INTERFEROMETERY II

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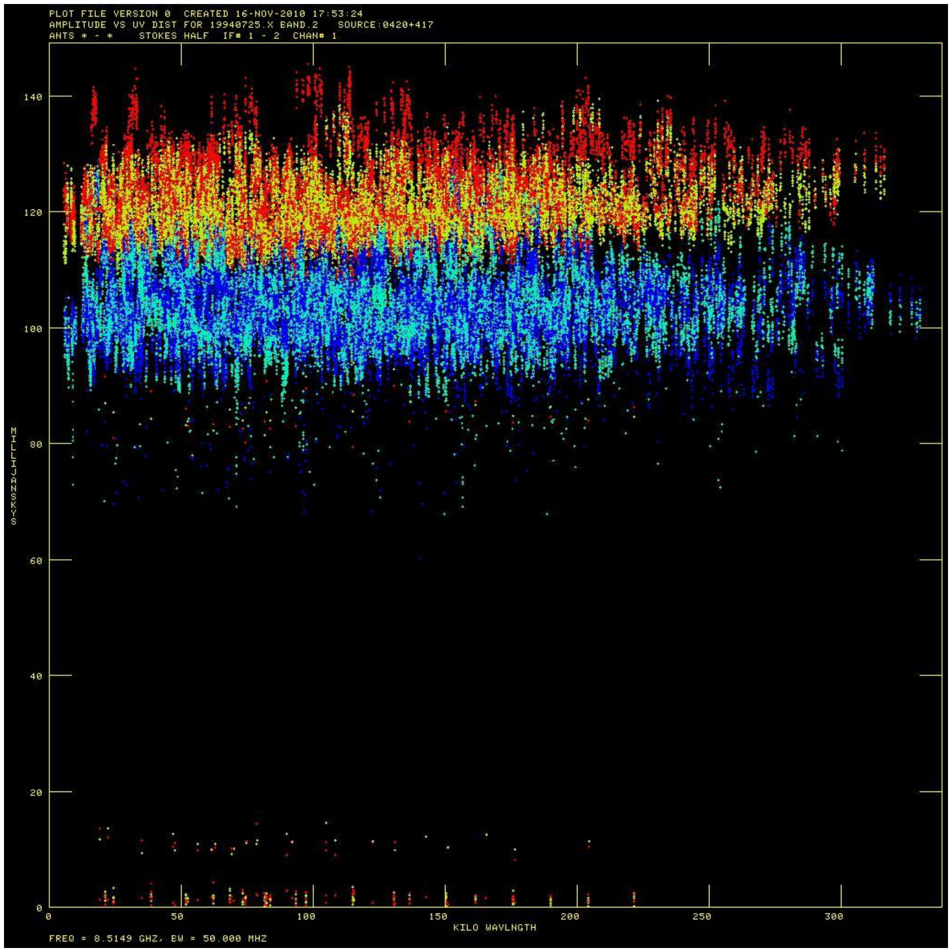
NOW THAT I'VE GOT THE DATA….



Synthesized beams

Calibration

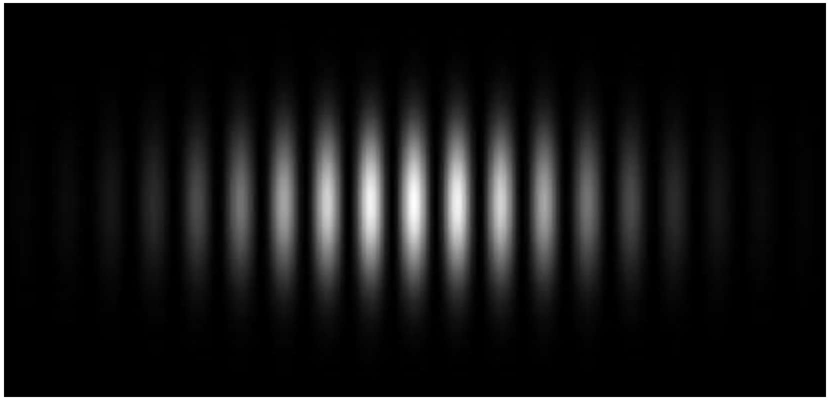
Making maps

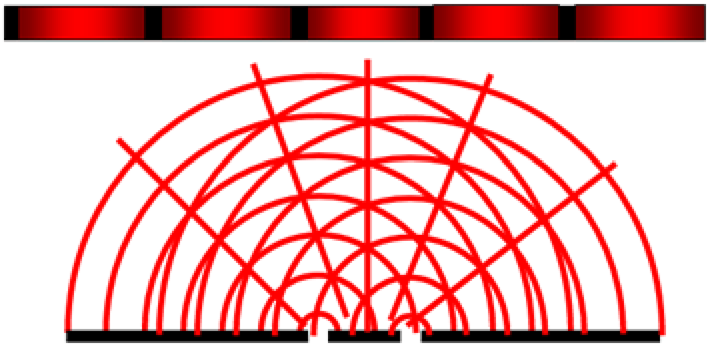
Deconvolution

Self-calibration

INTERFEROMETERS AS A TWO-SLIT EXPERIMENT

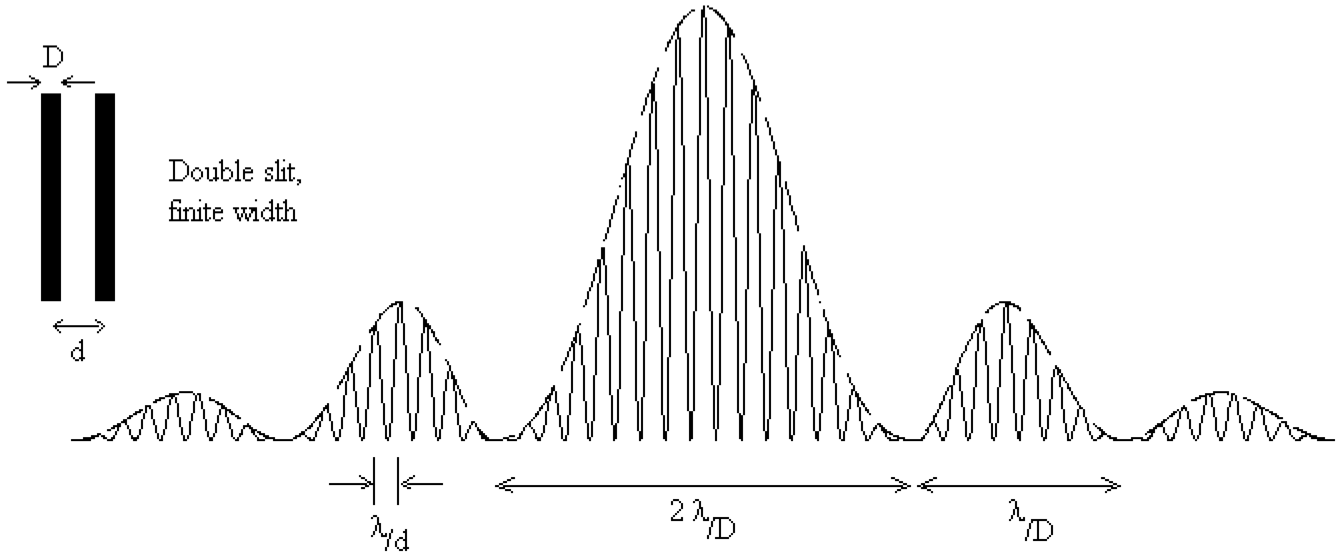
Recall that the reception pattern is the same as the transmission pattern.
The two-element radio interferometer then acts just like the two slit transmission pattern---FRINGES.





(resourcefulphysics.org)

THE RECEPTION PATTERN



A Sinusoid modified by the diffraction pattern of the slits.

Fringe spacing given by λ/d (d = baseline length)

Diffraction lobes given by λ/D (D = dish diameter)

An interferometer measures Fourier components of the brightness distribution, modified by the single dish reception pattern, or the "primary beam" pattern.

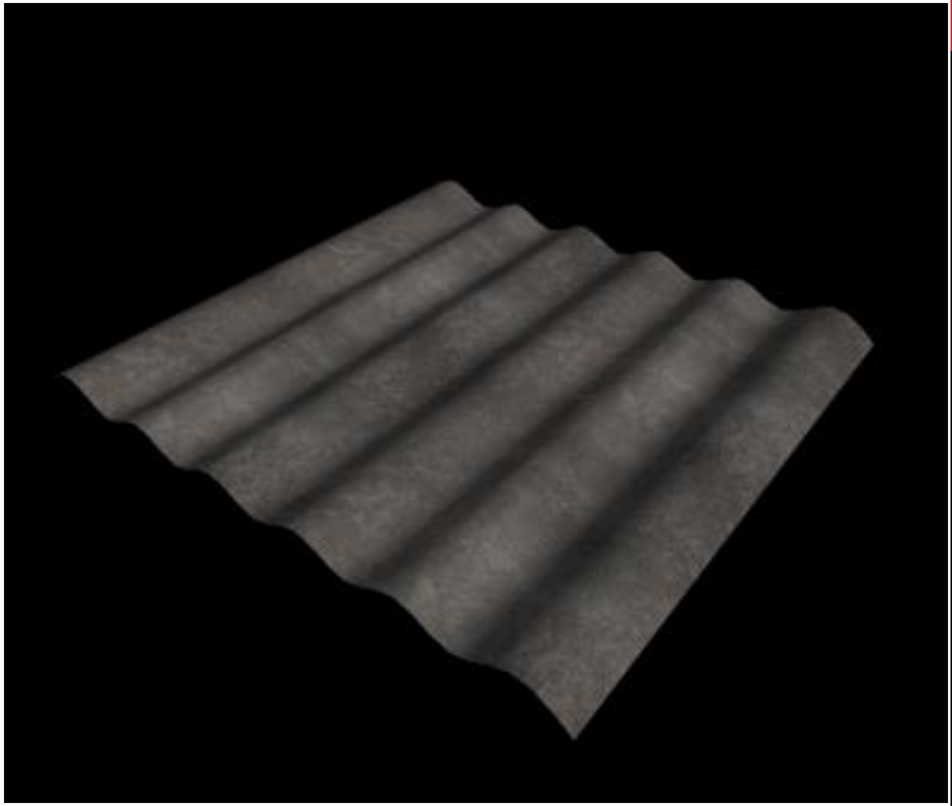
U,V DATA AND BRIGHTNESS

Each u,v point therefore measures a particular Fourier component of the brightness distribution B(α,δ).

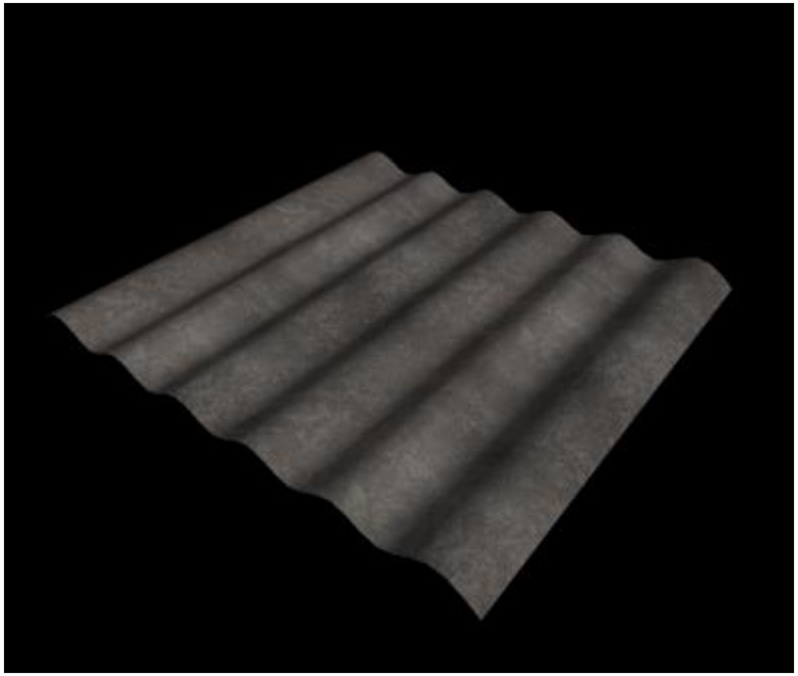
The amplitude says how bright it is.

