

# RA basics, noise, measurements ...

Essential Radio Astronomy:  
J. J. Condon and S. M. Ransom  
<http://www.cv.nrao.edu/course/astr534/ERA.shtml>

<http://www.naic.edu/~astro/sdss5/>

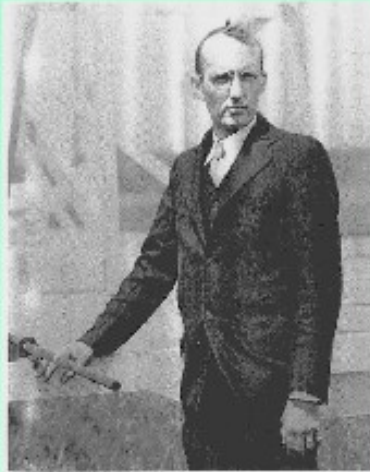
Single Dish Radio Astronomy, ASPCS 278

J. D. Kraus: Radio Astronomy

K. Rohlfs and T. L. Wilson: Tools of Radio Astronomy



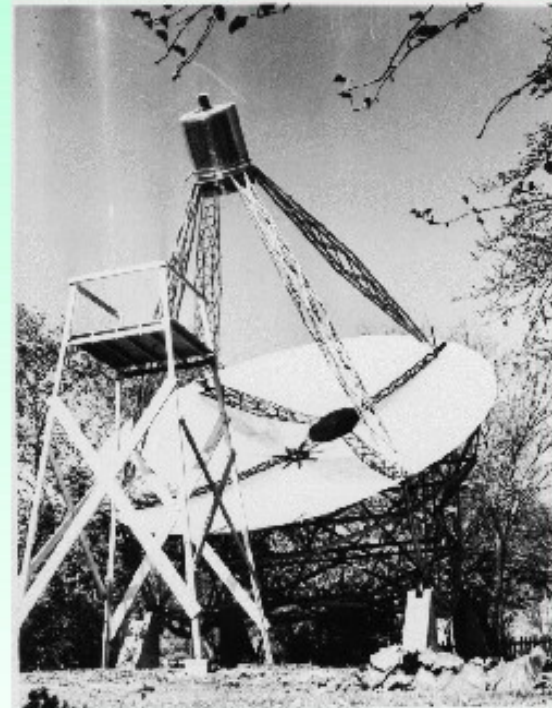
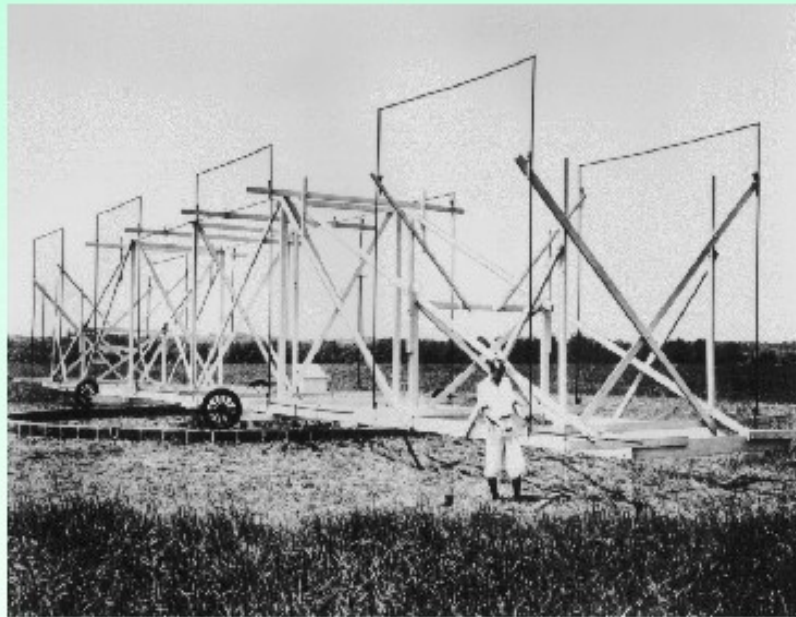
# Pioneers of radio astronomy



Karl Jansky  
1932



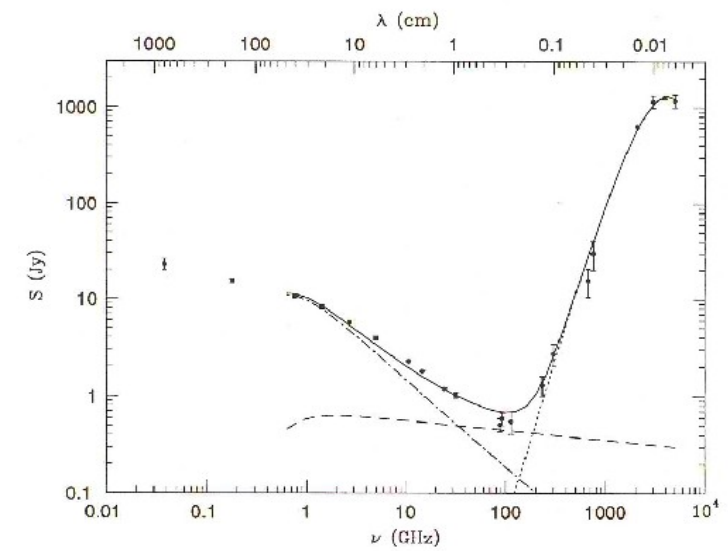
Grote Reber  
1938

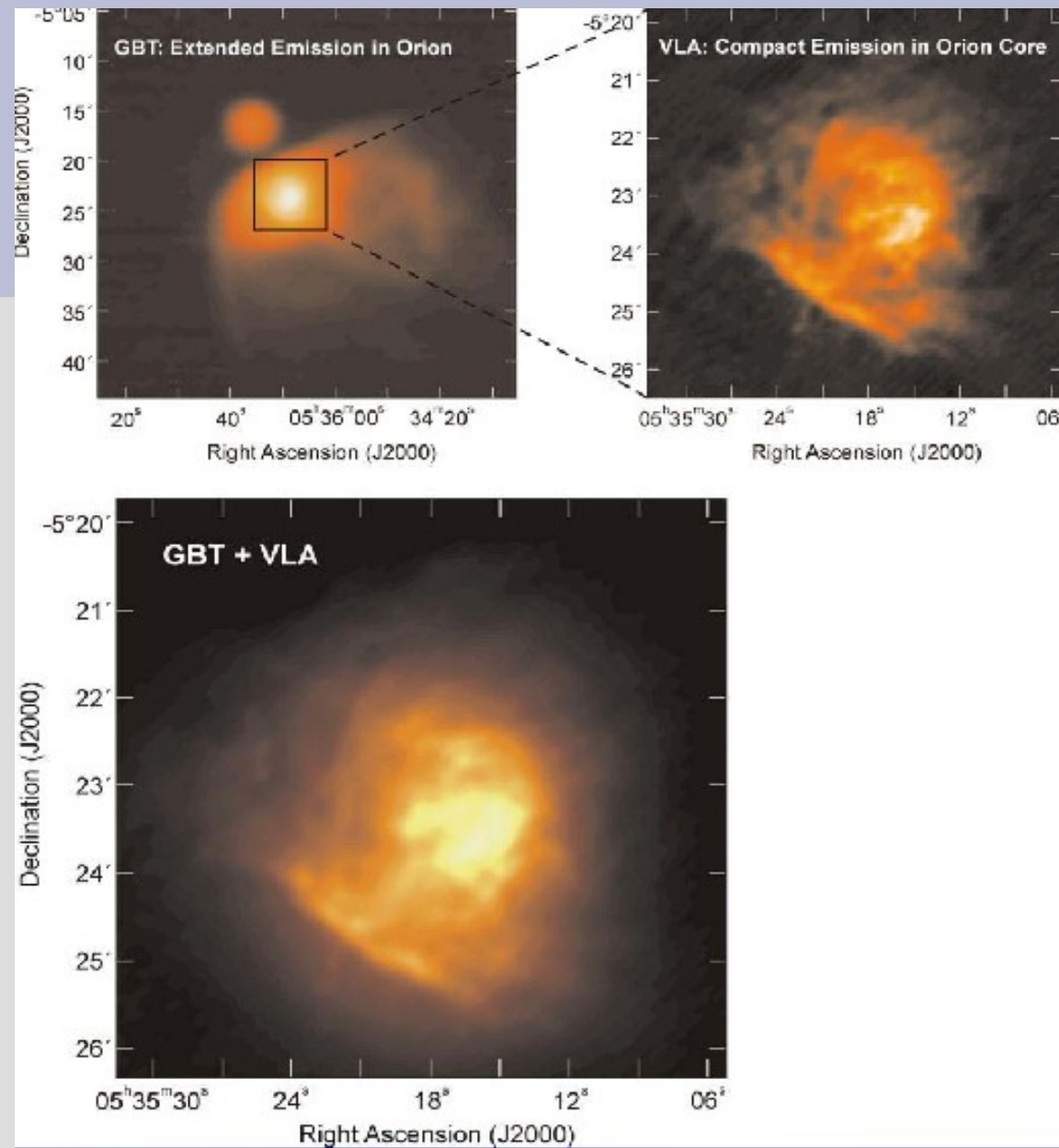




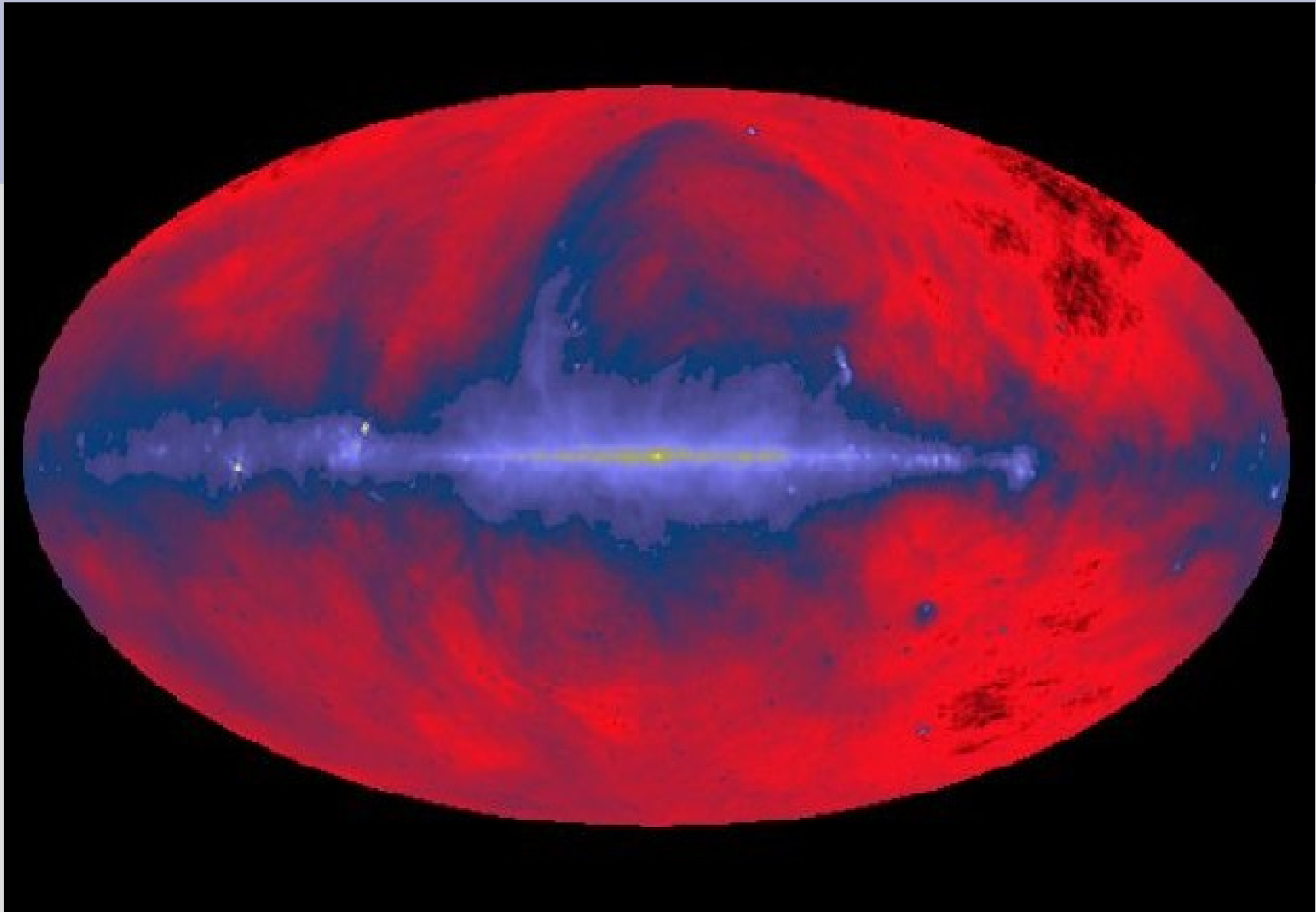
M82: starburst galaxy with a superwind  
Radio spectrum: thermal + non-thermal

Photo: NASA, ESA and Hubble Heritage Team  
Spectrum: Condon, 1992, ARA&A, 30, 575





## Orion nebula (HII region): GBT + VLA



## A 408-MHz all-sky continuum survey

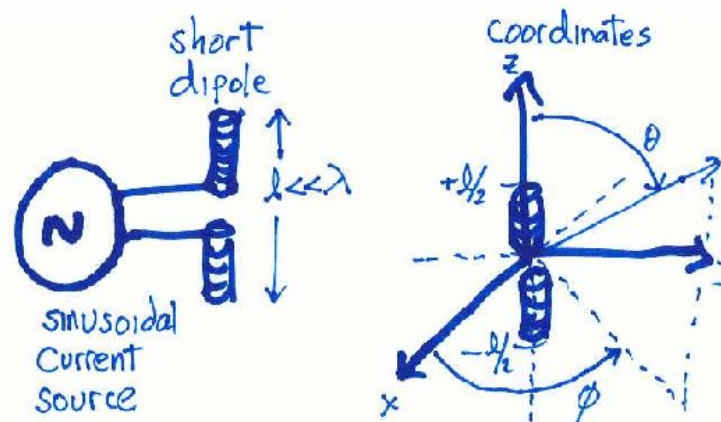
# Revision: antenna fundamentals

Antenna: device for converting electromagnetic radiation in to currents (receiving) or vice versa (radiating)

in radio astronomy we use antennas for receiving radiation

A simple antenna: two collinear conductors driven at the gap by a current source (transmitter)

Essential Radio Astronomy: J. J. Condon and S. M. Ransom



The coordinate system used to describe the radiation from a short dipole.

