

# POLARISATION I: *FOR THE LOVE OF STOKES*

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Thanks to Jimi Green

# Outline

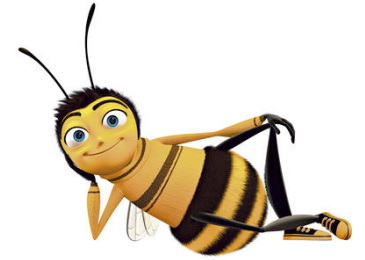


- Why study Polarimetry and what is Polarisation?
- Poincare and his spheres
- Jones and his vectors (and matrices)
- Stokes and his parameters
- How do we measure Polarisation?
- Leakages and Mueller's Matrix
- Polarised beam effects
- Science with Polarisation:
  - ▣ Masers and Zeeman splitting



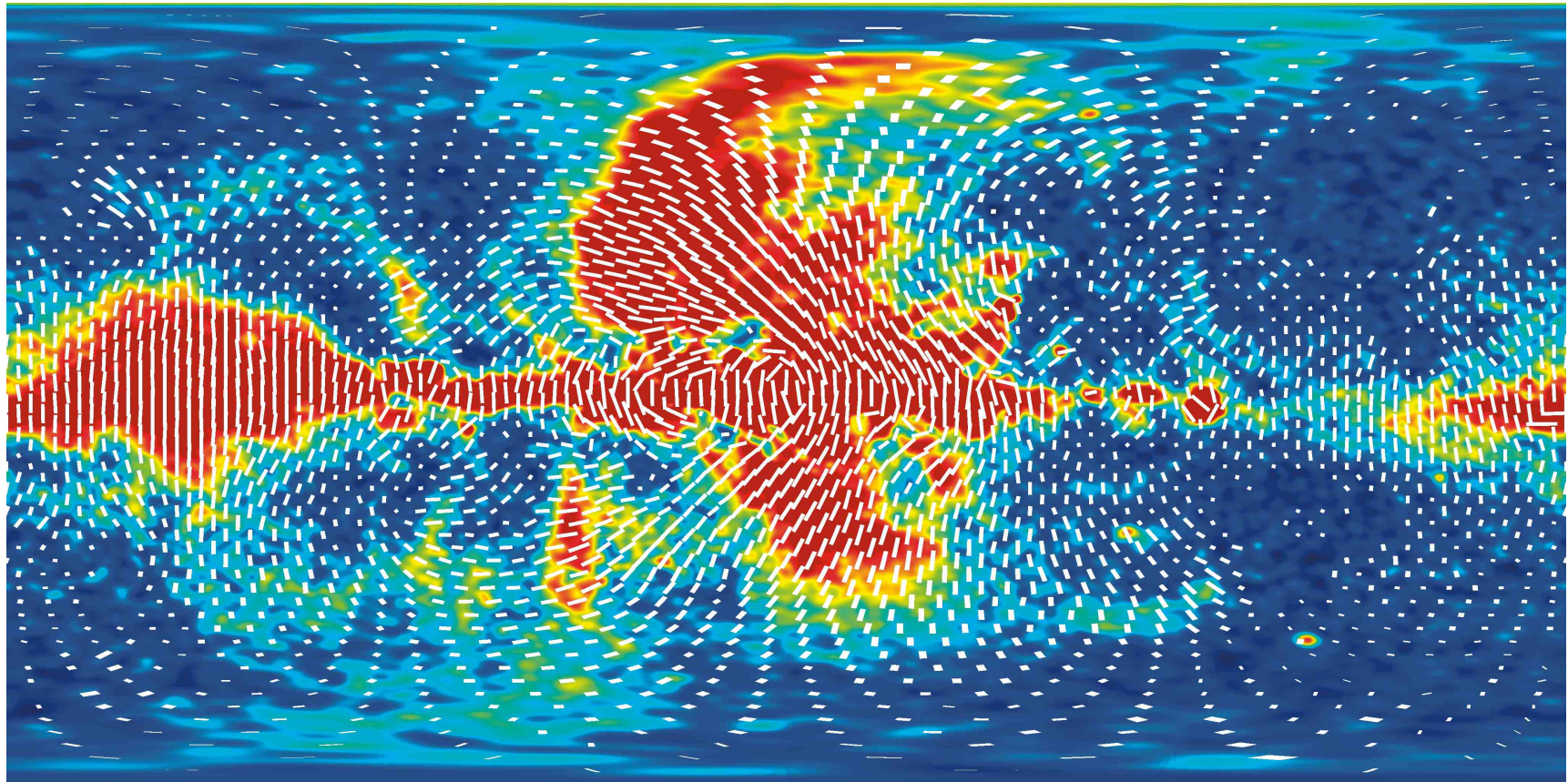
# Why Polarimetry?

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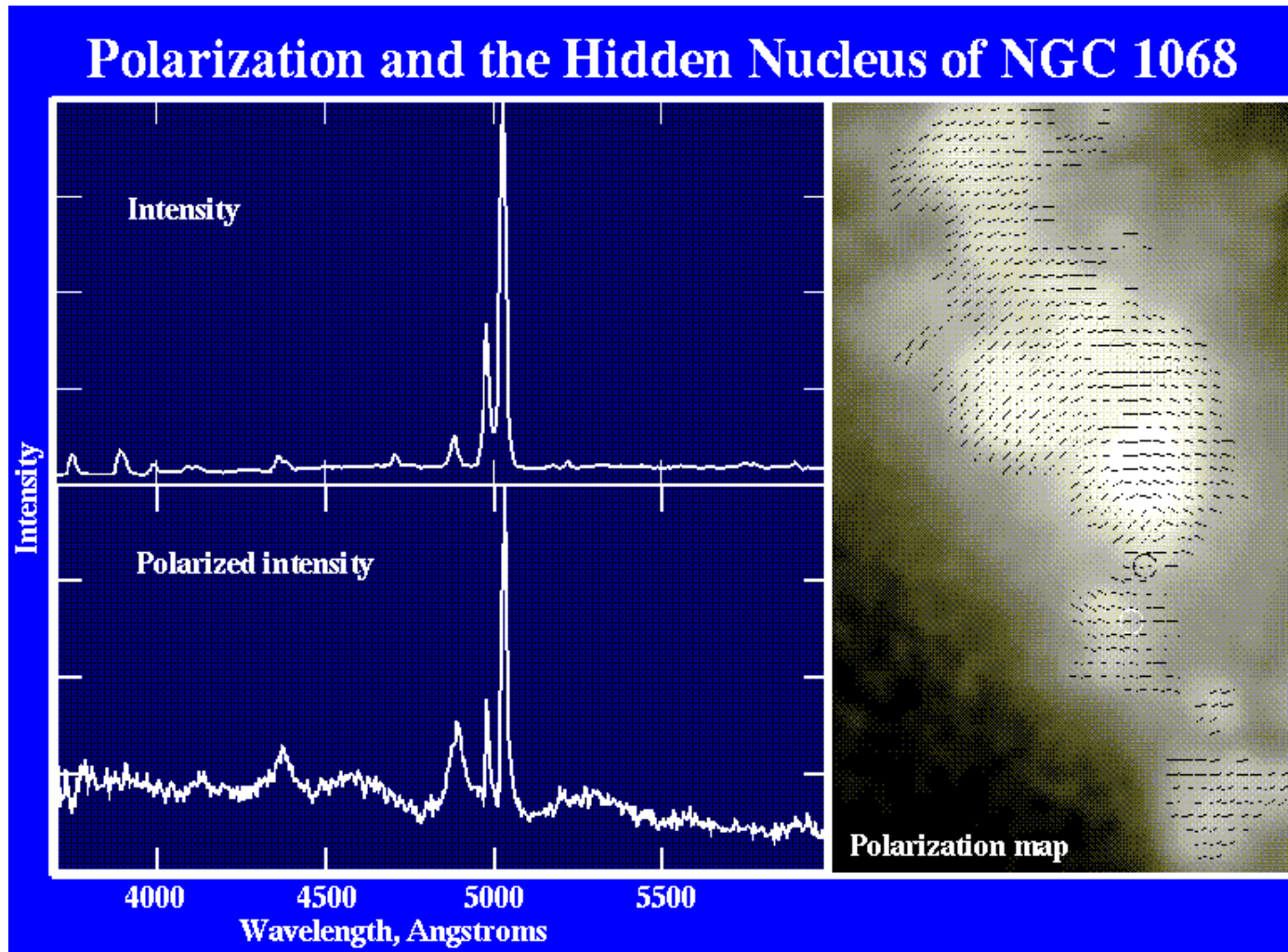


