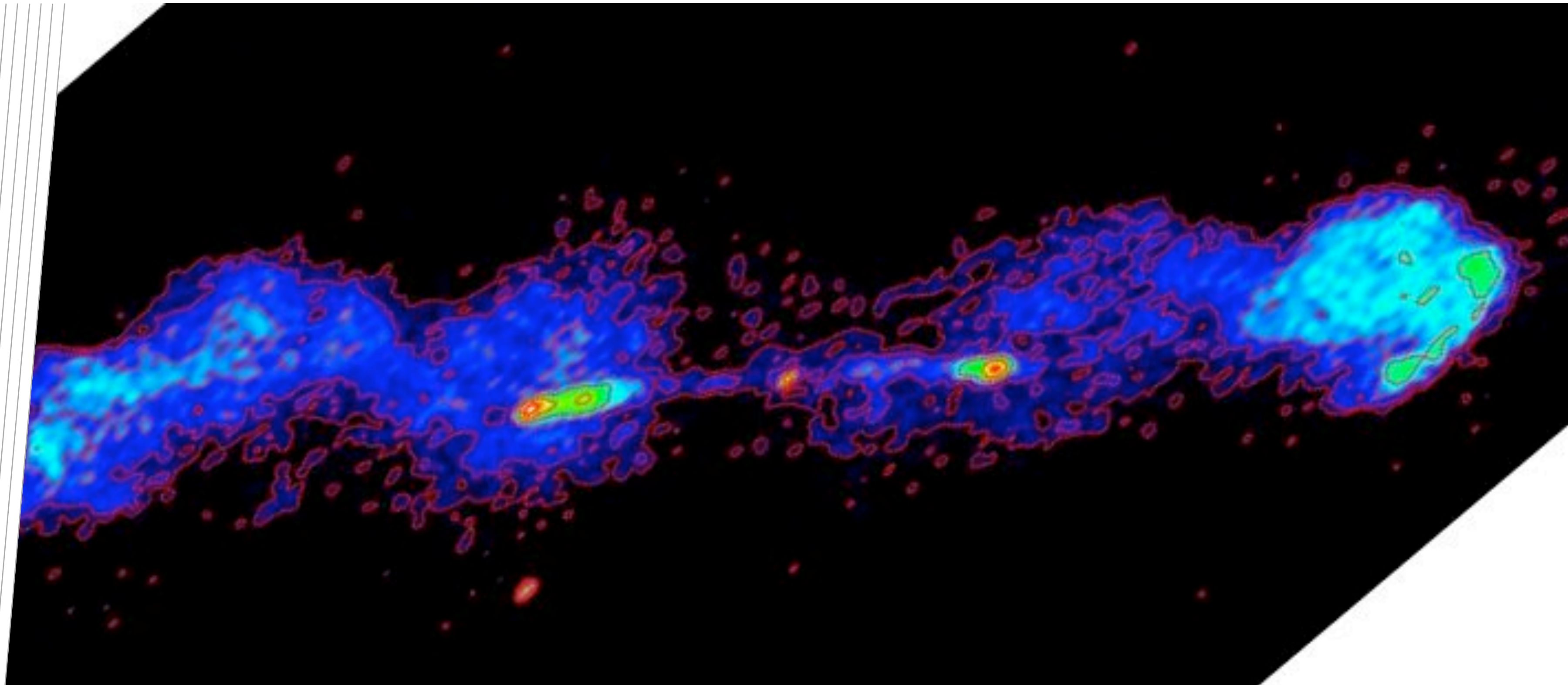

Interferometry I

Parkes Radio School 2011

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2011-09-28

References

- This talk will reuse material from many previous Radio School talks, and from the excellent textbook “Interferometry and Synthesis in Radio Astronomy” by Thompson, Moran and Swenson

Introduction

- Simple/early interferometry
- The theoretical interferometer
- The Fourier transform
- The action of the correlator
- Coordinate systems
- Practical Interferometers

