Radio Interferometry

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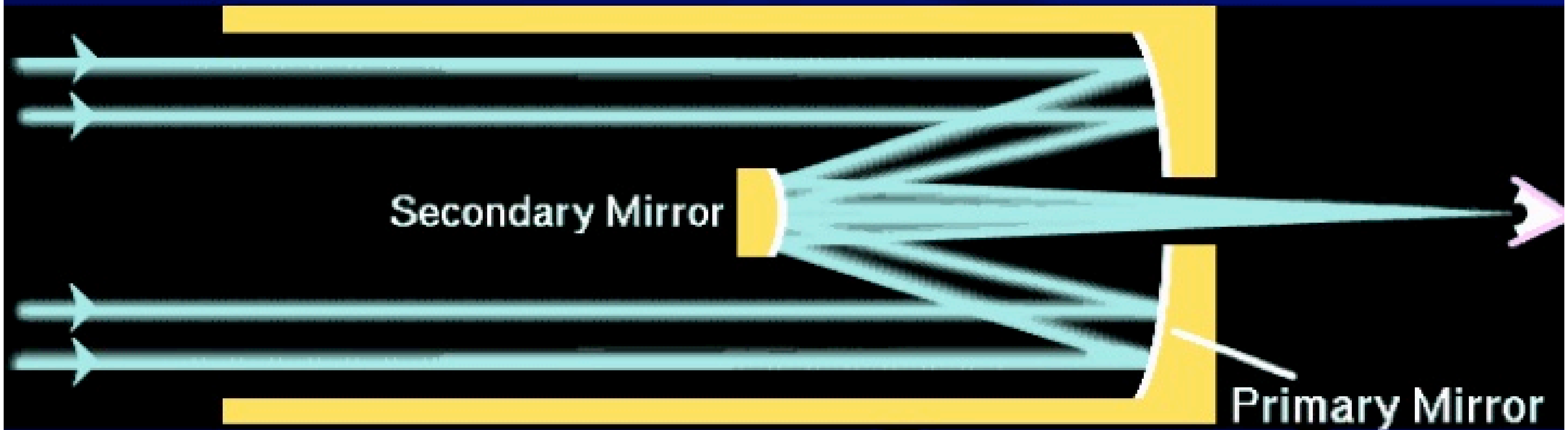
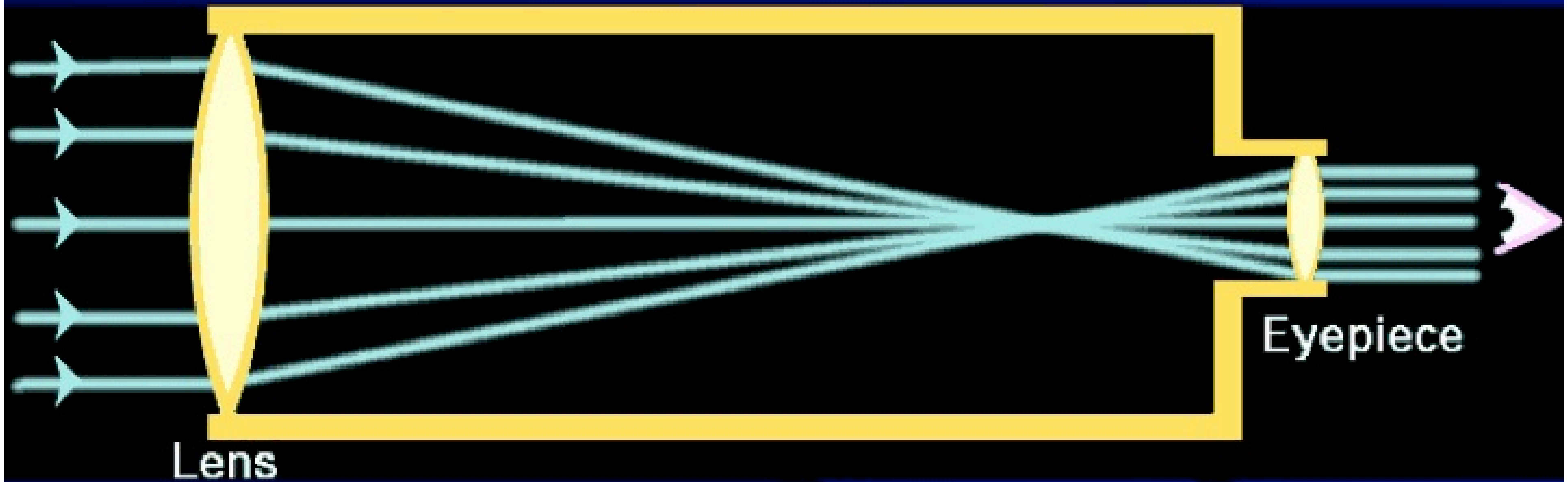
CSIRO ATNF

How telescopes work

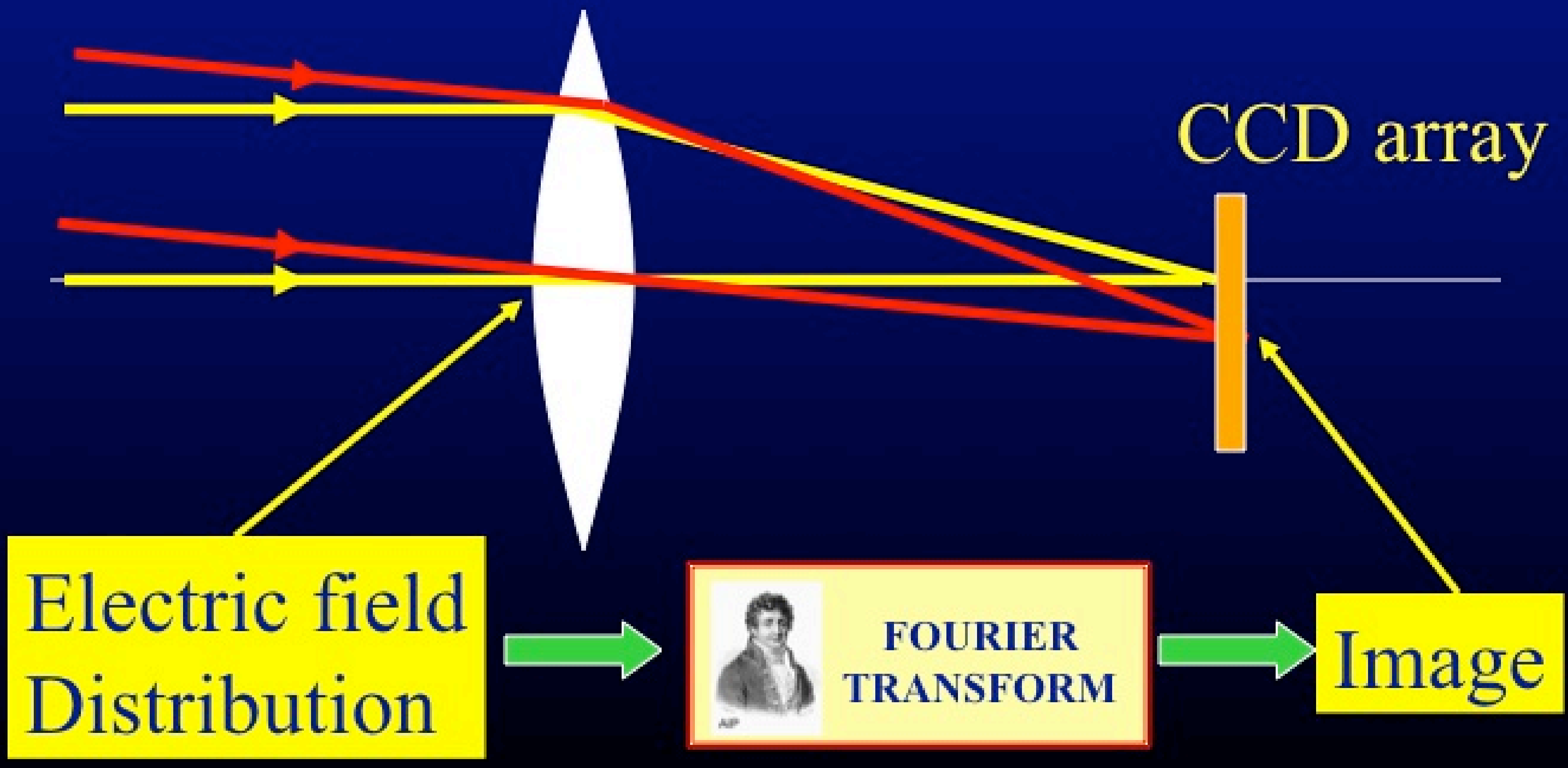
An optical Cassegrain telescope



An optical refracting telescope



What does the lens do (version 2)?



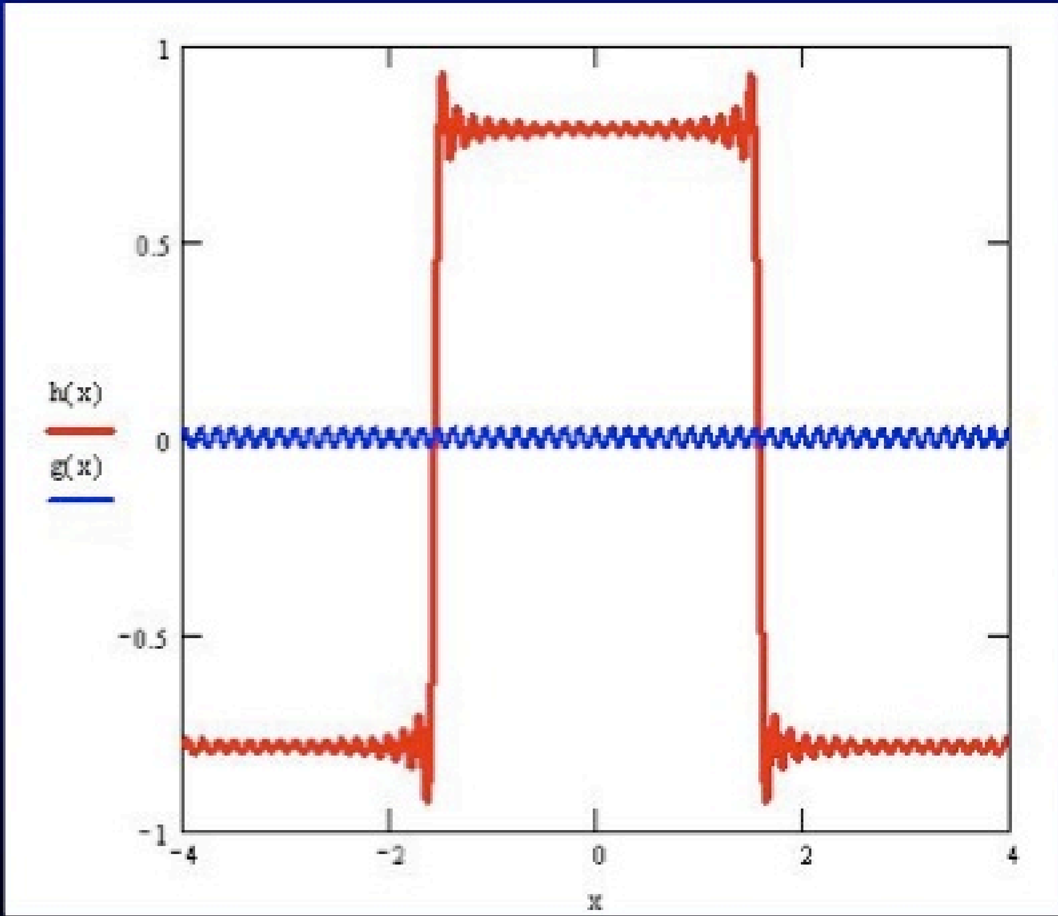
The Fourier Transform

Comte Jean Baptiste Joseph Fourier
1768-1830

Relates:

- time distribution of a wave - frequency distribution
- amplitude of a wavefront - image producing it
- many other pairs of quantities