- Polarisation is fundamentally important to understanding the Universe
  - ▣ Provides insight into magnetic fields
- In optical astronomy, it's difficult to make polarimetric observations; in radio astronomy, they can be made easily, so why not use this to our advantage!
- (It's also very important to the Birds & the Bees, navigationally speaking, c.f. Rossel & Wehner, 1984)

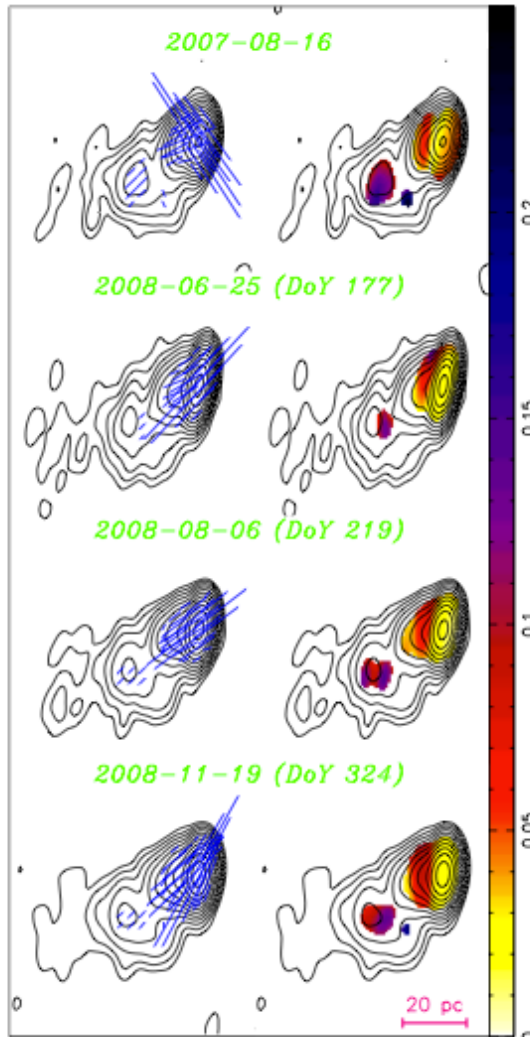
# Why Polarimetry?



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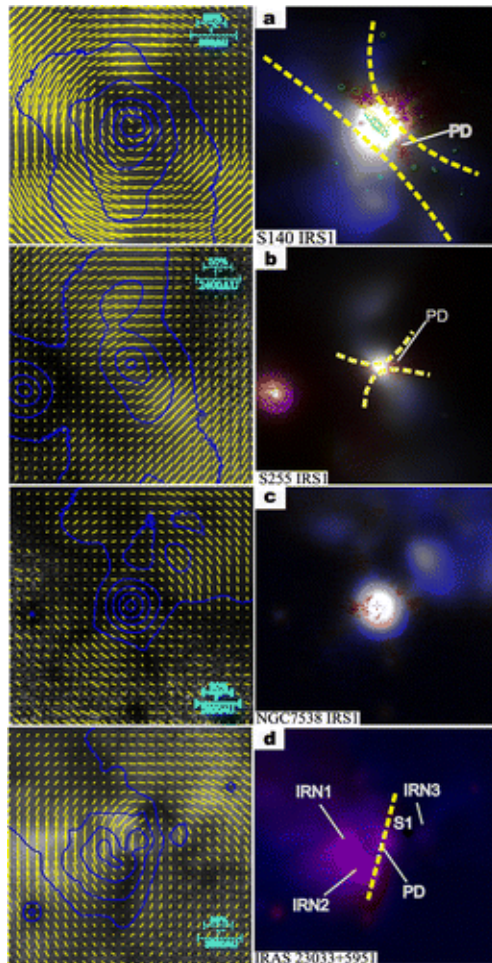


# Why Polarimetry?



- 15 GHz VLBA polarization observations of PKS 1502+106 (Abdo et al. 2010).
- E-field vectors shown on left, fractional linear polarization shown on right, contours are total intensity.

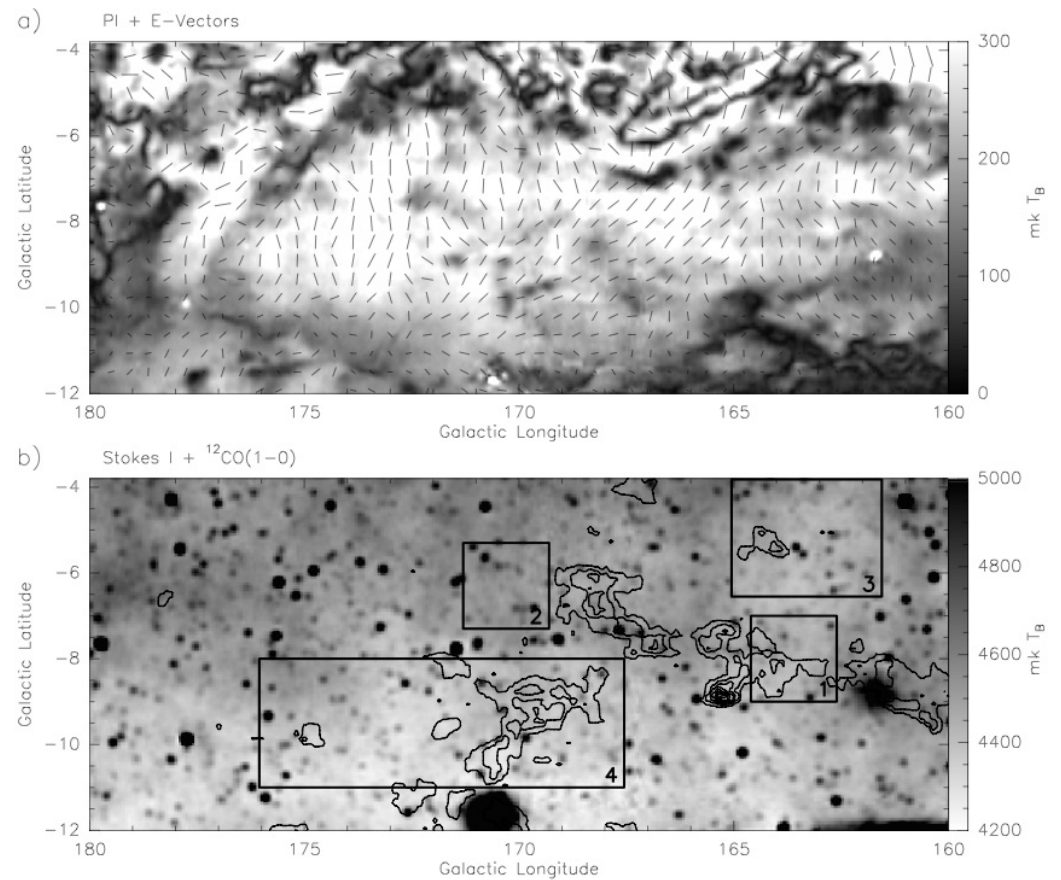
# Why Polarimetry?



- *Left panels:* Polarization degree images overlaid by polarization vectors (*yellow dashes*) and total intensity contour (*blue curves*).
- *Right panels:* Pseudocolor images composed of pure brightness images (*red*) and polarized brightness images (*blue*).

# Why Polarimetry

Faraday Screens have  
only a polarised  
structure, no standard  
(Stokes I) emission!



Haverkorn et al.

