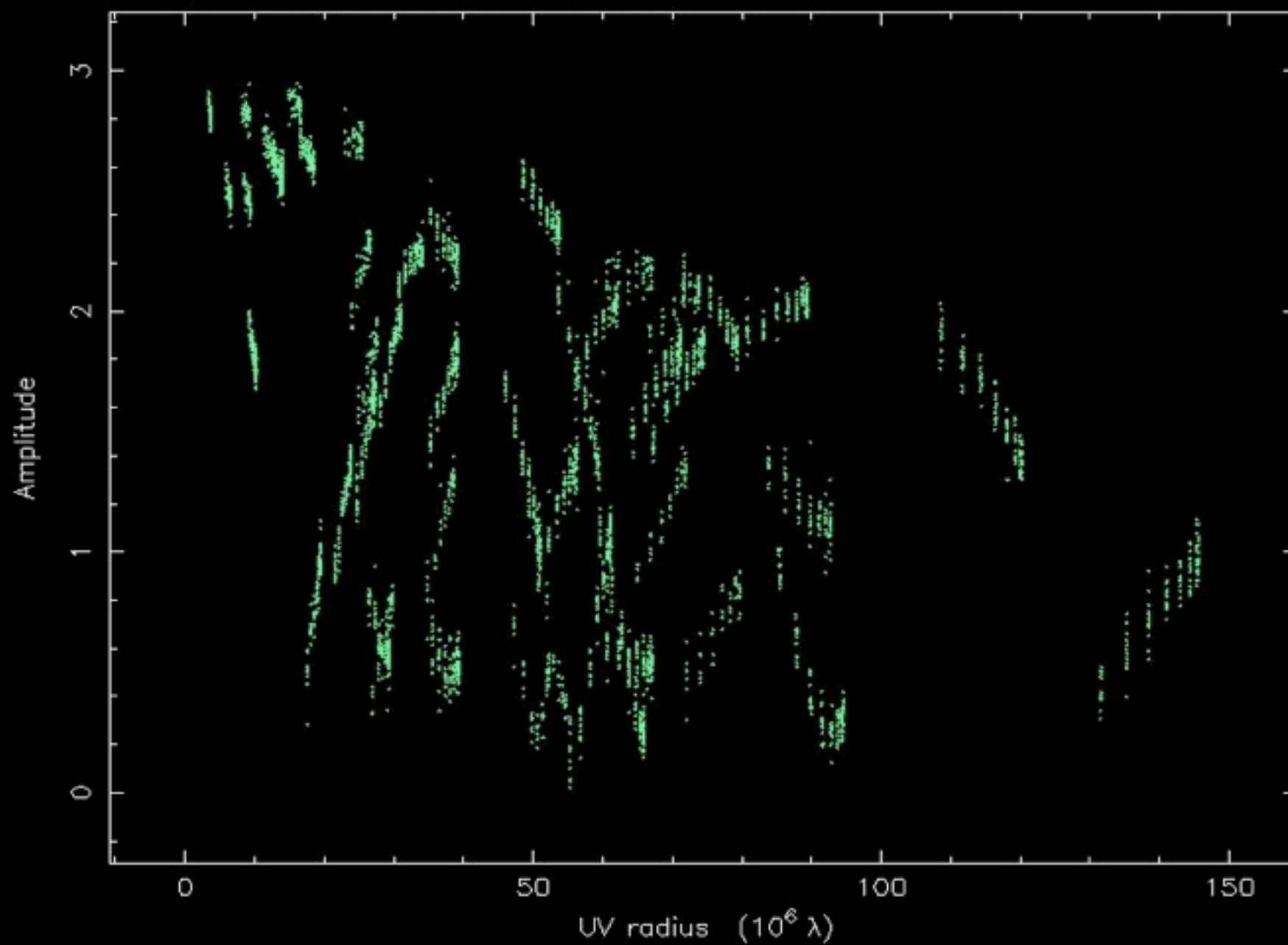
(ii) 0.8 Jy

$a < 0.7$  mas

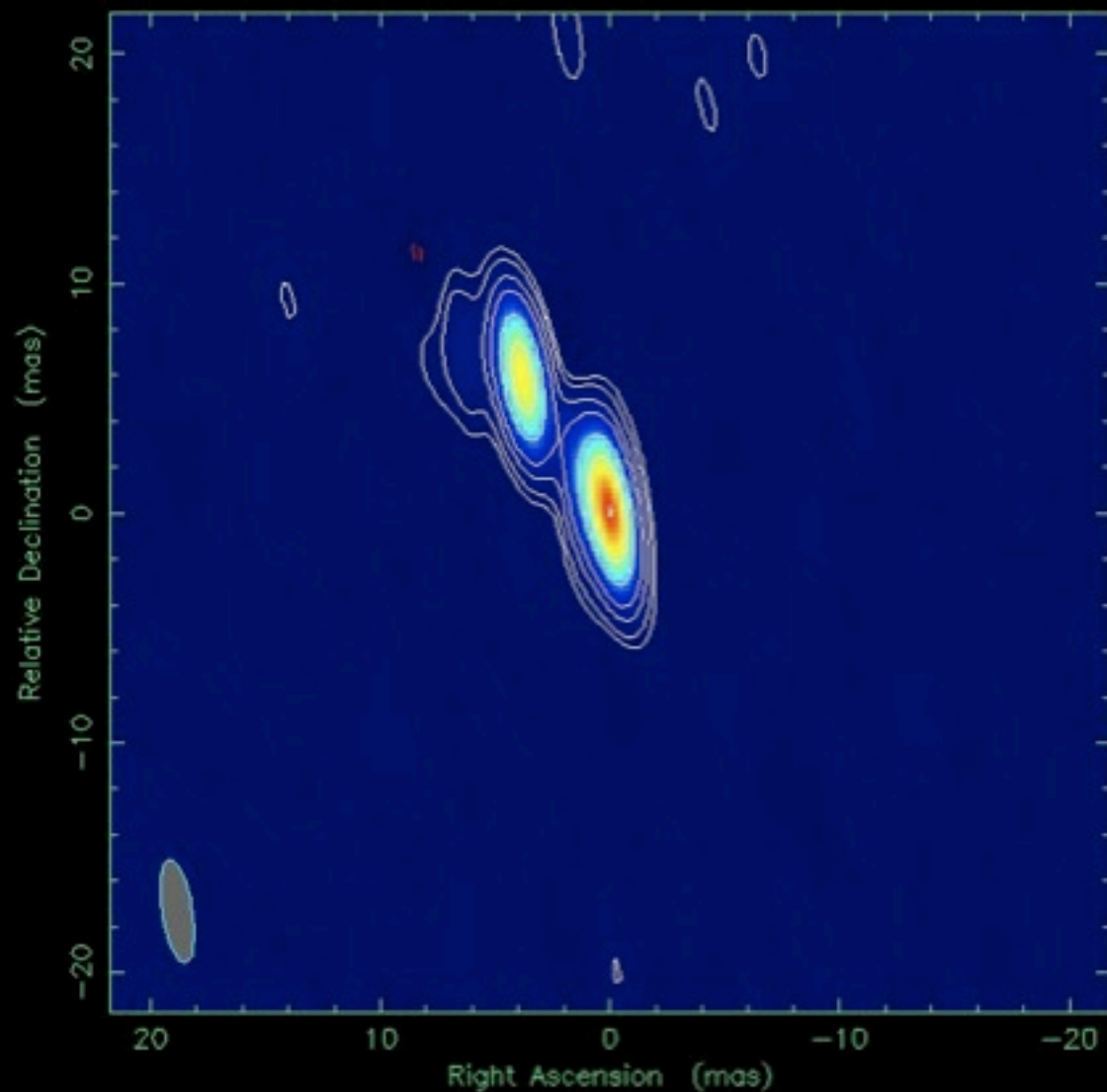
$b < 0.3$  mas

Separation 1.1 mas

2022+613 at 4.916 GHz 1996 Jun 05



Clean map. Array: BFHKLMNOPS  
2022+613 at 4.916 GHz 1996 Jun 05

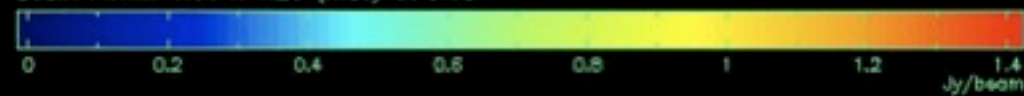


Map center: RA: 20 22 06.682, Dec: +61 36 58.804 (2000.0)