All waveforms are formed from a sum of sine waves

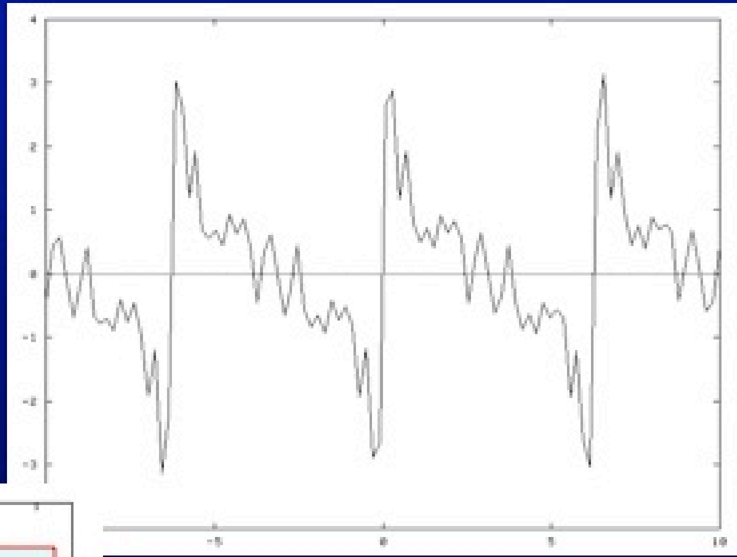


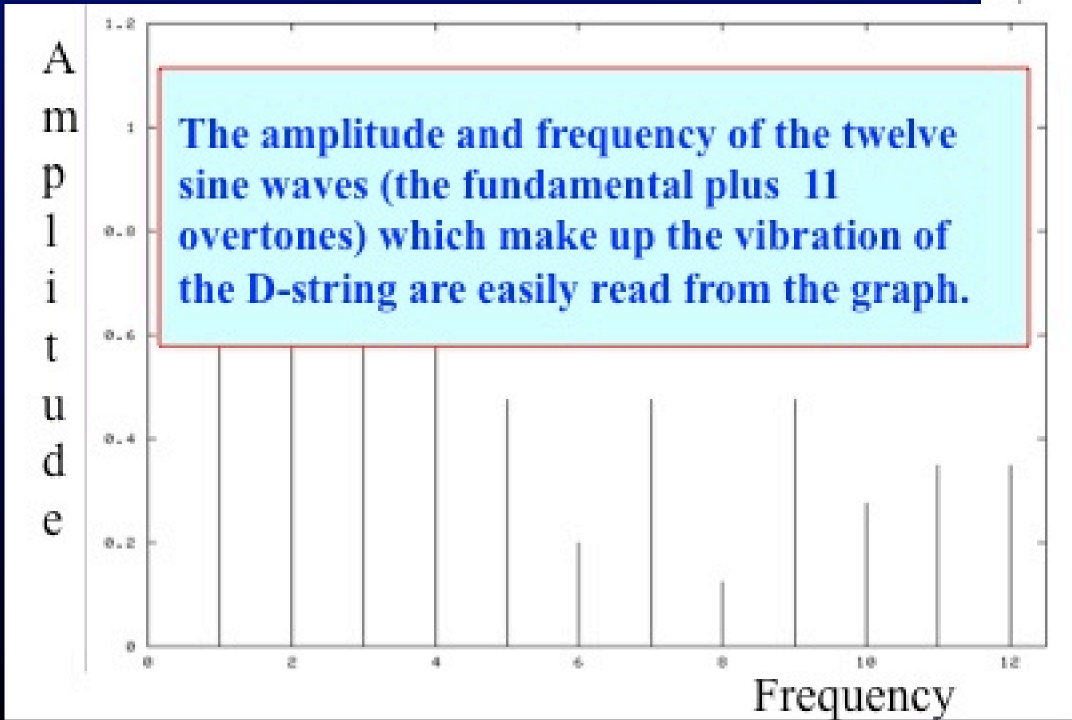
In general, **any** function can be composed -"*synthesised*" - from a number of sines and cosines of different periods and amplitudes.

Image courtesy Dave McConnell

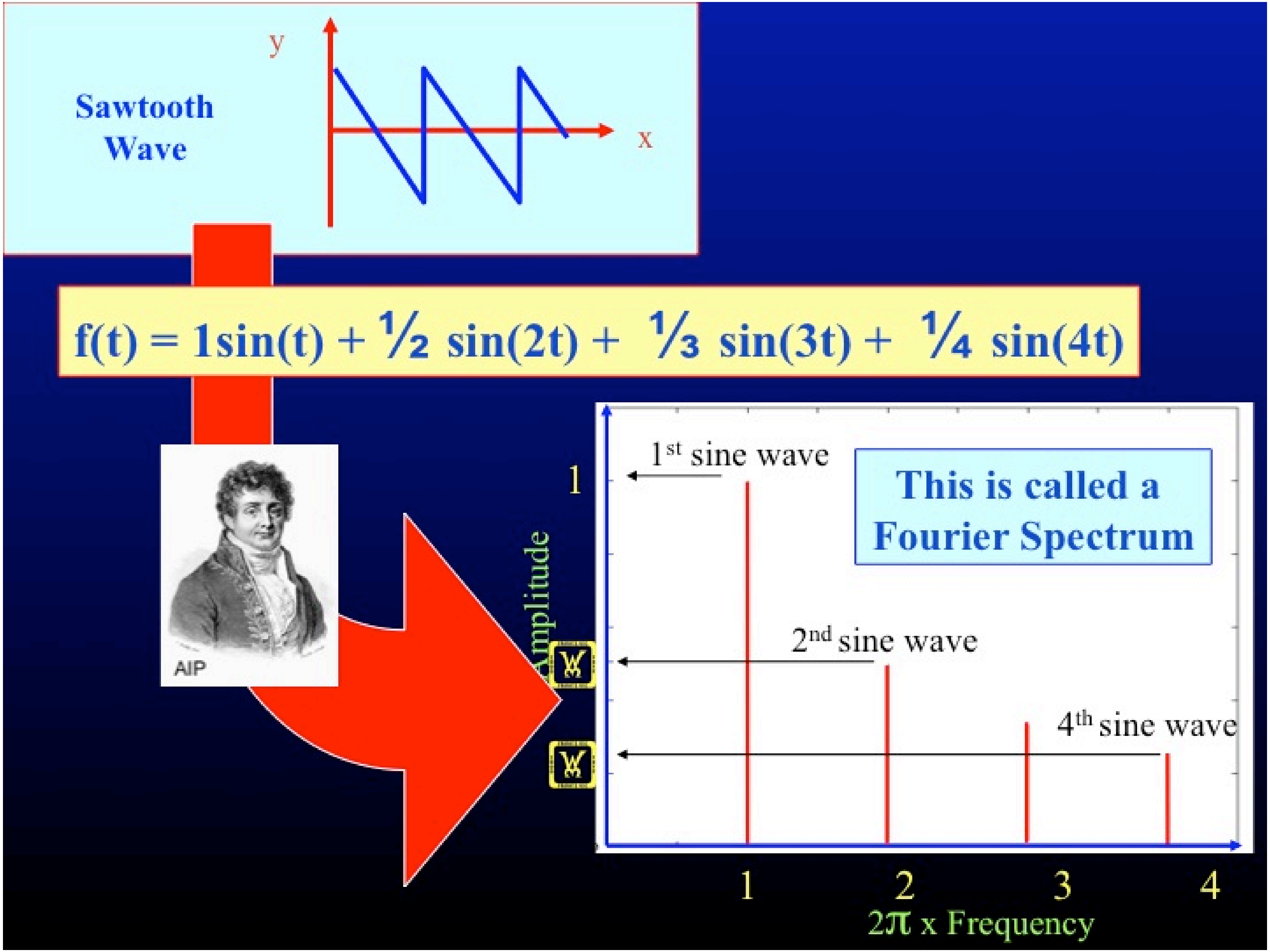
A Violin String

The open D string of a violin has the following waveform in the time domain:



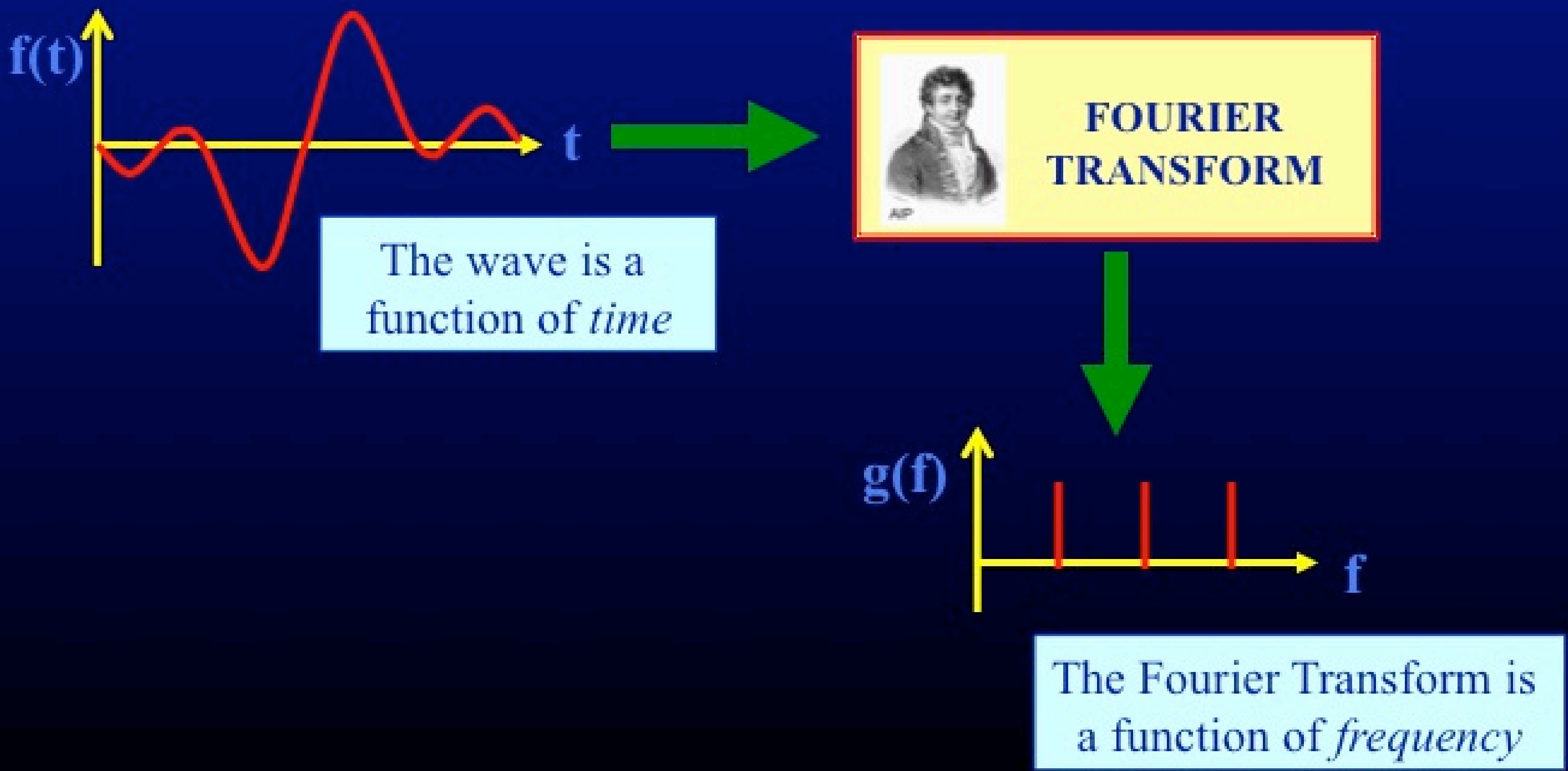


The Fourier Transform gives this frequency domain representation

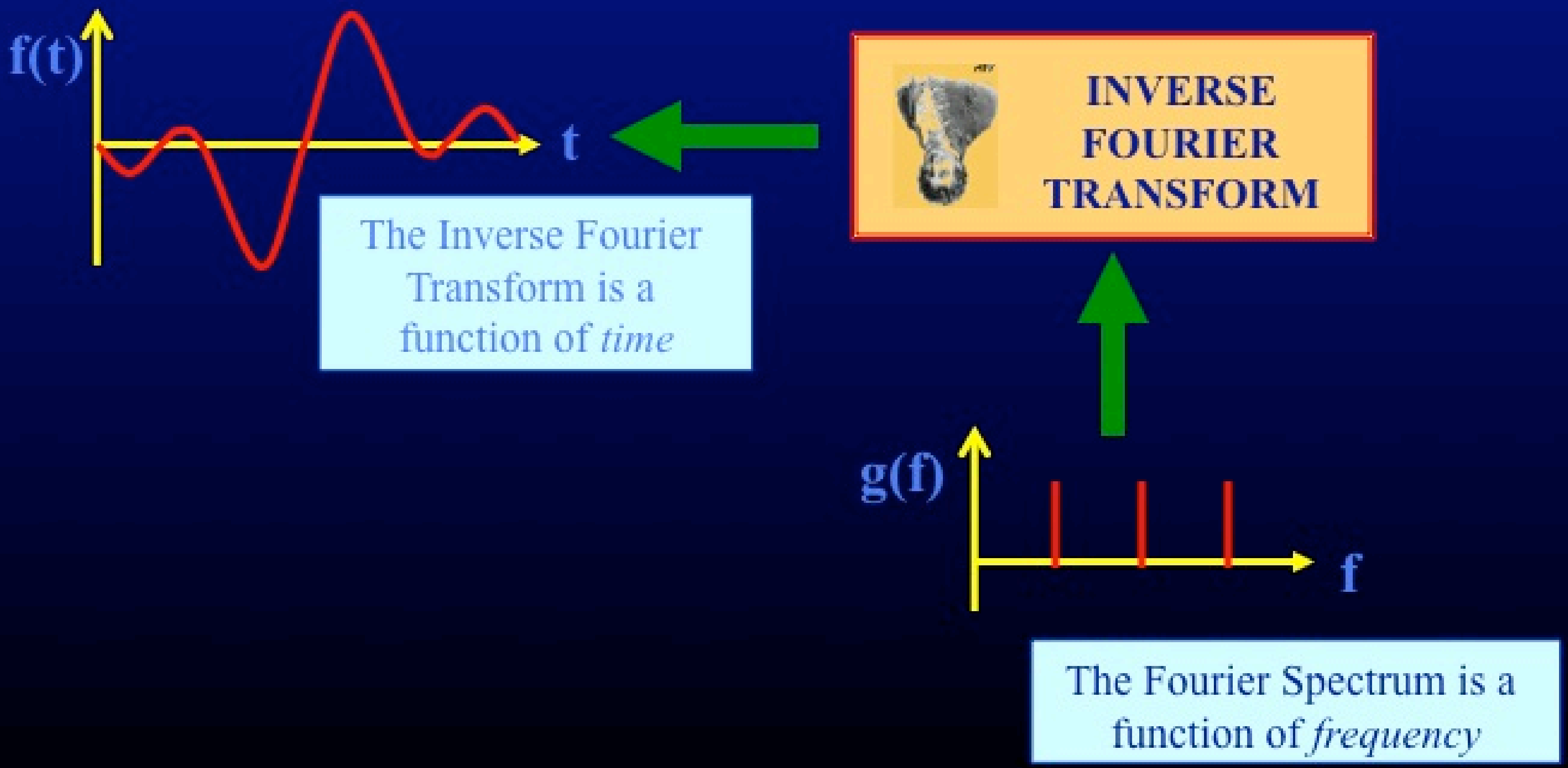


Sawtooth Wave
y
x
f(t) = 1sin(t) + ½ sin(2t) + ⅓ sin(3t) + ¼ sin(4t)
AIP
1st sine wave
This is called a Fourier Spectrum
1
Amplitude
2nd sine wave
4th sine wave
1
2
3
4
2π x Frequency