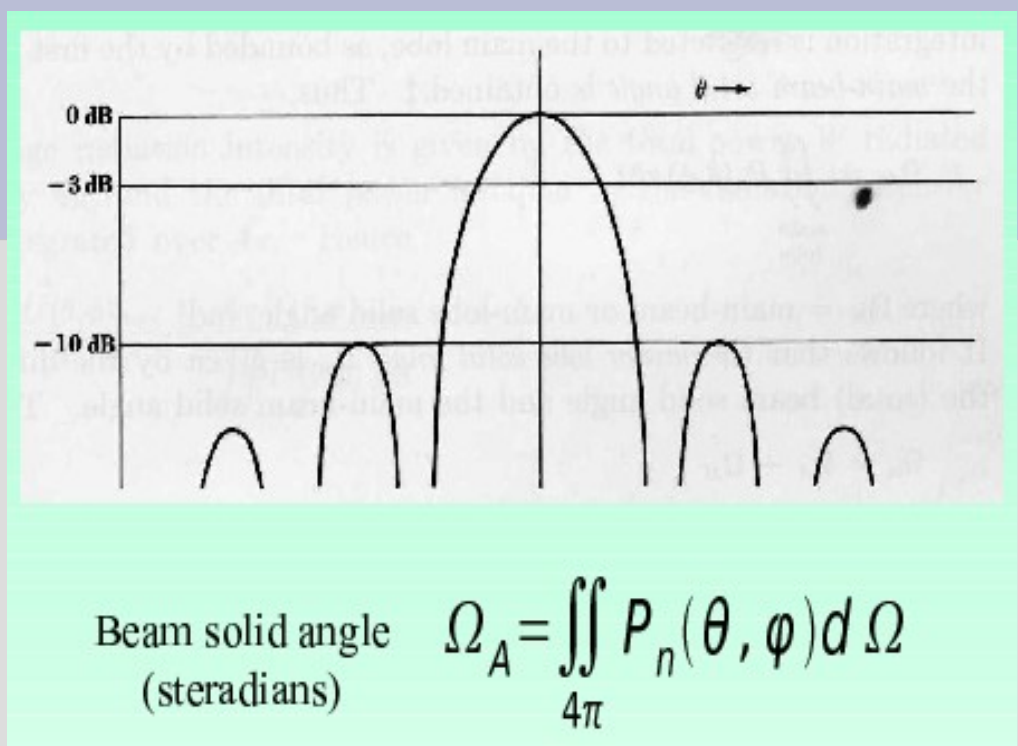
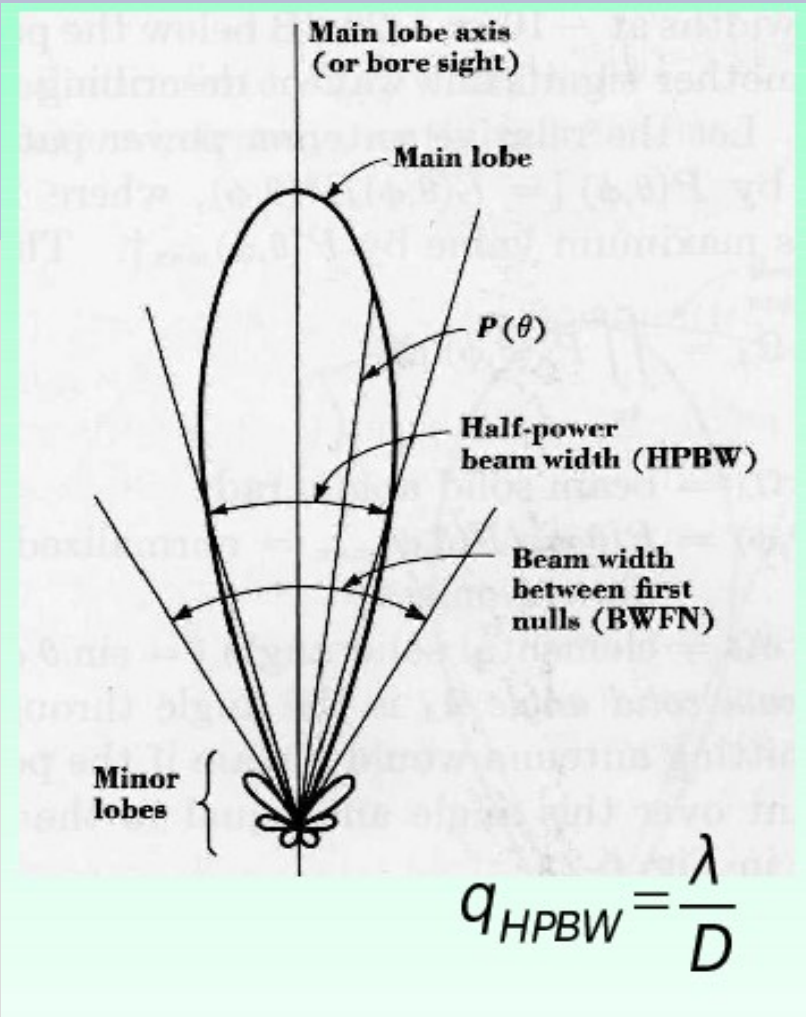
# Revision: antenna fundamentals

Power pattern: angular distribution of radiated power, usually normalised to unity at the peak

$$P \propto \sin^2 \theta$$

dipole: power pattern similar to that of an accelerated charge

all the charges in the dipole are being accelerated along one short line



## Antenna beam pattern or power pattern

## Some definitions and relations

Main beam efficiency,  $\epsilon$

$$\epsilon_M = \frac{\Omega_M}{\Omega_A}$$

Antenna theorem

$$\Omega_A = \frac{\lambda^2}{A_e}$$

Aperture efficiency,  $\epsilon_{ap}$

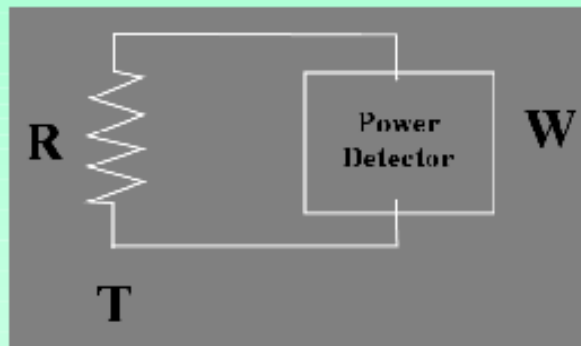
Effective aperture,  $A_e$

Geometric aperture,  $A_g$

$$\epsilon_{ap} = \frac{A_e}{A_g}$$

$$\epsilon_{ap} = \epsilon_{pat} \epsilon_{surf} \epsilon_{block} \epsilon_{ohmic} \dots$$

Detected power ( $W$ , watts) from a resistor  $R$   
 at temperature  $T$  (kelvin) over bandwidth  $\beta$ (Hz)



$$W = kT\beta$$

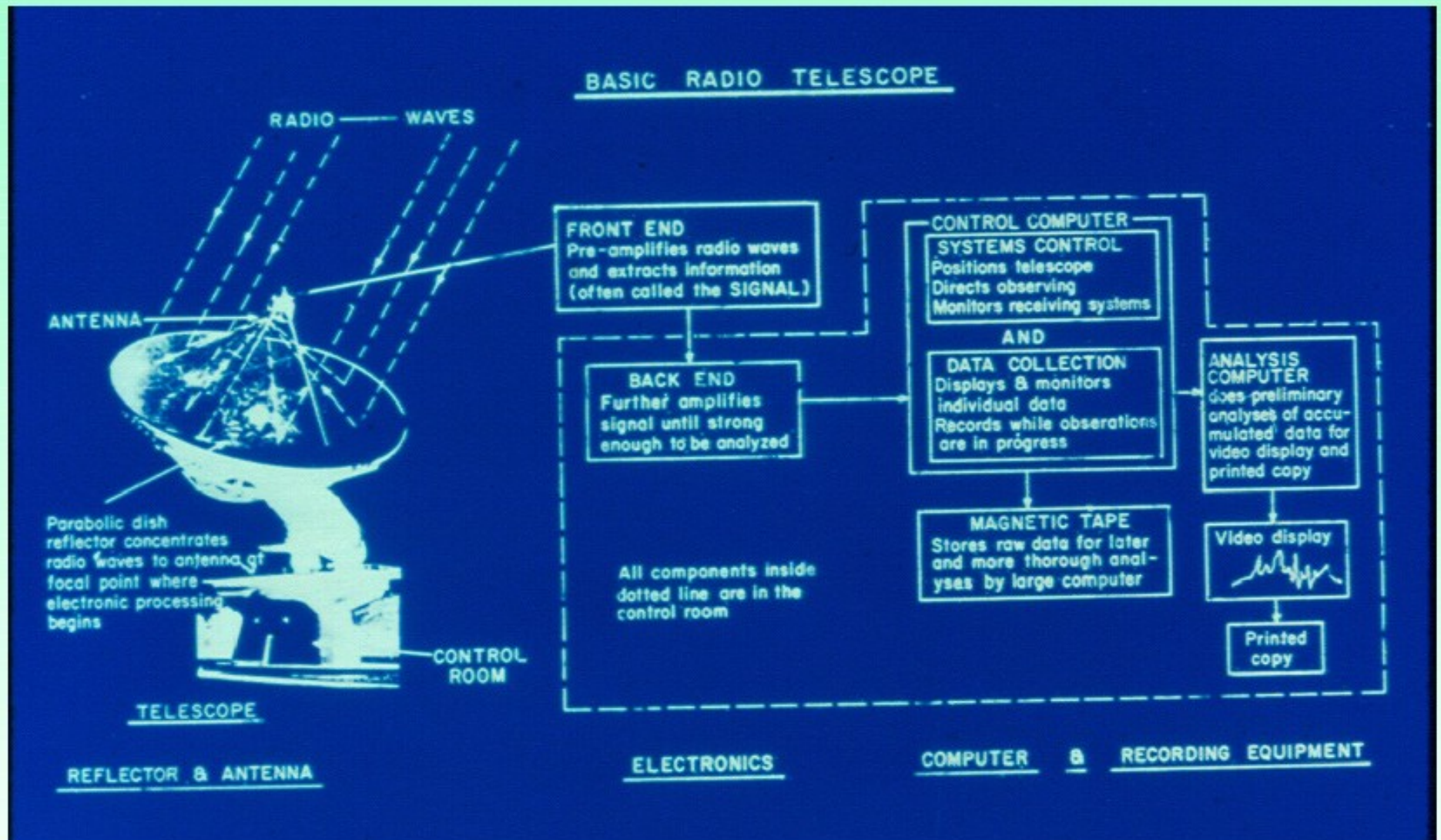
Power  $W_A$  detected in a radio telescope  
 Due to a source of flux density  $S$

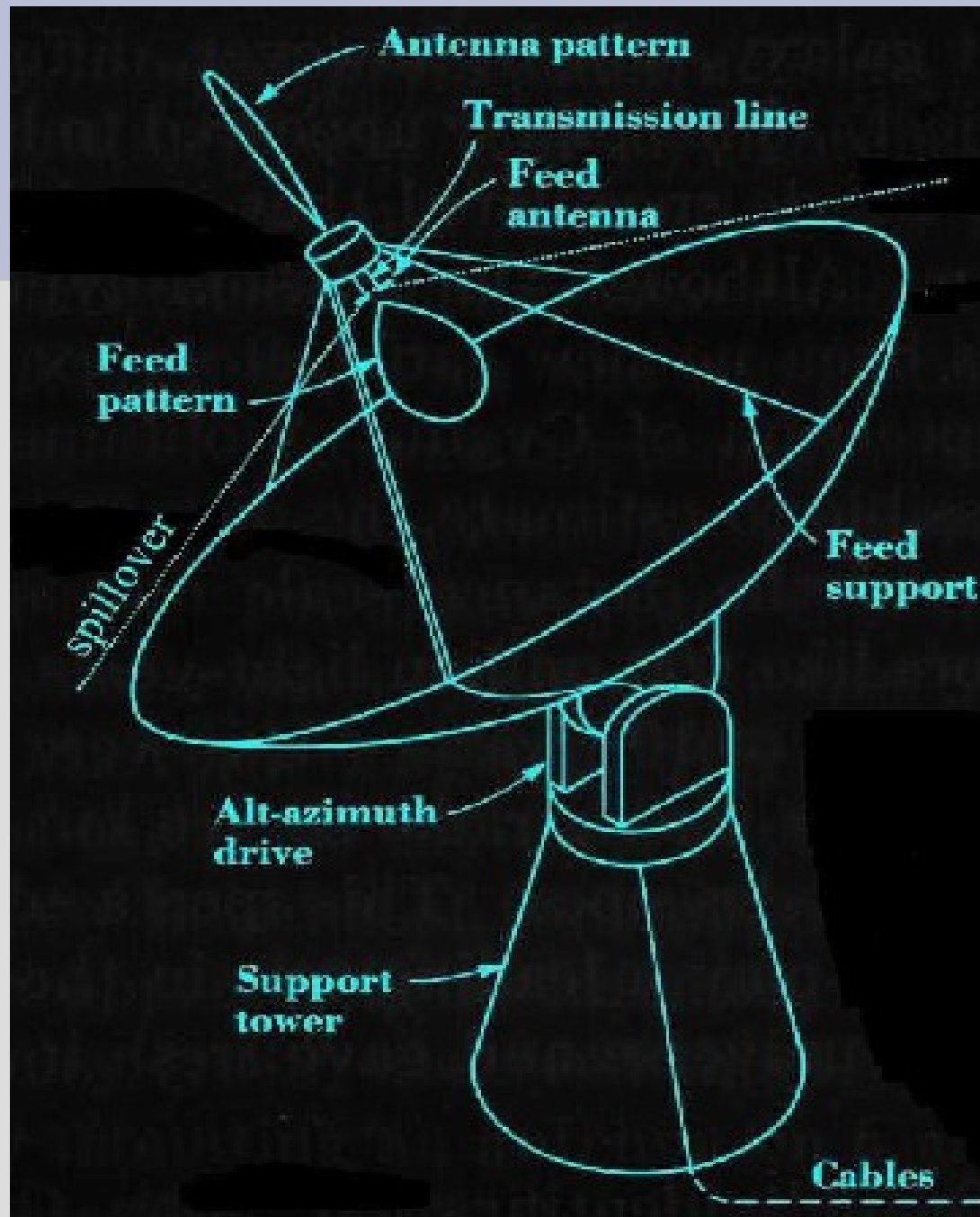
$$W_A = \frac{1}{2} AS\beta$$

power as equivalent temperature.  
 Antenna Temperature  $T_A$   
 Effective Aperture  $A_e$