# What is Polarisation?

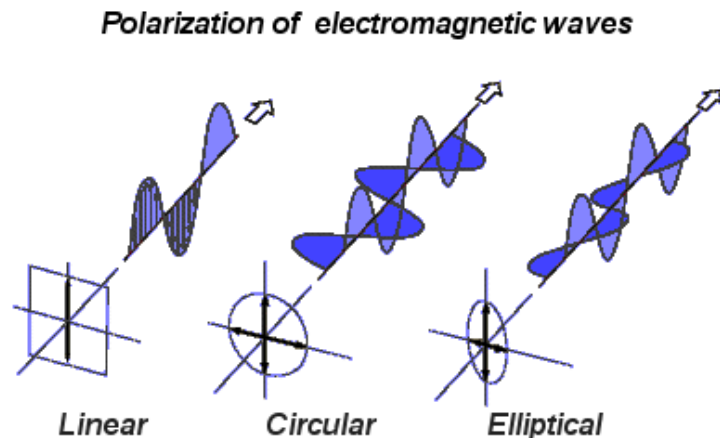


# What is Polarimetry & Polarisation?



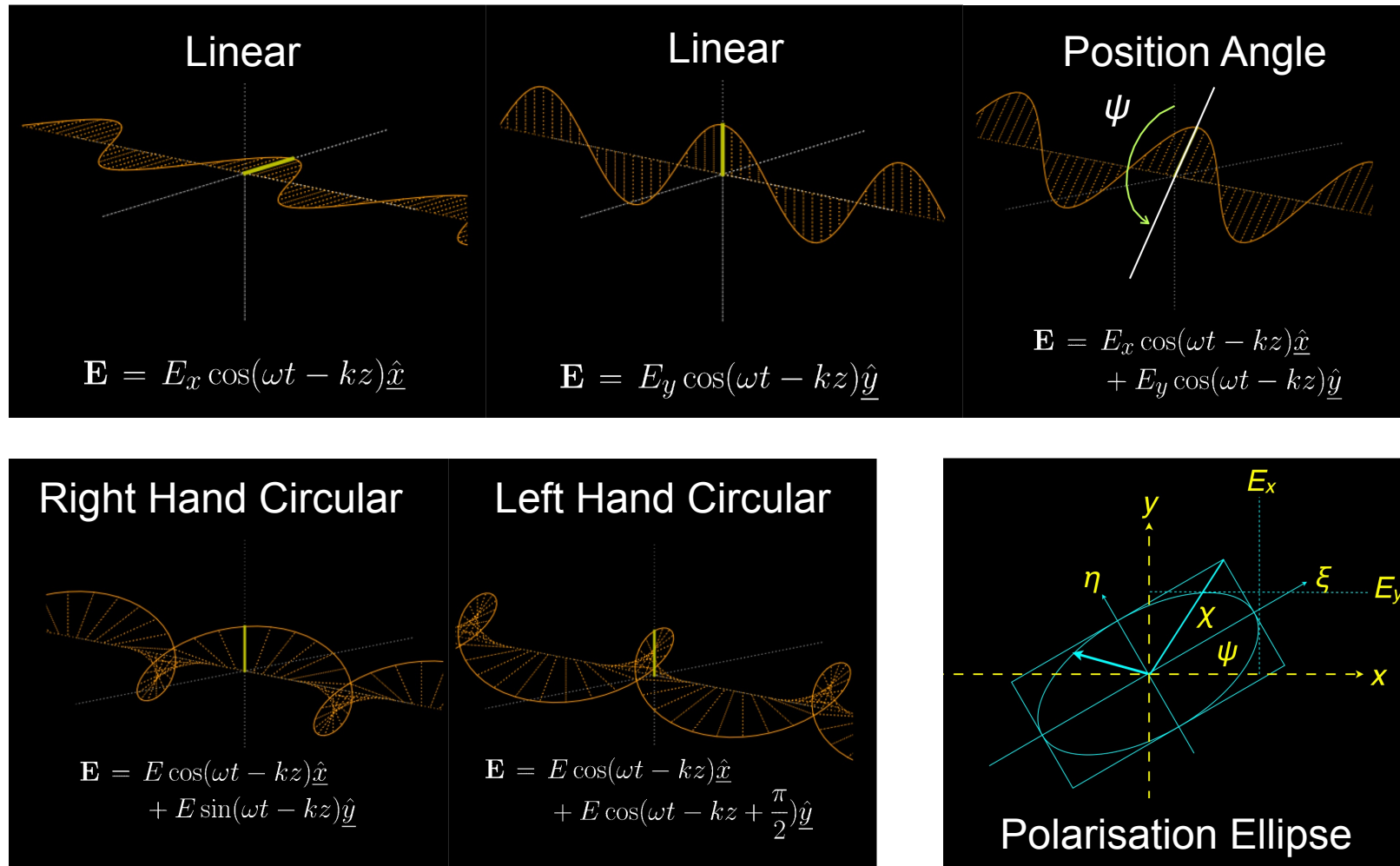
- ❑ Polarisation is the behaviour of the electric field with time.
- ❑ To simplify things we will start by considering monochromatic radiation.
- ❑ Astrophysical processes like synchrotron radiation can emit partially polarised emission, but *never* fully polarised.
- ❑ Interstellar matter can polarise random background emission or de-polarise polarised background emission.

# What is Polarisation?



- Linear: orthogonal components in phase with constant ratio of strengths giving constant direction of electric vector.
- Circular: orthogonal components  $90^\circ$  out of phase with equal amplitudes – electric vector traces circle.

# What is Polarisation?



# What is Polarisation?

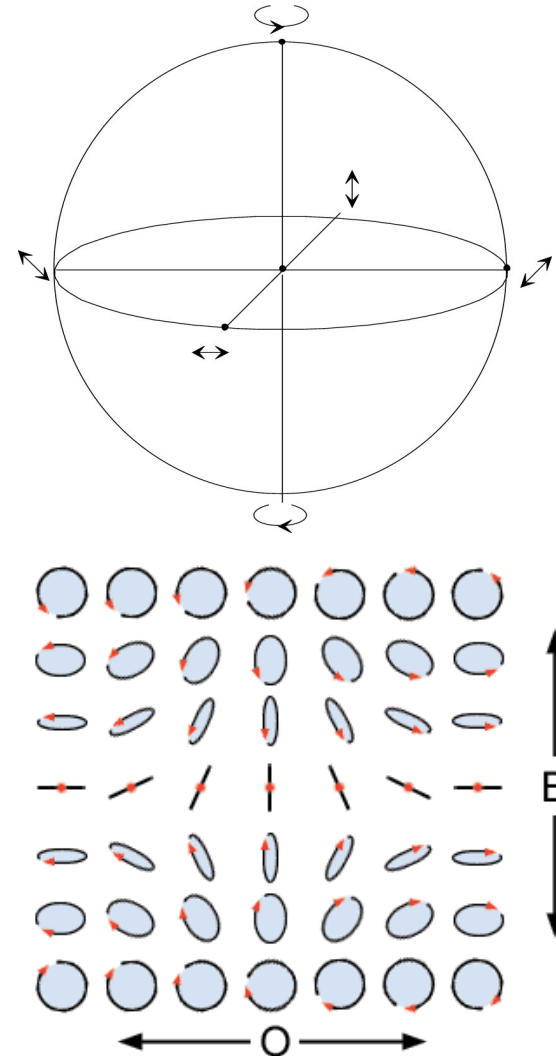
- Linearly polarised wave can be decomposed into two opposite handed circular waves.
- Conversely a circularly polarized wave can be decomposed into two orthogonal linear waves.
- Sum of two circular waves of unequal amplitude is elliptical.
- Sum of two orthogonal linears with a phase difference of between 0 and  $\pi/2$  is also elliptical.



# Jules Henri Poincaré

..and his sphere

- the spherical surface occupied by completely polarised states in the space of the vector
- Poles represent circular polarisations
  - ▣ Upper-hemisphere LHCP
  - ▣ Lower-hemisphere RHCP
- Equator represents linear polarisations with longitude representing tilt angle
- Latitude represents axial ratio



Now for the maths





# Robert Clarke Jones

..and his vectors

- Jones calculus is a matrix-based means of relating observed to incident fields.
- Vectors describe incident radiation and matrices the response of the instrument.
- The Jones Vector: 
$$\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}$$
- Examples:
  - ▣ Linearly (x-direction) polarised wave: 
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
  - ▣ Left-Hand Circularly polarised wave: 
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

# Robert Clarke Jones



..and his matrices

- Effect of instrument described by 2x2 matrix:

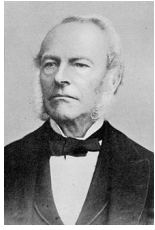
$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_0 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

- Simple Examples:

- Linear polariser:  $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

- Left-Hand Circular polariser:  $\frac{1}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$

- In practice matrix elements complex.
- Important: Only applicable to **completely** polarised waves.



# Sir George Gabriel Stokes

..and his parameters

- Defined by George in 1852
- Adopted for astronomy by Chandrasehkar in 1947.
- Can be used for partially polarised radiation.
- Not a vector quantity! Deals with power instead of electric field amplitudes.
- The correlator can produce ALL Stokes parameters simultaneously (not so easy in optical astronomy!)

# Stokes Parameters

- I – total intensity and sum of any two orthogonal polarisations
- Q and U – completely specify linear polarisation
- V – completely specifies circular polarisation

$$I = E_{0x}^2 + E_{0y}^2$$

$$Q = E_{0x}^2 - E_{0y}^2$$

$$U = 2E_{0x}E_{0y} \cos \delta$$

$$V = 2E_{0x}E_{0y} \sin \delta$$

$$I = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle$$

$$Q = \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle$$

$$U = \langle E_x E_y^* \rangle + \langle E_x^* E_y \rangle$$

$$V = i \left( \langle E_x E_y^* \rangle - \langle E_x^* E_y \rangle \right)$$

*(For linear feeds)*

# Fractional Polarisation

- The total linearly polarised intensity is defined as:

$$P = \sqrt{U^2 + Q^2} \quad \text{[for native linear feed]}$$

- A linearly polarised source will have an intrinsic position angle on the sky that is given by:

$$\Theta = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right) \quad \text{[for native linear feed]}$$

- The circular polarisation will be just Stokes V.
- Stokes parameters often presented as percentages of the total intensity.
- Since radio sources are *never* fully polarised, then the fractional linear and circular polarisation will always be  $< 1$

How do we measure it?