Fourier Transforms



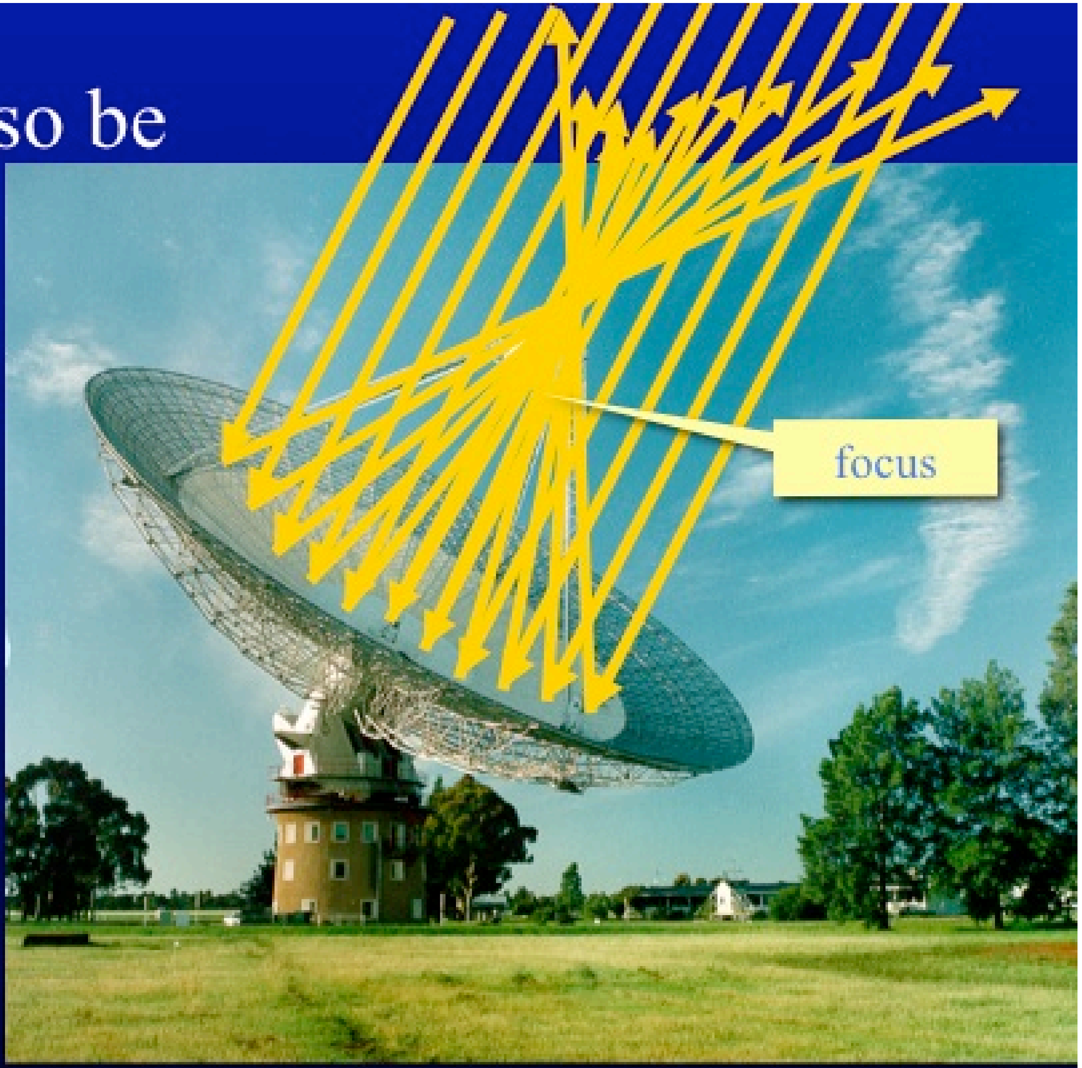
Fourier Transforms



Back to telescopes

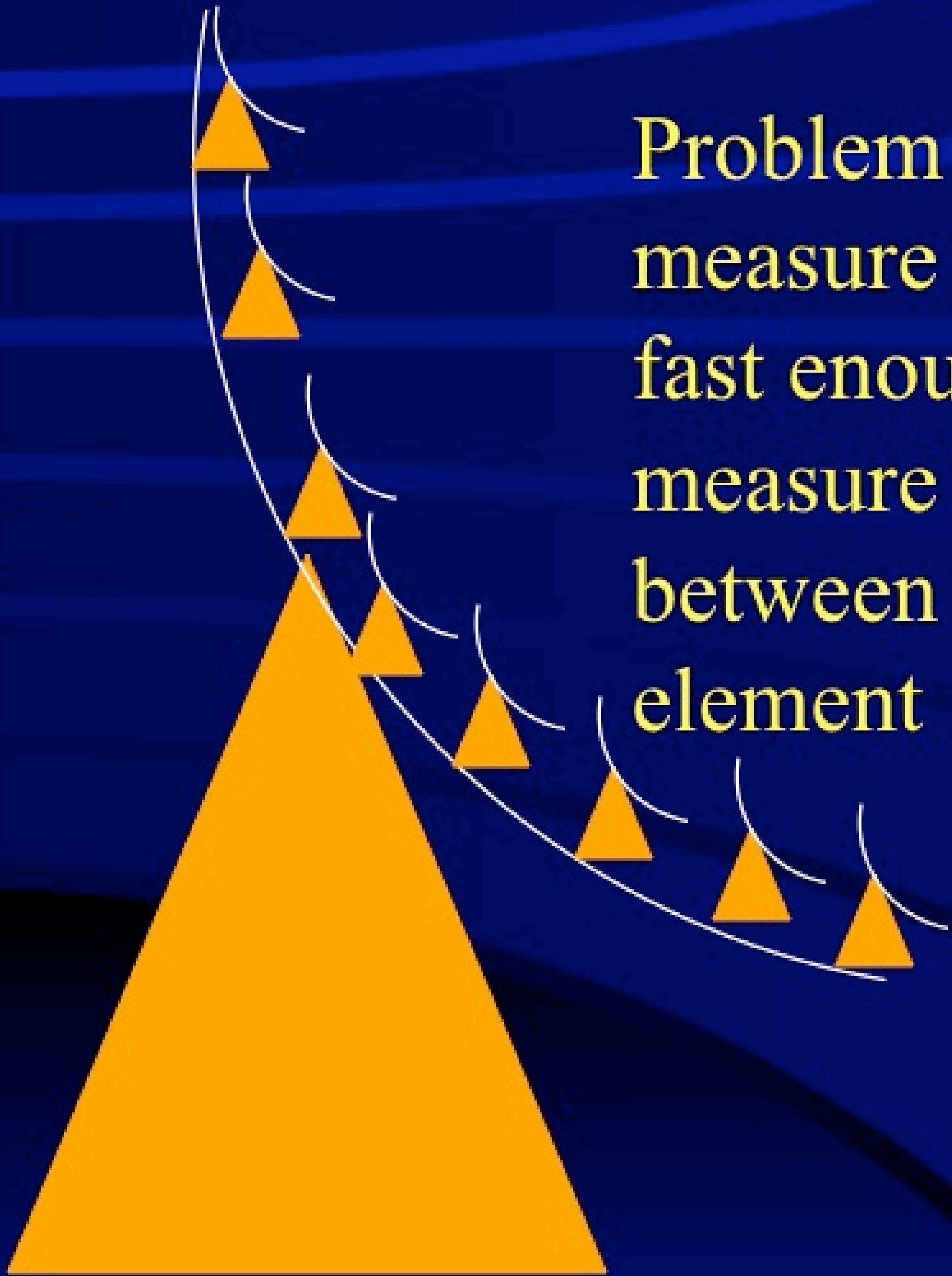
A parabolic dish can also be viewed in this way

- The shape of the dish delays different rays so they are in step at one place (the focus)
- The image formed at the focus is the Fourier transform of the wavefront



So how can we synthesise a really large telescope?

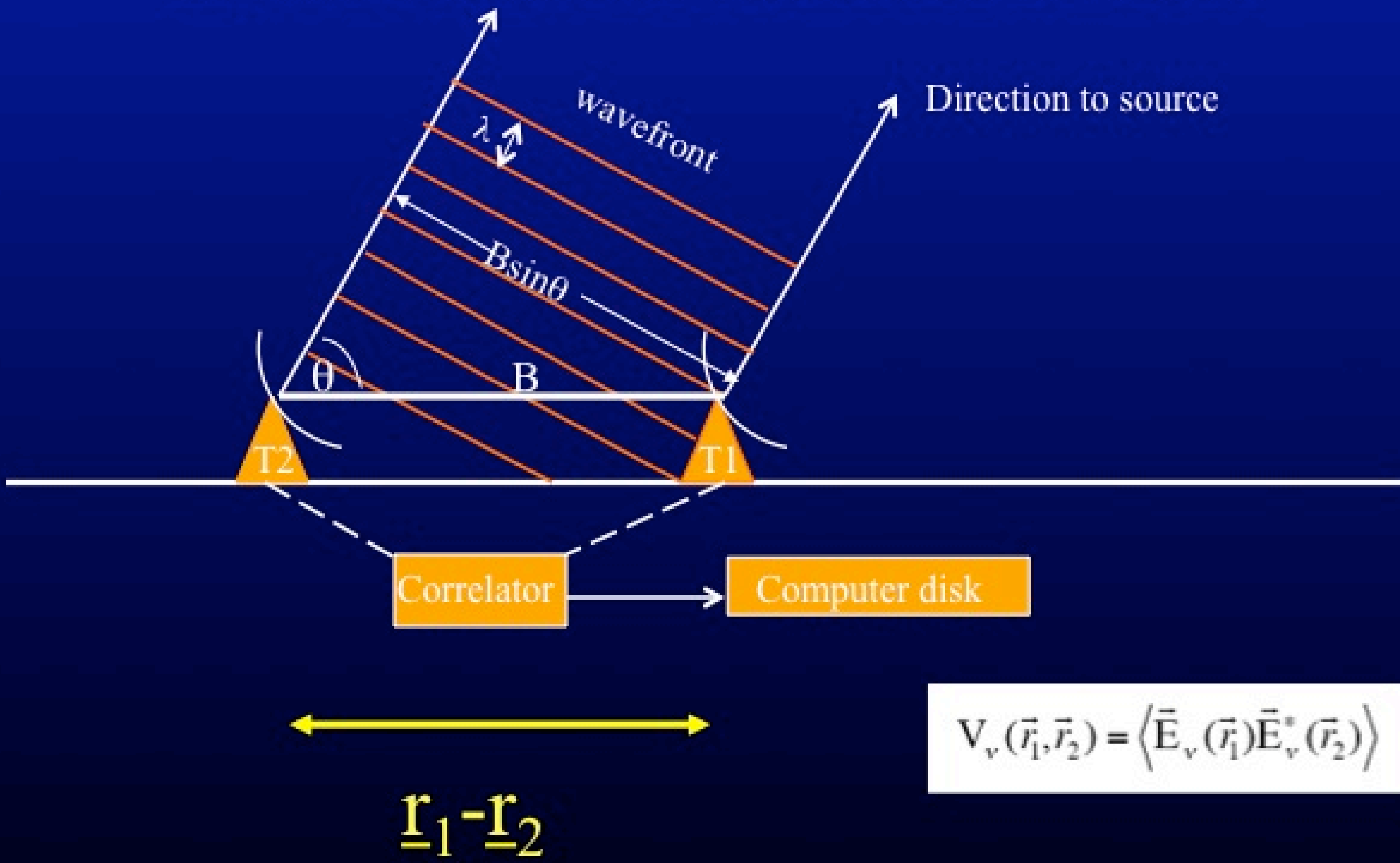
- View 1: we capture the rays at different places, and then delay them by the right amount, bring them together to form an image
- View 2: we measure the electric field in various places, and then calculate the Fourier transform of that distribution



Problem 1: we can't actually measure the electric field fast enough. Instead we measure the cross-correlations between the signals at each element