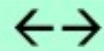
$$S = \frac{2kT_A}{A_e}$$

# Basic Radio Telescope





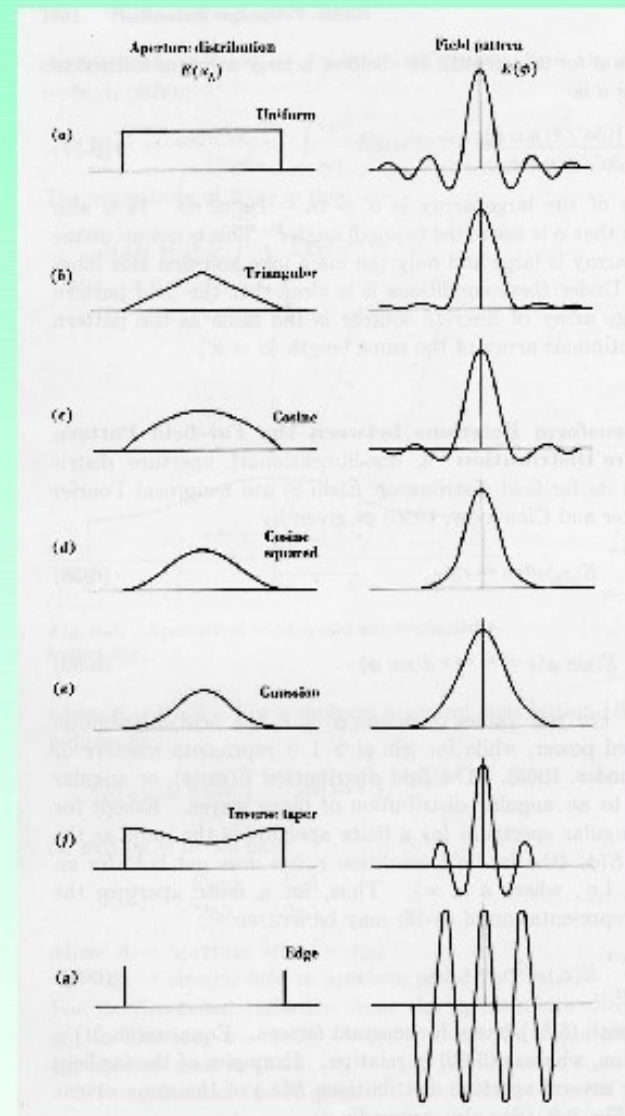
## Aperture Illumination Function



## Beam Pattern

A gaussian aperture illumination gives a gaussian beam:

$$\epsilon_{pat} \approx 0.7$$



Kraus, 1966. Fig.6-9, p. 168.

# Gain(K/Jy) for the GBT

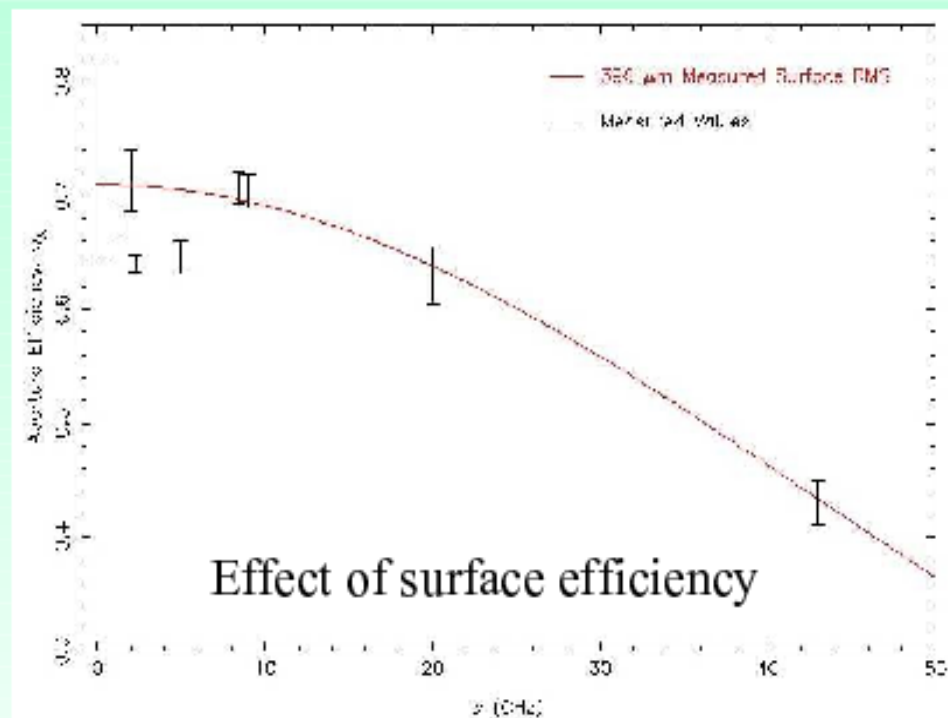
$$S = \frac{2kT_A}{A_e}$$

$$G = \frac{T_A}{S} = \frac{e_{ap} A_g}{2k}$$

$$G(K/Jy) = 2.84 \cdot \epsilon_{ap}$$

Including atmospheric absorption:

$$S = \frac{2kT_A}{A_e} e^{-\tau a}$$



## Power Received:

$$P_{rec}(\nu) = \frac{1}{2} A_e \int_{\Omega} I_{\nu}(\theta, \varphi) P_n(\theta, \varphi) d\Omega \quad \text{watts Hz}^{-1}$$

Effective Area  $\hat{\quad}$  Source  $\hat{\quad}$  Antenna Pattern

Power Received in bandwidth  $\Delta\nu$  :

$$P_{rec} = \int_{-\Delta\nu/2}^{\Delta\nu/2} P_{rec}(\nu) d\nu \quad \text{watts}$$

For thermal sources (Planck's Law):

$$I_{\nu}(\theta, \varphi) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT(\theta, \varphi)} - 1} \quad \text{watts m}^{-2} \text{ ster}^{-1} \text{ Hz}^{-1}$$

$$I_\nu(\theta, \varphi) = 2kT(\theta, \varphi)/\lambda^2$$

Substituting, this gives the following Rayleigh-Jeans approximation:

$$P_\nu = \frac{kA_e}{\lambda^2} \int_{\Omega} T(\theta, \varphi) P_n(\theta, \varphi) d\Omega \quad \text{watts Hz}^{-1}$$

# Antenna Temperature

Power available at terminal of a resistor:

$$W = kT \quad \text{watts Hz}^{-1}$$

Replace the antenna with a matched resistor at a physical temperature that gives the same response:

$$P_\nu = kT_A \quad \text{watts Hz}^{-1}$$

$$\begin{aligned} \text{and } T_A &= \frac{A_e}{\lambda^2} \int_{\Omega} T(\theta, \varphi) P_n(\theta, \varphi) d\Omega \\ &= \frac{1}{\Omega_A} \int_{\Omega} T(\theta, \varphi) P_n(\theta, \varphi) d\Omega. \end{aligned}$$

$T_A$  is defined as the ANTENNA TEMPERATURE.

# Flux Density

Spectral flux density for a discrete source  
(one with a clear boundary):

$$S = \int_{\text{source}} I_{\nu}(\theta, \varphi) d\Omega \quad \text{watts m}^2 \text{ Hz}^{-1}$$

Observed Flux Density:

$$S_o = \int_{\text{source}} I_{\nu}(\theta, \varphi) P_n(\theta, \varphi) d\Omega \quad \text{watts m}^{-2} \text{ Hz}^{-1}$$

This is  $\leq S$  depending on the size of the source.

## Large Source:

$$S_o = I_\nu(\theta, \varphi) \int P_n(\theta, \varphi) d\Omega \approx I_\nu(\theta, \varphi) \Omega_A \quad \text{watts m}^{-2} \text{ Hz}^{-1}$$

## Small Source

$$\begin{aligned} S = S_o &= P_n(0, 0) \int_{\text{source}} I_\nu(\theta, \varphi) d\Omega \\ &= \int_{\text{source}} I_\nu(\theta, \varphi) d\Omega \quad \text{watts m}^{-2} \text{ Hz}^{-1} \end{aligned}$$

The standard unit of spectral flux density in radio astronomy is the Jansky:

$$1 \text{ Jy} = 10^{-26} \text{ watts m}^{-2} \text{ Hz}^{-1}$$

From earlier equations:

$$P_\nu = kT_A = 1/2 A_e S_o \text{ watts Hz}^{-1}$$

which gives:

$$T_A/S_{oJ} = A_e 10^{-26}/2k \text{ Kelvins/Jy}$$

An effective aperture of 2760 m<sup>2</sup> is required to give a sensitivity of 1.0 K/Jy.



*The Sun in three imaginary photos taken from a long distance (left), medium distance (center), and short distance (right) would have a constant brightness but increasing angular size.*

Brightness of a source (Sun) does not depend  
on the distance

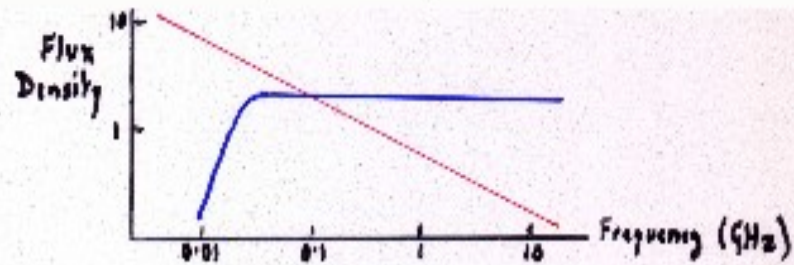
Flux density of a source (Sun) depends  
inversely on the square of its distance

# Radio continuum and line emission

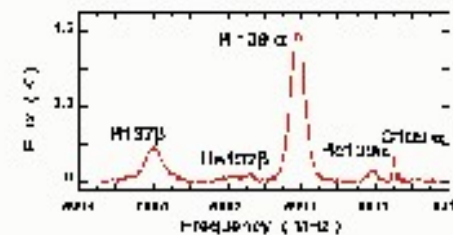
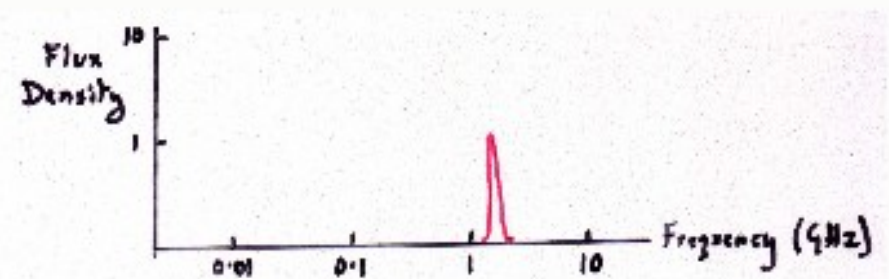
Celestial radio emission can be subdivided into two main spectral categories:

## 1. Broadband or Continuum Emission.

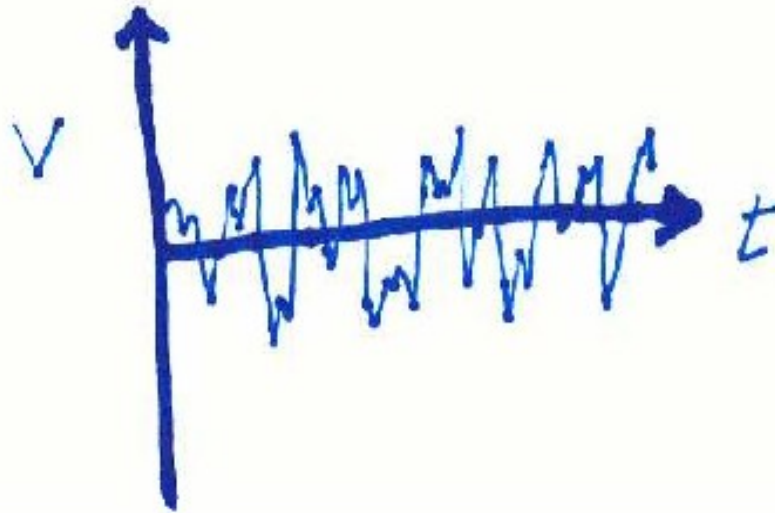
Typical examples: (a) planetary surfaces, (b) HII regions, (c) the Cosmic Microwave Background & (d) those Galactic and extragalactic sources that emit via the synchrotron process.