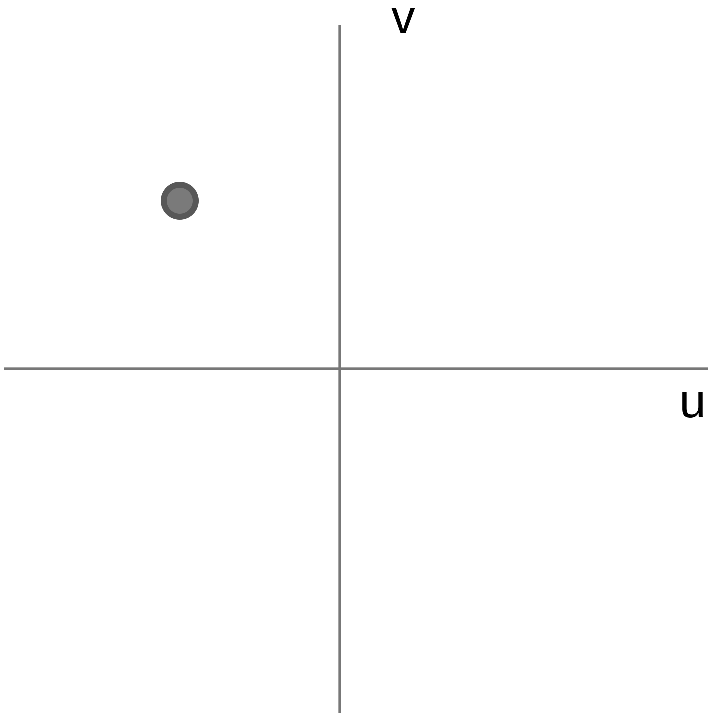
The phase gives the positional information.

The baseline gives the spatial frequency.

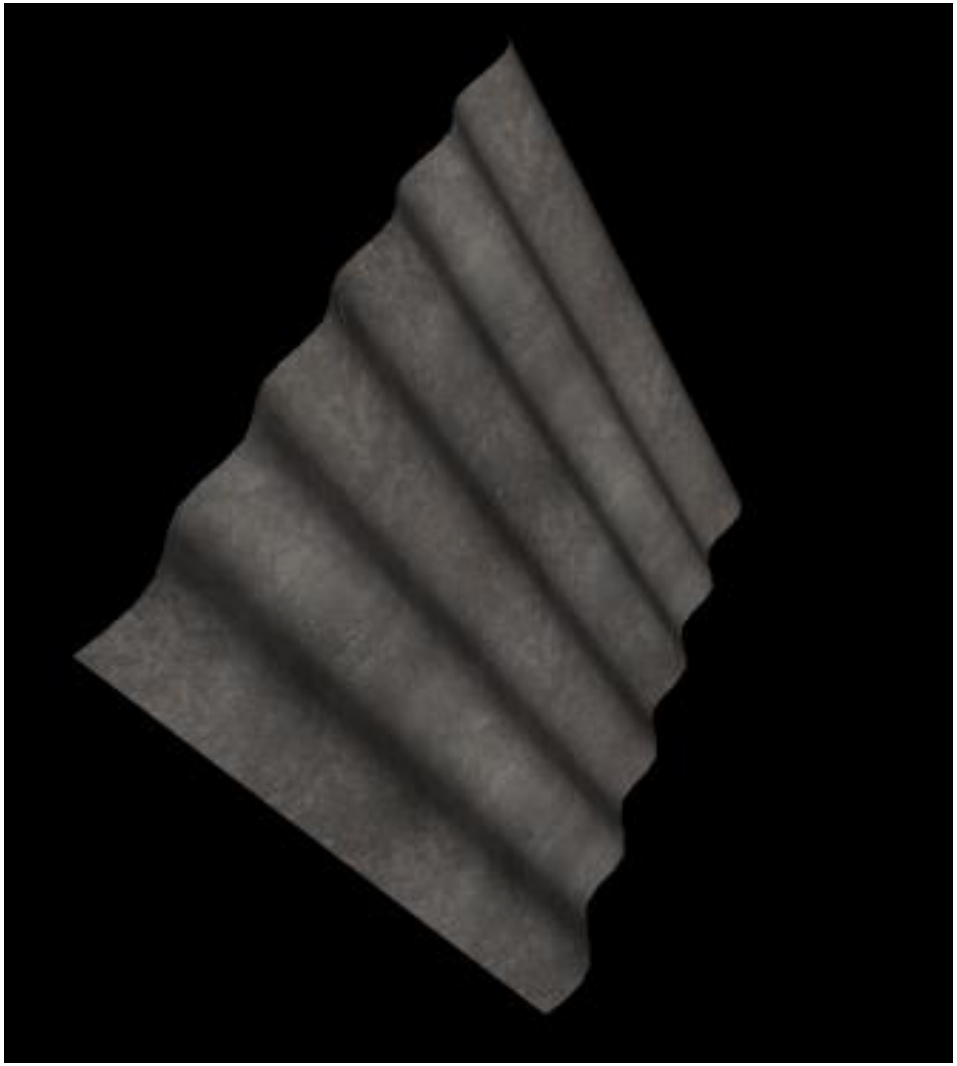


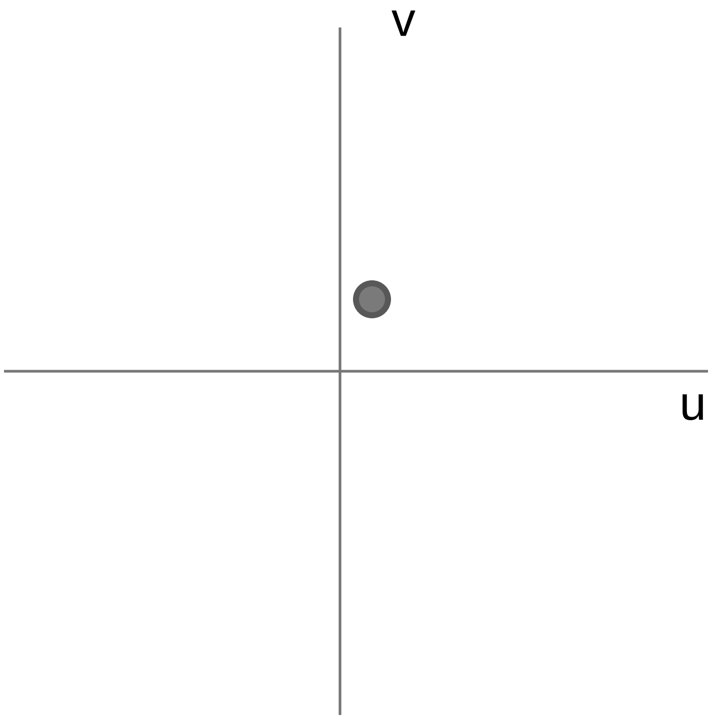
TIN ROOFS AND U,V DATA





TIN ROOFS AND U,V DATA





The final image is the sum of all of the corrugations.

SYNTHESIZED BEAM

The response of an interferometer to a point source is the sum of all of the corrugations with zero phase at the phase center.

Mathematically, it is Fourier Transform of the Sampling Function.

$$b(\theta, \phi) = FT\big(S(u, v)\big)$$

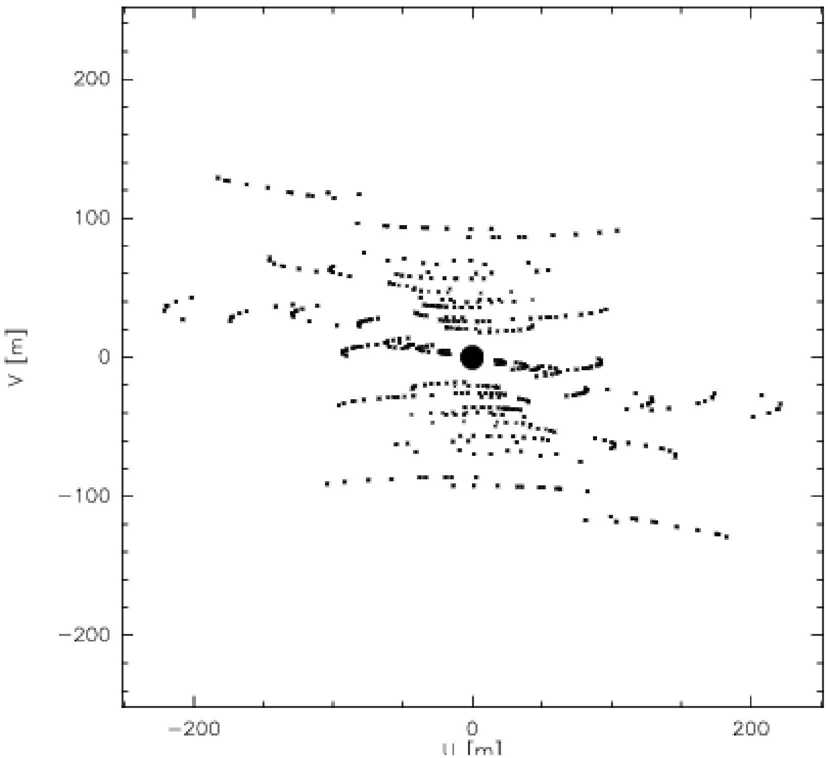
where $S(u, v) = \sum_i \delta(u_i, v_i)$ is the sampling function.

$b(\theta, \phi)$ is called the **Synthesized Beam**.

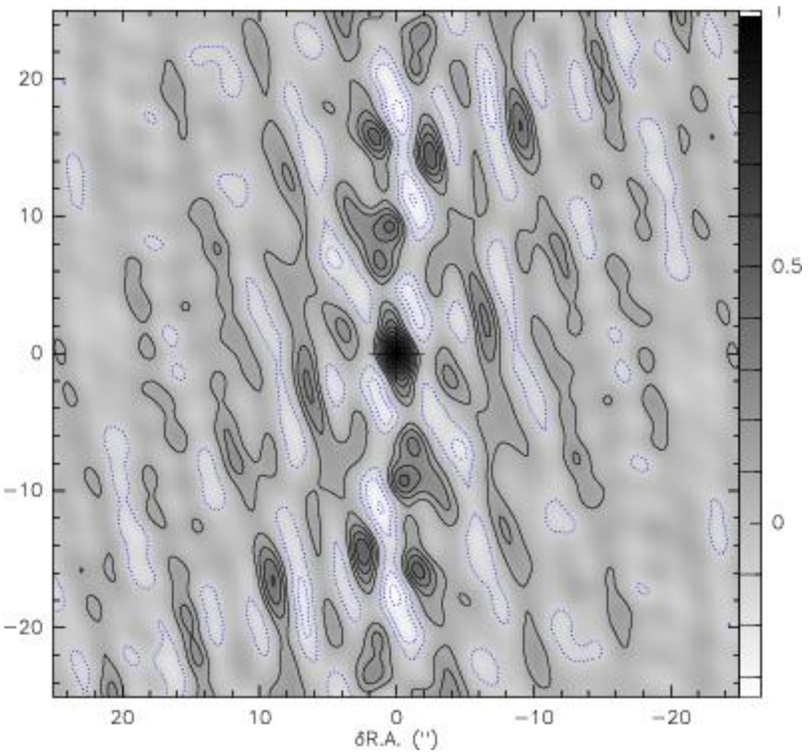
Its size is determined by the maximum baseline.

Its shape is determined by the u,v sampling.