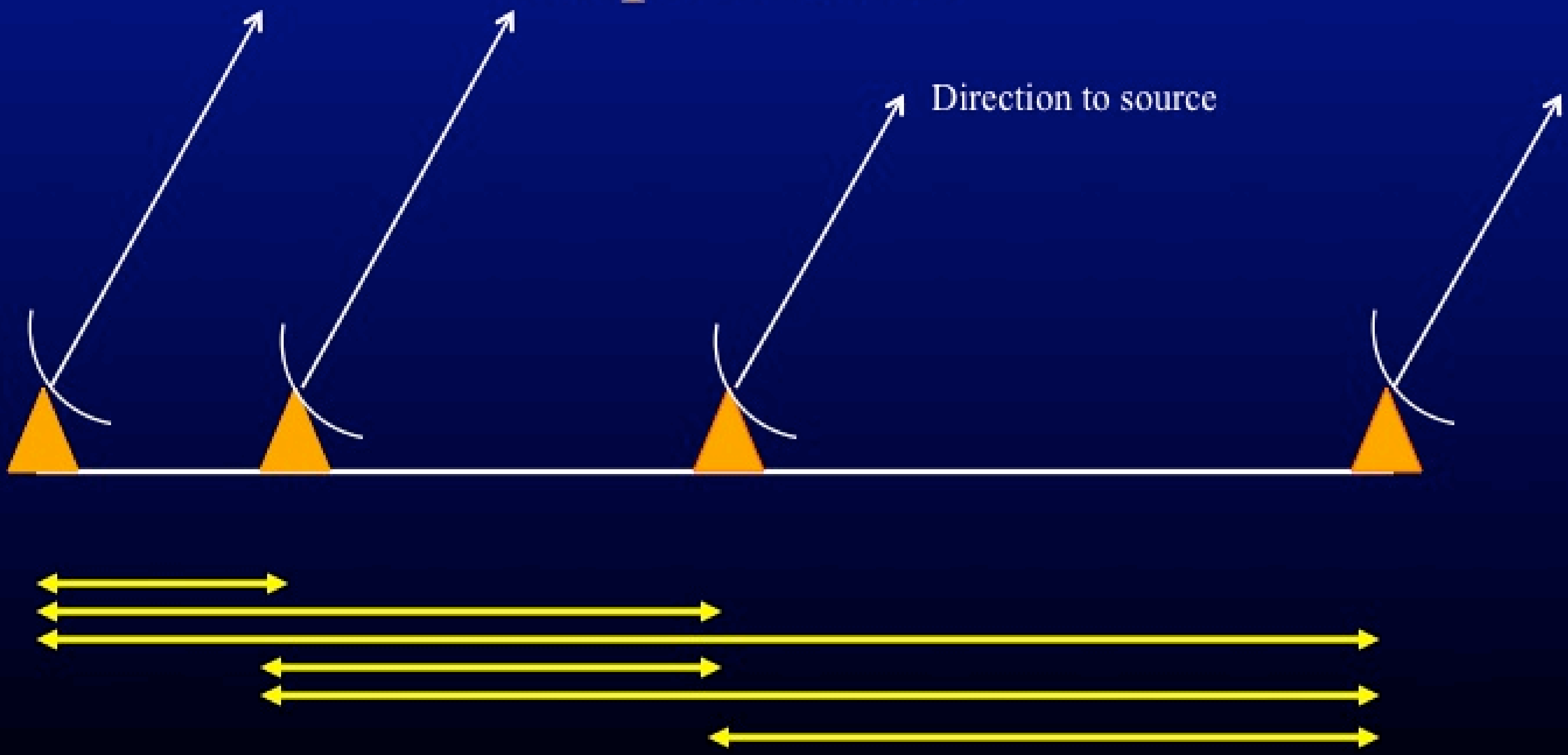
A synthesised radio telescope

A two-element Interferometer



$$V_{\nu}(\vec{r}_1,\vec{r}_2) = \left\langle \vec{E}_{\nu}(\vec{r}_1)\vec{E}^{*}_{\nu}(\vec{r}_2) \right\rangle$$

Need to measure the cross-correlation over as many spacings as possible

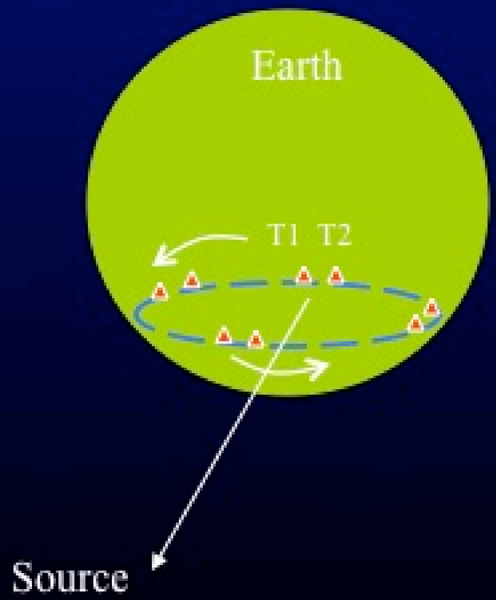


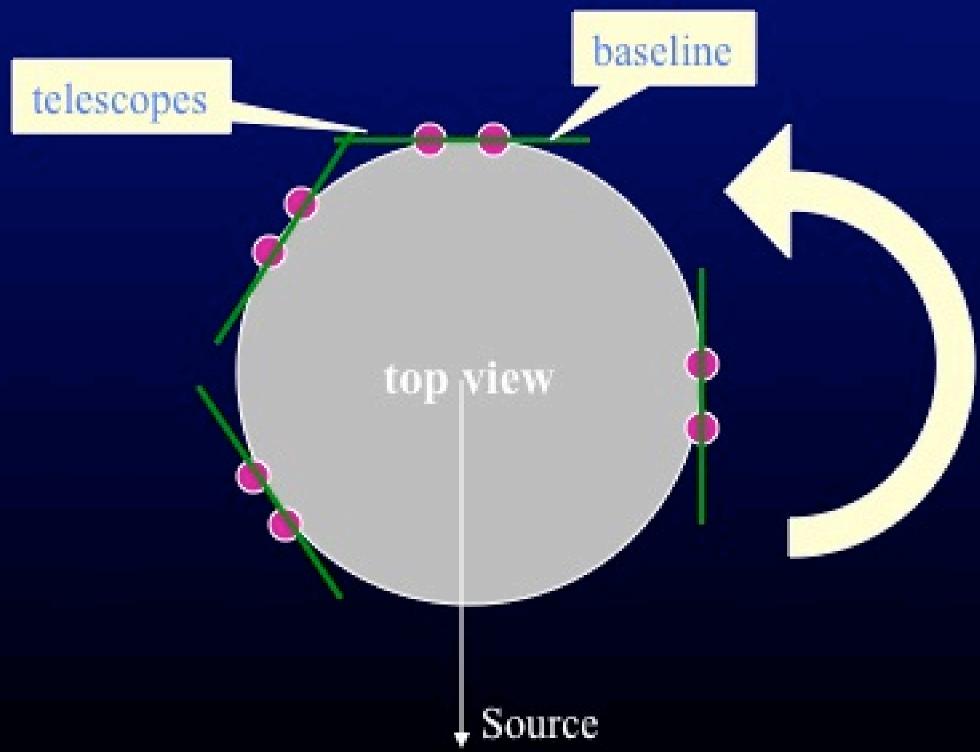
For n antennas, we get n(n-1)/2 spacings

Problem 2

- We can’t fill the aperture with millions of small radio telescopes.
- Solution: let the Earth’s rotation help us

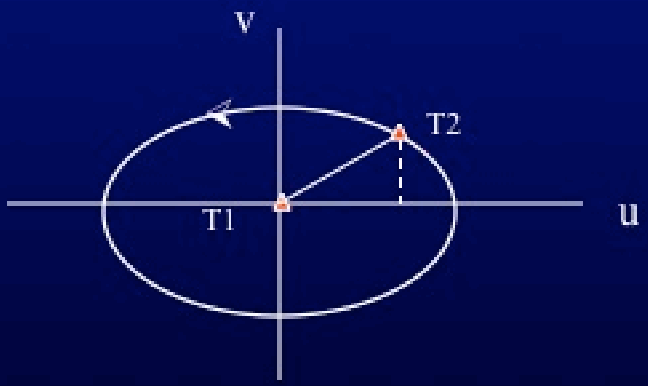
Using the Earth’s Rotation





The u-v Plane

- As seen from the source, each baseline traces out an ellipse with one telescope at the centre of the ellipse:



The projected baseline can be specified using u-v coordinates, where

- u gives the east-west component of the baseline; and
- v gives the north-south component of the baseline.

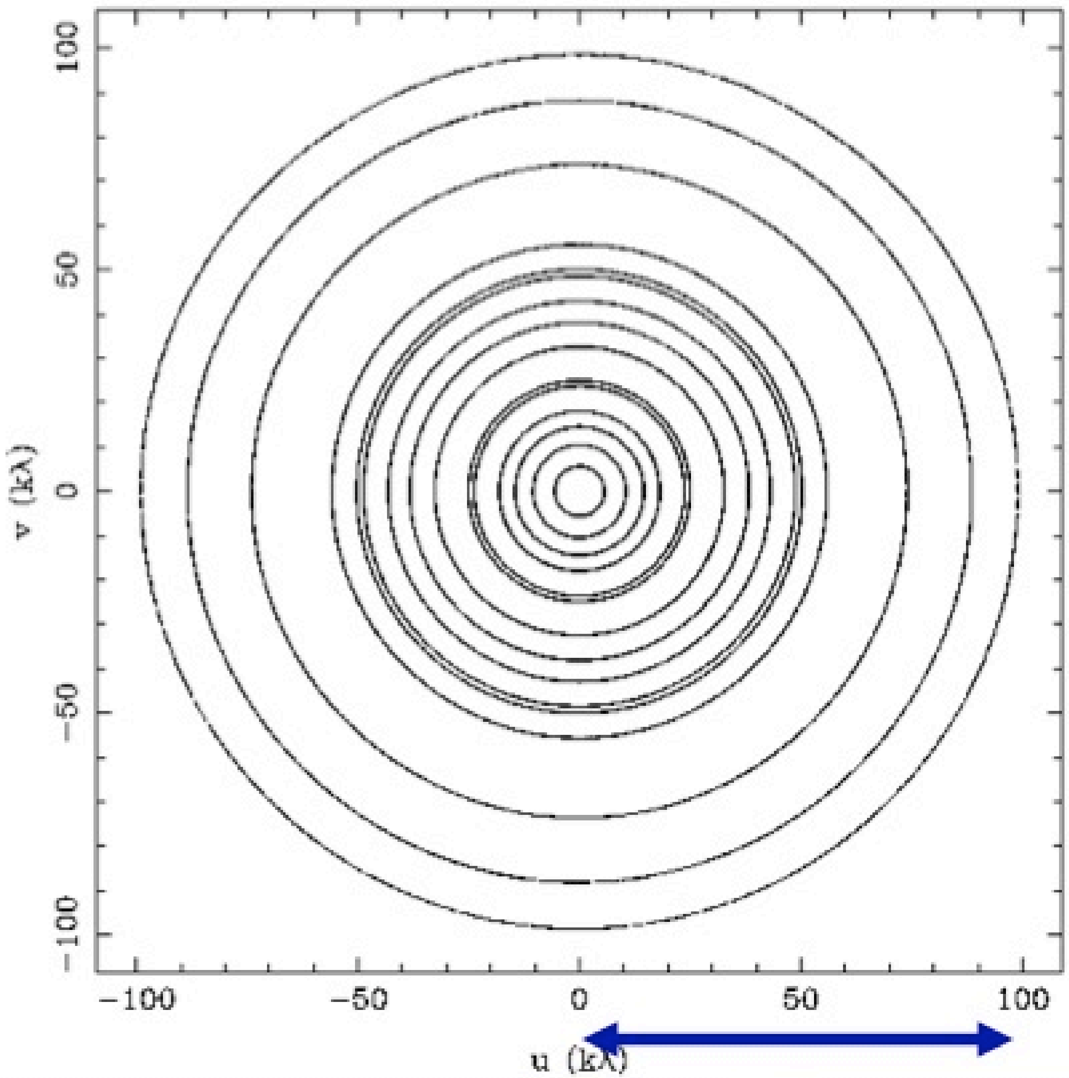
The projected baseline $B \sin\theta = (u^2 + v^2)^{1/2}$

$$V(u,v) = \int I(x,y) e^{-2\pi i(ux+vy)} dxdy$$
$$I(x,y) = \int V(u,v) e^{2\pi i(ux+vy)} dudv$$

So is that all?

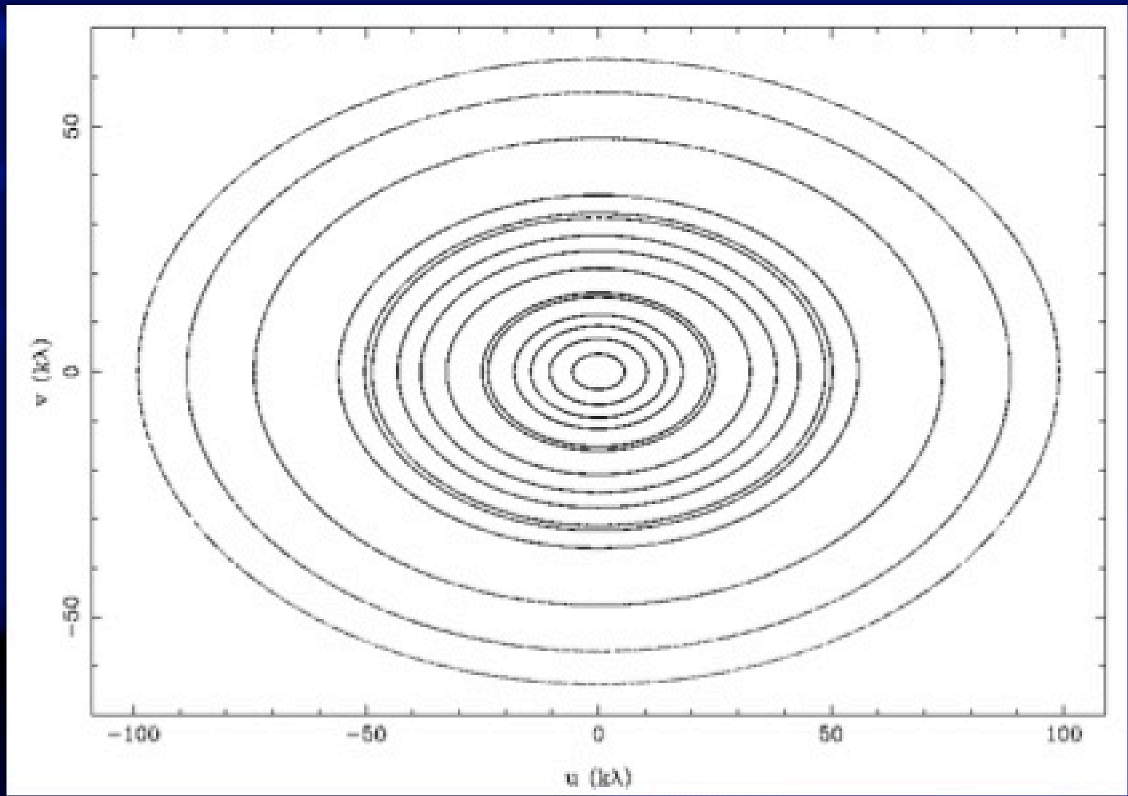
- If we were able to cover all the u-v plane with spacings, we could in principle get a perfect image by Fourier transforming all the measured correlated voltages.
- In practice there are gaps.
- The field distribution over the u-v plane is only sampled in some places, so what we actually measure is the field multiplied by a sampling function