# How do we measure it?



- Stokes parameters are the auto-correlation & cross-correlation products returned from the correlator, but input to the correlator can come from different feed types.
- Feeds normally designed to approximate pure linear or circular (known as ‘native linear’ or ‘native circular’ )
  - ▣ Linear Feeds – intrinsically accurate & provide true linear response.
  - ▣ Circular Feeds – less accurate & frequency dependent response.

# How do we measure it?

- Output of native linear feed is  $E_x$  and  $E_y$  field voltages, so:
  - I from  $XX+YY$
  - Q from  $XX-YY$
- Native circular adds  $90^\circ$  phase to  $X$ , so:
  - I from  $XX+YY$  (or  $LL+RR$ )
  - V from  $XX-YY$  (or  $LL-RR$ )

# Stokes Parameters

- For circular feeds  $Q$  and  $V$  swap round..

Linear

$$XX = I + Q$$

$$YY = I - Q$$

$$XY = U + iV$$

$$YX = U - iV$$

Circular

$$RR = I + V$$

$$LL = I - V$$

$$RL = Q + iU$$

$$LR = Q - iU$$

# But is it really that simple?

- Do we just plug in our computer and get  $\{I, Q, U, V\}$  out of the correlator?
- No, there are leakages!
  - ▣ The total intensity can leak into the polarised components (I into  $\{Q, U, V\}$ ).
  - ▣ The linear polarisation can leak into the circular ( $\{Q, U\}$  into V).
  - ▣ ... and all combinations and permutations are allowed!
- Without correcting for leakage, you're not going to get proper Stokes parameters!



# Hans Mueller

..and his matrix

- The leakage of each polarisation into the other can be measured and quantified in a 4x4 matrix first proposed by Mueller in 1943.

$$M = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{iI} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix}$$

# The Mueller Matrix

Correlator Output

Incoming Radiation

$$\begin{bmatrix} XX + YY \\ XX - YY \\ XY \\ YX \end{bmatrix} = M \cdot \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

# Example (simple) Mueller Matrices

- If feeds were perfect:

- ▣ Dual linear feed:  $M$  is unitary

- ▣ Dual linear feed rotated  $45^\circ$ :  $Q$  and  $U$  interchange and sign change for rotation:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- ▣ A dual linear feed rotated  $90^\circ$ : signs of  $Q$  and  $U$  reversed:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- As Alt-Az telescope tracks source, feed rotates on sky by the parallactic angle (PA):

$$M_{sky} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2PA & \sin 2PA & 0 \\ 0 & -\sin 2PA & \cos 2PA & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# The more general Mueller Matrix

- For a (realistic) dual linear feed:

$$M = \begin{bmatrix} 1 & \left(-2\varepsilon \sin\phi \sin 2\alpha + \frac{\Delta G}{2} \cos 2\alpha\right) & 2\varepsilon \cos\phi & \left(2\varepsilon \sin\phi \cos 2\alpha + \frac{\Delta G}{2} \sin 2\alpha\right) \\ \frac{\Delta G}{2} & \cos 2\alpha & 0 & \sin 2\alpha \\ 2\varepsilon \cos(\phi + \varphi) & \sin 2\alpha \sin\varphi & \cos\varphi & -\cos 2\alpha \sin\varphi \\ 2\varepsilon \sin(\phi + \varphi) & -\sin 2\alpha \cos\varphi & \sin\varphi & \cos 2\alpha \cos\varphi \end{bmatrix}$$

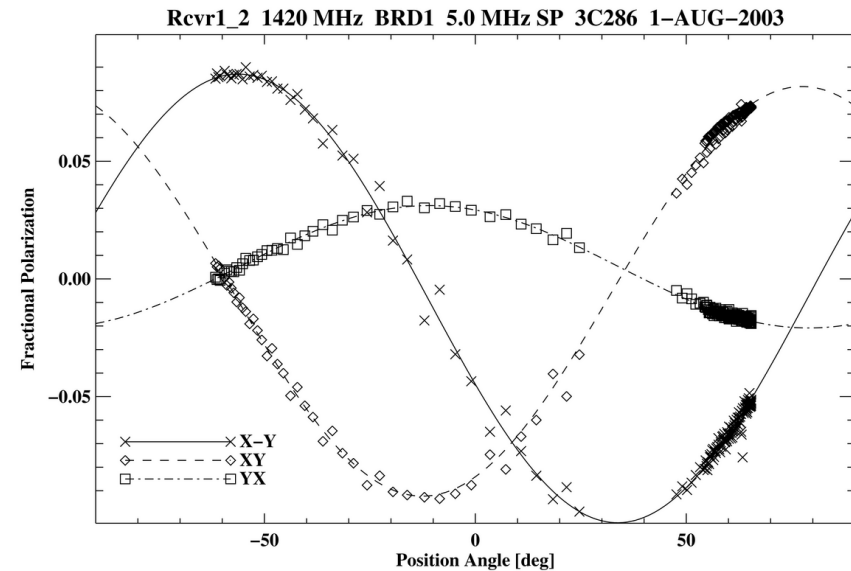
- The Mueller matrix has 16 elements, but **ONLY 7 INDEPENDENT PARAMETERS**. The matrix elements are not all independent.

# Calculating the Mueller Matrix

- For a perfect system, as we track a polarised source across the sky the parallactic angle changes and this should produce:
  - ▣ For XX-YY:  $\cos 2(\text{PA}_{\text{az}} + \text{PA}_{\text{src}})$ , centred at zero.
  - ▣ For XY:  $\sin 2(\text{PA}_{\text{az}} + \text{PA}_{\text{src}})$ , centred at zero
  - ▣ For YX: zero (most sources have zero circular polarisation)

# Calculating the Mueller Matrix

□ But, what we find is:



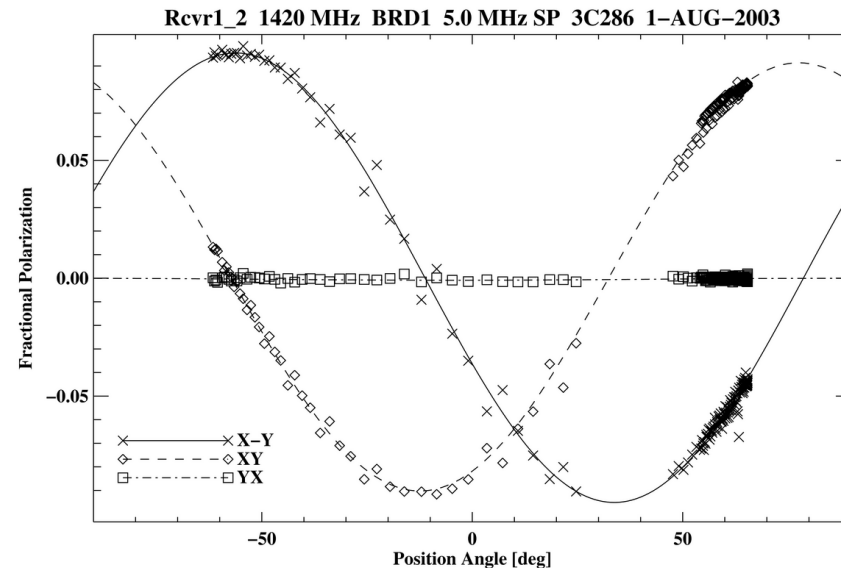
$\Delta\text{TAG} = -0.017 \pm 0.006$   
 $\text{PSI} = -16.7 \pm 1.8$   
 $\text{ALPHA} = +0.4 \pm 0.9$   
 $\text{EPSILON} = +0.004 \pm 0.001$   
 $\text{PHI} = +152.3 \pm 23.0$   
 $Q_{\text{SRC}} = -0.037 \pm 0.002$   
 $U_{\text{SRC}} = -0.085 \pm 0.002$   
 $\text{POL}_{\text{SRC}} = +0.093 \pm 0.002$   
 $\text{PA}_{\text{SRC}} (**\text{UNCORRECTED FOR } M_{\text{ASTRO}} **) = -56.81 \pm 0.64$   
 NR GOOD POINTS: X-Y = 149 XY = 158 YX = 160 / 160

Mueller Matrix:

1.0000	-0.0083	-0.0065	0.0033
-0.0083	0.9999	0.0001	0.0145
-0.0053	-0.0042	0.9581	0.2865
0.0052	-0.0139	-0.2865	0.9580

# Calculating the Mueller Matrix

- Which enables the matrix to be calculated and the observations corrected to give what we expect:



DELTAG =  $0.000 \pm 0.006$   
 PSI =  $+0.3 \pm 1.8$   
 ALPHA =  $-0.0 \pm 0.9$   
 EPSILON =  $+0.000 \pm 0.001$   
 PHI =  $-39.8 \pm 235.4$   
 $Q_{\text{SRC}} = -0.037 \pm 0.002$   
 $U_{\text{SRC}} = -0.085 \pm 0.002$   
 $POL_{\text{SRC}} = +0.093 \pm 0.002$   
 $PA_{\text{SRC}} (**\text{UNCORRECTED FOR } M_{\text{ASTRO}} **) = -56.82 \pm 0.64$   
 NR GOOD POINTS: X-Y = 149 XY = 158 YX = 158 / 160

Mueller Matrix:

1.0000	0.0002	0.0006	-0.0005
0.0002	1.0000	0.0000	-0.0012
0.0006	-0.0000	1.0000	-0.0044
-0.0005	0.0012	0.0044	1.0000

# Putting it all together



- In the end what we are trying to do is relate products from our correlator to the intrinsic polarized radiation from the source.
- So we need to correct the raw correlator outputs for
  - ▣ Imperfections in the receiver (leakages).
  - ▣ The orientation of the receiver with respect to the telescope structure (maybe).
  - ▣ The changing parallactic angle (maybe).
  - ▣ Any measured propagation related polarization effects (e.g. Faraday rotation).



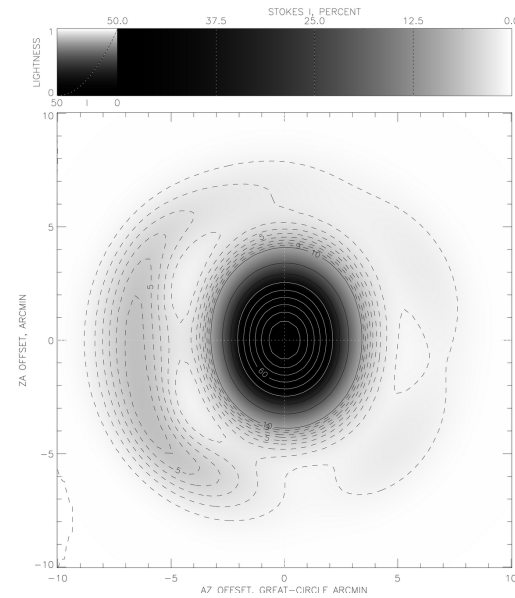
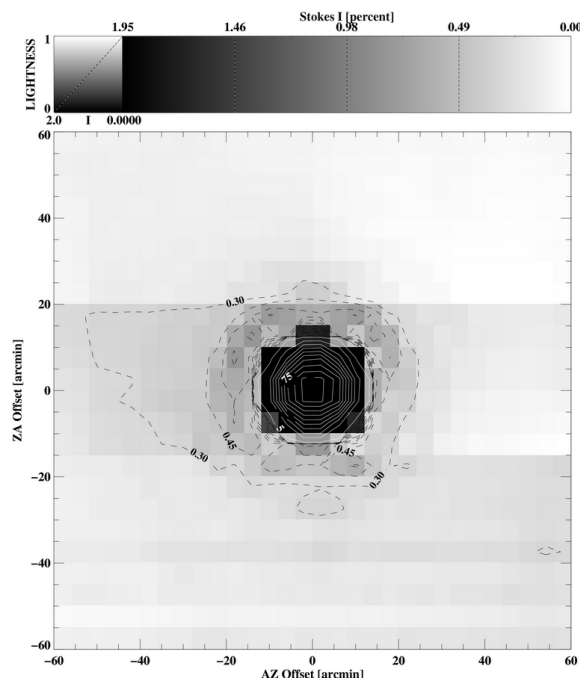
# Beam Effects

# Beam Effects



- For point sources, all of the previous is fine.
- What if the source you're looking at is extended compared to the telescope beam?
- There are instrumental beam effects that can confuse the measurement of extended polarised signals. They are...
  - ▣ Squint
  - ▣ Squash

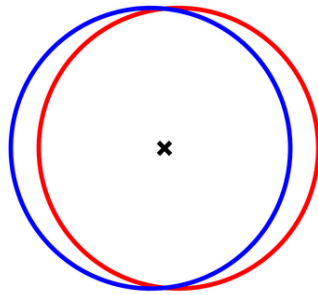
# Stokes I response



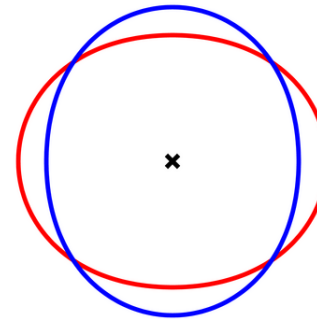
Heiles et al. 2001

# Beam Squint & Squash

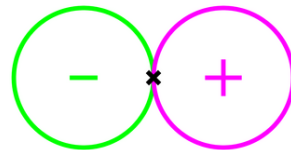
**BEAM SQUINT**  
RHCP  
LHCP



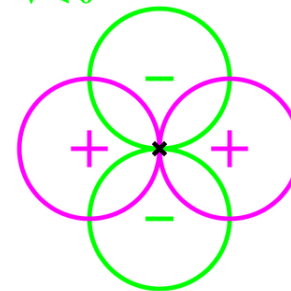
**BEAM SQUASH**  
RHCP  
LHCP



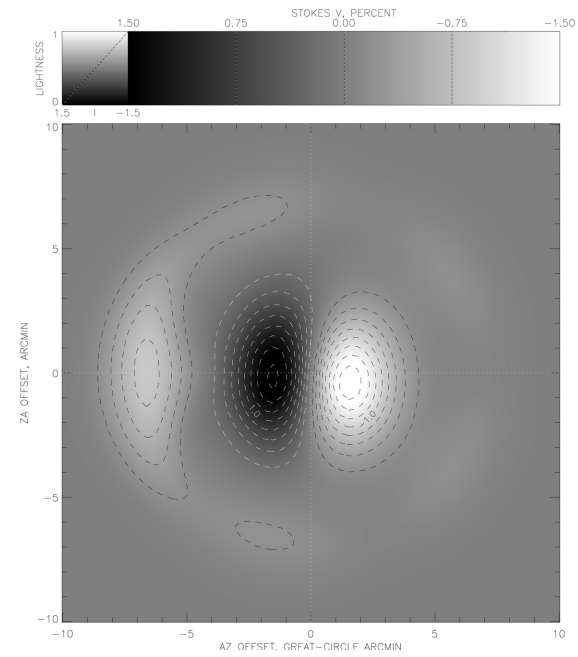
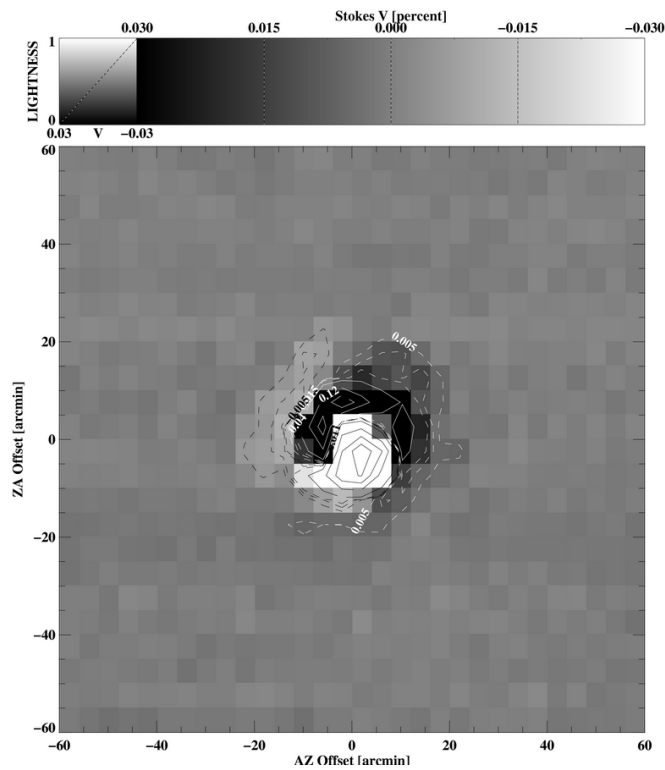
**V = RHCP - LHCP**  
V > 0  
V < 0