## 2. Line emission due to low-energy transitions within atoms and molecules in space.



Terzian & Lewis,  
(unpublished)



*The antenna output voltage produced by an astronomical continuum source varies rapidly on short time scales, but the longer-term average power is steady.*

The purpose of the simplest **total-power radiometer** is to measure the time-averaged power of this noise in some well-defined radio frequency (RF) range

$$\nu_{\text{RF}} - \frac{\Delta\nu_{\text{RF}}}{2} \text{ to } \nu_{\text{RF}} + \frac{\Delta\nu_{\text{RF}}}{2},$$

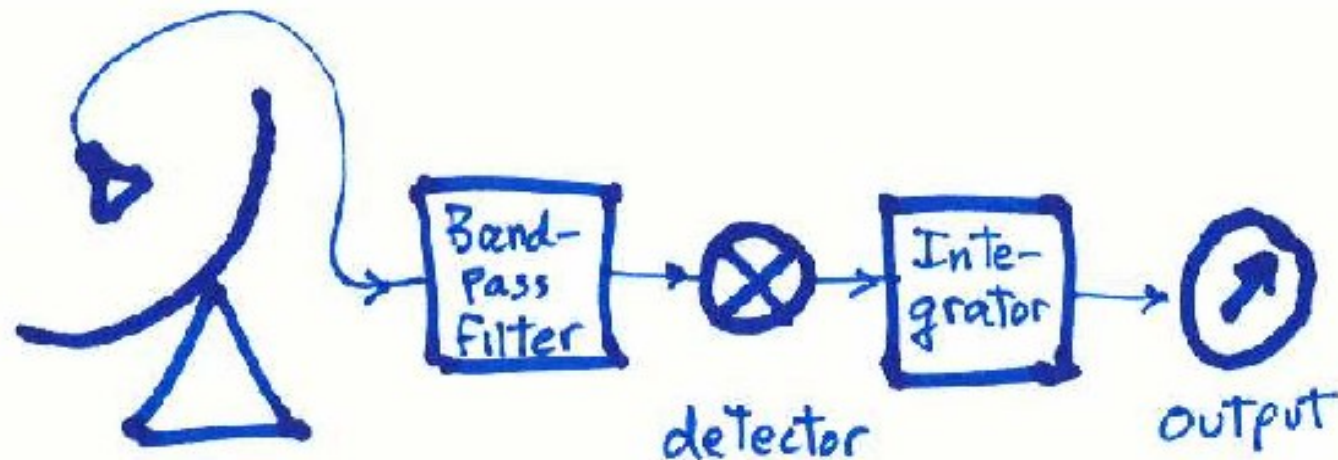
Radio emission from a celestial source is essentially random noise; nearly indistinguishable from noise generated by a warm resistor. Normally this noise is also stationary; its time-averaged power does not change during the course of the observations. Instantaneous power produced by the source varies erratically on time scales as short as the inverse of the receiver bandwidth.

$$T_N \equiv \frac{P_v}{k} \quad (3F1)$$

where  $k \approx 1.38 \times 10^{-23}$  Joule K<sup>-1</sup> is the Boltzmann constant.

The conceptually simplest radiometer consists of three stages in series:

- (1) an ideal (lossless) **bandpass filter** that passes input noise only in the desired frequency range,
- (2) an ideal **square-law detector** whose output voltage is proportional to the square of its input voltage; that is, its output voltage is proportional to its input power,
- (3) and a signal averager or **integrator** that smoothes out the rapidly fluctuating detector output.



*The simplest radiometer filters the broadband noise coming from the telescope, multiplies the signal voltage by itself (square-law detection), and smoothes the detected voltage, which can be read by a meter as shown.*

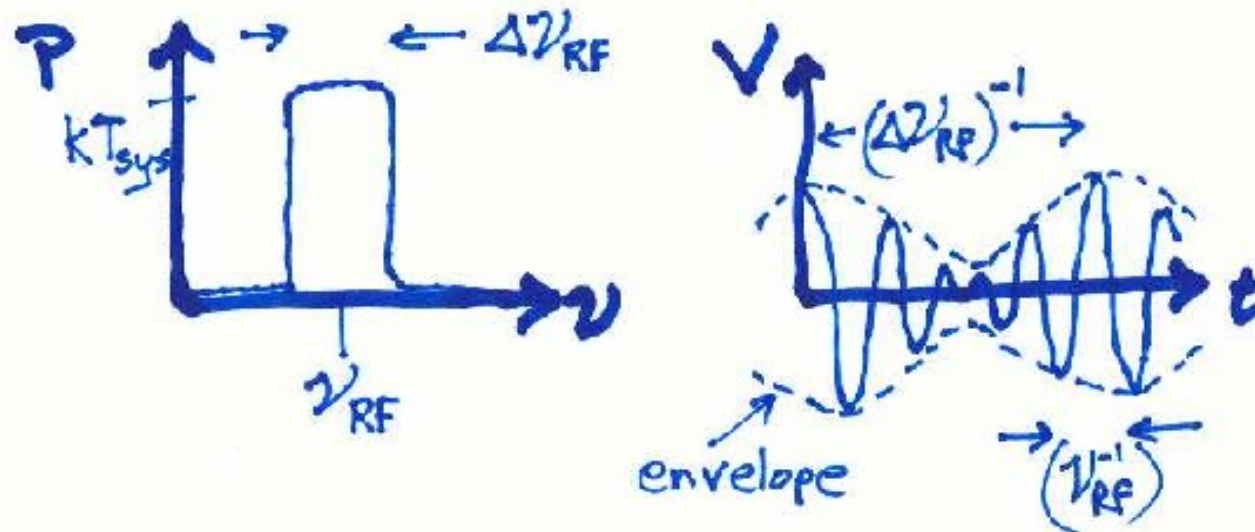
The temperature equivalent to the *total* noise power from all sources referenced to the input of an ideal receiver input is called the **system noise temperature**:

$$T_{\text{sys}} = T_{\text{cmb}} + \Delta T_{\text{source}} + T_{\text{atm}} + T_{\text{spillover}} + T_{\text{recv}} + \dots \quad (3F2)$$

The contributions listed explicitly in Equation 3F2 are  $T_{\text{cmb}} \approx 2.73$  K from the cosmic microwave background,  $\Delta T_{\text{source}}$  from the astronomical source being observed,  $T_{\text{atm}}$  from atmospheric emission in the telescope beam,  $T_{\text{spillover}}$  to account for radiation that the feed picks up in directions beyond the edge of the reflector, and  $T_{\text{recv}}$  to represent the noise power generated by the receiver itself, referenced to the receiver input. The desired signal  $\Delta T_{\text{source}}$  was written with a  $\Delta$  to emphasize that it is usually much smaller than the total system noise:  $\Delta T_{\text{source}} \ll T_{\text{sys}}$ . For example, in the  $\nu_{\text{RF}} \approx 4.85$  GHz sky survey made with the 300-foot telescope, the system noise was  $T_{\text{sys}} \approx 60$  K, but the faintest sources detected contributed only  $\Delta T_{\text{source}} \approx 0.01$  K.

Contribution from the Milky Way galaxy and the CMB changes greatly with frequency. At 38 MHz  $T_{\text{Gal}} \sim 10^{**4} - 10^{**5}$  K. At 327 MHz  $T_{\text{Gal}} \sim 20$  to 2000 K, and negligible above  $\sim 5$  GHz. At higher frequencies, contribution from the CMB dominates.

After passing through an input filter of width  $\Delta\nu_{\text{RF}} < \nu_{\text{RF}}$  the noise signal is no longer completely random; it becomes more like a sine wave of frequency  $\approx \nu_{\text{RF}}$  whose amplitude envelope is **modulated** (varies) on time scales  $\Delta t \approx (\Delta\nu_{\text{RF}})^{-1} > \nu_{\text{RF}}^{-1}$ . The positive and negative envelopes are similar so long as  $\Delta\nu_{\text{RF}} \ll \nu_{\text{RF}}$ .



The noise voltage output of a filter with center frequency  $\nu_{\text{RF}}$  and bandwidth  $\Delta\nu_{\text{RF}}$  is a sinusoid with frequency  $\nu_{\text{RF}}$  whose envelope fluctuates on time scales  $(\Delta\nu_{\text{RF}})^{-1} > (\nu_{\text{RF}})^{-1}$ .

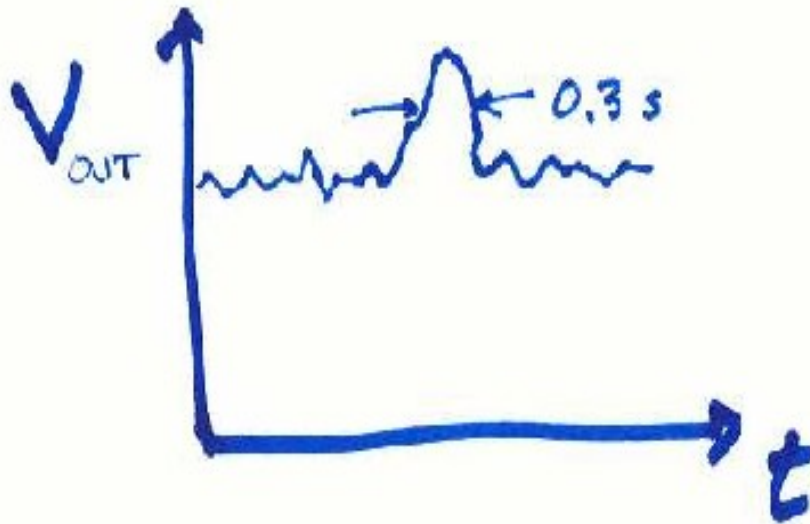
Since the input filter does not pass signals with zero frequency (DC), the time-averaged voltage is nearly zero. However, the average power at the filter output is not zero; it is

$$P \approx P_{\nu} \Delta\nu_{\text{RF}}$$

The filter output is sent to a square-law detector, a device whose output voltage is proportional to the square of the input voltage, which in turn is proportional to the input power. The detector

The scanning time between half-power points was thus  $\approx 0.3$  s.

---



*A point source appears as a Gaussian with FWHM duration 0.3 s in the receiver output.*

---

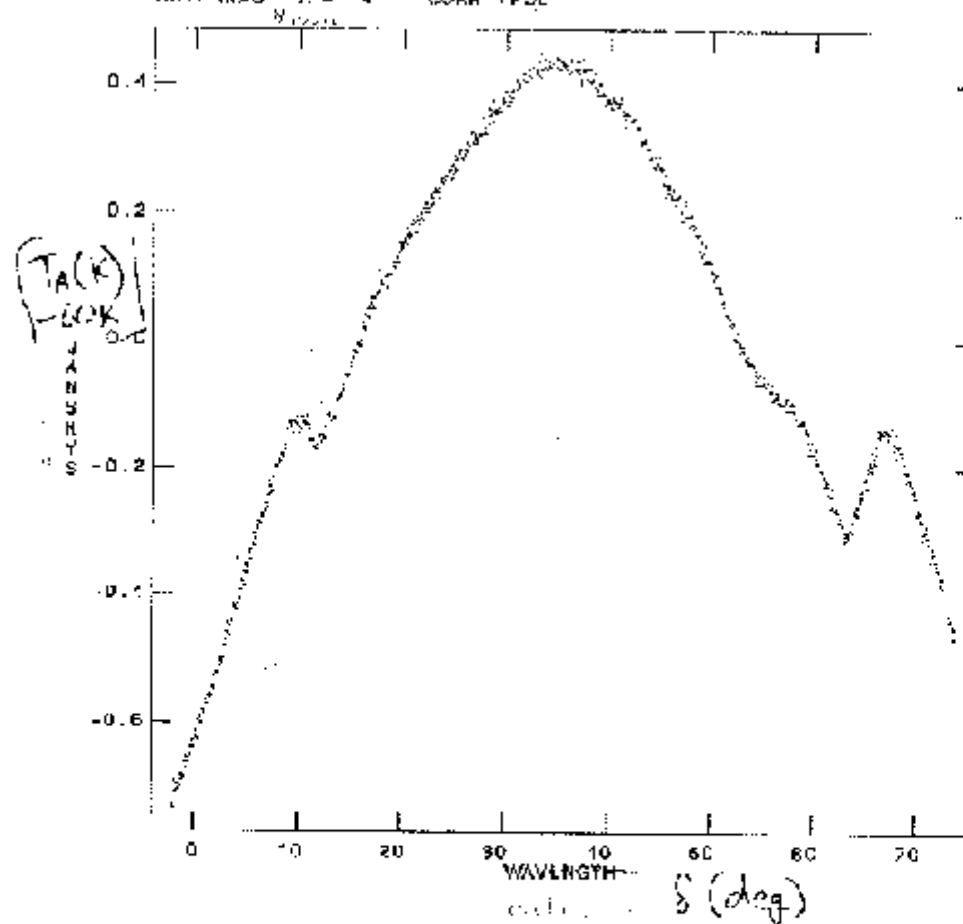
The data were integrated and sampled every  $\tau = 0.1$  s, so there were  $\approx 3$  samples per half-power beamwidth. A subset of the samples taken from one receiver during one scan covering the declination range  $\delta \approx -2^\circ$  to  $\delta \approx +73^\circ$  is shown.