Example 1: ATCA at declination = -85 degrees

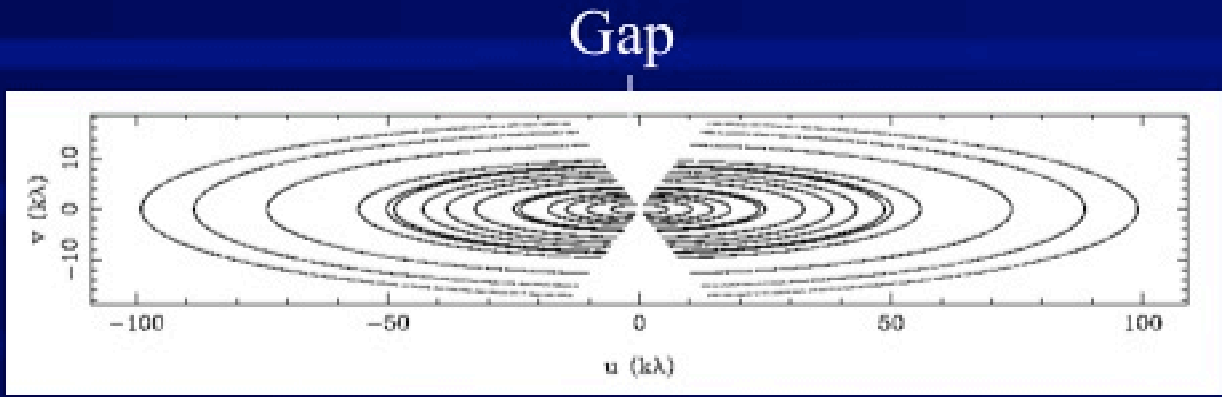


100 kλ = 6 km at 6 cm

Example 2: declination = -40 degrees



Example 3: declination = -10 degrees



The sampling function and dirty image

$$I^D(x,y) = \int S(u,v)V(u,v)e^{2\pi i(ux+vy)}dudv$$

Dirty image

Sampling function
Or "uv coverage"

Convolution theorem

Fourier Transform

FT (A.B) = FT(A)*FT(B)

multiply convolve

$$B = FT\{S\}$$

where $$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau) \cdot g(t-\tau)\, d\tau$$

Deconvolution

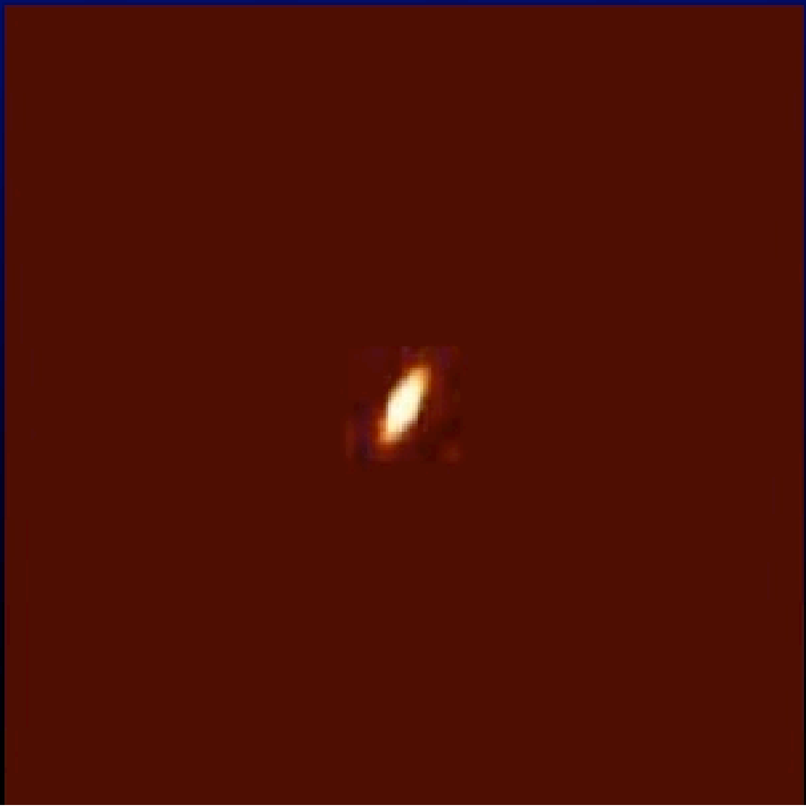
Perfect Image = FT (Electric field)
Dirty Image = FT(Electric Field.uv coverage)
= FT(Electric Field)*FT(uv coverage)
= Perfect Image * Dirty Beam

So to obtain a perfect image, we need to "deconvolve" the Dirty image.

- This turns out to be really hard - you can't just divide both sides!
- No analytical solution.
- Instead we could guess at a result and work backwards to refine our guess.
- Well –known Deconvolution techniques include CLEAN, MEM, etc.

Cleaning dirty images

- A process was designed by Högbom in the early 1970s to clean dirty images.

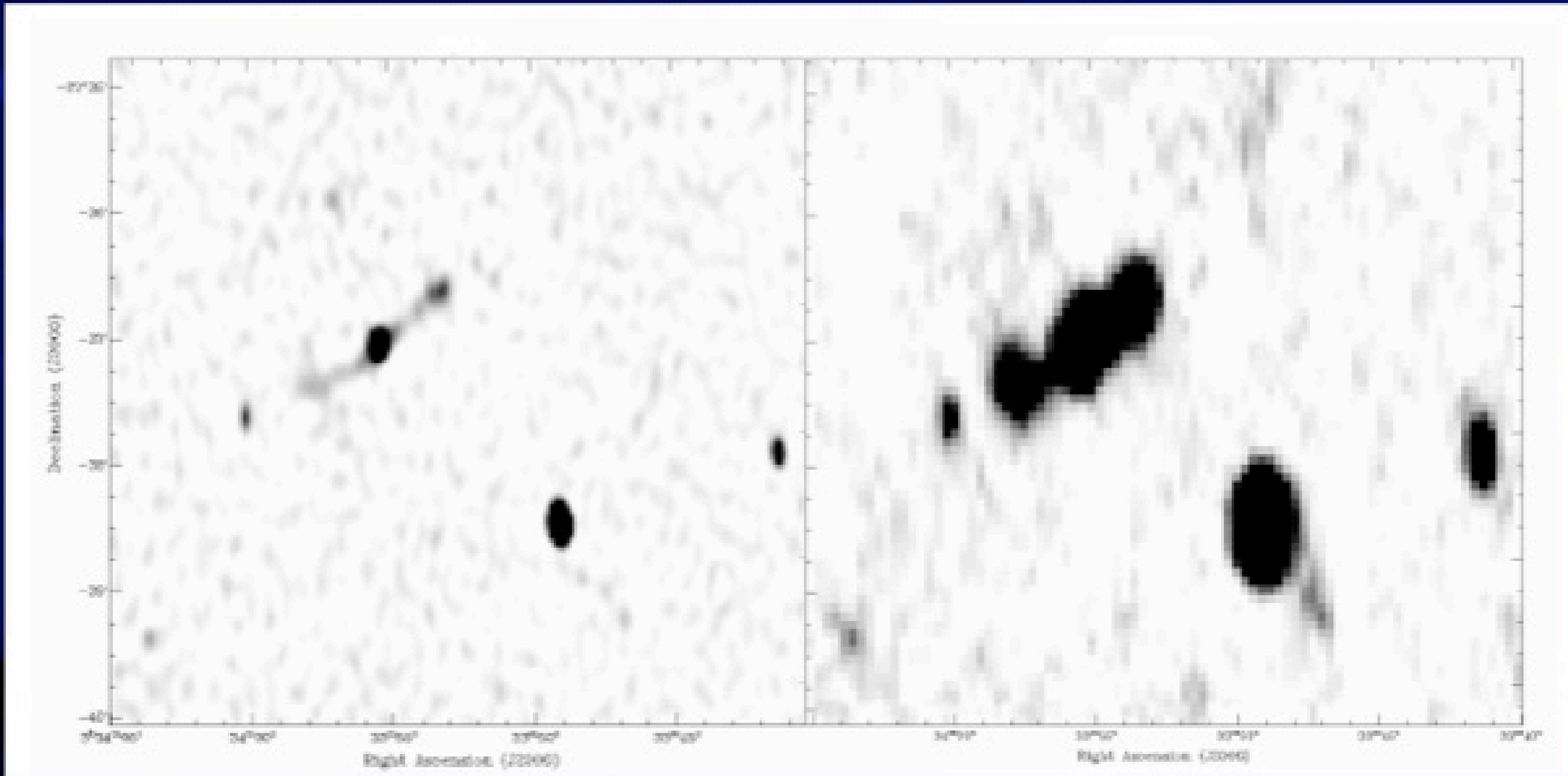


- Estimate value and position of peak
- Subtract off the 'dirty beam' due to a point source of this flux
- repeat until only the noise is left on the image.
- Add back the flux, convolving each point source with an ideal "clean beam"
- The result is the 'cleaned image'.

Weighting (=tapering)

- Leave data as they are = "natural" weighting
  - No loss of information
  - Maximises sensitivity
- Weight data so that uv plane is uniformly sampled
  - i.e. reduce the weight of regions of the uv plane containing many measurements
  - Reduces sensitivity
  - Improves resolution (by effectively giving more weight to longer baselines)
  - Often improves sidelobe levels
- Various "robust" weighting schemes

The effect of weighting



Uniform weighting

beam-size = 11 x 5 arcsec

σ = 25μJy

Natural weighting

beam-size = 28 x 16 arcsec

σ = 17 μJy

The process of synthesis observing

- Observe the source for some hours, letting the Earth rotate the baseline
- Correlate the signals between telescopes, and store the results of those multiplications on disk
- At the end of the observation, assign the results of the multiplications to the correct position on the u-v disk (Gridding)
- Weight the “visibilities” (=measurements) as appropriate
- Fourier Transform the uv plane to produce a Dirty Image
- Deconvolve the Dirty Image to obtain a Clean Image

So is that all?

Other problems include

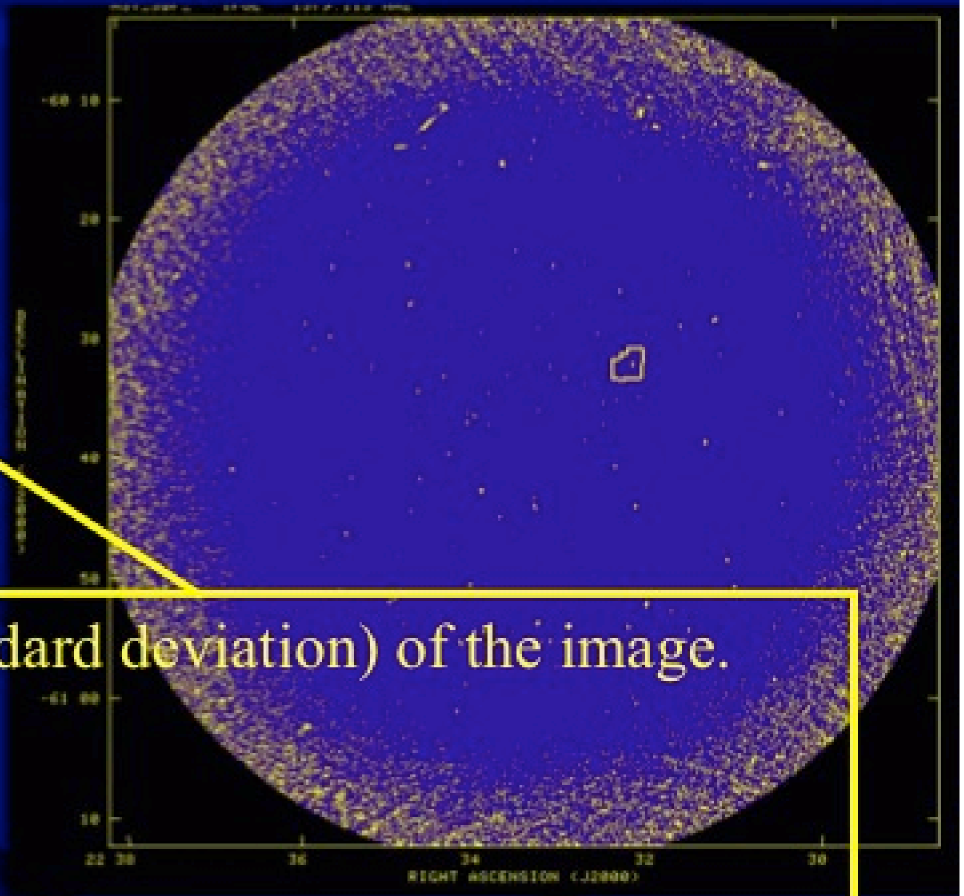
- calibration errors
  - use calibrator sources
- bad data
  - edit the data
- “twinkling” in the atmosphere or ionosphere
  - use techniques such as “Selfcal” and phase referencing

Self-calibration (=selfcal)

- Do all the above to get the best possible image
- Use that image to calibrate the data
- Re-image the data
- Repeat this process until there is no further improvement
- Typically use one or two iterations of phase selfcal followed by one or two amplitude/phase selfcals
- Must be used with extreme caution!!!!