**V = RHCP - LHCP**  
V > 0  
V < 0

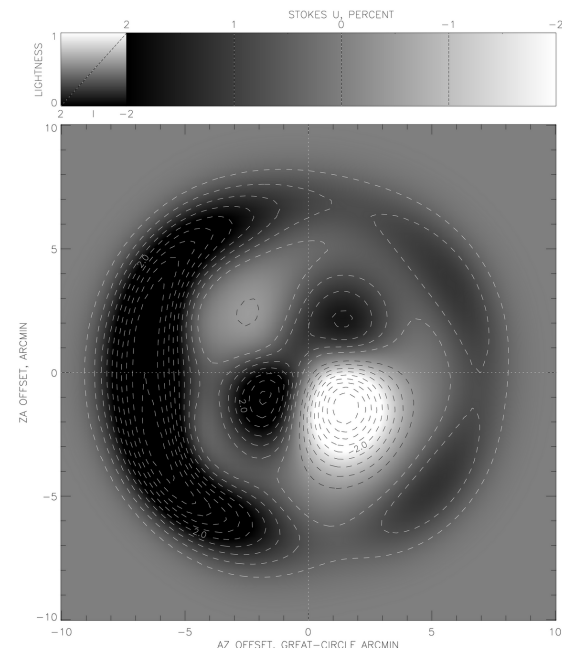
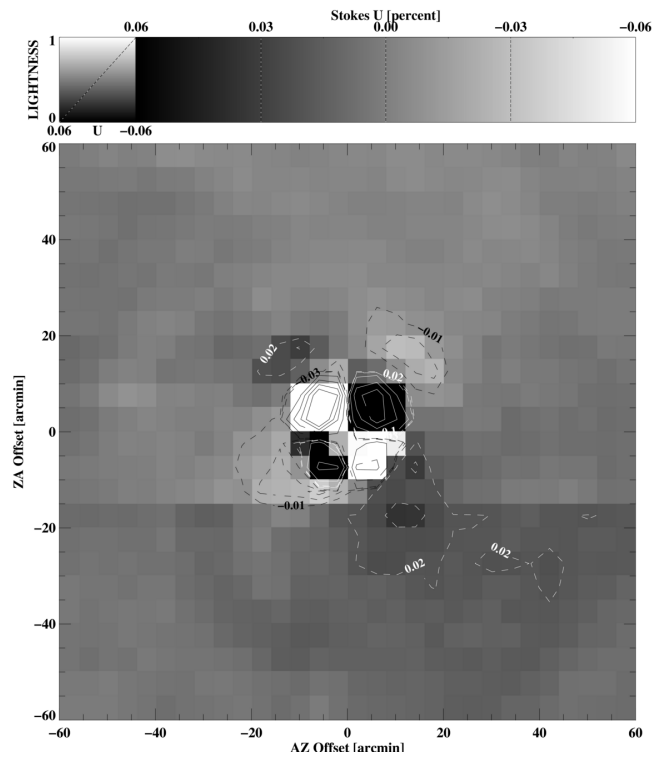


# Squint in action



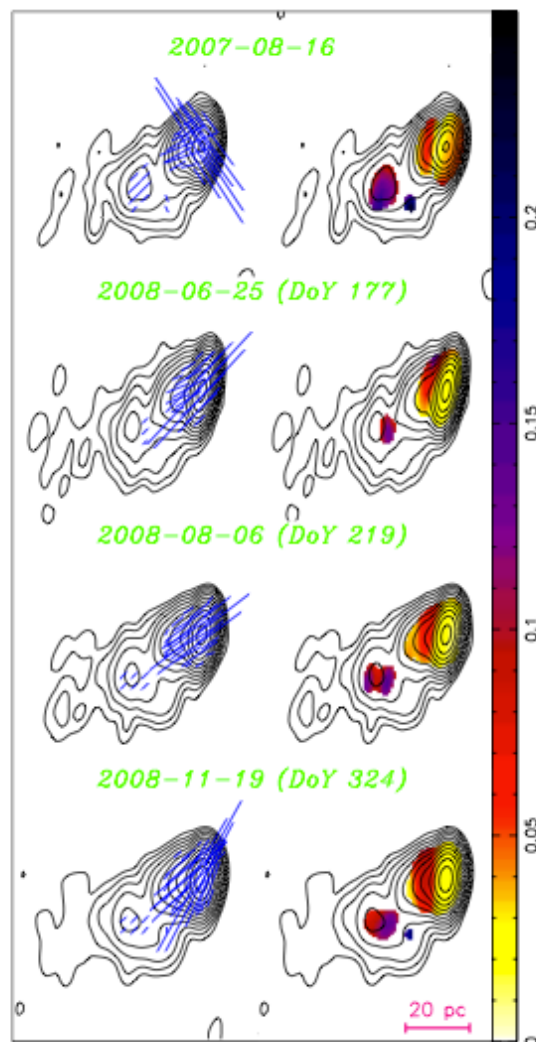
Heiles et al. 2001

# Squash in action



Heiles et al. 2001

# Depolarization!

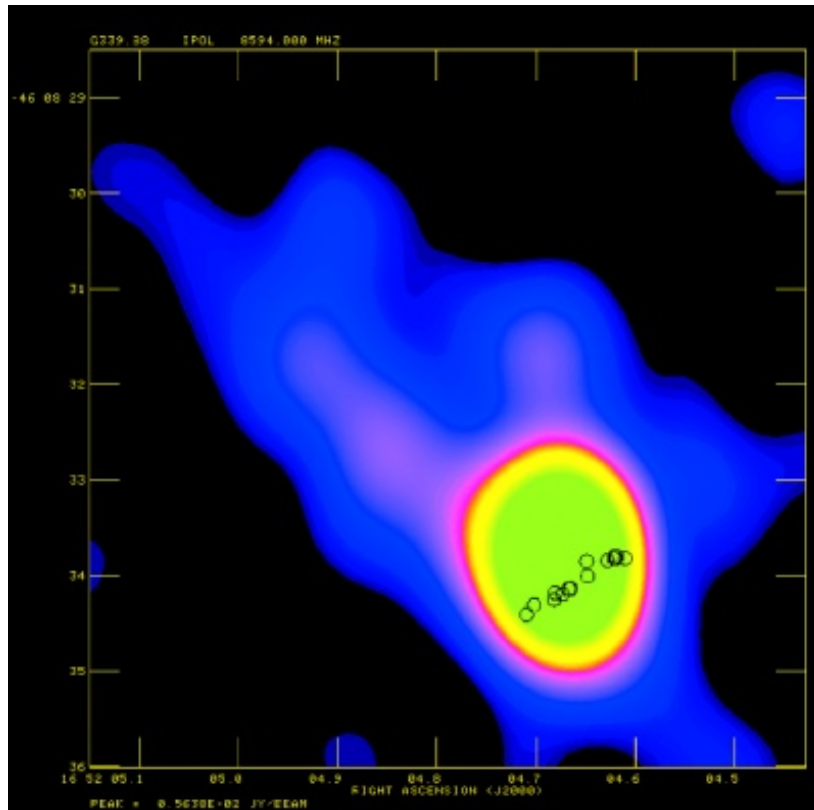


- 15 GHz VLBA polarization observations of PKS 1502+106 (Abdo et al. 2010).
- E-field vectors shown on left, fractional linear polarization shown on right, contours are total intensity.