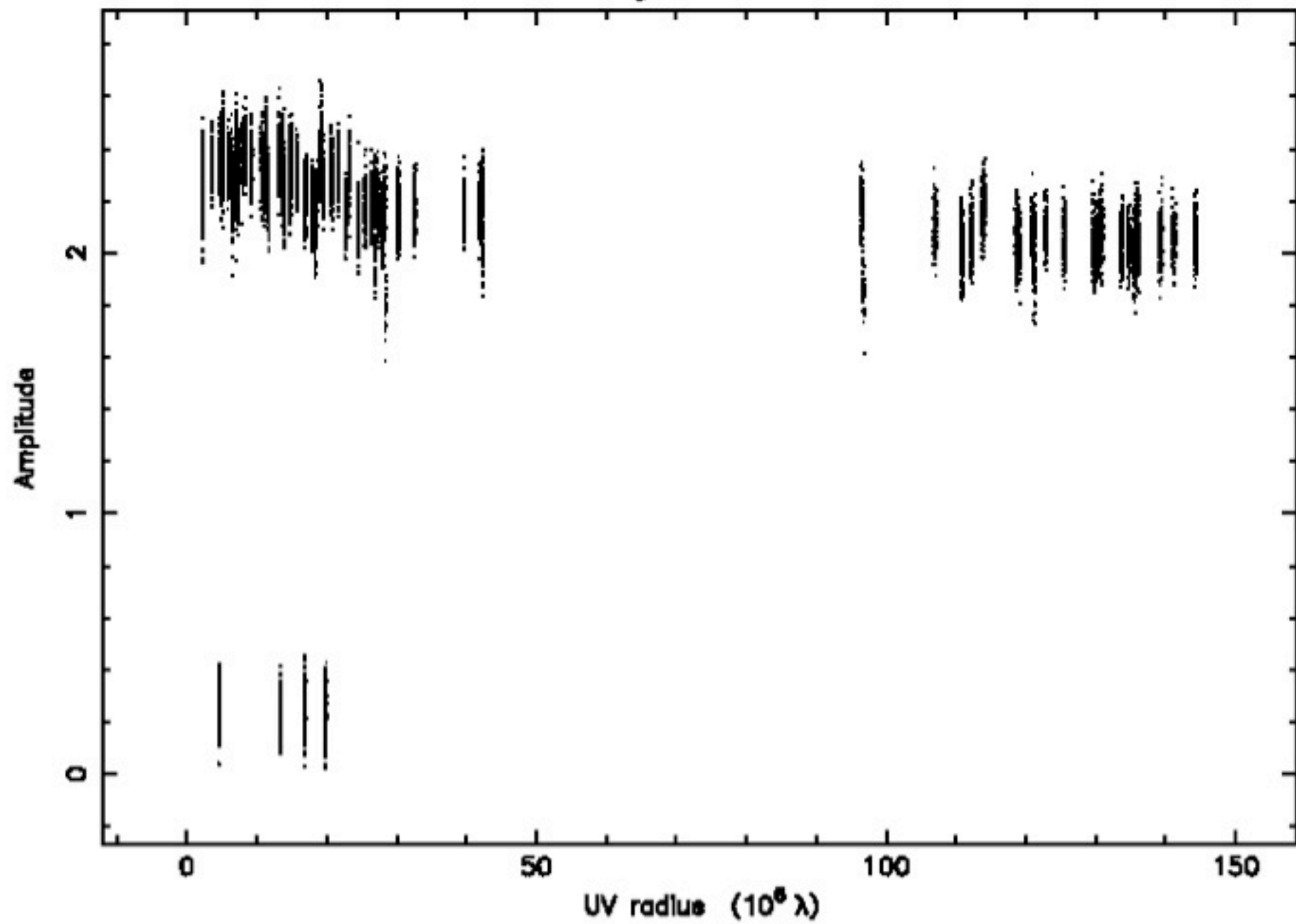
Map peak: 1.43 Jy/beam

Contours %: -1 1 2 5 10 25 50 99

Beam FWHM: 4.56 x 1.29 (mas) at 8.95°



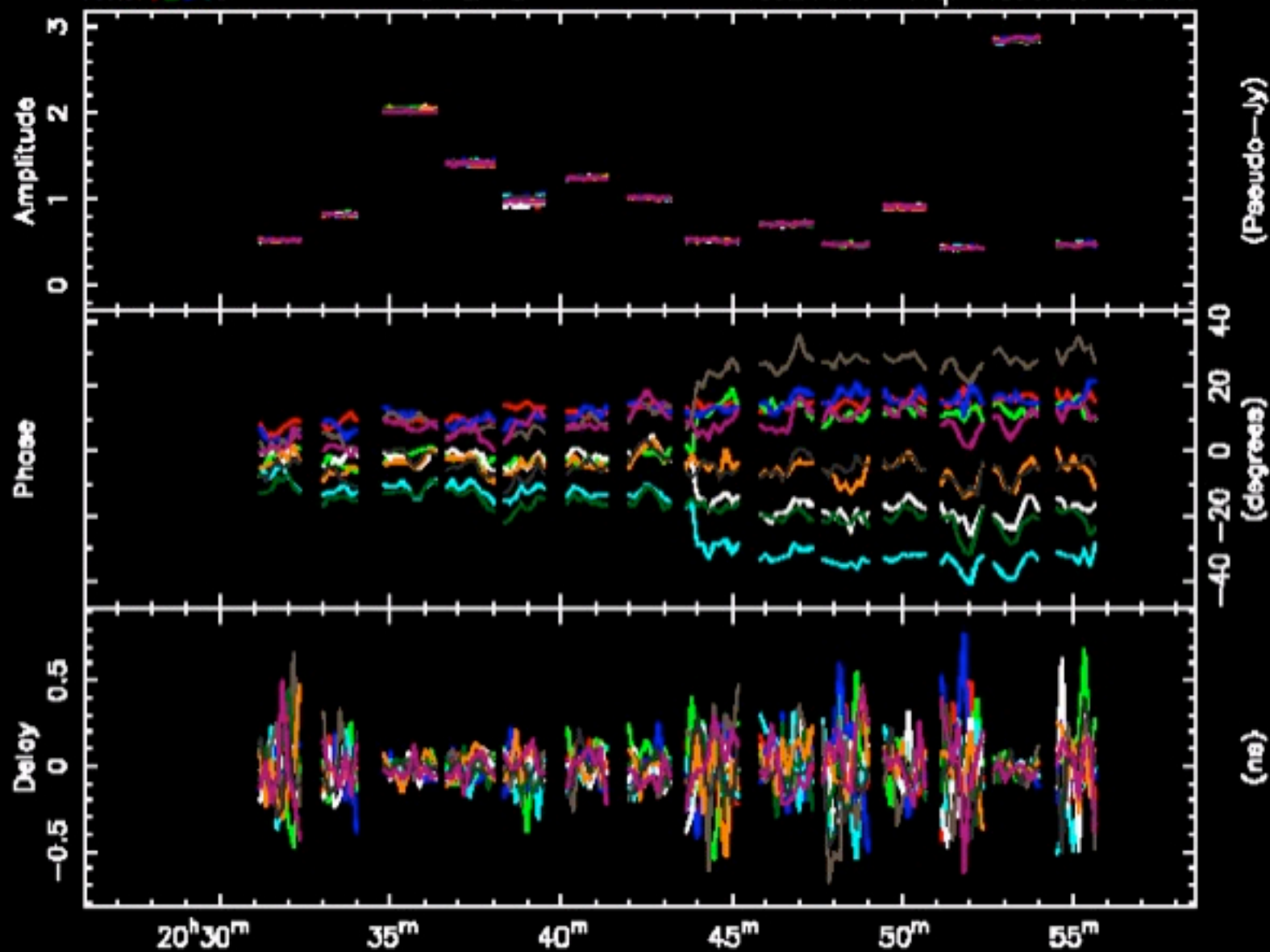
J2258-27 at 8.421 GHz In RR 2005 Aug 29



Ants: 12345-

$\Delta$   $\square$   $\oplus$   $\Gamma$

1532+018 Frq: 4800. & 8640.



45AA 23AA 12AA 34AA 35AA 13AA 24AA 25AA 14AA 15AA

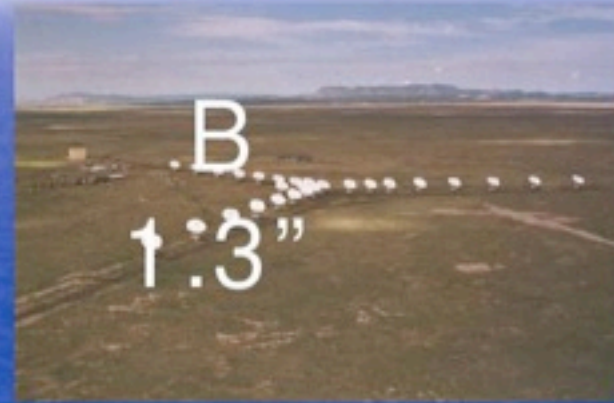
# Interferometry in practice

- Baseline calibration
  - Done in advance by Observatory staff
- Delay calibration
  - Do at beginning of observation
- Amplitude calibration
  - Scan(s) of PKS 1934-638 or Uranus
- Phase calibration
  - Regular scans of bright, nearby, compact source
- Bandpass calibration
  - Especially for spectral line observations
- Pointing calibration
  - Regular pointing updates for mm observations