---

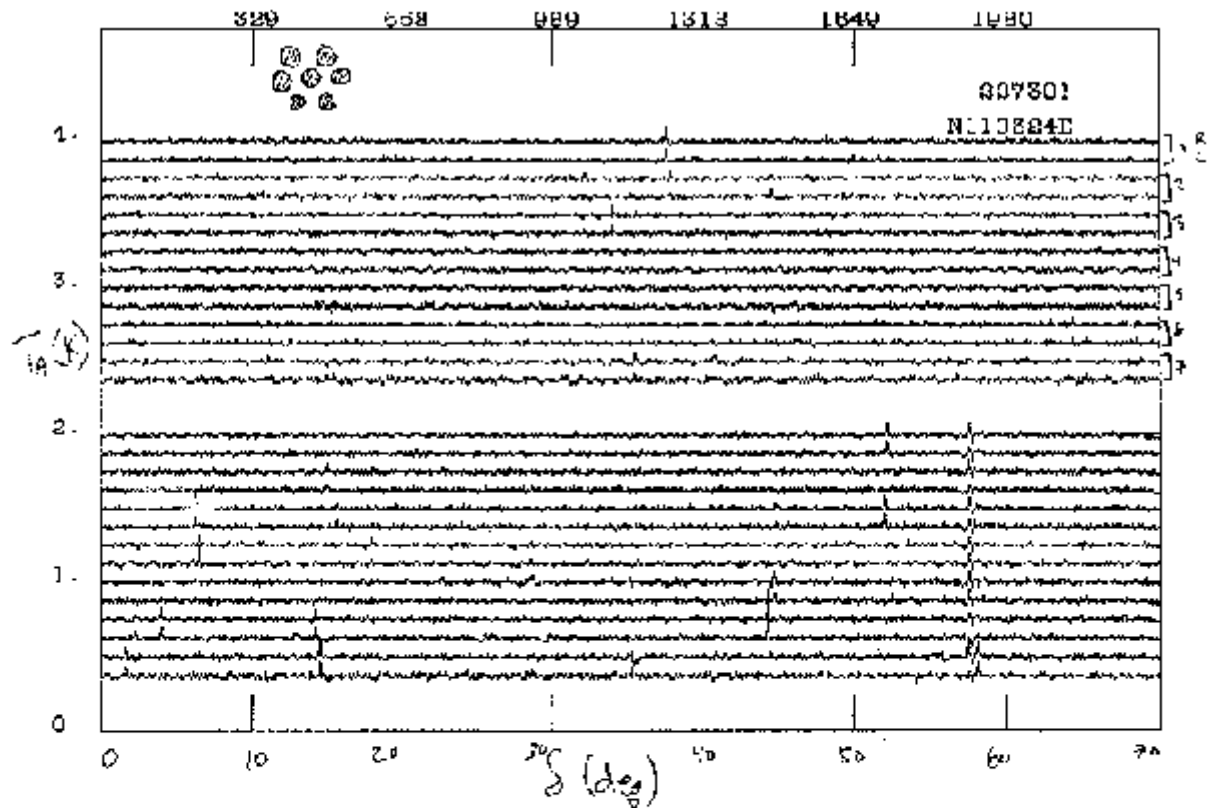
PLOT FILE VERSION 2 CREATED 29-OCT-1987 15:34.25

REAL YS V FOR CONDON SD CALIB. 1

ANTENNAS 1 4 4 CORN 1 POL



The intensity scale has been calibrated in Kelvins, and the large mean  $T_{\text{sys}} \approx 60$  K has been subtracted. By far the biggest time-dependent signal (spanning a range of about 1 K) is caused by ground radiation entering the prime-focus feed via leakage through the reflector mesh and spillover. Fortunately, this unwanted ground signal varies smoothly with telescope elevation, so subtracting a short (about 40 arcmin long) running-median baseline takes out the spillover signal without removing compact radio sources. The outputs from all 14 receiver channels (7 beams  $\times$  2 polarizations/beam) after baseline subtraction are shown in the next viewgraph. Only now are the faint radio sources visible above the noise fluctuations.



Data from all 14 receivers after subtraction of running-median baselines. Sources appear as spikes in both polarization channels (R and L) of one or two beams. Interference is usually visible in all 14 receivers simultaneously.

The rms noise observed is consistent with the prediction of the total-power radiometer equation:

$$\sigma_T \approx \frac{T_{\text{sys}}}{\sqrt{\Delta\nu_{\text{RF}}\tau}} \approx \frac{60\text{ K}}{\sqrt{6 \times 10^8 \text{ Hz} \times 0.1\text{ s}}} \approx 0.008\text{ K}$$

Note that the output of a total-power receiver scales in proportion to the overall gain  $G$  of the receiver:

$$P_\nu = GkT_{\text{sys}}$$

If  $G$  isn't perfectly constant, the change in output

$$\Delta P_\nu = \Delta GkT_{\text{sys}}$$

caused by a **gain fluctuation**  $\Delta G$  produces a change

$$\Delta T_G = T_{\text{sys}} \left( \frac{\Delta G}{G} \right)$$

which is indistinguishable from a comparable change  $\Delta T$  in the system noise temperature produced by an astronomical source. Since gain fluctuations and input noise fluctuations are independent random processes, their **variances** (the variance is the square of the rms) add, and the total receiver output fluctuation becomes:

$$\begin{aligned} \sigma_{\text{total}}^2 &= \sigma_{\text{noise}}^2 + \sigma_G^2 \\ \sigma_{\text{total}}^2 &= T_{\text{sys}}^2 \left[ \frac{1}{\Delta\nu_{\text{RF}}\tau} + \left( \frac{\Delta G}{G} \right)^2 \right] \end{aligned}$$

The **practical total-power radiometer equation** is thus:

$$\sigma_T \approx T_{\text{sys}} \left[ \frac{1}{\Delta\nu_{\text{RF}}\tau} + \left( \frac{\Delta G}{G} \right)^2 \right]^{1/2} \quad (3F4)$$

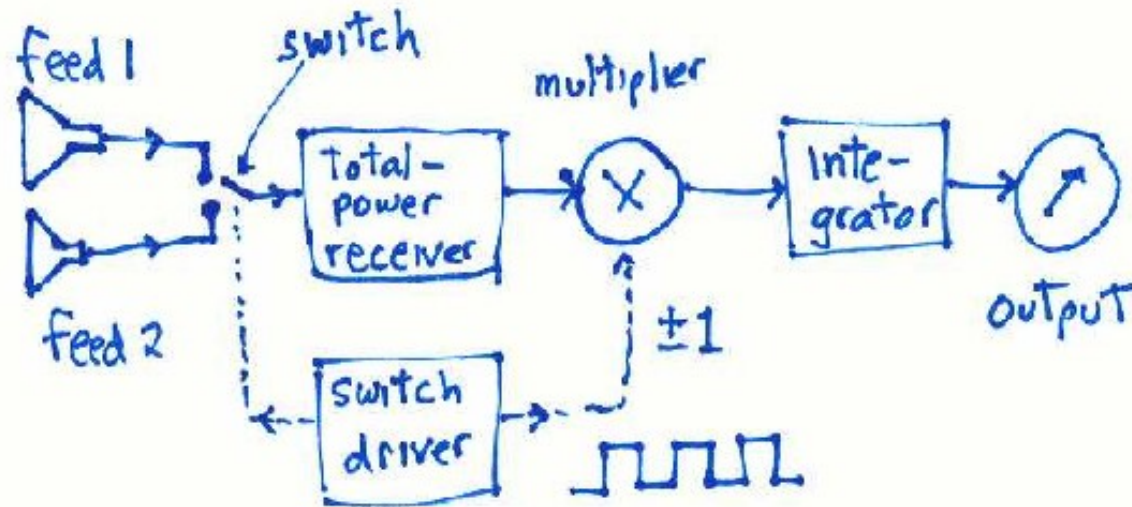
Clearly, gain fluctuations will significantly degrade the sensitivity unless

$$\left( \frac{\Delta G}{G} \right) \ll \frac{1}{\sqrt{\Delta\nu_{\text{RF}}\tau}}$$

For example, the 5 GHz receiver used to make the sky survey with the 300-foot telescope had  $\Delta\nu_{\text{RF}} \approx 6 \times 10^8 \text{ Hz}$  and  $\tau \approx 0.1 \text{ s}$ , so the fractional gain fluctuations on time scales up to a few seconds (the time to scan one baseline length) had to satisfy

$$\frac{\Delta G}{G} \ll \frac{1}{\sqrt{6 \times 10^8 \text{ Hz} \times 0.1 \text{ s}}} = 1.3 \times 10^{-4}$$

This is difficult to achieve in practice.



Block diagram of a beamswitching differential radiometer.

If the system temperatures are  $T_1$  and  $T_2$  in the two positions of the switch, then the receiver output is proportional to  $T_1 - T_2 \ll T_1$  and the effect of gain fluctuations is only

$$\Delta T_G \approx (T_1 - T_2) \frac{\Delta G}{G} \ll T_1 \frac{\Delta G}{G}.$$

Likewise, the atmospheric emission in two nearly overlapping beams through the troposphere is nearly the same, so most of the tropospheric fluctuations cancel out. The main drawback with Dicke switching is that the receiver output fluctuations, relative to the source signal in a single beam, are doubled, so the radiometer equation for a Dicke switching receiver is:

$$\sigma_T = \frac{2T_{\text{sys}}}{\sqrt{\Delta\nu_{\text{RF}}\tau}} \quad (3F5)$$

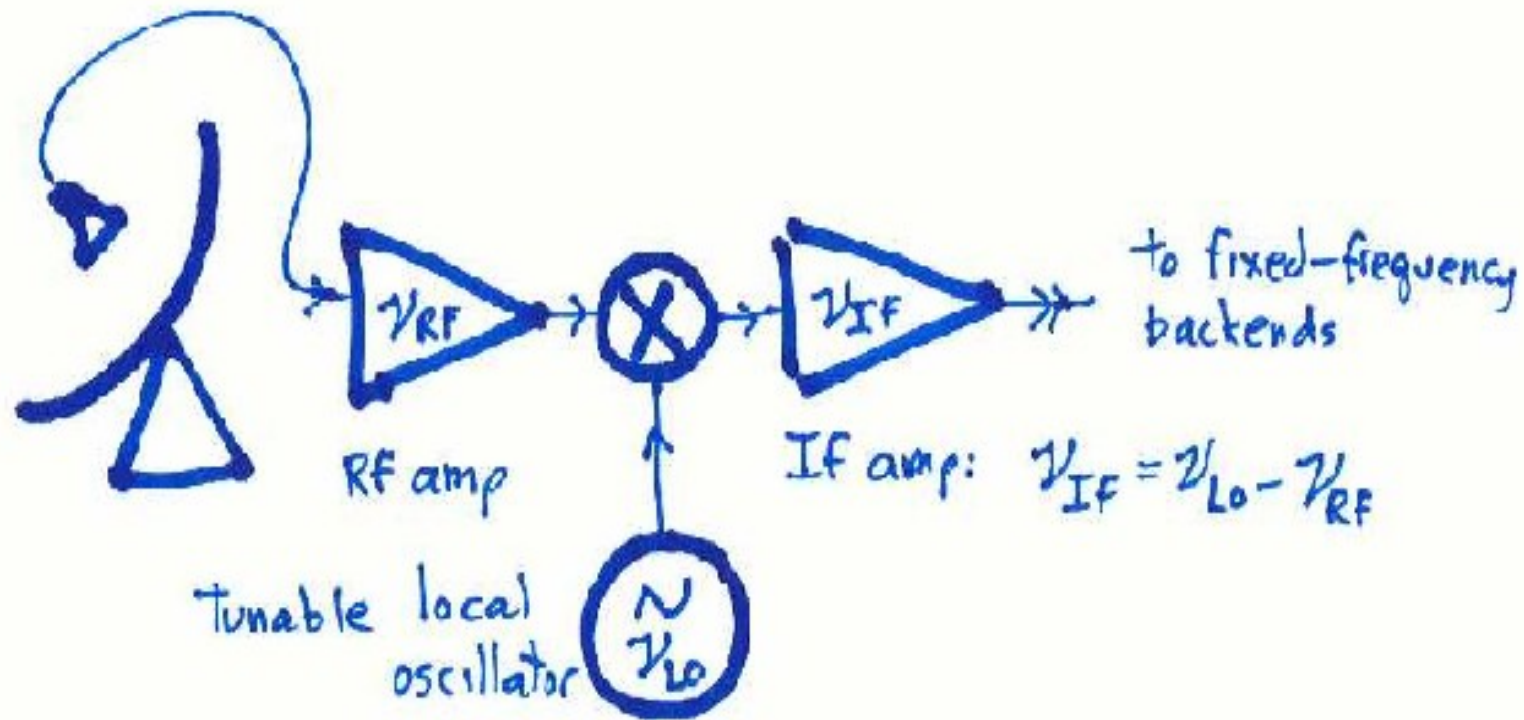
Few actual radiometers are this simple. Nearly all practical radiometers are **superheterodyne** receivers, in which a **mixer** multiplies the RF signal by a sine wave of frequency  $\nu_{LO}$  generated by a **local oscillator**. The product of two sine waves contains the sum and difference frequency components

$$2 \sin(2\pi\nu_{LO}t) \times \sin(2\pi\nu_{RF}t) = \cos[2\pi(\nu_{LO} - \nu_{RF})t] - \cos[2\pi(\nu_{LO} + \nu_{RF})t] .$$

The difference frequency is called the **intermediate frequency** (IF). The advantages of superheterodyne receivers include doing most of the amplification at lower frequencies ( $\nu_{IF} < \nu_{RF}$ ), which is usually easier, and precise control of the  $\nu_{RF}$  range covered via tuning *only*

- Long cables less lossy at lower frequencies (nowadays one uses optical optical fibres
  - Fear of positive feedback causing receiver oscillations
- If  $\nu_{LO}$  can be varied over a wide range, a standard fixed frequency IF chain can service a wide range of front ends

the local oscillator so that **back-end** devices following the untuned IF amplifier, multichannel filter banks or digital spectrometers for example, can operate over fixed frequency ranges.