AN EXAMPLE

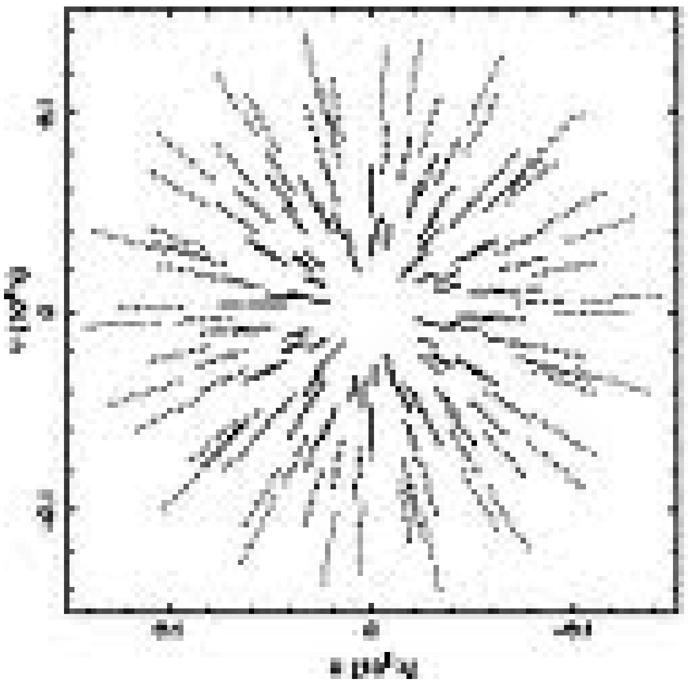


u,v coverage

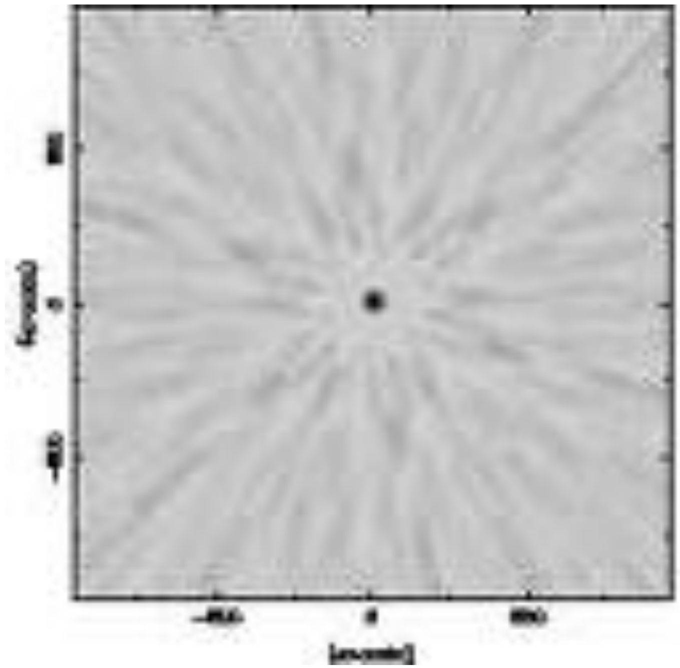


Synthesized beam

U,V COVERAGE



u,v coverage



Synthesized beam

The more densely sampled the u,v plane, the "cleaner" the synthesized beam. Sidelobes are due to undersampling in u,v.

CALIBRATION

The interferometer is a complex device operating in the real world. The data collected are imperfect. Fluctuations in temperature and weather can and do affect the measurements.

Calibration of the data is required:

1. PHASE
2. AMPLITUDE
3. FLUX
4. PASSBAND

PHASE CALIBRATION

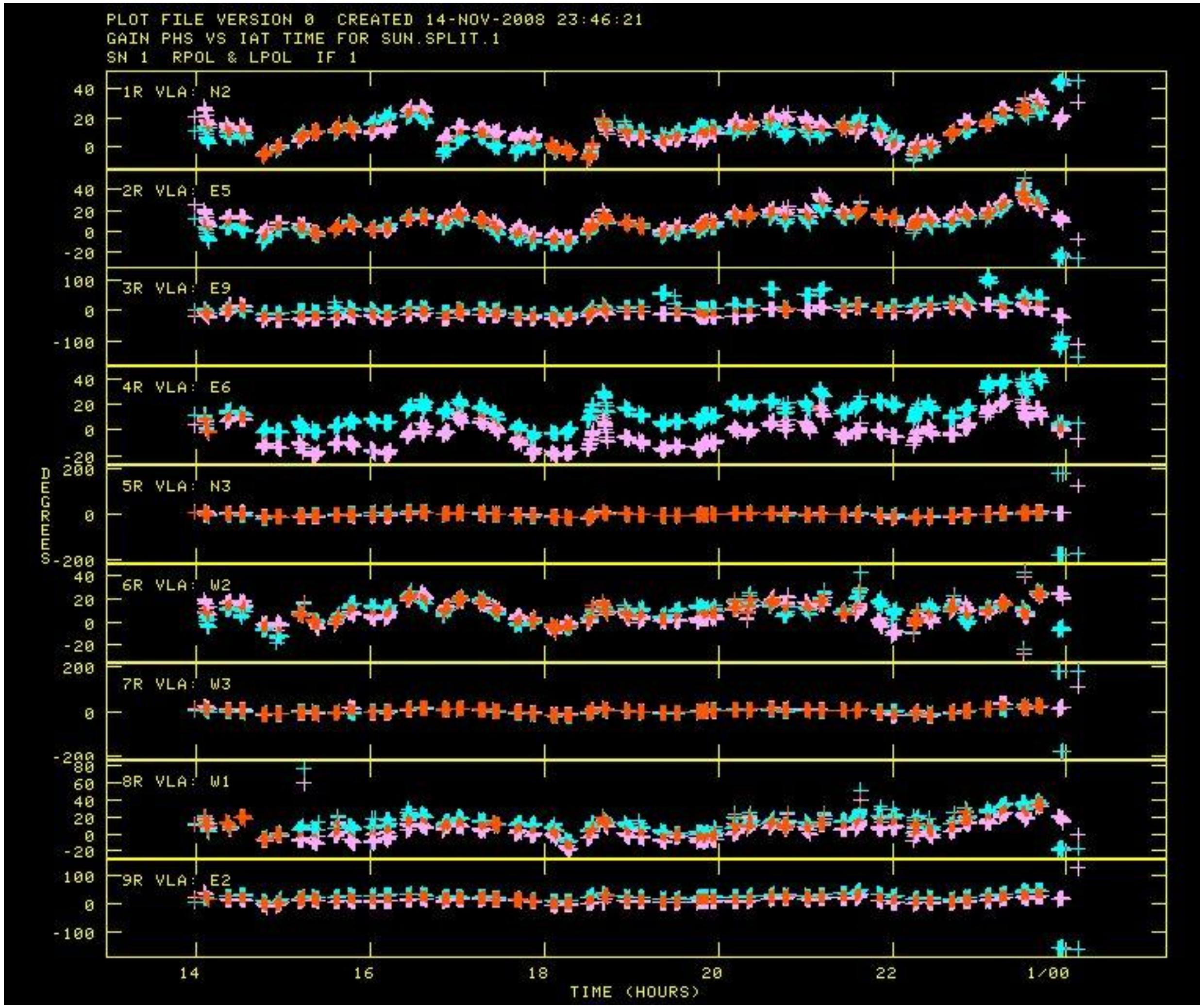
The visibilities V(u,v) are measurements of amplitude and phase. The phase of a point source at the "phase center" (exact middle of the observed field) is known exactly from geometry.

After compensating for the group delay, the phase for a point source at the phase center should be zero for every baseline.

METHOD: Observe a strong point source ("phase calibrator") periodically during the observations. The measured non-zero phases are interpolated in time. The interpolated phases are then subtracted from the data.

PHASE CALIBRATION

phase



time

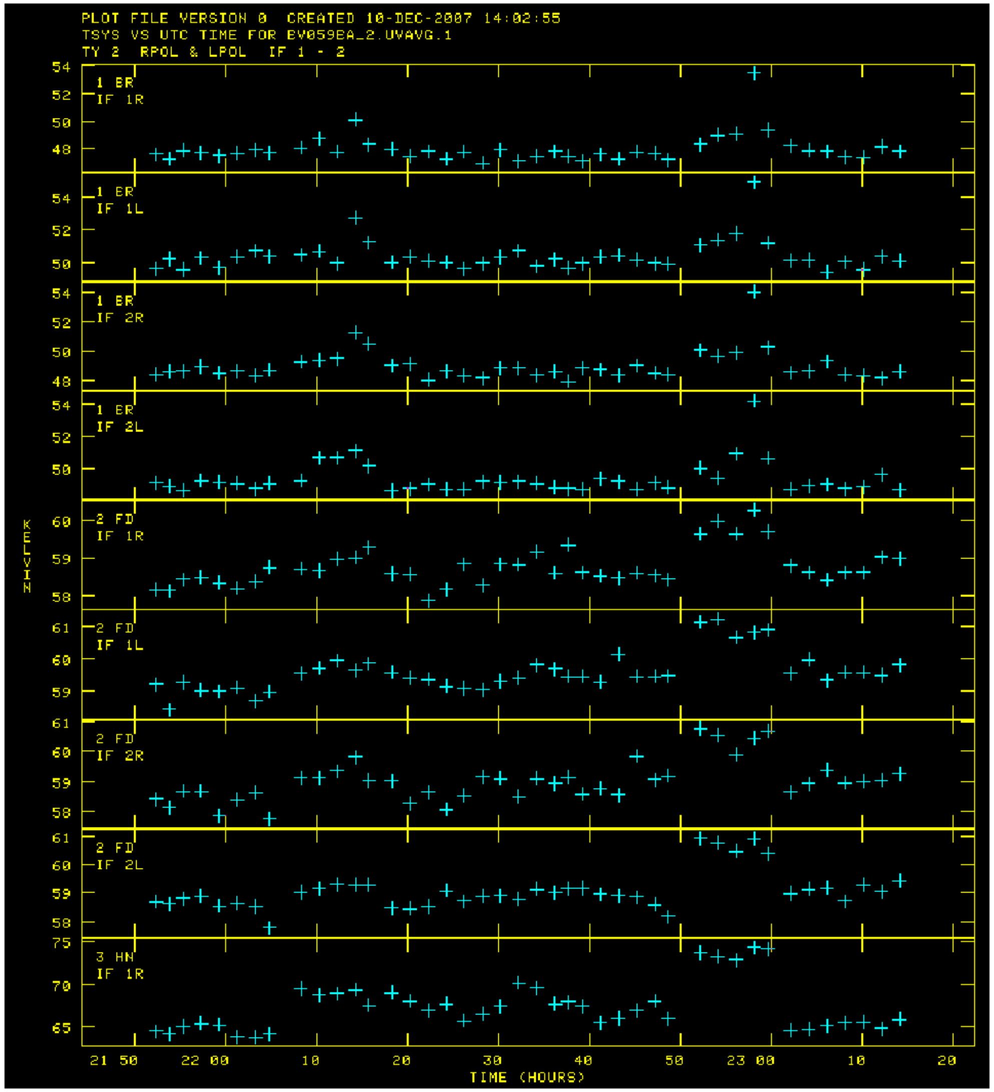
AMPLITUDE CALIBRATION

Since the phase calibrator’s amplitude remains constant during the course of the observations, its measured amplitude provides a measure of the system gain for each baseline.

METHOD: The measured amplitudes of the “phase calibrator” are measured and interpolated in time. They are then applied to the data by dividing all the data by the interpolated amplitudes.

AMPLITUDE CALIBRATION

AMPLITUDE



TIME

FLUX CALIBRATION

The visibility amplitudes must then be converted to real astronomical units (Jy or K).

Careful calibration has established the fluxes of a few bright sources whose fluxes are non-varying and well-measured at all observing frequencies. For example, the EVLA uses 3C286 and 3C48.

METHOD: Observe a strong "flux calibrator" and apply its known flux to all observations.

For high frequencies, sufficiently strong point sources are often unavailable, and flux calibration is tied to observations of planets, e.g., the ATCA uses Uranus at 3 mm. This is a bit more complicated since it is a disk rather than a point source.