Sensitivity

- Sensitivity = a measure of the weakest detectable radio emission
- E.g. this image has a 5-σ sensitivity of 10 μJy/beam



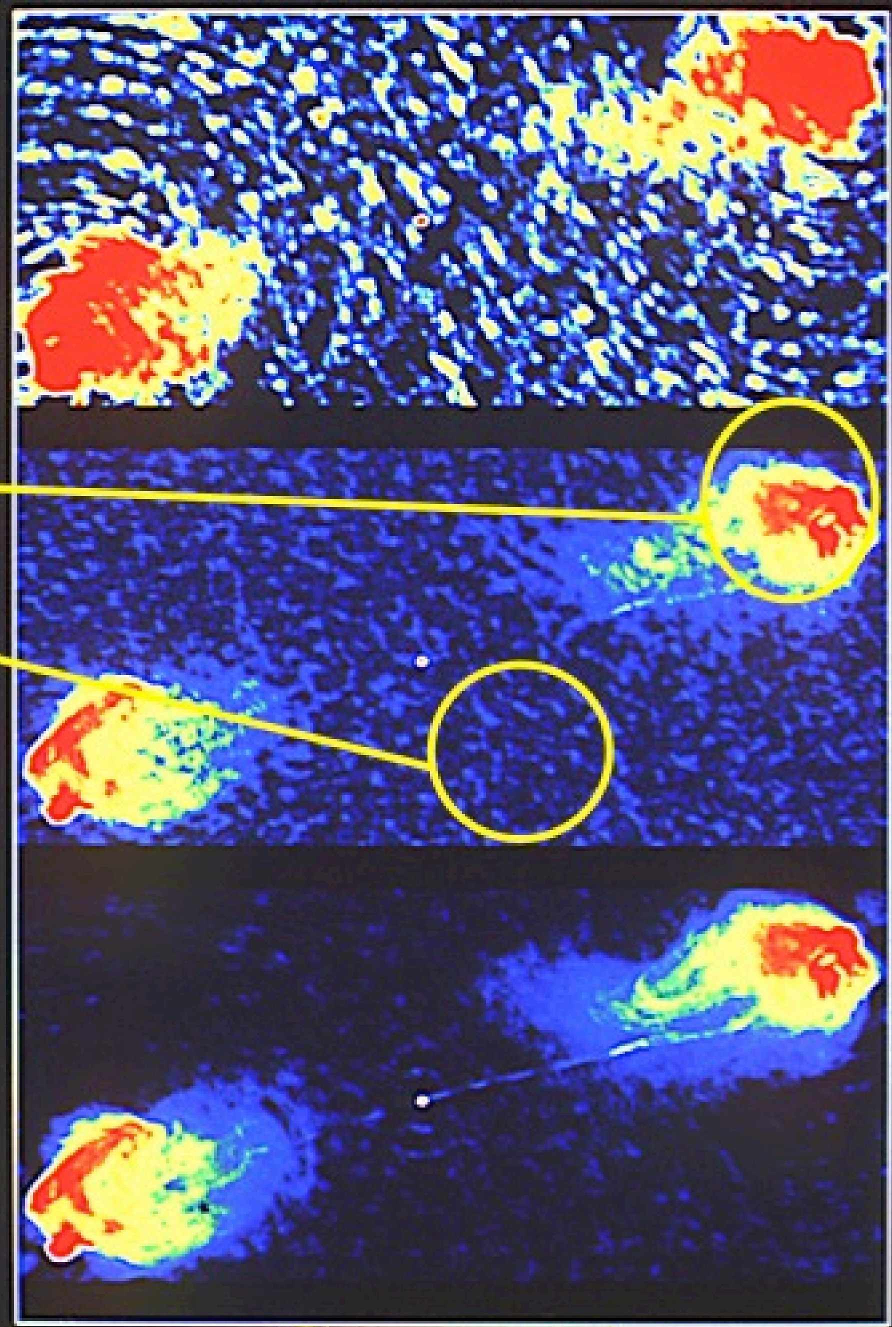
Units:
I Jansky (Jy) = 10^{-26} W m^{-2} Hz^{-1}
More commonly use
1 mJy = 10^{-3} Jy, or
1 μJy = 10^{-6} Jy

1 σ = rms noise (or standard deviation) of the image.
This assumes:
- Noise is Gaussian
- Noise is uniform

Are these good assumptions?

Dynamic range

- Usually defined as peak flux ~~in the image~~ divided by the rms in a source-free (and artefact-free) region of the image.
- Dynamic ranges of 100's are common
- Dynamic ranges of 10^5 have been achieved with great care
- e.g. rms =10 μJy/beam, peak = 1 Jy/beam



Cygnus A

Sensitivity of a synthesis array (of n identical antennas)

$$\text{Sensitivity } \Delta S = \frac{1}{\eta_s} \frac{T_{sys}}{G_a \sqrt{(n(n-1).\tau.\Delta\nu)}}$$

E.g. old ATCA @ 20 cm, $\Delta\nu$ = 128 MHz,

- T = 10 min, ΔS = 190 μJy/beam
- T = 12 hr, ΔS = 22 μJy/beam
- T = 30 x 12 hr, ΔS = 4 μJy/beam

How to increase the sensitivity
(i.e. reduce ΔS)

How to increase the sensitivity (i.e. reduce ΔS)

$$\Delta S = \frac{1}{\eta_s} \frac{T_{sys}}{G_a \sqrt{(n(n-1).\tau.\Delta\nu)}}$$

- Increase the system efficiency

E.g. increase the number of bits of digitisation
- CABB does this

How to increase the sensitivity (i.e. reduce ΔS)

$$\Delta S = \frac{1}{\eta_s} \frac{T_{sys}}{G_a \sqrt{(n(n-1).\tau.\Delta\nu)}}$$

- Increase the system efficiency
- Reduce the system temperature

  - New receivers/polarisers can do this
  - But already approaching limit at most frequencies

How to increase the sensitivity (i.e. reduce ΔS)

$$\Delta S = \frac{1}{\eta_s} \frac{T_{sys}}{G_a\sqrt{(n(n-1).\tau.\Delta\nu)}}$$

- Increase the system efficiency
- Reduce the system temperature
- Increase the number of antennas

Expensive!

How to increase the sensitivity (i.e. reduce ΔS)

$$\Delta S = \frac{1}{\eta_s} \frac{T_{sys}}{G_a \sqrt{(n(n-1).\tau.\Delta\nu)}}$$

- Increase the system efficiency
- Reduce the system temperature
- Increase the number of antennas
- Increase the area or efficiency of individual antennas

Expensive! And efficiency already near-optimum

How to increase the sensitivity (i.e. reduce ΔS)

$$\Delta S = \frac{1}{\eta_s} \frac{T_{sys}}{G_a\sqrt{(n(n-1).\tau.\Delta\nu)}}$$

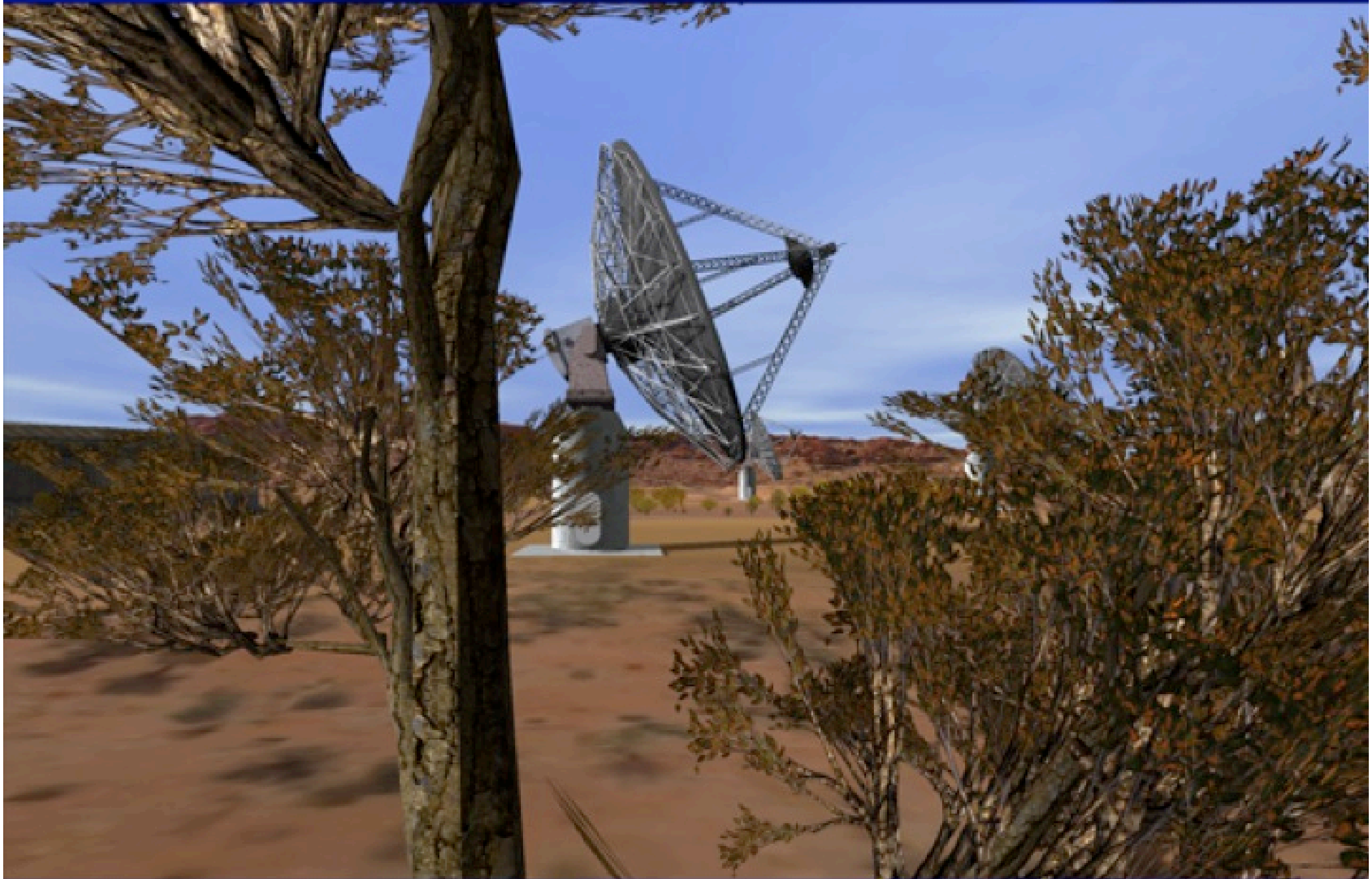
- Increase the system efficiency
- Reduce the system temperature
- Increase the number of antennas
- Increase the area or efficiency of individual antennas
- Increase the integration time

How to increase the sensitivity (i.e. reduce ΔS)

$$\Delta S = \frac{1}{\eta_s} \ldots$$

- Increase the system
- Reduce the system t
- Increase the number
- Increase the area or
- Increase the integrat

- Write better proposals!
- Large surveys already approaching practical limits.
- And note √ law of diminishing returns
- Use focal-plane array to observe more sky, so more time can be spent on each point (ASKAP)



How to increase the sensitivity (i.e. reduce ΔS)

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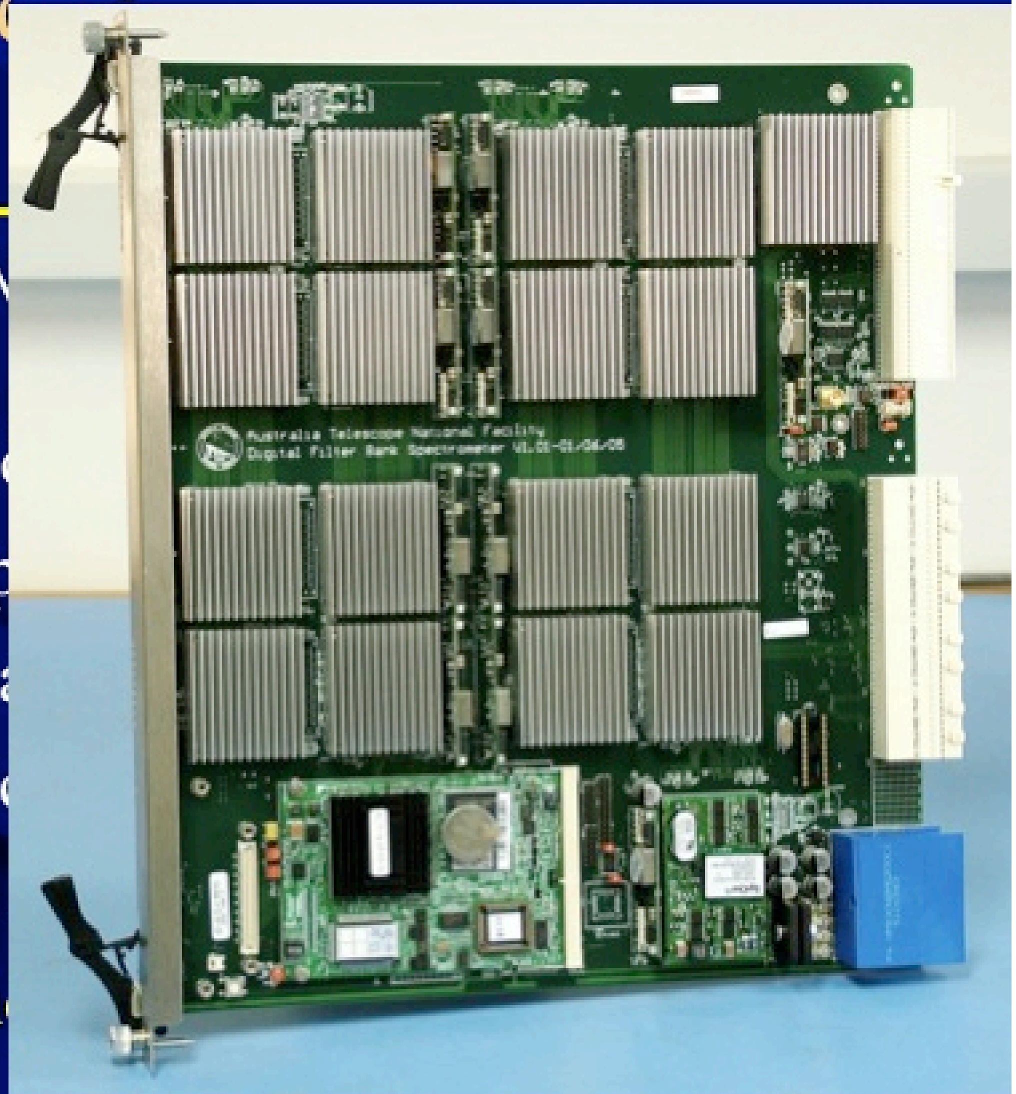
- Increase the system efficiency
- Reduce the system temperature
- Increase the number of antennas
- Increase the area or efficiency of individual antennas
- Increase the integration time
- Increase the system bandwidth (CABB)