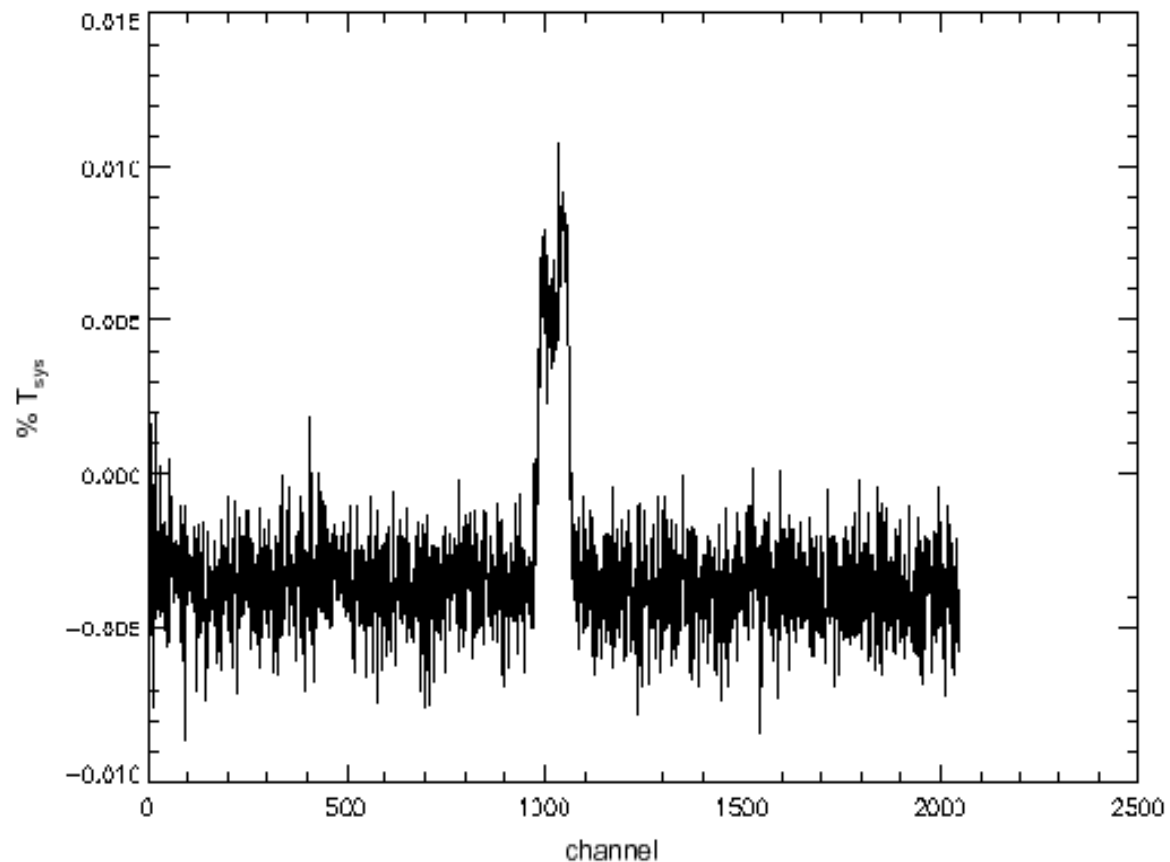


*Block diagram of a simple superheterodyne receiver. Only the local oscillator is tuned to change the observing frequency range.*

# Determining $T_{\text{source}}$

$$(\text{ON} - \text{OFF})/\text{OFF}$$

$$[(T_{\text{source}} + T_{\text{everything else}}) - (T_{\text{everything else}})] / T_{\text{everything else}}$$



# Determining $T_{\text{source}}$

$$\frac{(\text{ON} - \text{OFF})/\text{OFF}}{[(T_{\text{source}} + T_{\text{everything else}}) - (T_{\text{everything else}})]/ T_{\text{everything else}}}$$

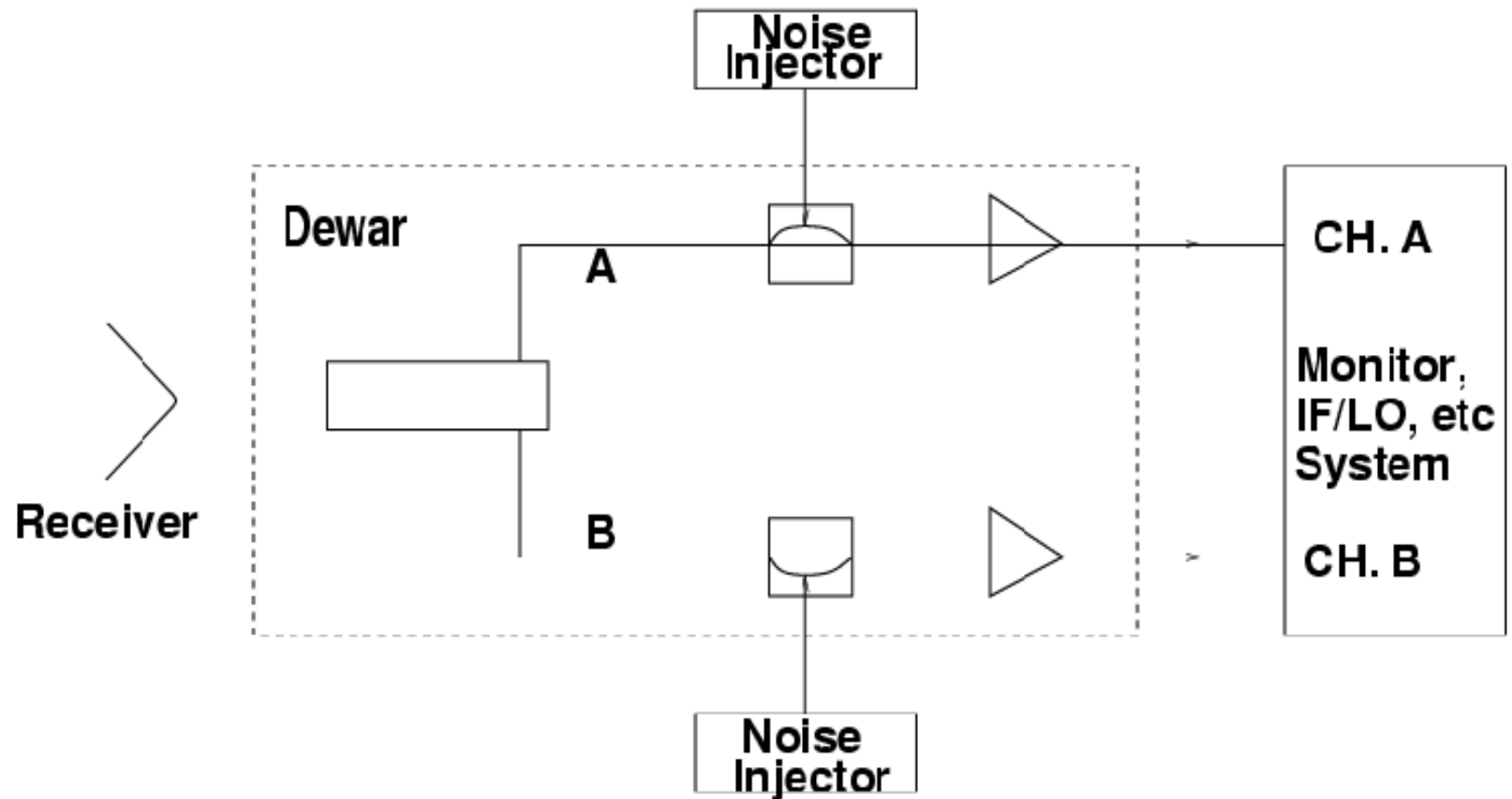
$$\text{Result} = \frac{T_{\text{source}}}{T_{\text{system}}}$$

Units are % \* System Temperature

Need to determine system temperature to calibrate data

# Determining $T_{\text{sys}}$

## Noise Diodes



# Determining $T_{\text{sys}}$

## Noise Diodes

$$T_{\text{src}}/T_{\text{sys}} = (\text{ON} - \text{OFF})/\text{OFF}$$

.

.

.

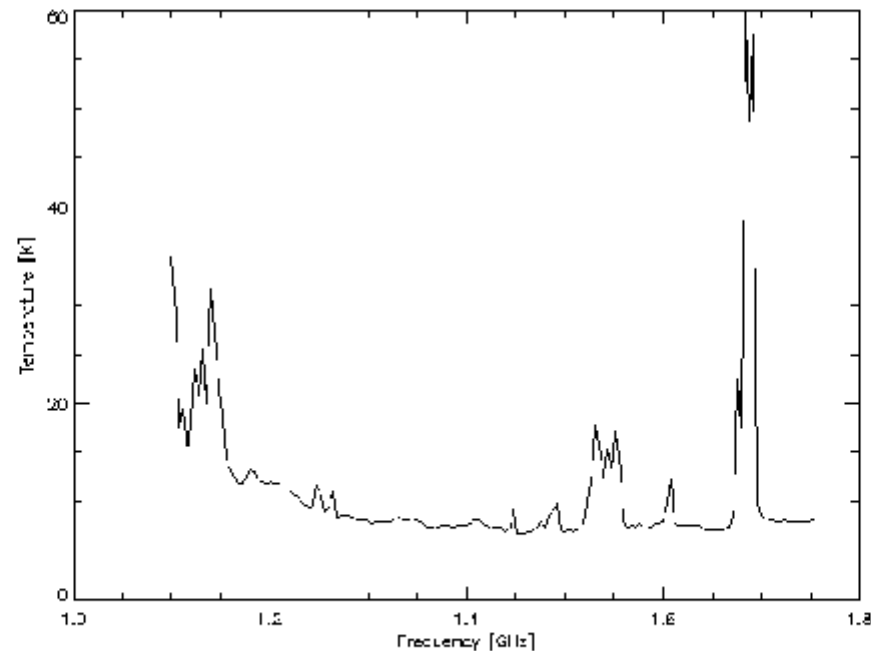
$$T_{\text{diode}}/T_{\text{sys}} = (\text{On} - \text{Off}) / \text{Off}$$

$$T_{\text{sys}} = T_{\text{diode}} * \text{Off}/(\text{On} - \text{Off})$$

# Determining $T_{\text{sys}}$

## Noise Diodes - Considerations

- Frequency dependence

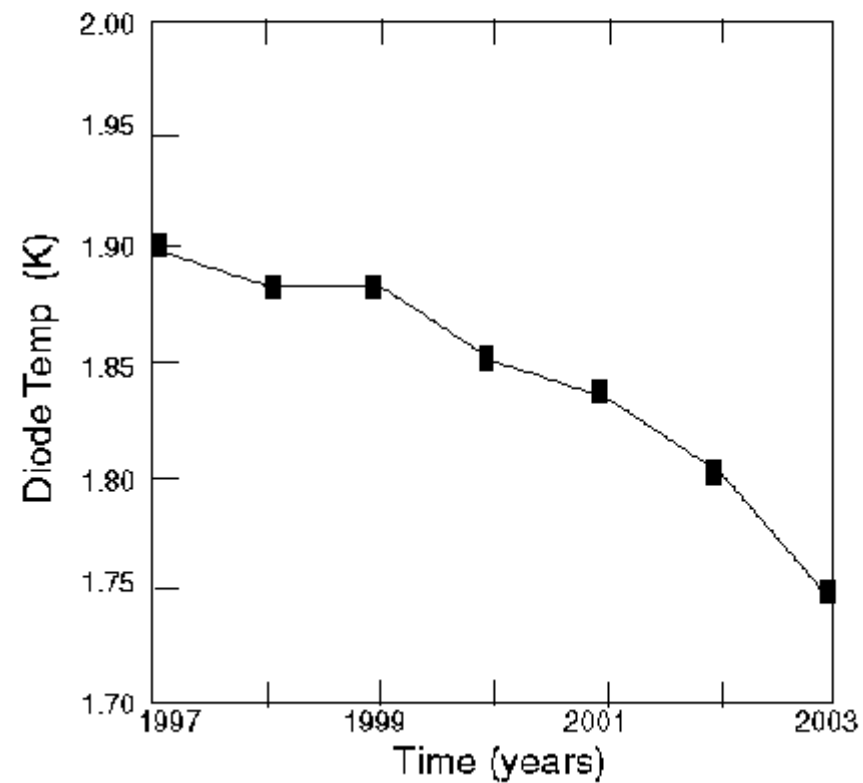


Lab measurements of the GBT L-Band calibration diode, taken from work of M. Stennes & T. Dunbrack - February 14, 2002

# Determining $T_{\text{sys}}$

## Noise Diodes - Considerations

- Frequency dependence
- Time stability



## Determining $T_{\text{source}}$

$$T_{\text{source}} = \frac{(\text{ON} - \text{OFF})}{\text{OFF}} T_{\text{system}}$$

Blank Sky or other

From diodes, Hot/Cold loads, etc.

Telescope response has not been accounted for!