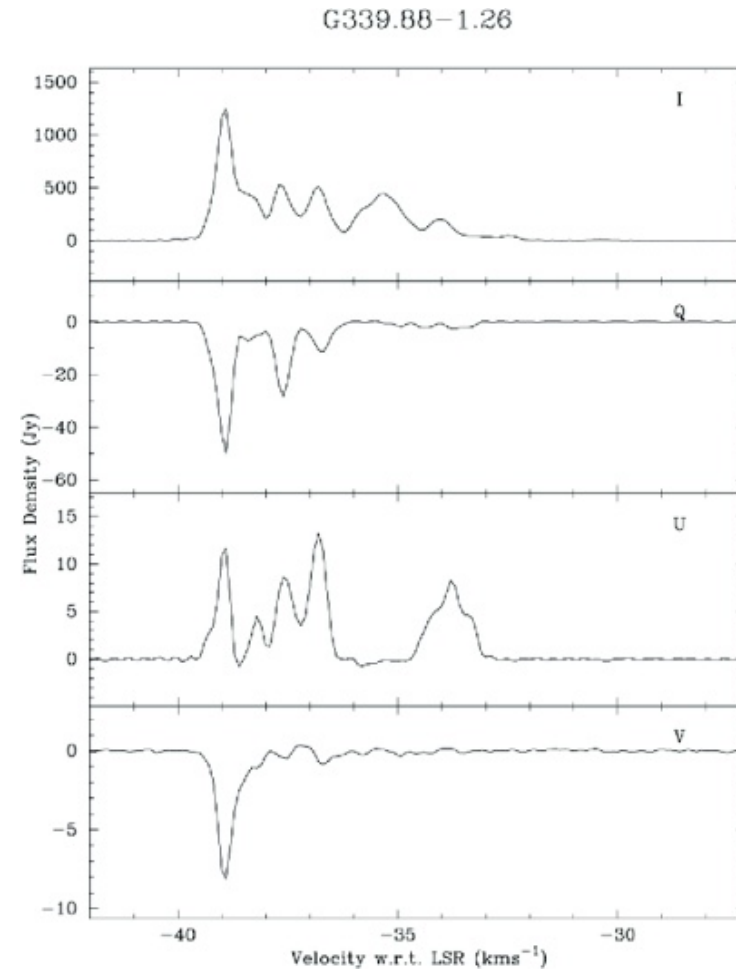
Here comes the science



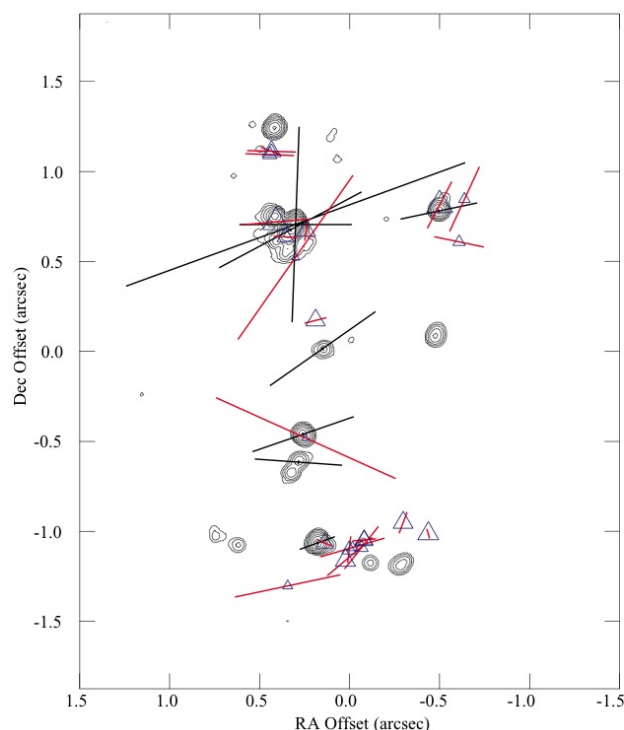
# Polarisation of Masers



Ellingsen (2002)



# Polarisation of Masers



**W3(OH)** Harvey-Smith & Cohen 2006, Vlemmings et al 2006.

- Filamentary maser structure.
- Linear polarisation up to 8%.
- Polarisation angles indicate north-south structure, and are consistent with OH.

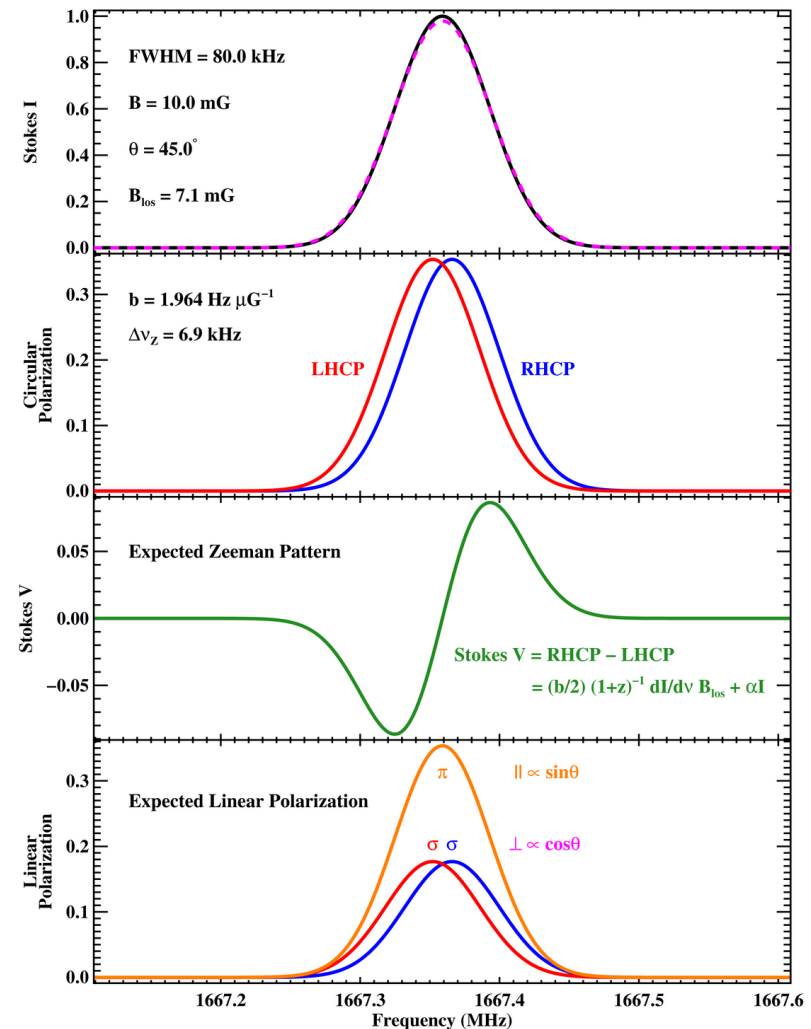
# Make like a banana:

## Zeeman Splitting

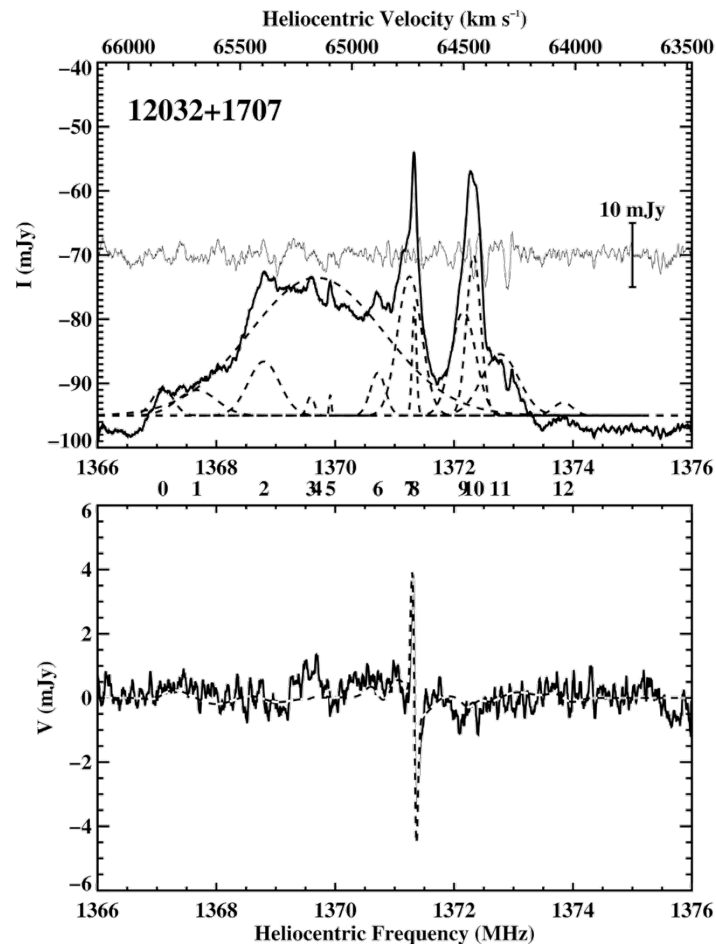


- Atoms & molecules in net magnetic moment will have their energy levels split in the presence of a magnetic field.
- Detected through frequency shift between right and left circularly polarised emission