How to increas
(i.e. re

$$\Delta S = \frac{1}{\eta_s} \frac{}{G_a \sqrt{}}$$

- Increase the system effic
- Reduce the system temp
- Increase the number of a
- Increase the area or effic
- Increase the integration
- Increase the system band

- This is the primary goal of CABB

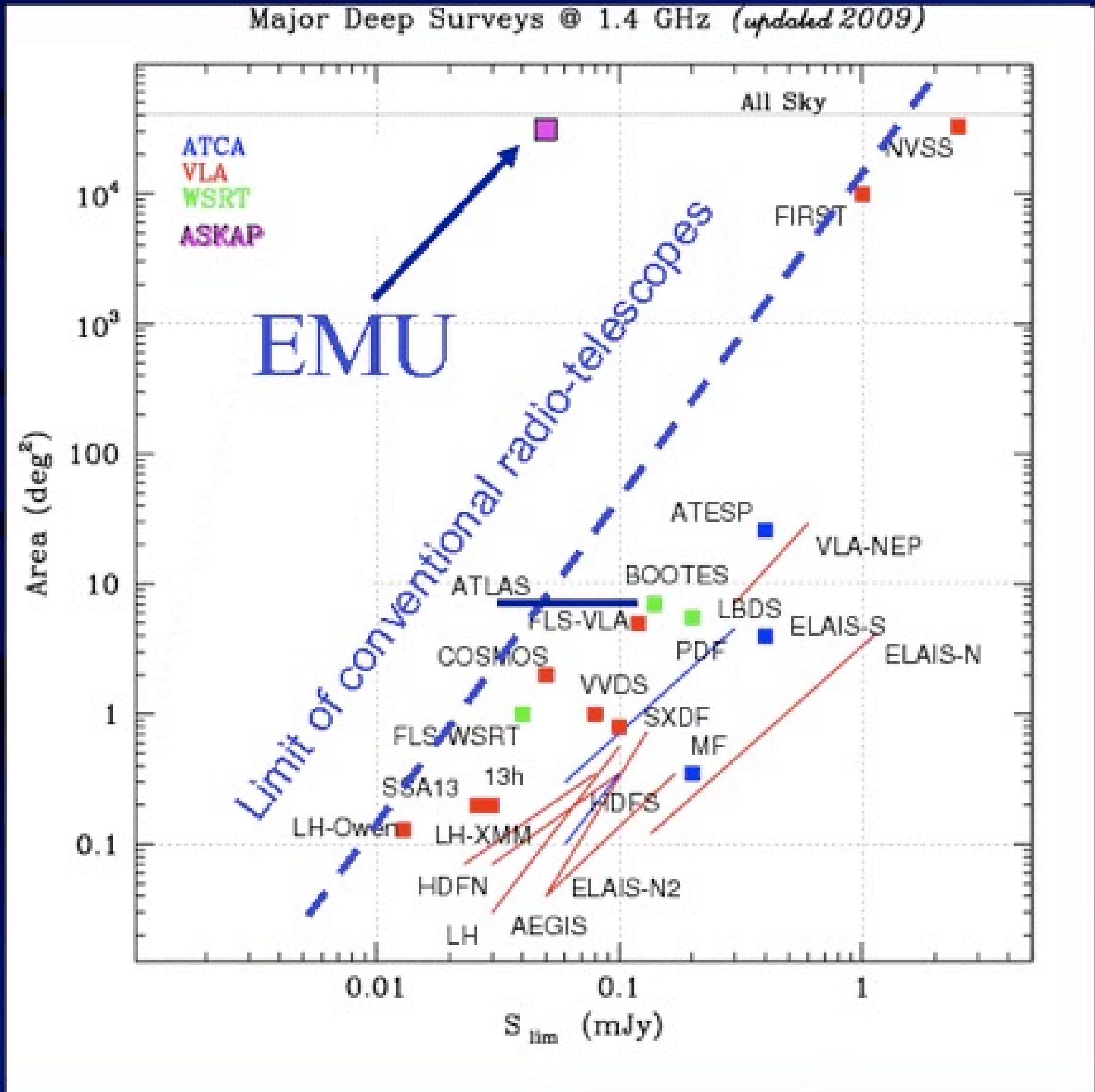


$$\Delta S = \frac{1}{\eta_s} \frac{T_{sys}}{G_a \sqrt{(n(n-1).\tau.\Delta\nu)}}$$

What does this mean in practice?

- This is the minimum 1-σ detectable signal obtained from a tied array
- It is also (in principle) the standard deviation (or rms noise) of an image made with the array.
- It is often called "the theoretical limit" or "the thermal limit".

Current major 20cm surveys



So why doesn't every image reach the "theoretical limit"?

Confusion (1)

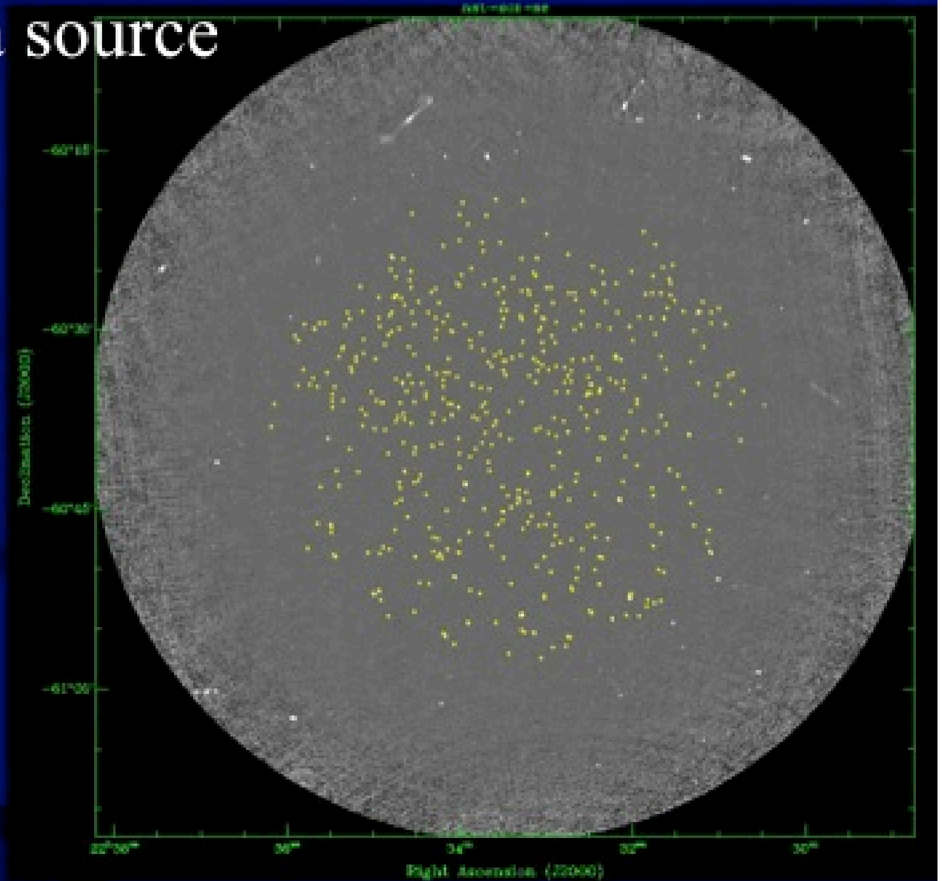
- Given a sufficiently sensitive observation, sources will start to overlap.
- Eventually, every beam will be filled with a source.
- This is the confusion limit, and depends on beam size and astrophysical source distributions.
- For the ATCA at 20 cm, $S_{\text{fundamental confusion}}$=0.05 μJy/beam

Confusion (2)

- Beam confusion
  - Sources are closer together than the instrumental resolution
- Fundamental (non-instrumental) confusion
  - sources themselves overlap,
  - cannot be separated by ANY instrument, regardless of beam size
- Sidelobe confusion
  - noise added to image because of sources in sidelobes (even with perfect calibration).

Confusion (3)

- A commonly adopted rule of thumb is that only 1% of beams should contain a source
- For the ATCA at 20 cm, $S_{instrumental} \sim 20\ \mu Jy/beam$
- We have already passed this point.



Dynamic range limited by incomplete uv coverage

- Missing uv information means that the solution is unconstrained in some regions of the image
- i.e. the imaging algorithm can "invent" sources.
- This often limits the sensitivity of snapshot images
- For deep surveys, ATCA in principle capable of "complete uv coverage" (but never used in this mode).
- Current deep surveys generally have good (but not perfect) uv coverage
- Not yet clear if this will be a serious limit for deep surveys.

Dynamic range limited by calibration errors

- Selfcal can correct any antenna-based gain errors (if there are strong sources in the field)
- What about gain errors that vary across primary beam?
- In 2005, HDFS paper noted that this may limit us to $\sigma \sim 10\ \mu$Jy
- In 2006, our CDFS/ELAIS observations were limited at about $\sigma \sim 20\ \mu$Jy by a combination of effects such as these.
- Work in progress (Emil Lenc) to fix this


Deciding what level to “believe”

Deciding what level to "believe" (1)

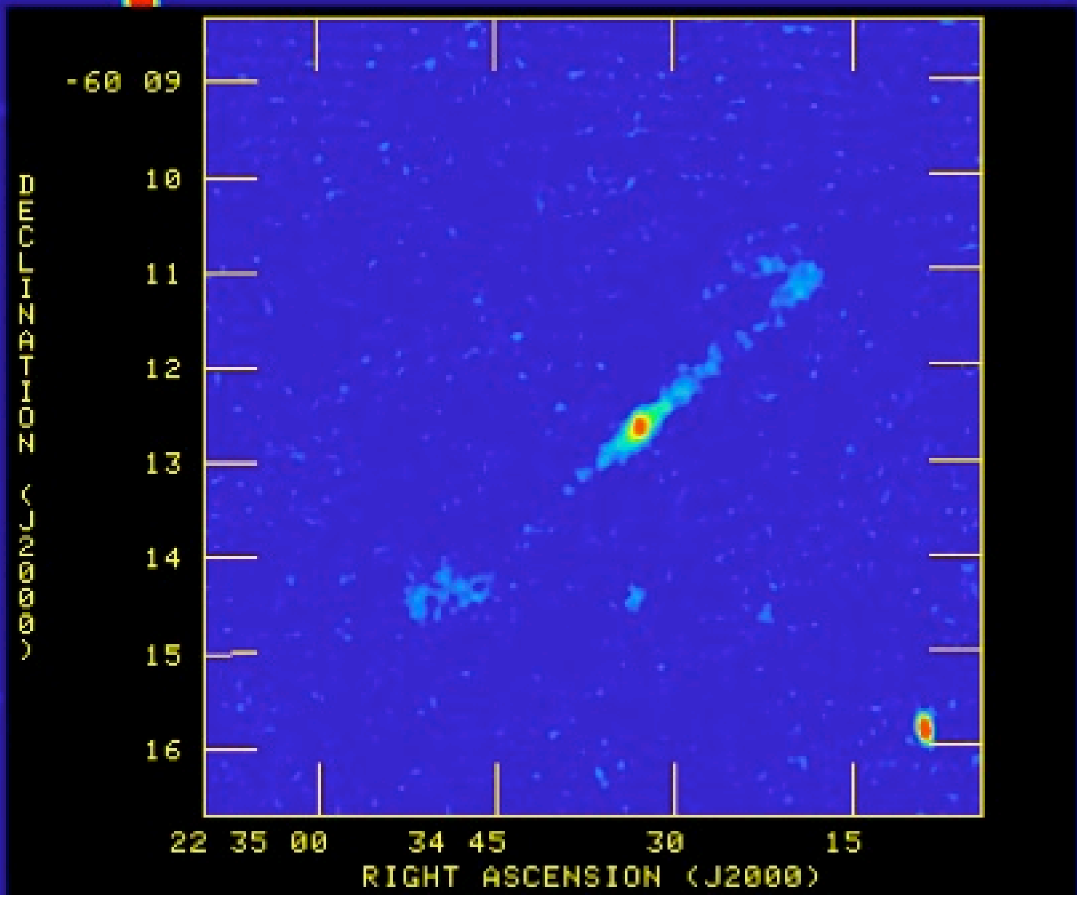
This image has ~ 4.10^4 independent beams, so Gaussian noise would suggest

- **50 peaks > 3σ**
- **~1 peak > 4σ**
- **No peaks > 5σ**

- In practice, probably several noise peaks > 5σ
- Most astronomers will not "believe" anything at about this level.
- The trouble is: noise in synthesis images is NOT Gaussian!
- So Normal distribution statistics do not apply

Deciding what level to "believe" (2)

- So what level of source do you believe?
- Problem is not trivial, and a simple cut-off is unlikely to be useful.