# Determining $T_{\text{source}}$

$$T_{\text{source}} = \frac{(\text{ON} - \text{OFF})}{\text{OFF}}$$

Blank Sky or other

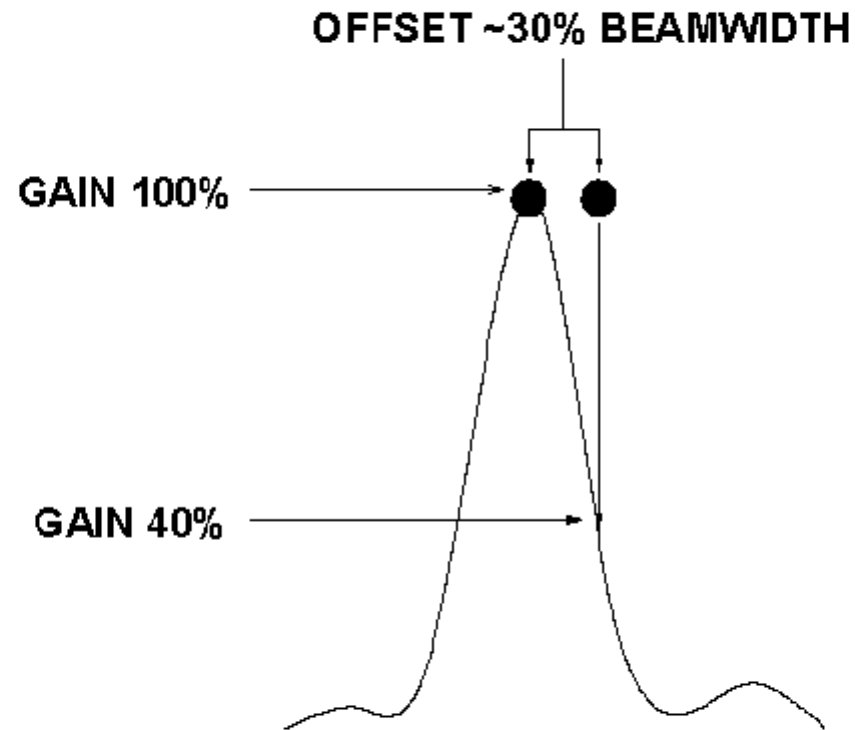
From diodes, Hot/Cold loads, etc.

$T_{\text{system}}$

$$\frac{1}{\text{GAIN}}$$

Theoretical,  
or  
Observational

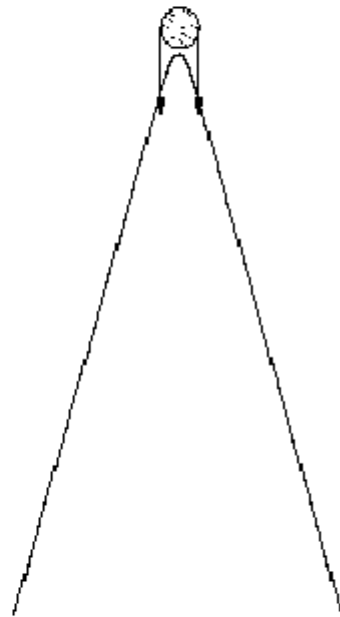
# Other Issues: Pointing



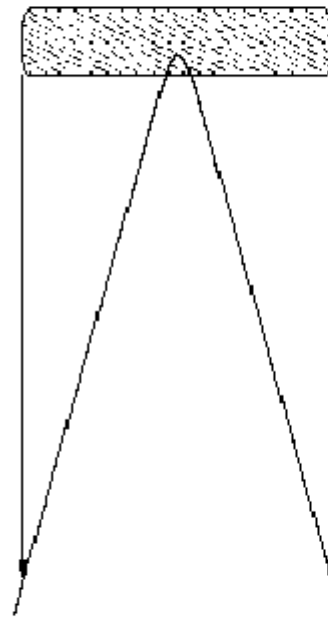
Results in reduction of telescope gain  
Typically can be corrected in software

# Other Issues: Focus

**Well Focused  
Beam**



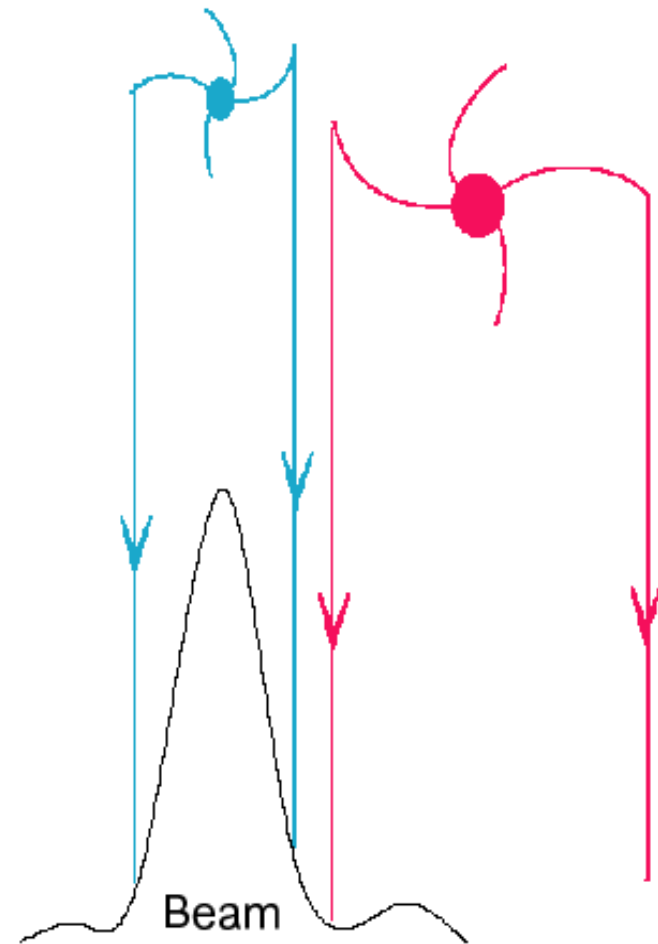
**Poorly Focused  
Beam**



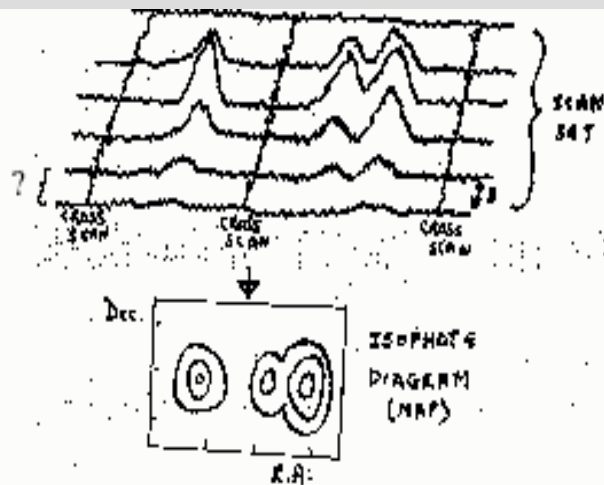
Results in reduction of telescope gain  
Corrected mechanically

## Other Issues: Side Lobes\*

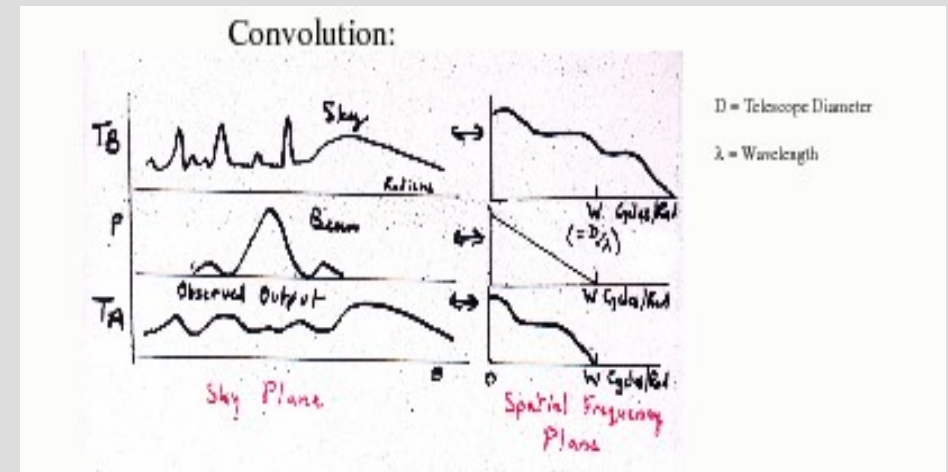
- Allows in extraneous or unexpected radiation
- Can result in false detections, over-estimates of flux, incorrect gain determination
- Solution is to fully understand side lobes



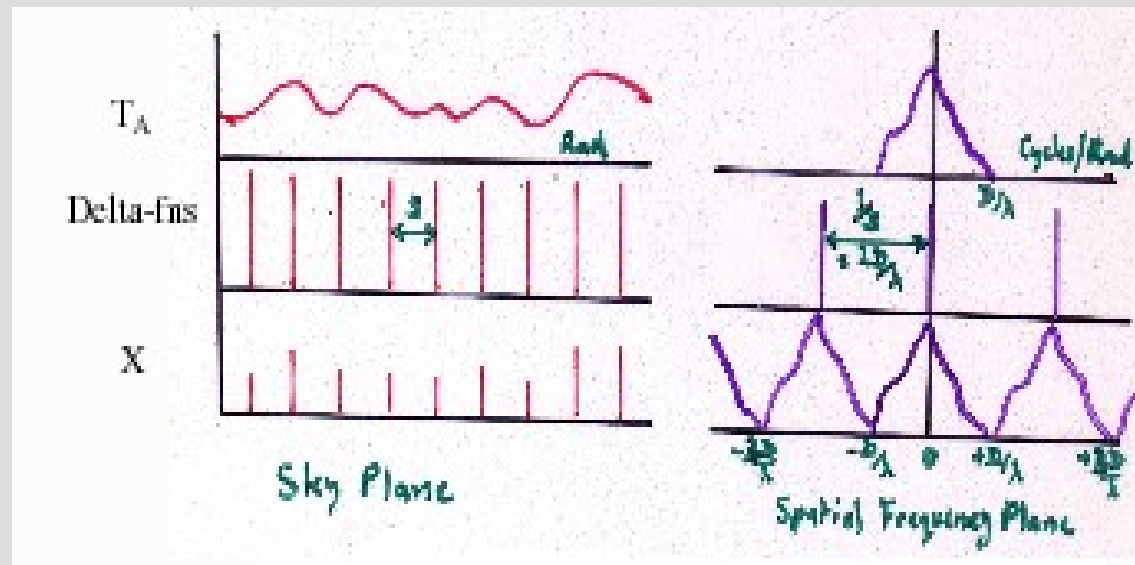
# mapping



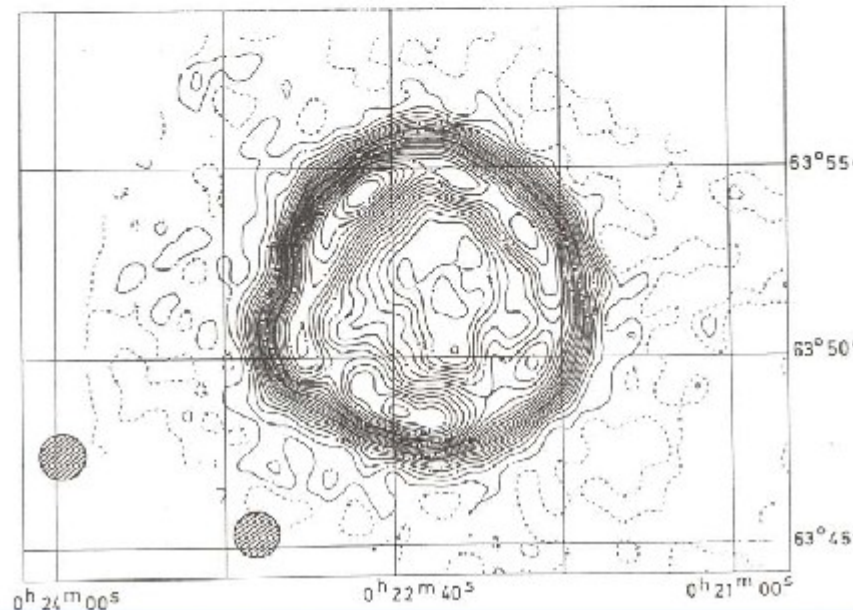
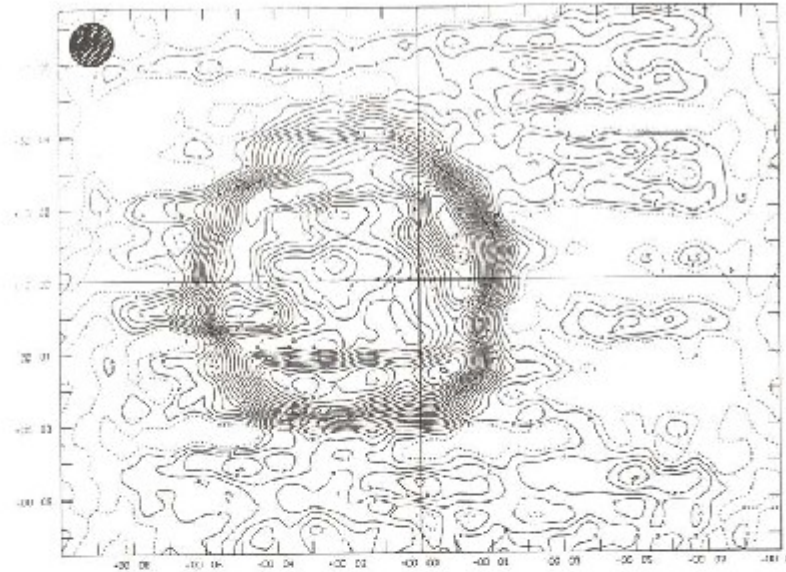
How far apart should we place our scans (and how often should we sample along a scan) in order to lose no information which our telescope is capable of passing?