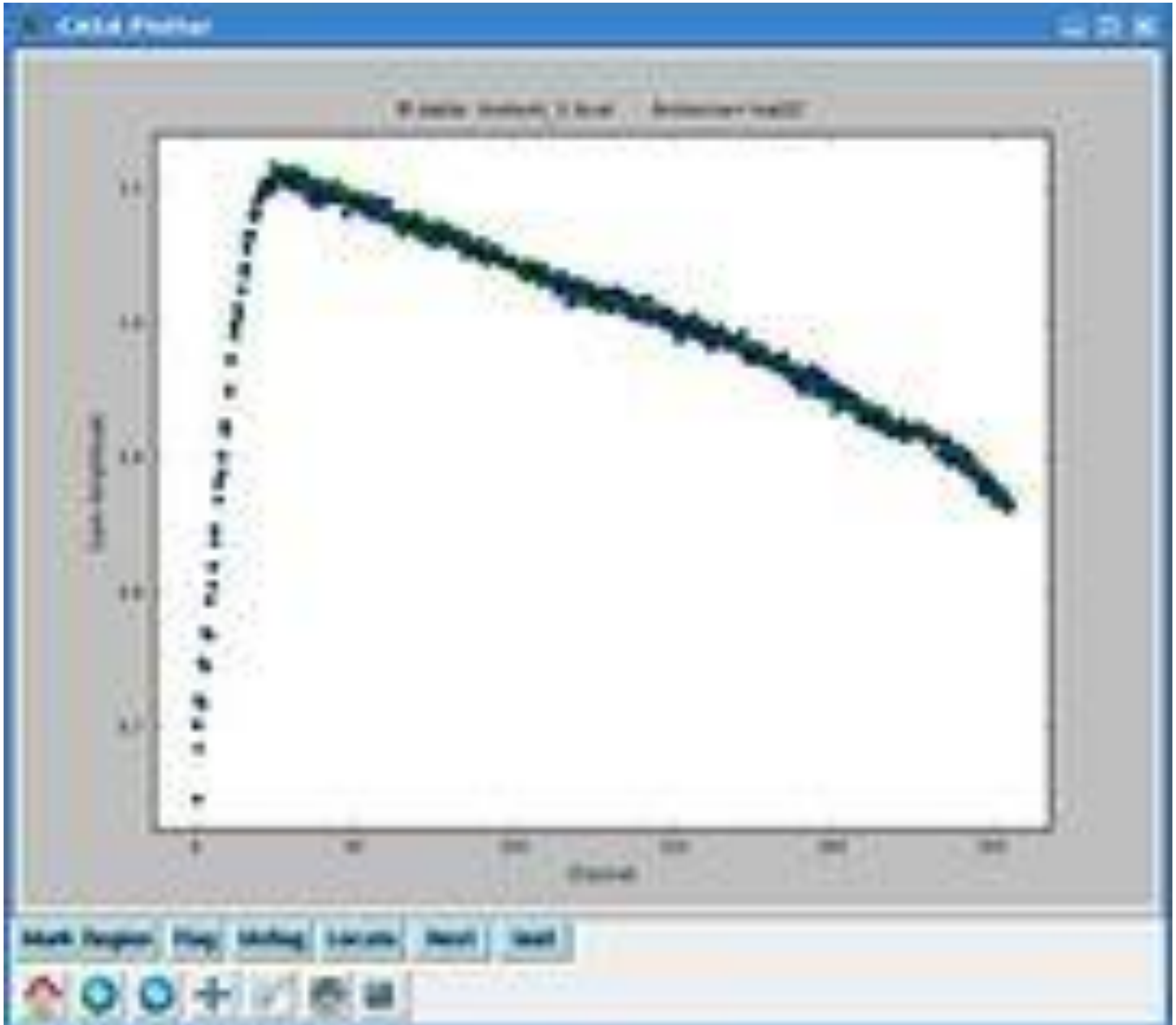
PASSBAND CALIBRATION

For spectral line observations, the varying amplitude and phase response across the spectrometer must also be calibrated.

Many bright point sources are essentially constant in flux over small fractional bandwidths $\Delta\nu/\nu$. Thus, these strong sources provide a featureless spectral reference for calibrating the spectral phase and amplitude response of the system.

METHOD: Observe a strong, spectrally featureless continuum source with the same instrumental set-up as the spectral line source. Apply the measured phases and amplitudes for each spectral channel to all spectral line data.

PASSBAND CALIBRATION



EDITING U,V DATA

Not all of the data points are perfect!

In addition to noise, glitches often occur due to instrumental hiccups or radio interference (microwaves, radars, airplanes, sparkplugs, lightning, etc…)

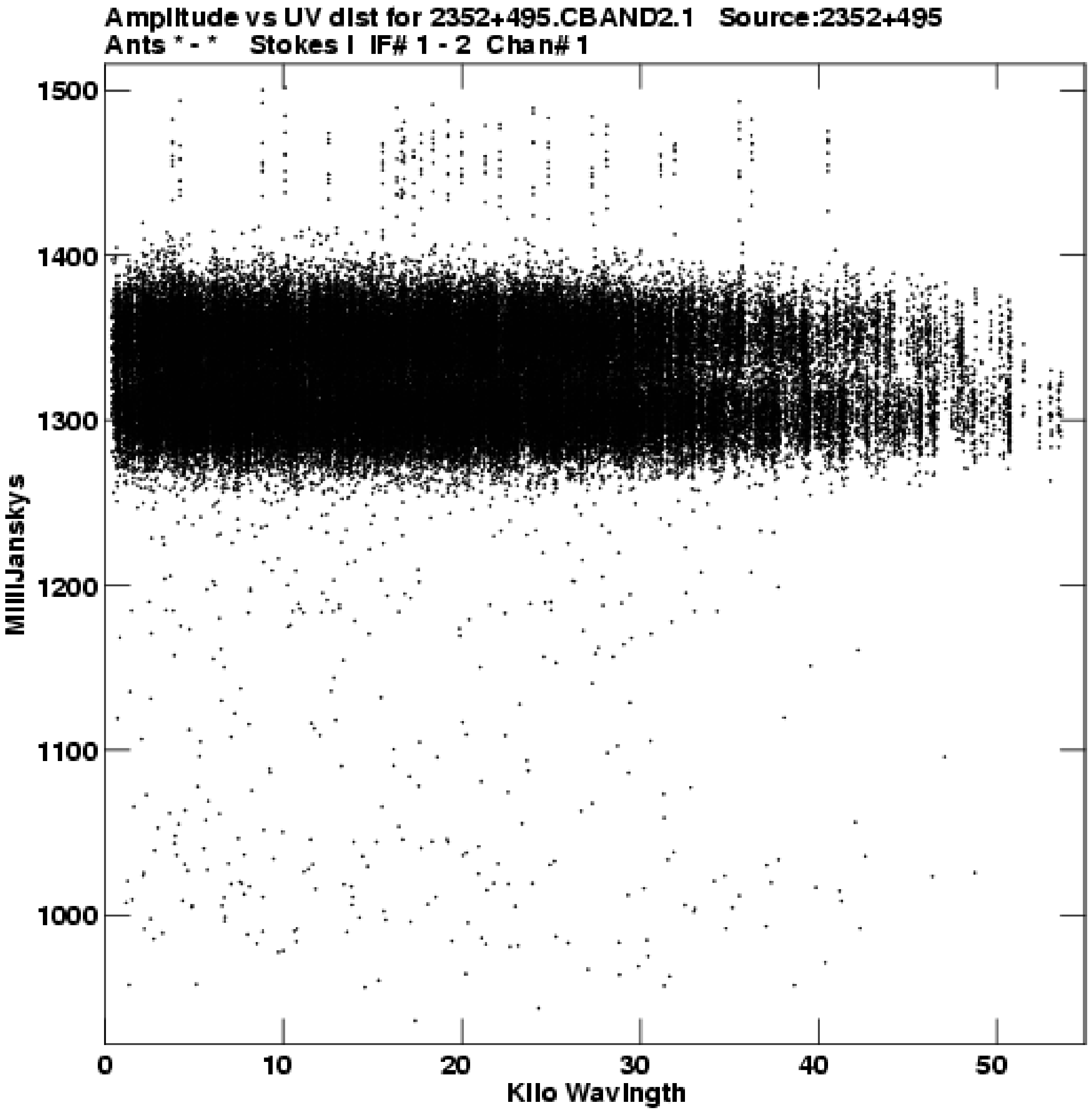
The first step in data processing is to identify and remove obviously bad data. BAD DATA ARE WORSE THAN NO DATA!

It is usually easiest to inspect data in the u,v plane, plotting amplitude as a function of u,v distance from the origin (the length of the projected baseline).

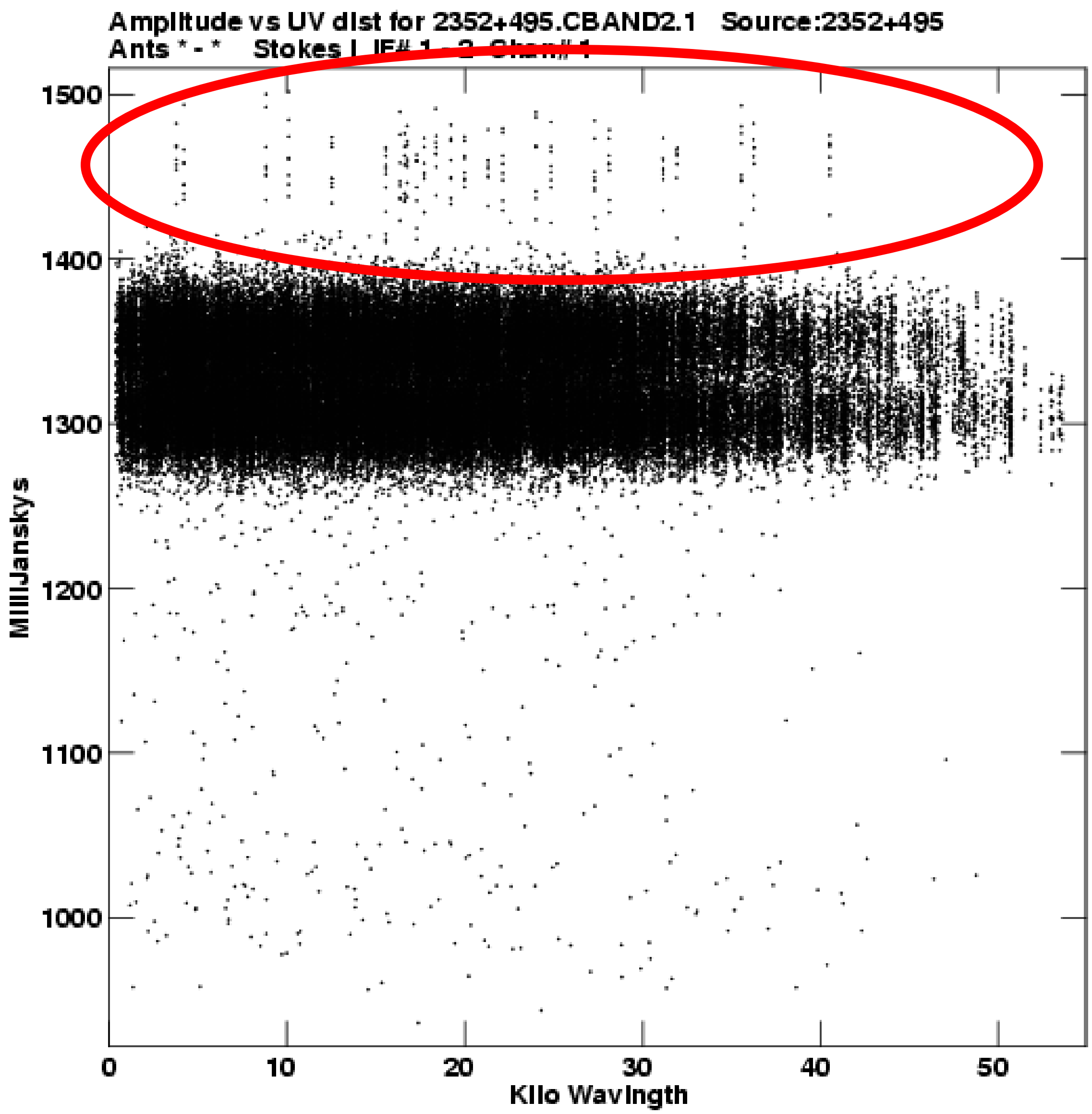
The data usually follow an envelope with some outliers. These outliers are "flagged" as bad and removed from analysis.

This can be done by hand, or by specifying a clip level, say 10 sigma.

FLAGGING BAD DATA



FLAGGING BAD DATA



MAPPING

The measured visibilities consist of amplitudes and phases at the sampled u,v points.