## Extra considerations for wide fields

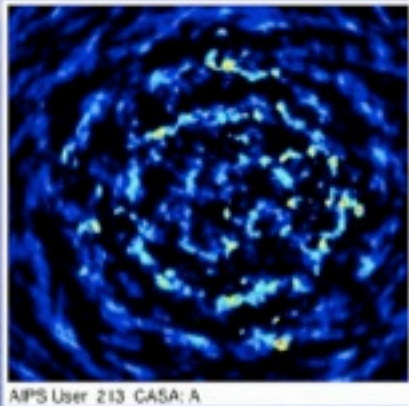
- Data is averaged (typically over 10 sec) before being output
- For wide fields:
  - Temporal (azimuthal) smearing
  - Bandwidth (radial) smearing
  - Primary beam effects
  - Mosaicing

# A real-life example

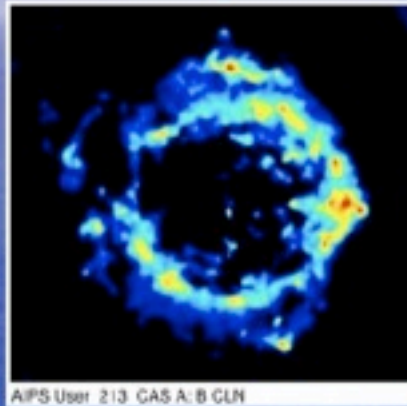


# Cas A: four VLA configurations

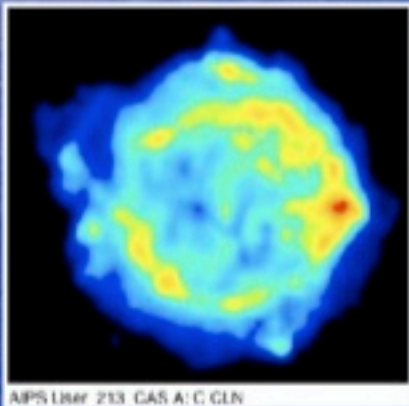
A  
0.3''



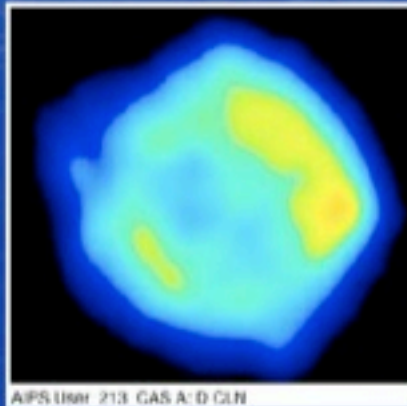
B  
1.3''



C  
4''

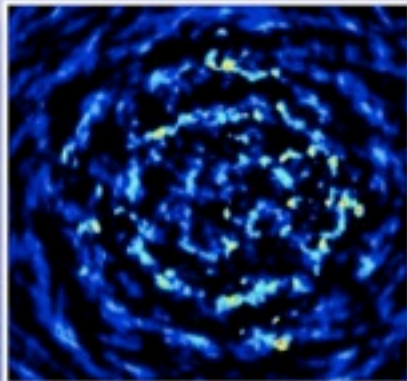


D  
15''

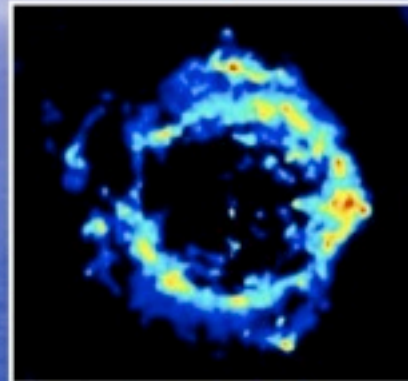


# Cas A: four VLA configurations

A  
0.3"