$$V = RHCP - LHCP \propto B_{los}$$



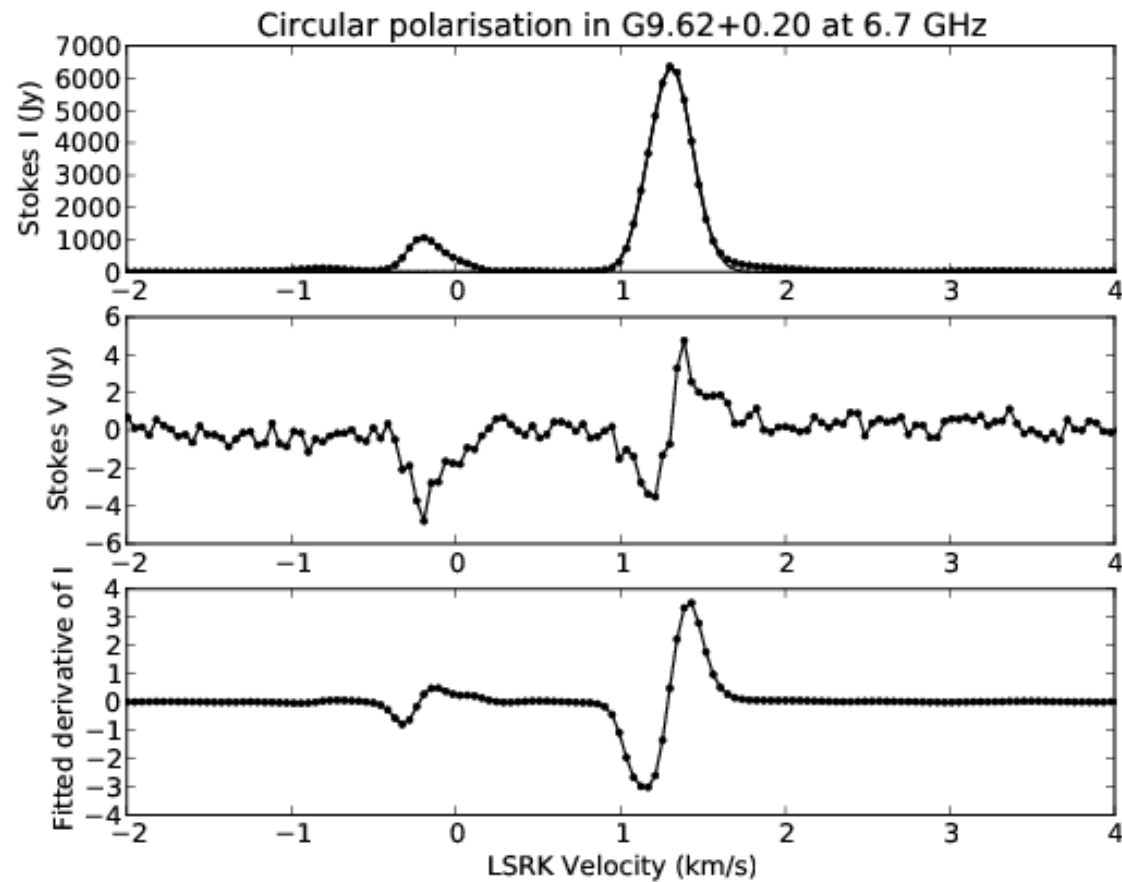
# Zeeman Splitting of Masers



- OH Gigamaser  
(luminosity greater than  $10^4$  solar luminosities)

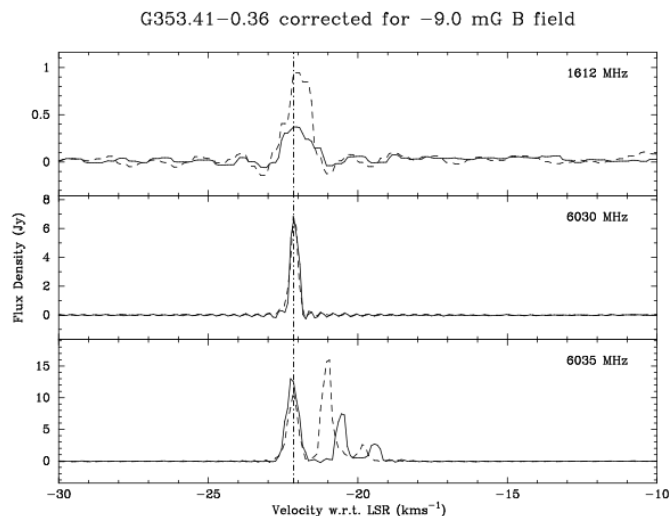
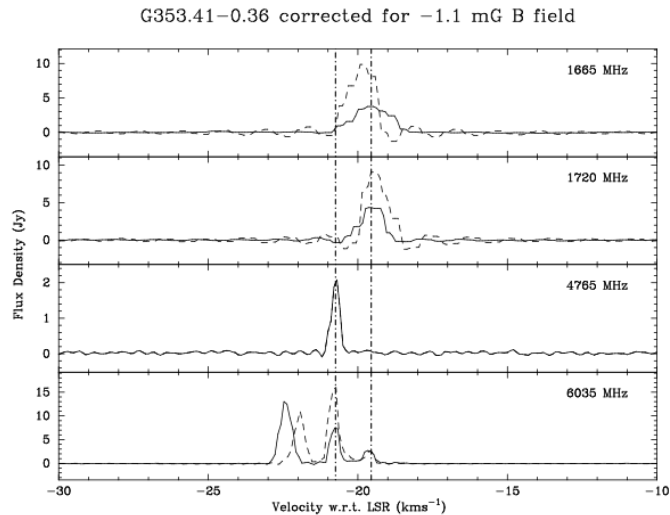


# Zeeman Splitting of Masers



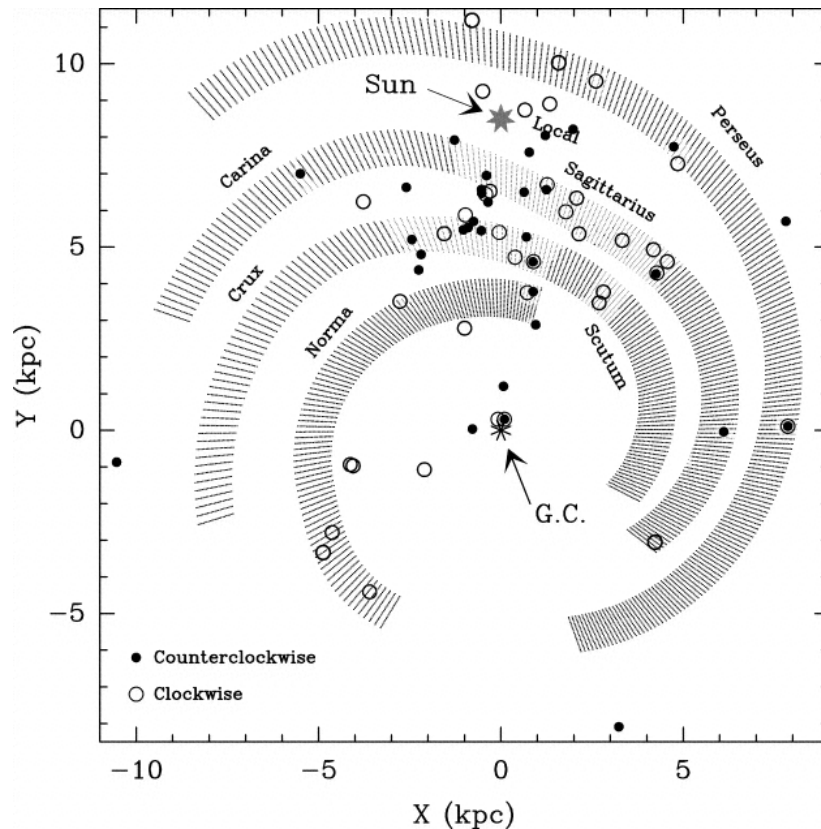
Zeeman splitting  
in a Galactic  
6.7 GHz  
methanol maser  
(Stack &  
Ellingsen 2011).

# Zeeman Splitting of Masers



- ATCA Zeeman splitting measured simultaneously in 6 different OH maser transitions shows variations of nearly an order of magnitude on sub-pc scales in high-mass star formation regions.

# Zeeman Splitting of Masers



Fish et al 2003

- Line-of-sight magnetic field directions deduced from OH maser Zeeman splitting.
- 74 star-forming regions:
  - ▣ 41 with an overall magnetic field oriented in a clockwise sense.
  - ▣ 33 with field oriented counterclockwise as viewed from above the Galactic center.
- Field consistency within 2-kpc of Sun.



In summary..

# Summary



- Polarisation in radio astronomy very important to improving our knowledge & understanding.
- Can describe polarisation with the Polarisation Ellipse and the Poincare Sphere.
- Dr. Jones offers a vector representation for ideal cases of completely polarised emission.
- Mueller and his matrices are the best option for real situations.
- There are Linear and Circular feed types, must account for which you are using.
- **Understanding the polarisation properties of your dish is fundamental to successful observations!**
- (Masers offer exciting science opportunities!)

# Useful References

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- Born and Wolf: 'Principle of Optics', Chapters 1 and 10