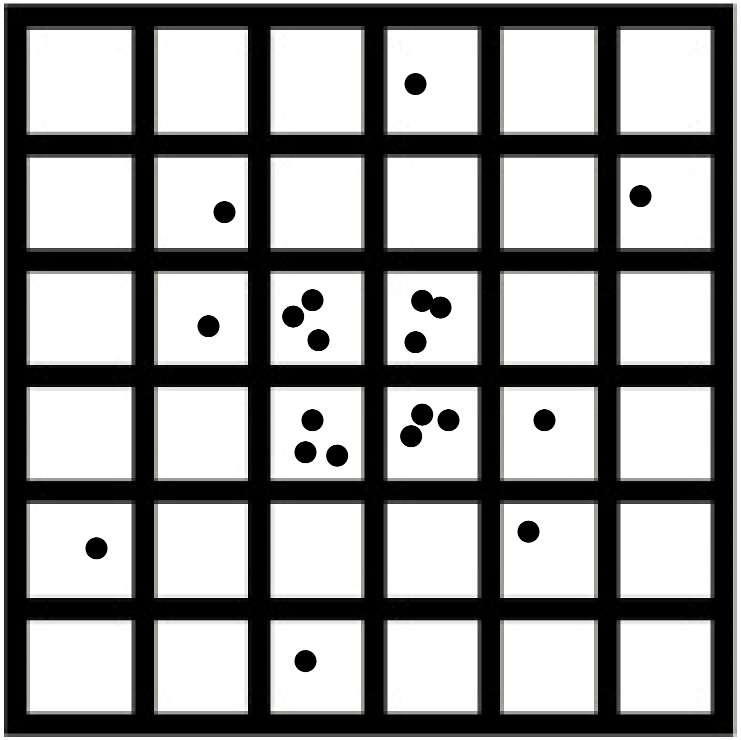
$$\mathcal{V}(u_i, v_i) = |V|e^{2\pi i\phi}$$

They measure the Fourier components of the brightness distribution on the sky.

The usual procedure is to do a Fast Fourier Transform (FFT) on the visibility data, since FFTs are computationally inexpensive.

The u,v data are placed onto a regular grid. Each "cell" in the u,v grid is given a value equal to the weighted average of all the data points in the cell.

GRIDDING AND WEIGHTING



<u>NATURAL WEIGHTING</u>:

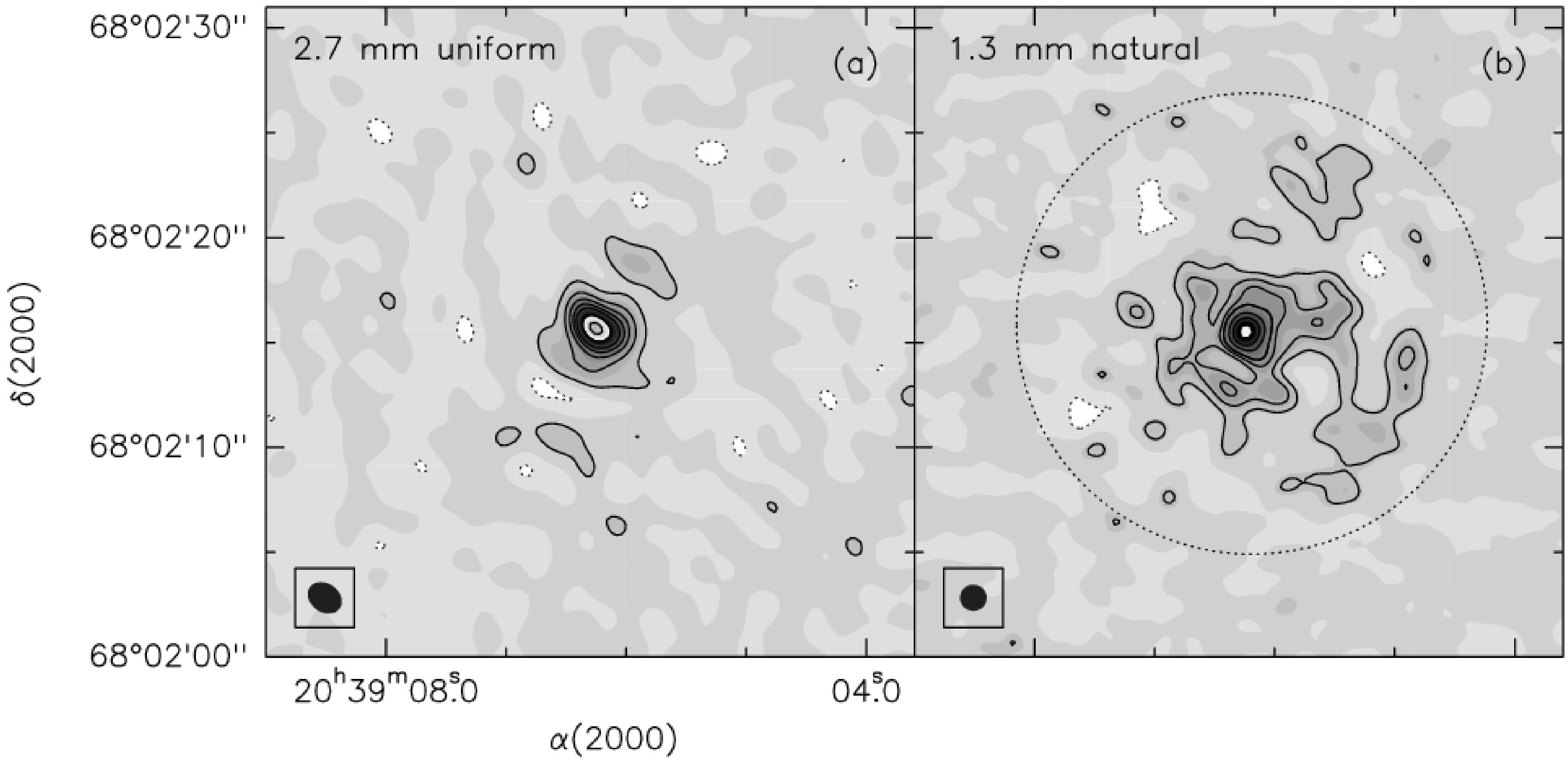
- **Weight = # points in cell**
- **Best signal to noise**
- **Emphasizes short spacings**
- **Poorer angular resolution**

<u>UNIFORM WEIGHTING</u>:

- **Weight = 1 (all cells equal)**
- **Poorer signal to noise**
- **Emphasizes long spacings**
- **Better angular resolution**

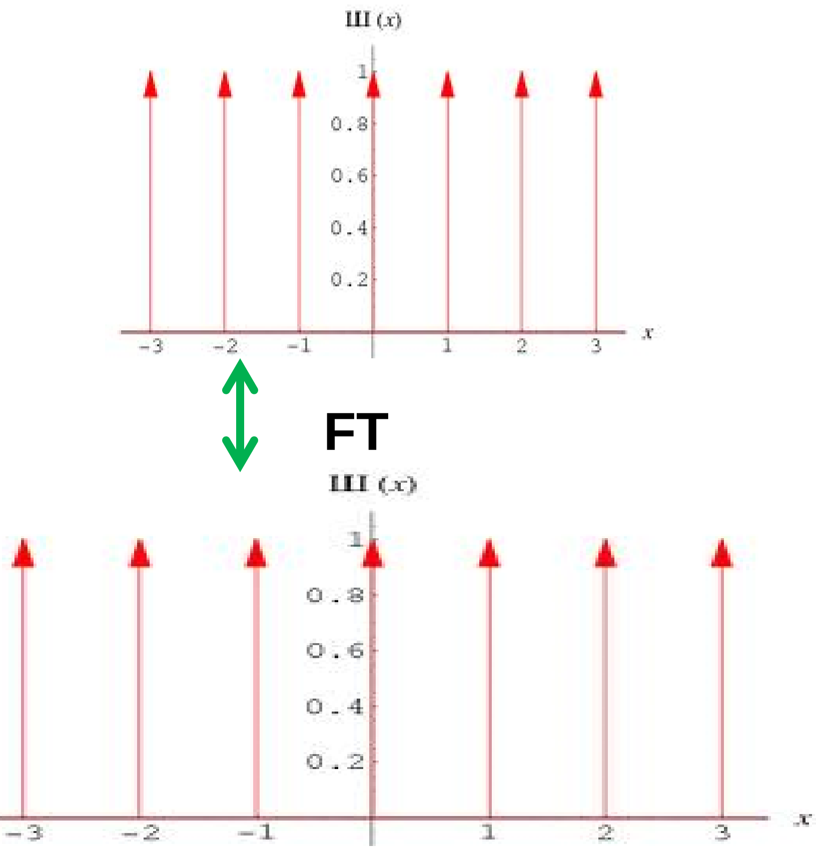
CELLS WITH NO U,V POINTS ARE SET TO ZERO.

UNIFORM VS. NATURAL



Use uniform weighting to emphasize small-scale structure of bright sources.
Use natural weighting to emphasize extended structure of faint sources.

ALIASING



The Fourier transform of the SHAH function (bed of nails) is another Shah function.

Since gridding in the u,v plane is the same as convolving with the Shah function, the FT in the image plane is also a Shah function. Thus the map is replicated periodically in the image plane on an infinite, uniform grid.

If a source is outside of the mapped area, in can creep in from the neighboring, replicated image!

This effect is called aliasing.

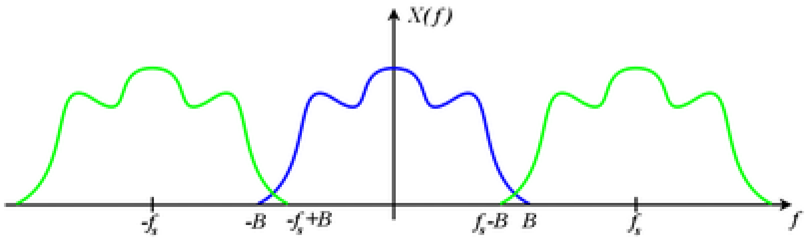
ALIASING

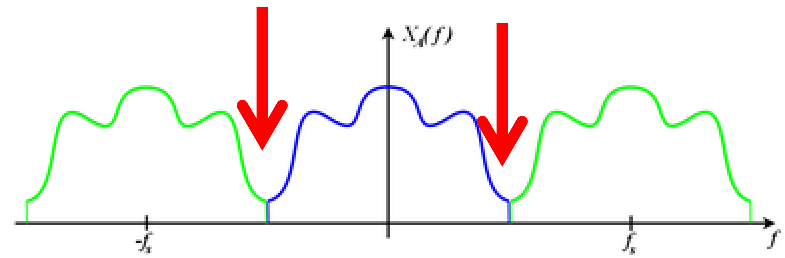
In one dimension, suppose we had a source brightness distribution like the blue line.

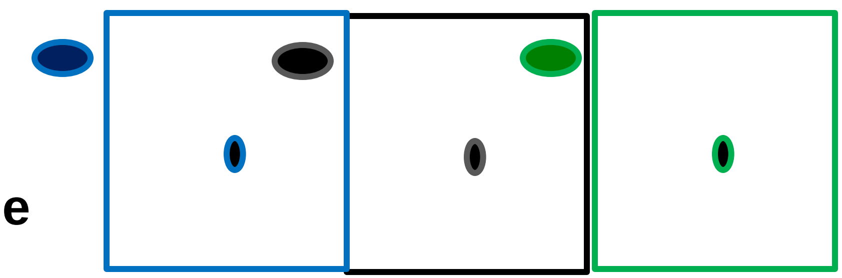
Then because of the gridding (bed of nails Shah function) it would repeat in adjacent maps.

If the map is too small the aliased images would bleed over from the next map and give nonsense results at the map edges. Even worse, sources could appear at the wrong place.

Always make a big map first.







INVISIBLE IMAGES

The FT of the visibilities produces an incomplete representation of the true brightness.