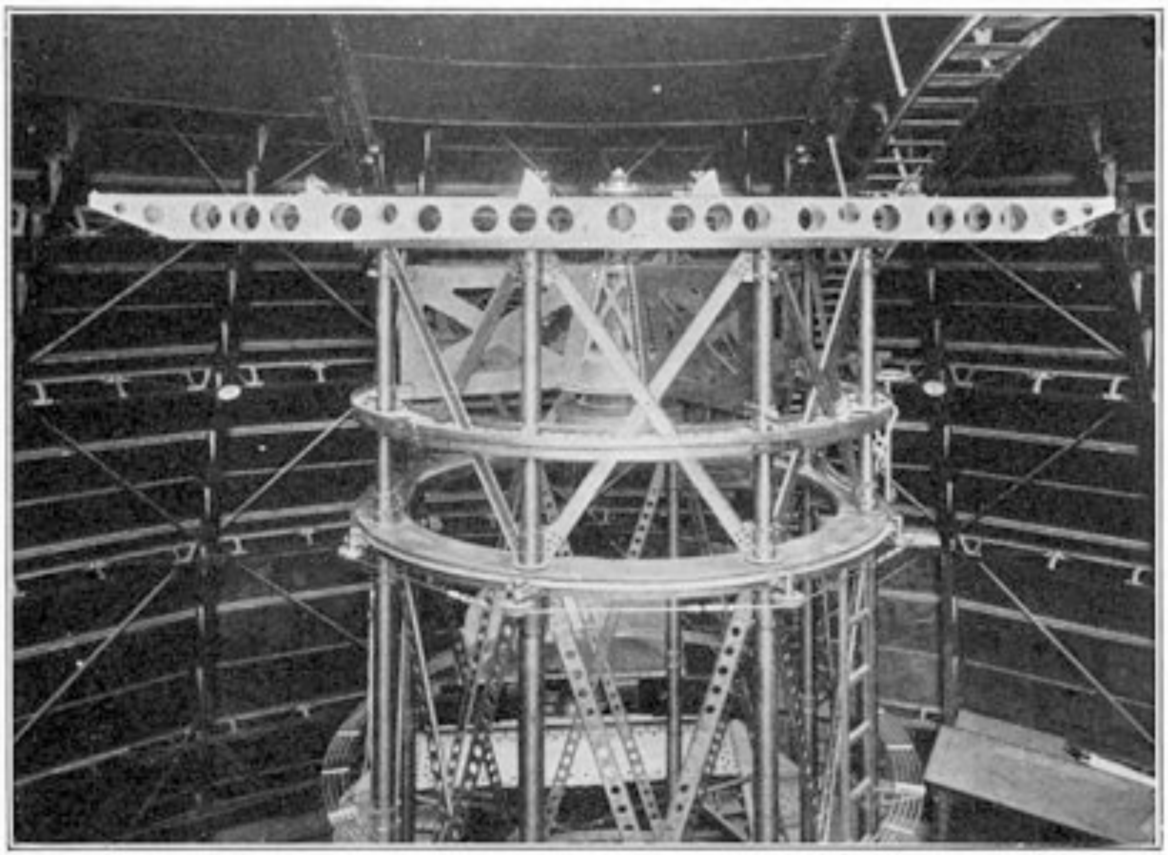
Implications for real observing will be shown in pull-out boxes like these.

Useful mathematical relations will be shown in pull-out boxes like these.

Why Interferometry?

- Interferometry is the quest for finer resolution
  - Better resolution allows for more precise studies of complex sources, better discrimination between multiple sources and allows for more accurate matching between observations at different wavelengths
  - Back to 1940: optical telescopes have resolutions of arcseconds; radio telescopes have arcminute resolution
  - We could build bigger radio telescopes, but becomes uneconomical quickly

The Michelson-Pease Interferometer



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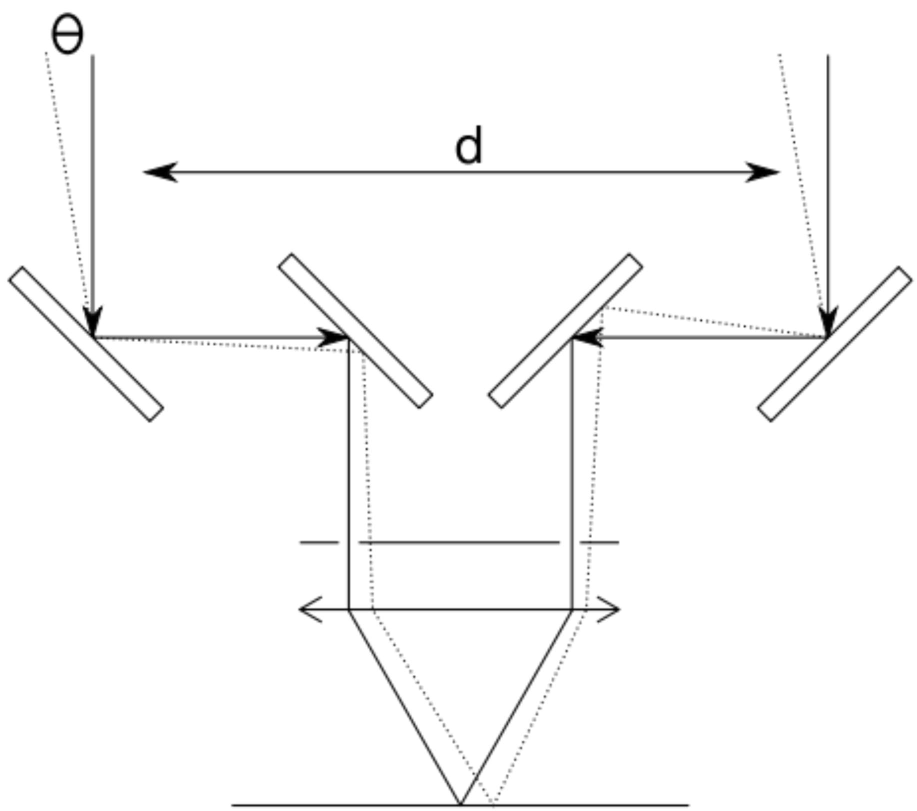