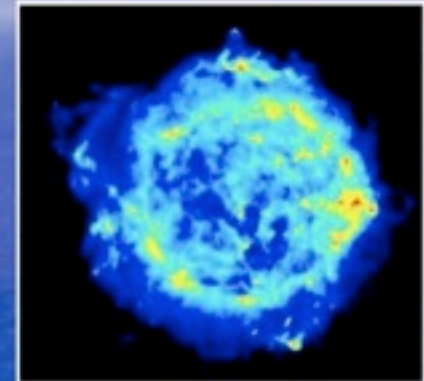


AIPS User 213 CASA: A



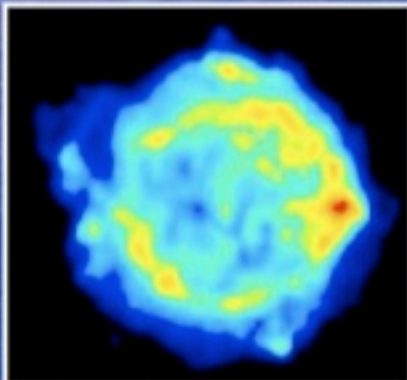
AIPS User 213 CAS A: B CLN

B  
1.3"

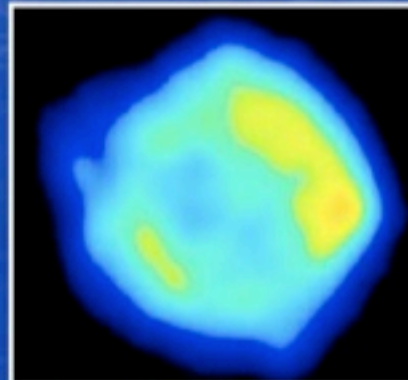


AIPS User 213 CASA: A+B+C

C  
4"



AIPS User 213 CAS A: C CLN

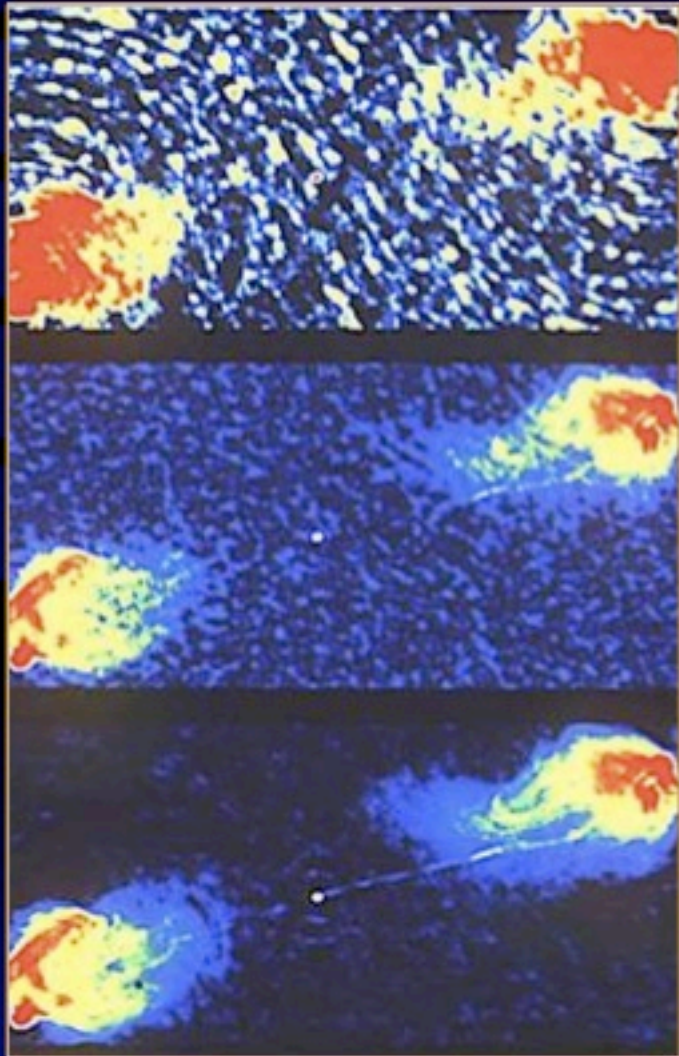


AIPS User 213 CAS A: D CLN

D  
15"

A+ B+ C+ D  
0.3" +  
total flux

# Cygnus A



← Raw data

VLA continuum

← Deconvolution

correcting for gaps between  
telescopes

← Self Calibration

adaptive optics

# Review

- Interferometry uses indirect imaging to gain in angular resolution
- Image plane is the FT of  $(u,v)$  plane
- Not all spatial frequencies are sampled in a single observation, which can lead to deficiencies in the image