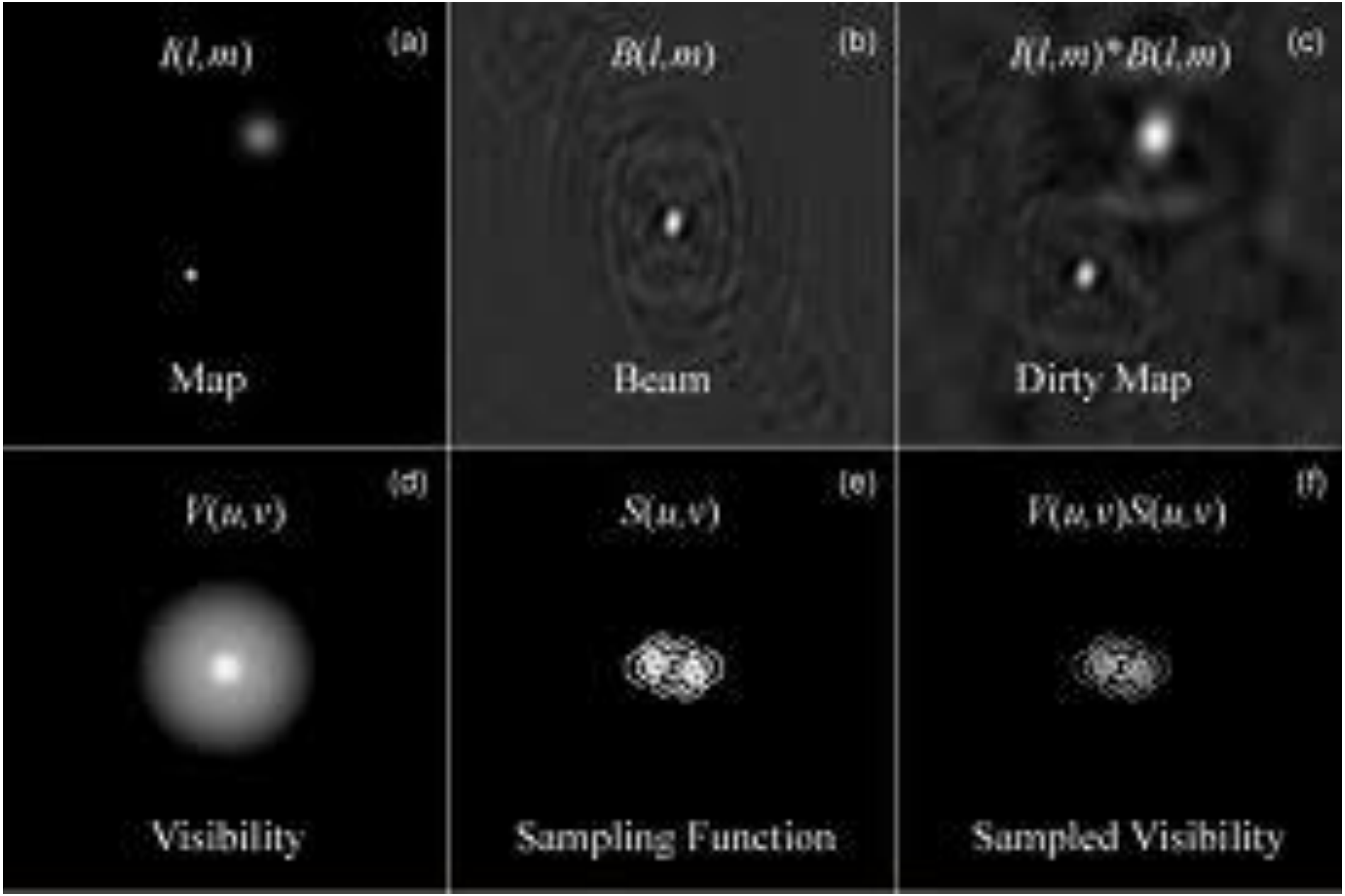
The observed brightness image is the convolution of the synthesized beam with the true brightness distribution.

$$B_{obs}(\alpha,\delta) = b(\alpha,\delta) ** B_{true}(\alpha,\delta)$$

But the u,v plane is only partially sampled, and therefore the observed image contains only a few of the infinite number of Fourier components required to generate B_{true}.

Since the visibility function is incompletely sampled, there is missing information, and the true brightness distribution cannot ever be completely measured.

MAPPING



I(l,m)
(a)
Map
B(l,m)
(b)
Beam
I(l,m)*B(l,m)
(c)
Dirty Map
V(u,v)
(d)
Visibility
S(u,v)
(e)
Sampling Function
V(u,v)S(u,v)
(f)
Sampled Visibility

HOW DO WE RECOVER B_{TRUE}?

Can we solve the convolution equation

$$B_{obs}(\alpha,\delta) = b(\alpha,\delta)**B_{true}(\alpha,\delta)$$

to recover B_{true}?

NO! There are an infinite number of solutions!

Take any function you want for the u,v points that are NOT measured. The interferometer CANNOT measure these Fourier components and will ALWAYS GIVE THE SAME DIRTY MAP!

The sampling function is---arbitrarily--- set to zero at the "missing" u,v spacings. The best we can do is make "reasonable" guesses to interpolate from the measured u,v data into the unsampled regions.

APPROACH #1

The image consists of empty sky + a bunch of point sources.

This is the basis for the CLEAN algorithm.

ONLY a source with a true brightness distribution that looks exactly like the dirty beam will be accurately recovered by the FT of the visibilities

The “reasonable” assumption is that sources do not look like dirty beams.

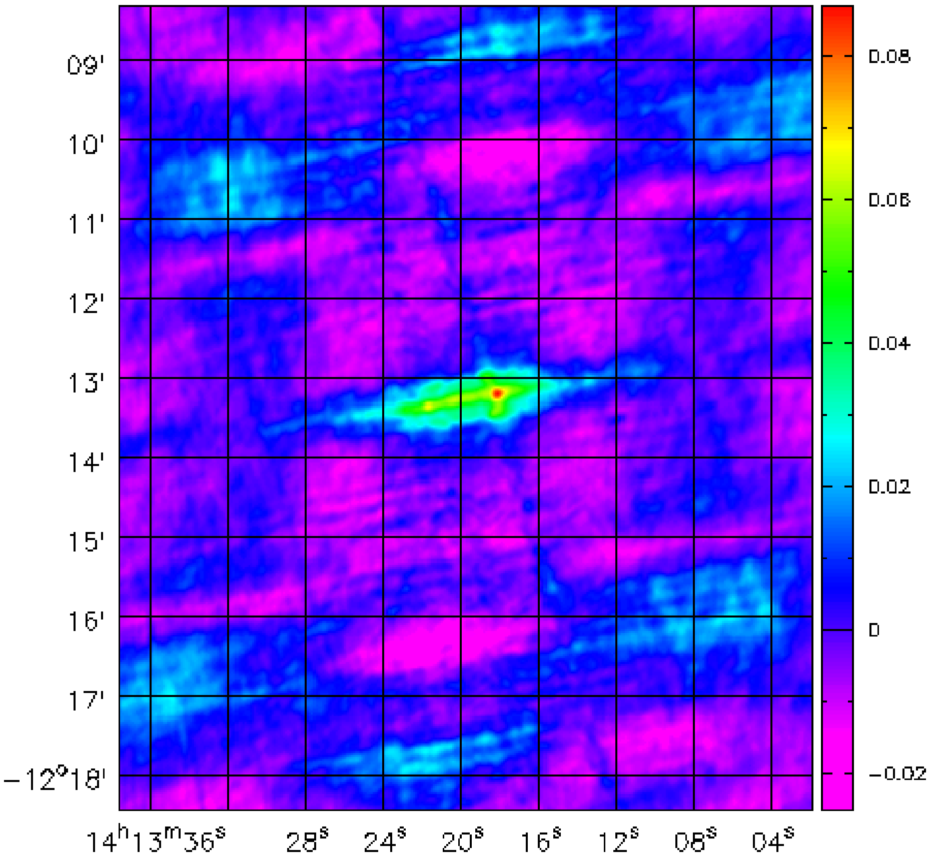
Anything that looks like a dirty beam is probably a point source that has been convolved with the dirty beam.

CLEAN ALGORITHM

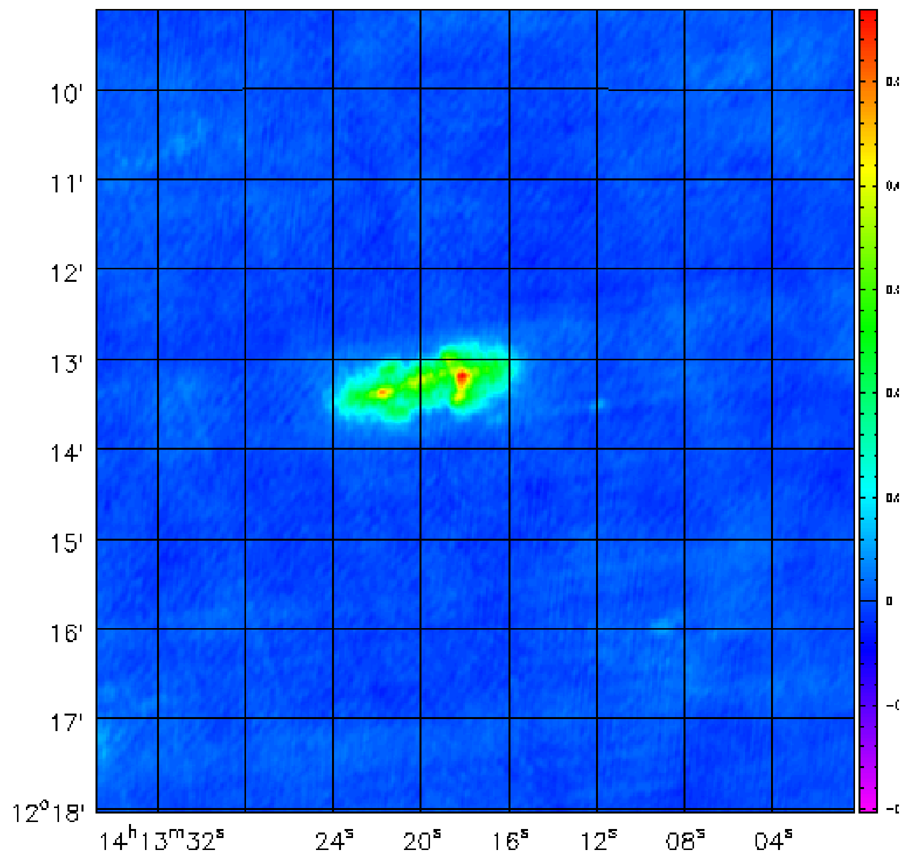
1. **Find the strength and location of brightest point in the image (or some defined portion of the image)**
2. **Subtract from the dirty image at this location the "dirty" synthesized beam b multiplied by the peak strength and the "loop gain" factor $\gamma \sim 0.05$ to 0.5**
3. **Consider these as point-sources; store their positions and the subtracted fluxes.**
4. **Go to (1) unless the algorithm reaches some termination conditions (e.g. remaining peak is below some user-specified level). What is left are the *residuals*.**
5. **Convolve the accumulated point source model with an idealized CLEAN beam (i.e. add back components) usually an elliptical Gaussian fit to the inner part of the dirty beam.**
6. **Add the residuals from step (3) to the CLEAN image.**