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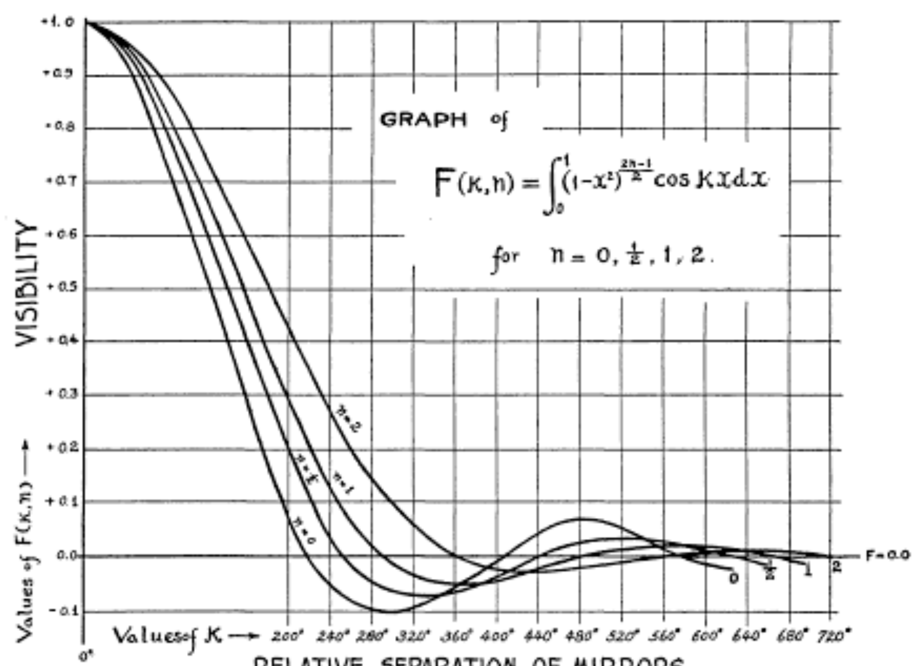
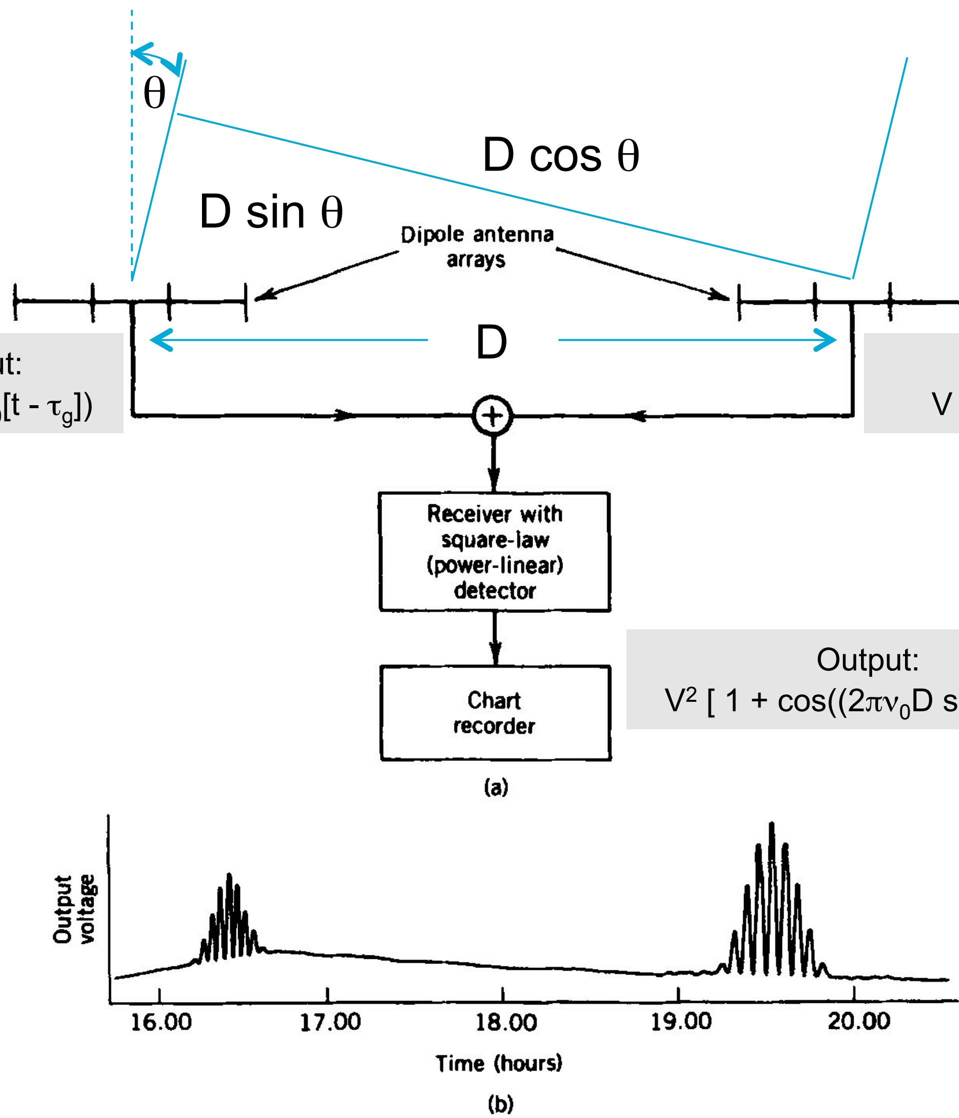