Deciding what level to "believe" (3)

Radio-astronomical noise is strongly non-Gaussian

- And yet, quoting an rms noise is still regarded as a useful rule-of-thumb guide to sensitivity.
- E.g. 10-σ detections are probably OK!

Other techniques:

- Look at the statistics of "negative sources"
- Look at the statistics of sources in the V image
- Compare with other wavelengths, or use other information
- Use cross-identifications with shifted data
- Use simulations

Deciding what level to “believe” (3)

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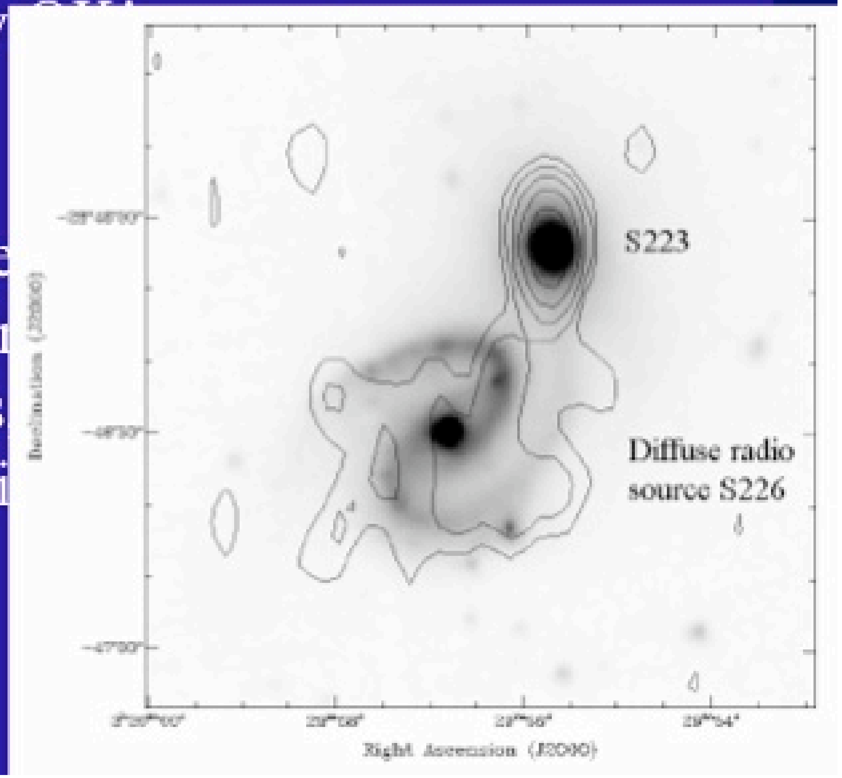
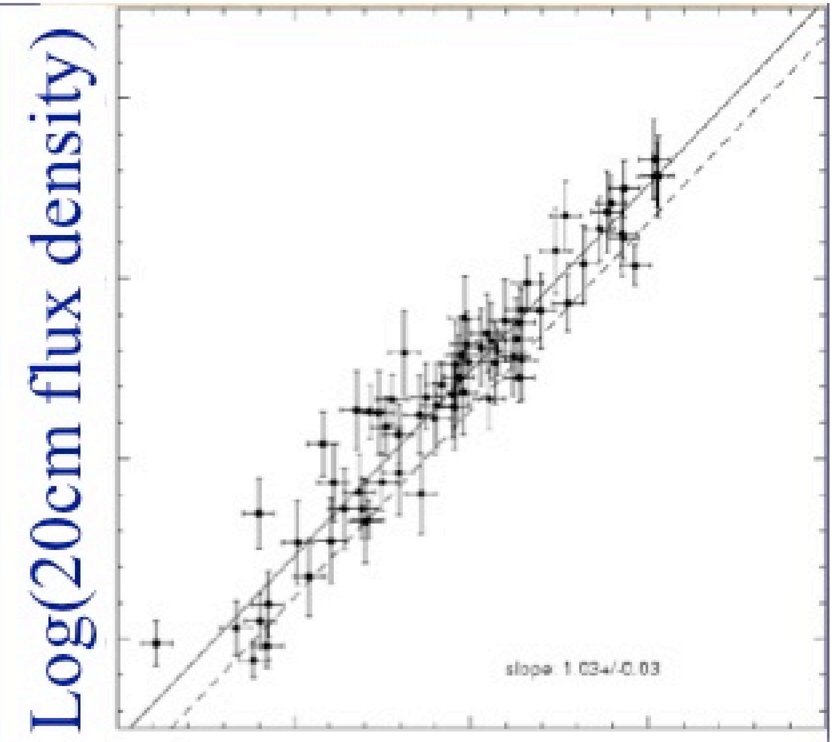


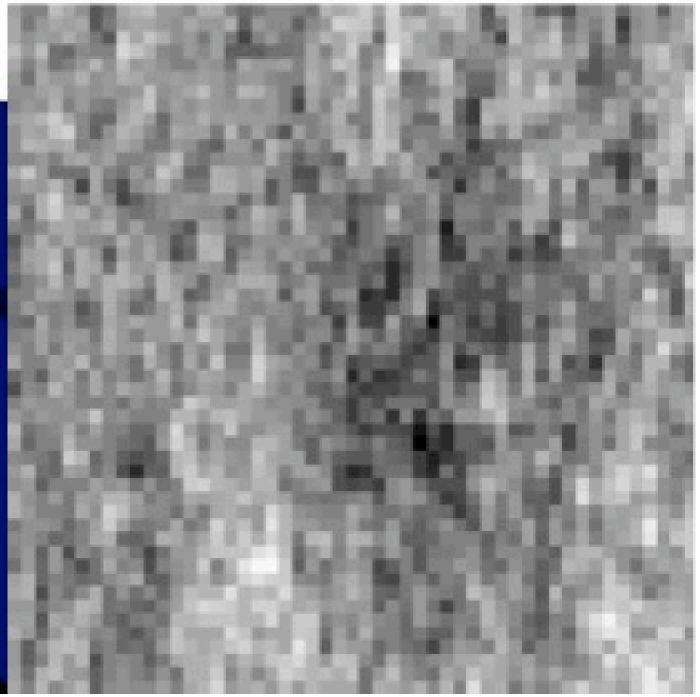
Obtaining the ultimate sensitivity: Stacking

Stacking (1)

Question: what happens to the radio-FIR correlation at low flux densities?

- Thousands of sources with FIR detections,
- Predicted radio flux is well below available radio sensitivity

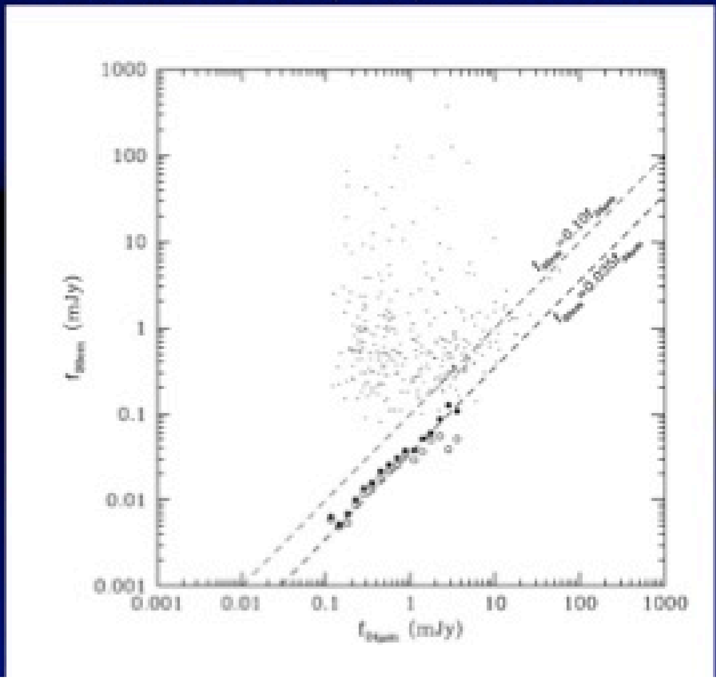


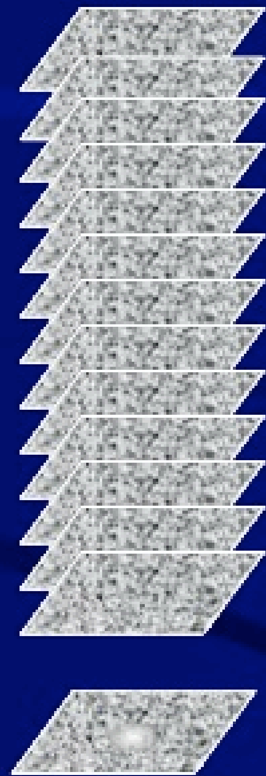


Stacking(2)

- Solution: take the radio image at the position of each FIR source, and add them together.

- E.g. 1800 FIR source -> increase radio sensitivity by √1800 ~ 42





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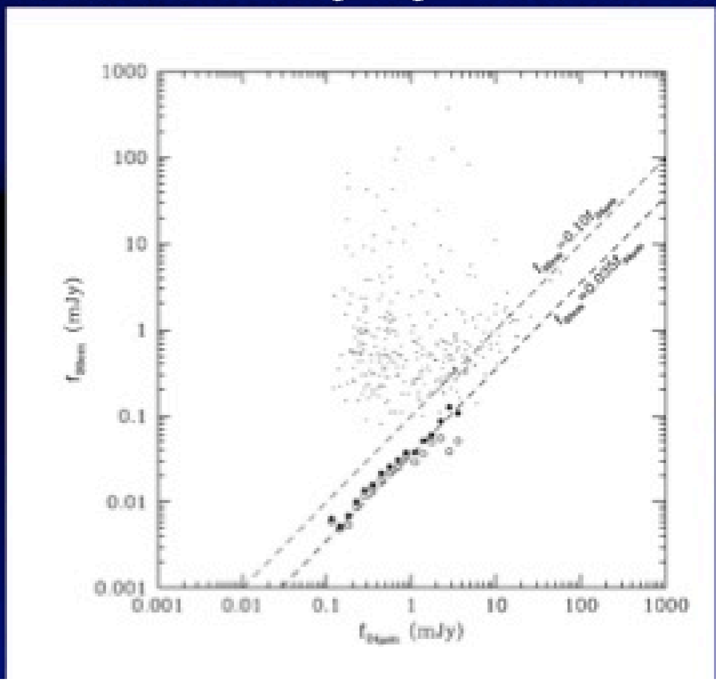

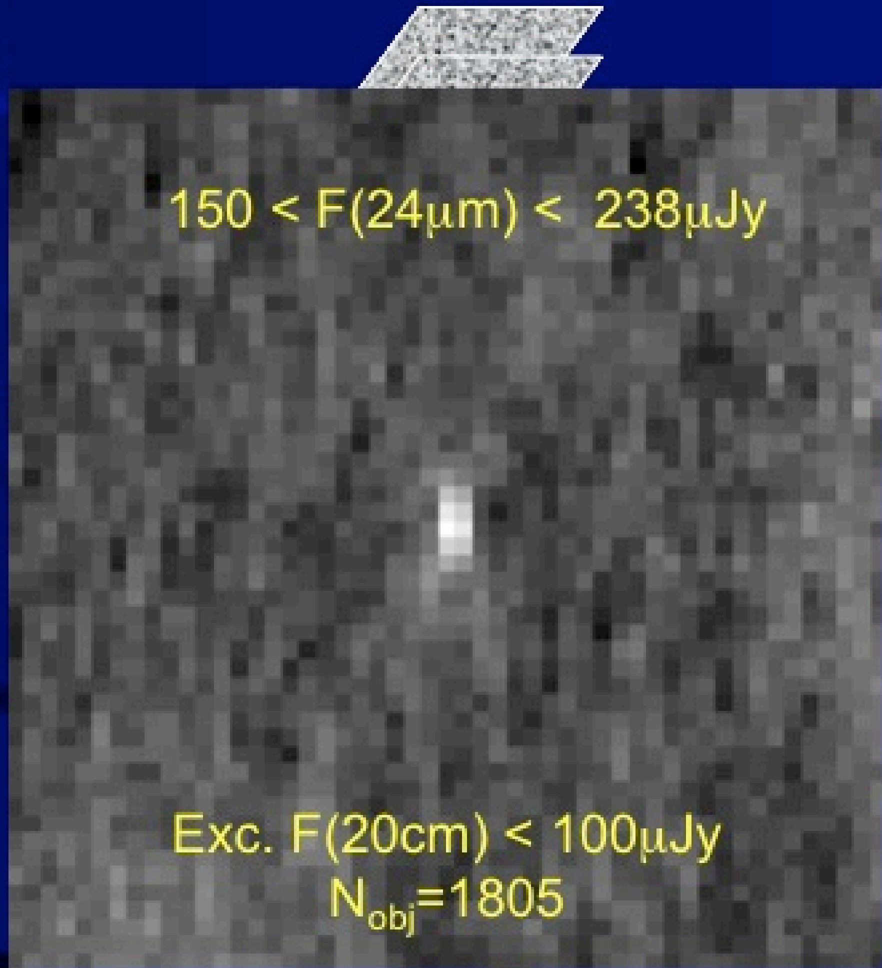

rms: 1.5μJy


The End