EXAMPLE OF CLEAN

Dirty image of Jupiter

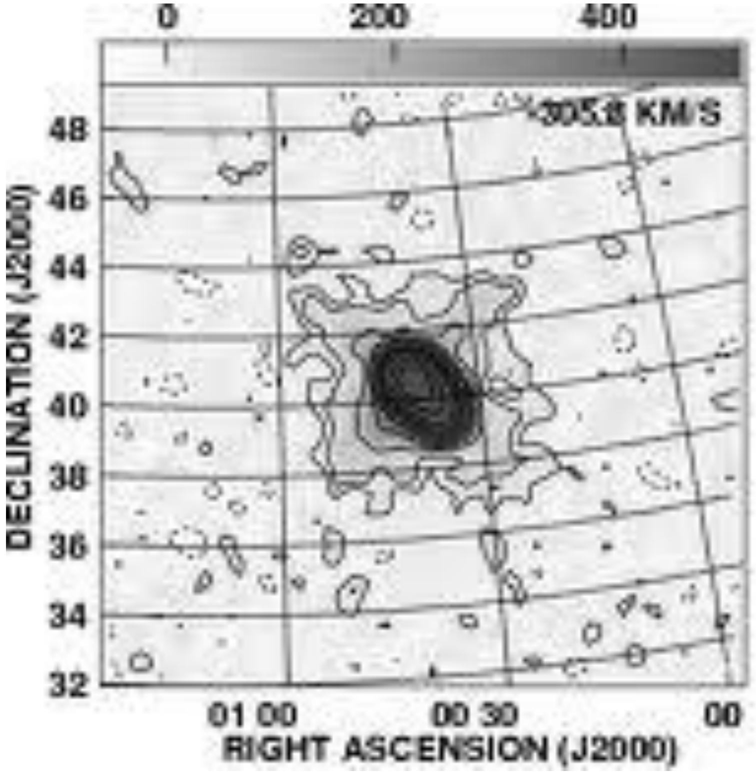


CLEAN image of Jupiter

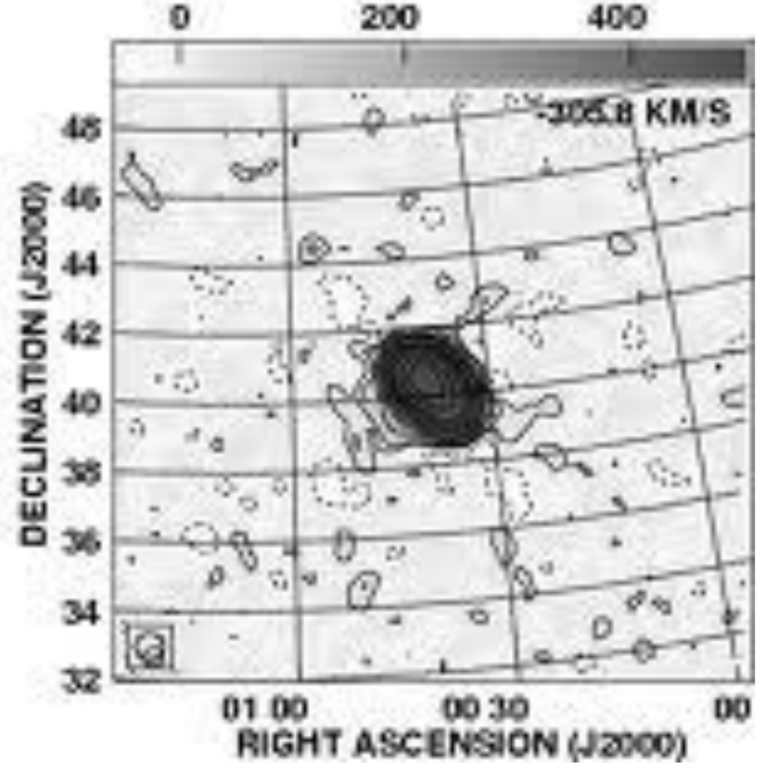


ANOTHER CLEAN EXAMPLE

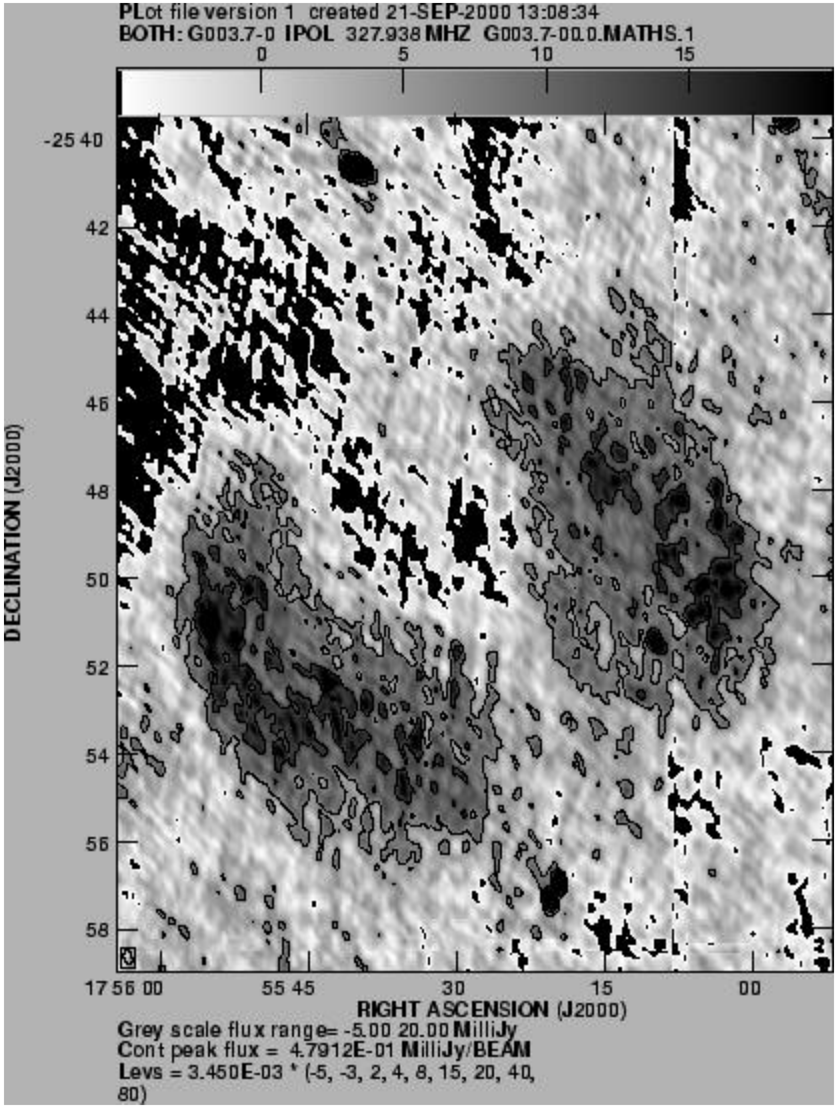
Dirty map



CLEAN map



CLEANER BEWARE!

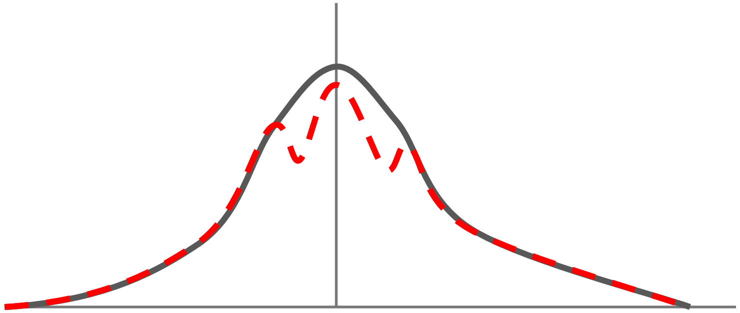


CLEAN is a non-linear process.

Results will be different with different values of loop gain and termination condition.

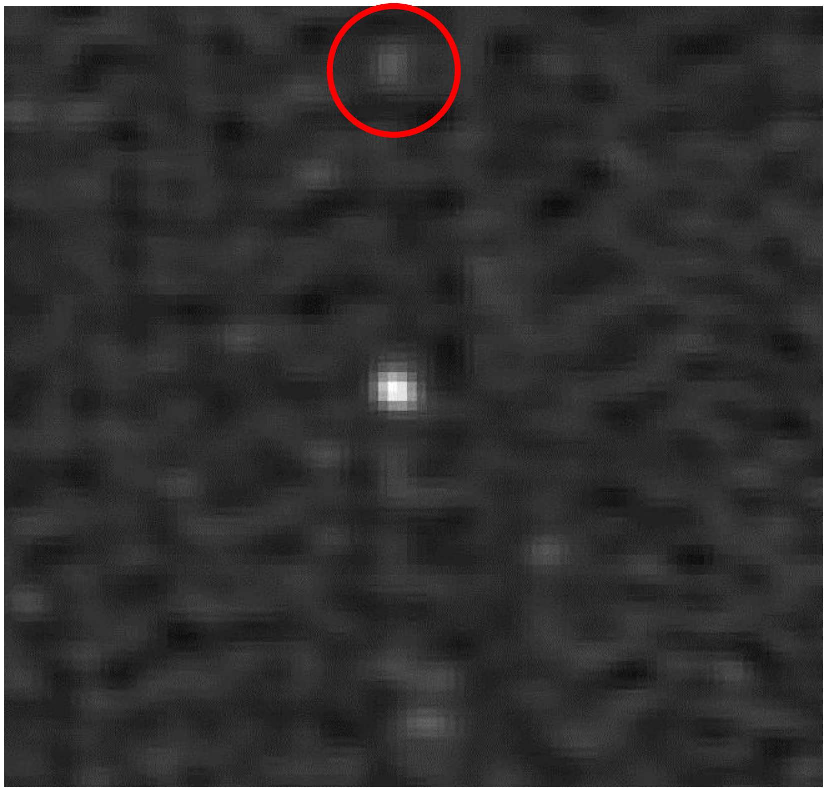
Does not do well on extended sources.

Often produce striped artifacts.

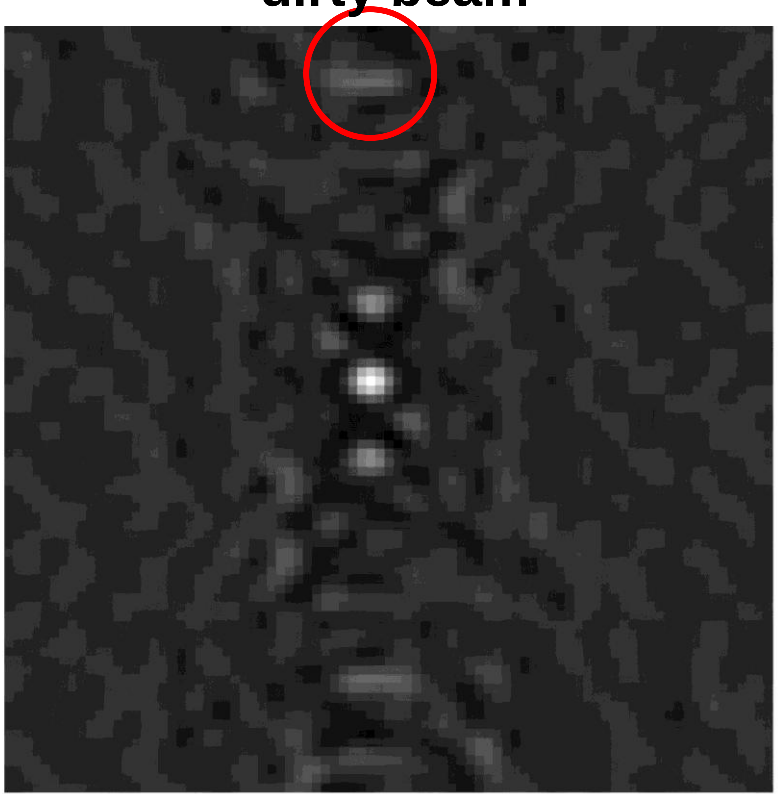


SOURCES ON SIDELOBES

CLEAN map



dirty beam



PRACTICAL ADVICE

Always make images of the full primary beam first to avoid aliasing.

You must make an image at least twice as large as the area you want to clean.

Clean around the bright sources first. You can change the clean regions later.

Use a large loop gain for a few compact sources; a small one for complex extended sources.

End the cleaning when you start getting a lot of negative clean components, or when the clean component values are comparable to the noise (typically a few hundred to a few thousand iterations).

APPROACH #2

The distribution on the sky is smooth and slowly varying.

This is certainly a “reasonable” assumption for extended sources!

Find a model consistent with the observed data that has the properties of being slowly varying.

MAXIMUM ENTROPY

Choose some function F(B') such that the deconvolved brightness distribution B' maximizes the function F(B'), with the constraint that B' must fit the observed visibilities.

Two common choices:

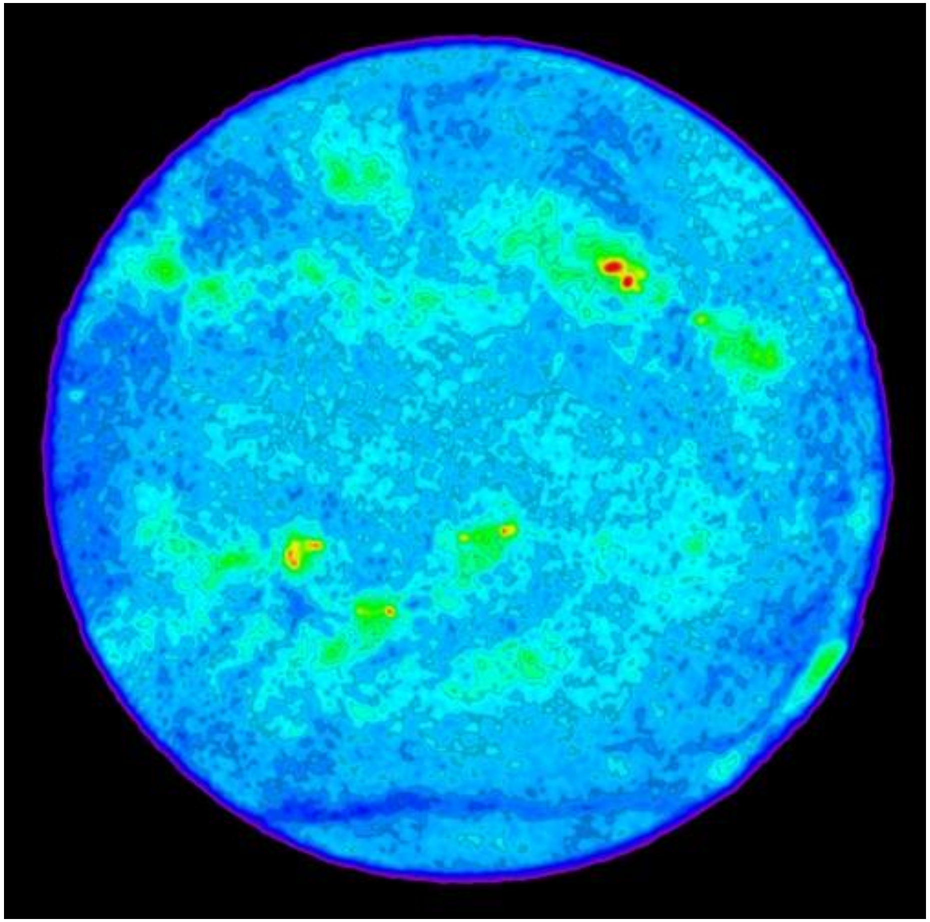
$$\boldsymbol{F} = \sum_{\boldsymbol{i}} \log B'_i$$

$$\boldsymbol{F} = -\sum_i \frac{B'_i}{B'_{tot}} \log \left(\frac{B'_i}{B'_{tot}}\right)$$

Where

$$B'_{tot} = \sum_{\boldsymbol{i}} \log B'_i$$

AN EXAMPLE OF MAXIMUM ENTROPY



Maximum entropy works best on extended emission.