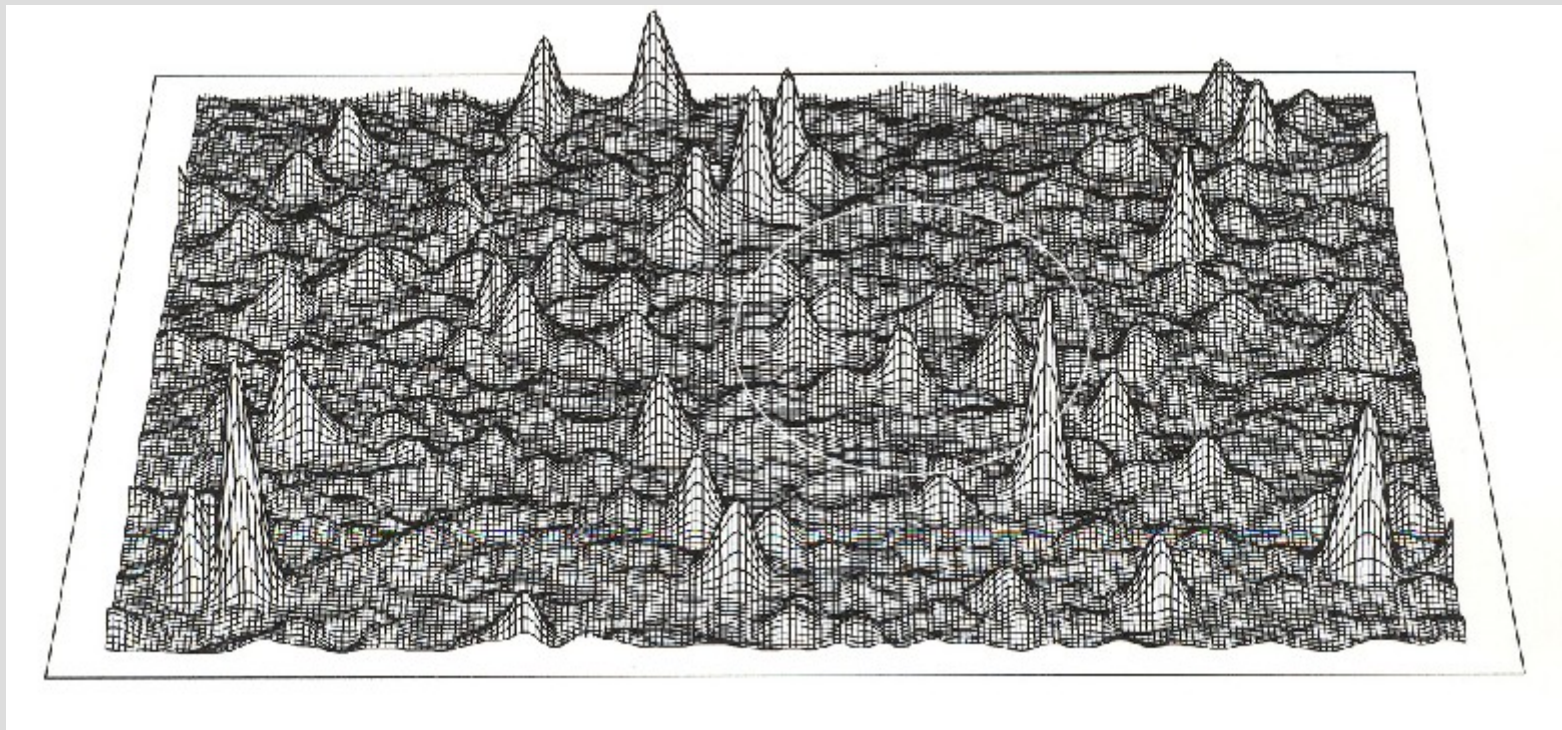
# mapping



The SNR 3C10 imaged  
at 10.7 GHz by  
horizontal scans (top)  
and basket-weaving  
(bottom)

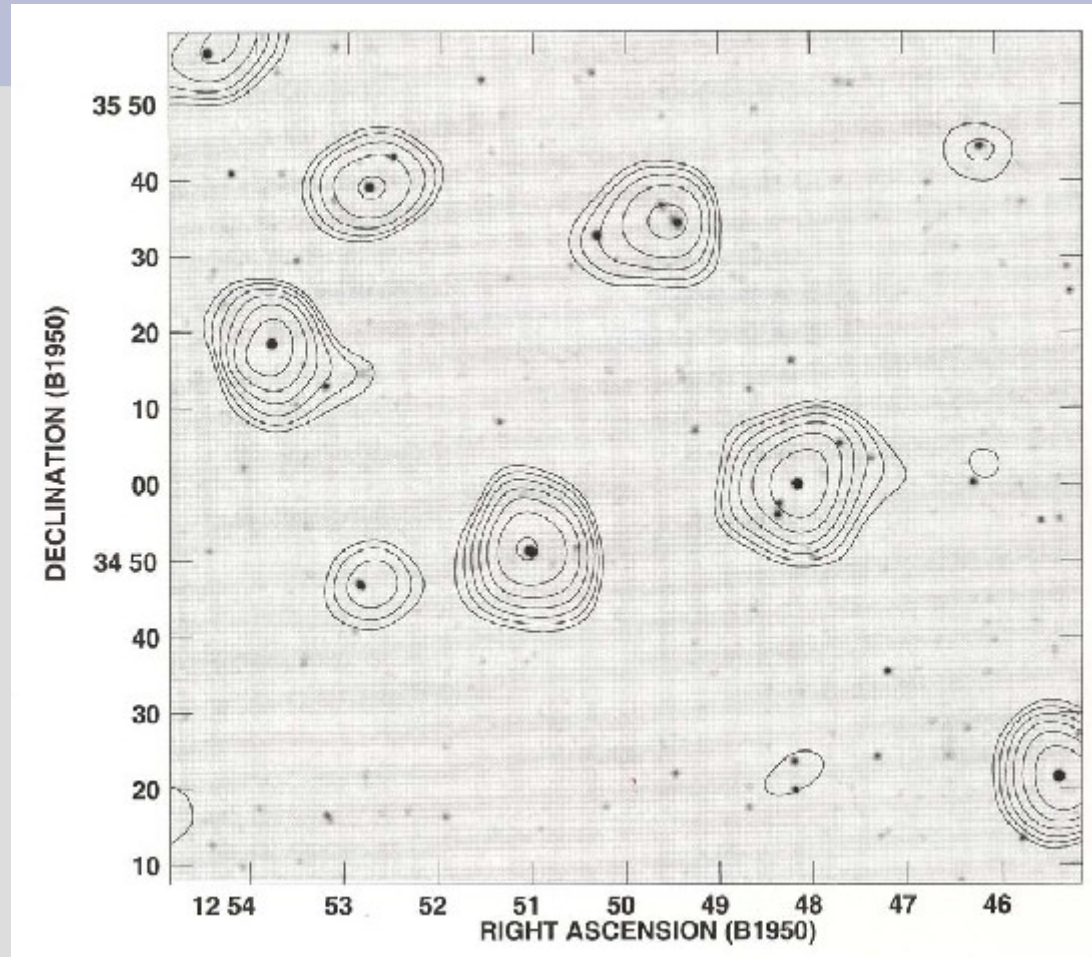


# confusion



Profile of 45 deg sq near the North Galactic Pole imaged with 12 arcmin resolution at 1.4 Ghz. The strongest source seen has a flux density of  $\sim 1.5$  Jy.

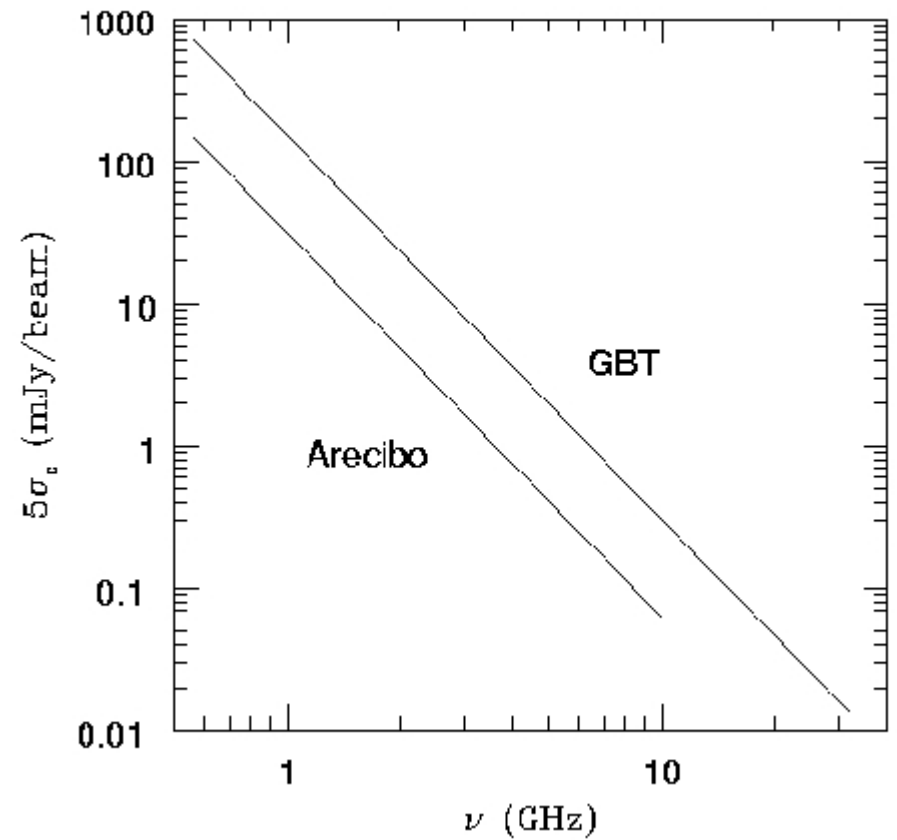
# confusion

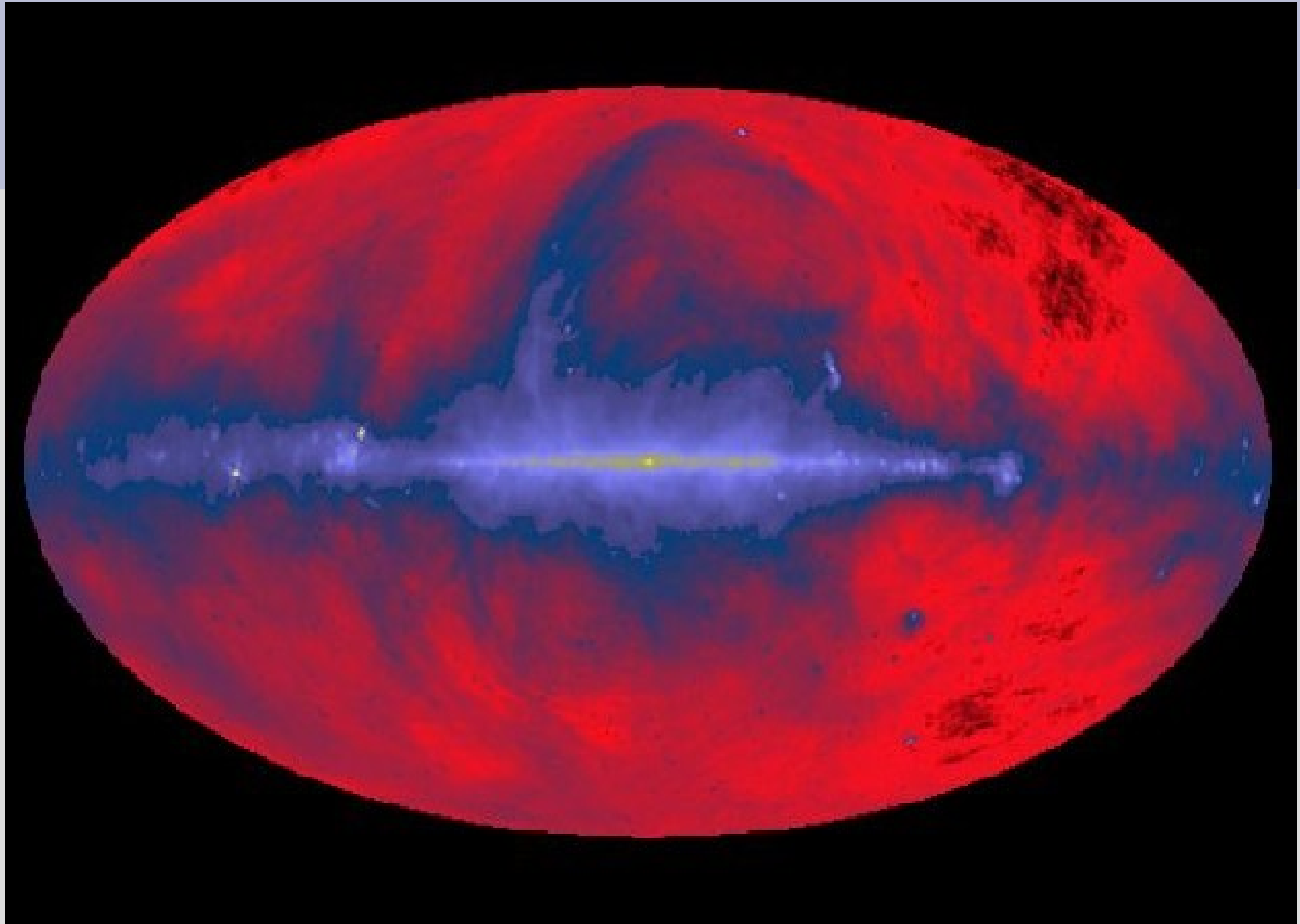


NVSS (45 arcsec) in gray scale and GB6 (12 arcmin) in contours shows source blending. Lowest contour: 45 mJy/beam

# confusion

$$\left( \frac{\sigma_c}{\text{mJy beam}^{-1}} \right) \approx 0.2 \left( \frac{\nu}{\text{GHz}} \right)^{-0.7} \left( \frac{\theta_M \theta_m}{\text{arcmin}^2} \right)$$





408-MHz all-sky image: Haslam, Salter, Stoffel, Wilson 1982