Maximum entropy deconvolved VLA image of the sun (NRAO).

ZERO SPACING FLUX

Since an interferometer can never measure the flux at 0 baseline (the dishes cannot crash into each other!), it can never measure the total power.

An interferometer is insensitive to extended structure.

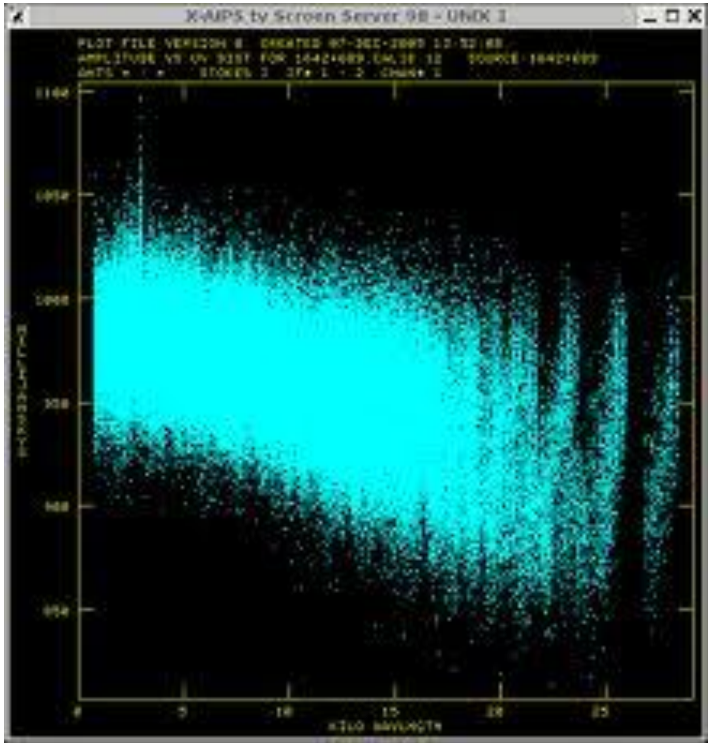
To add sensitivity to these larger scales, sometimes the "zero-spacing flux" is entered by hand, or a model is inserted.

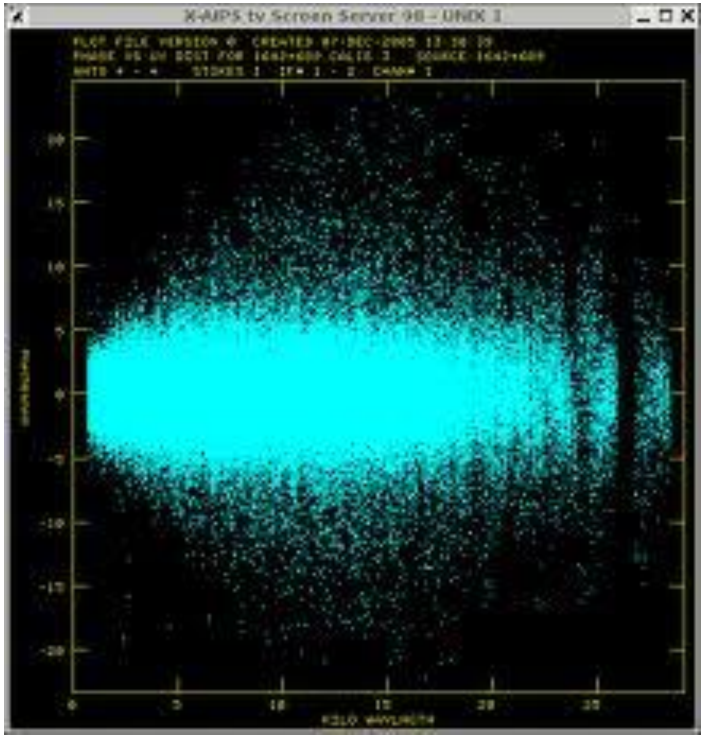
Sometimes interferometer data are combined with single dish data.

CAN YOU TELL IF YOU HAVE LARGE-SCALE STRUCTURE?

Extended Source **Point Source**

Flux





Baseline length

SELF CALIBRATION

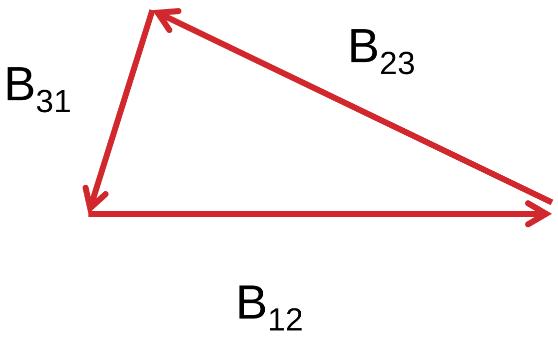
There are certain relationships among the phases and amplitudes in the visibility data. These provide additional constraints that can be exploited to improve the calibration.

For example, suppose we consider three antennas: 1,2, and 3.

Define the "closure phase" $C_{123} = \varphi_{12} + \varphi_{23} + \varphi_{31}$

Since $\phi = 2\pi i(\boldsymbol{B} \cdot \boldsymbol{\sigma})$, **and we are going around a triangle, the vector sum of the baseline vectors is zero, and so the closure phase is 0.**

This is true even in the presence of errors.



SELF CALIBRATION

The idea behind self-calibration is to treat the gain factors applied in the calibration process as free parameters that vary with time.

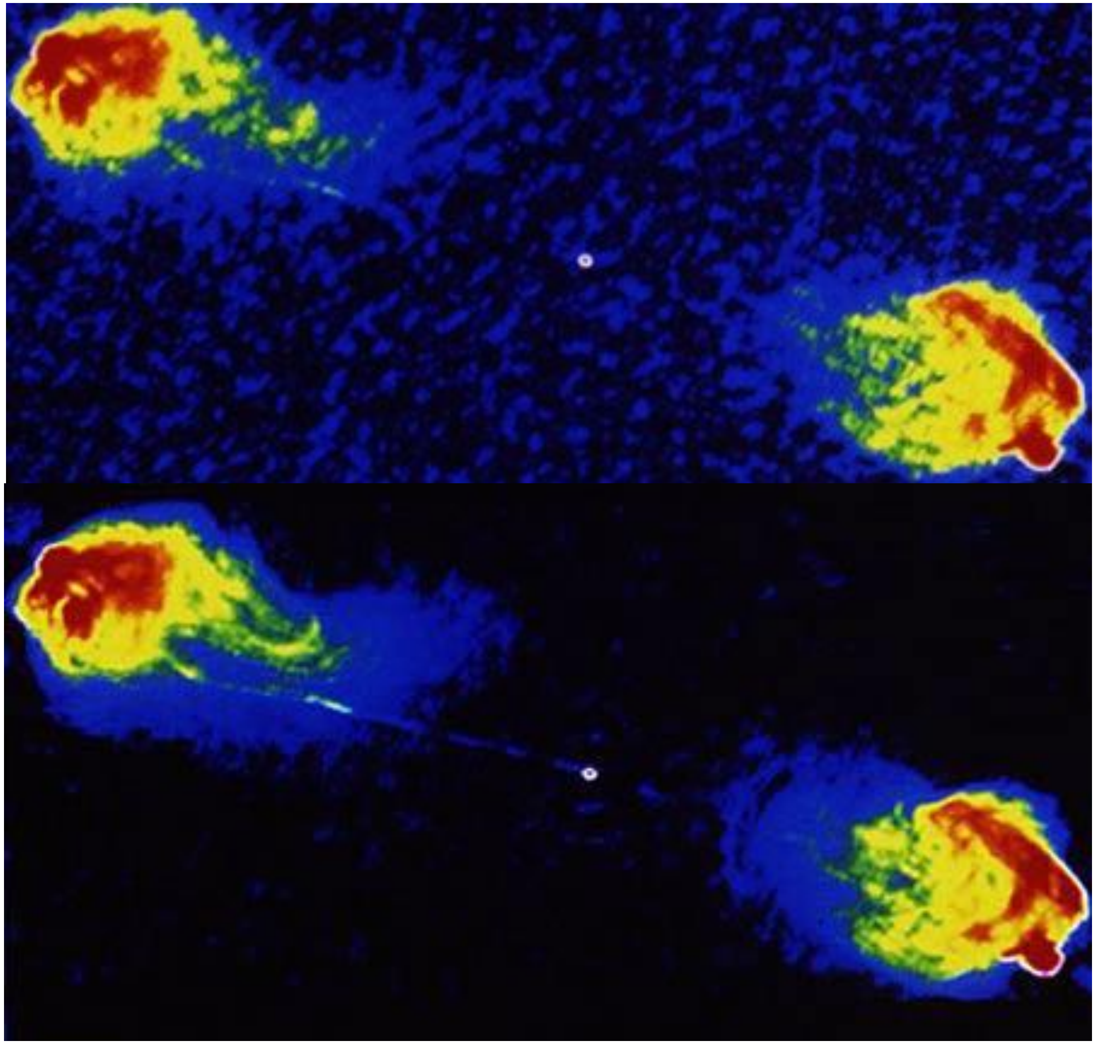
This is true! For example, a cloud drifting over an antenna will change its phase. The idea is to use the data itself, constrained by the closure relations (and a similar relation for amplitude), to adjust the gains for each individual antenna as a function of time.

The gain factors contain both amplitude and phase information.

SELF-CALIBRATION ALGORITHM

1. Make an initial map (typically using CLEAN)
2. Adjust the gain factors for each antenna, over some specified time interval, that minimizes the difference between the model and the measured visibilities, subject to the closure constraints.
3. Use these new gains to calibrate the observed visibilities and make a map.
4. Make a new map and repeat until the process converges (no noticeable difference in the maps).

SELF-CAL IMPROVES DYNAMIC RANGE



Dynamic range improvements of 10 – 1000 is not uncommon.

The biggest effect is on the phases (positional information). It takes flux from the wrong place and puts it where it belongs.

NRAO

TIPS ON SELF-CAL

- **Self calibration only works when there is plenty of signal. It will not work on faint sources.**
- **The starting model is not terribly important, but in later iterations, you should do a good job with CLEAN or MEM.**
- **Choose time intervals that are small enough to track the variations in the phases and amplitudes. Depending on the weather this could be ~minutes to an hour.**
- **Choose time intervals that are long enough to get enough signal-to-noise (say, at least 5 σ) for accurate calibration. This depends on your source brightness. Typical values are ~5 to 10 minutes for bright sources.**
- **If you have a strong spectral source, i.e. a maser, you can self-cal on the maser and pass the solutions along to your continuum data or spectral line data.**

TAKE HOME MESSAGES

The interferometer is analogous to a 2-slit interference pattern.

The synthesized "dirty" beam is the FT of the sampling function. The better the u,v coverage, the better the beam.

Calibrate your u,v data by looking at strong point continuum sources: phase, amplitude, flux, and passband.

Edit and flag your bad u,v data.

Mapping is done by the FFT, which causes aliasing: MAKE BIG MAPS FIRST

Deconvolution techniques: choose CLEAN for compact sources and Maximum Entropy for very extended sources.

Self-calibration will greatly improve the dynamic range for bright sources.