FIG. 5.—Visibility-curves for various sources

An interference pattern is created, which can be compared to models and thus used as a way of measuring structure smaller than the resolution afforded by the primary mirror!

The Ryle and Vonberg Radio Interferometer



Wave arrives at left antenna after right antenna by:
$\tau_g = (D/c) \sin \theta$

Output:
$V \sin(2\pi\nu_0[t - \tau_g])$

Output:
$V \sin(2\pi\nu_0 t)$

ν_0	= central frequency
c	= speed of light
V	= voltage

Output:
$V^2 [1 + \cos((2\pi\nu_0 D \sin\theta)/c)]$

The angular width of the fringes is proportional to the baseline distance D!

Now for the maths!

- The fringe function is a general result:

$$F = \cos(2\pi\nu\tau_g) = \cos\left(\frac{2\pi D l}{\lambda}\right)$$

where:

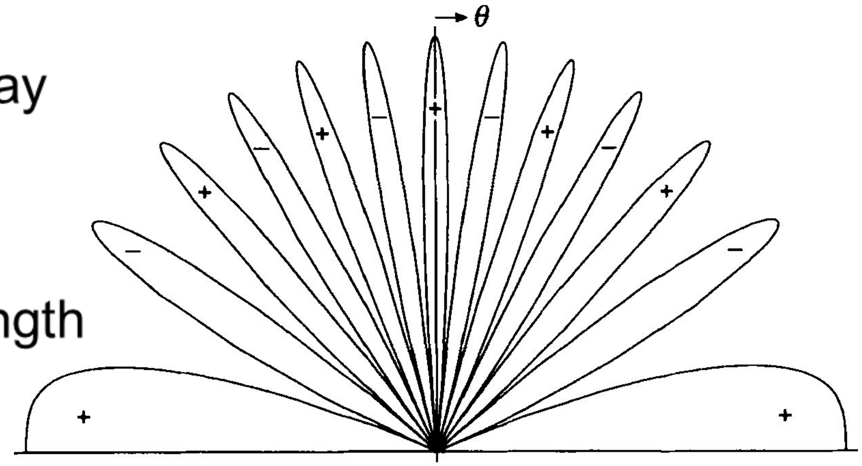
$l = \sin\theta$

τ_g is the geometric path length delay

ν is the frequency

λ is the wavelength = c/ν

D is the interferometer baseline length

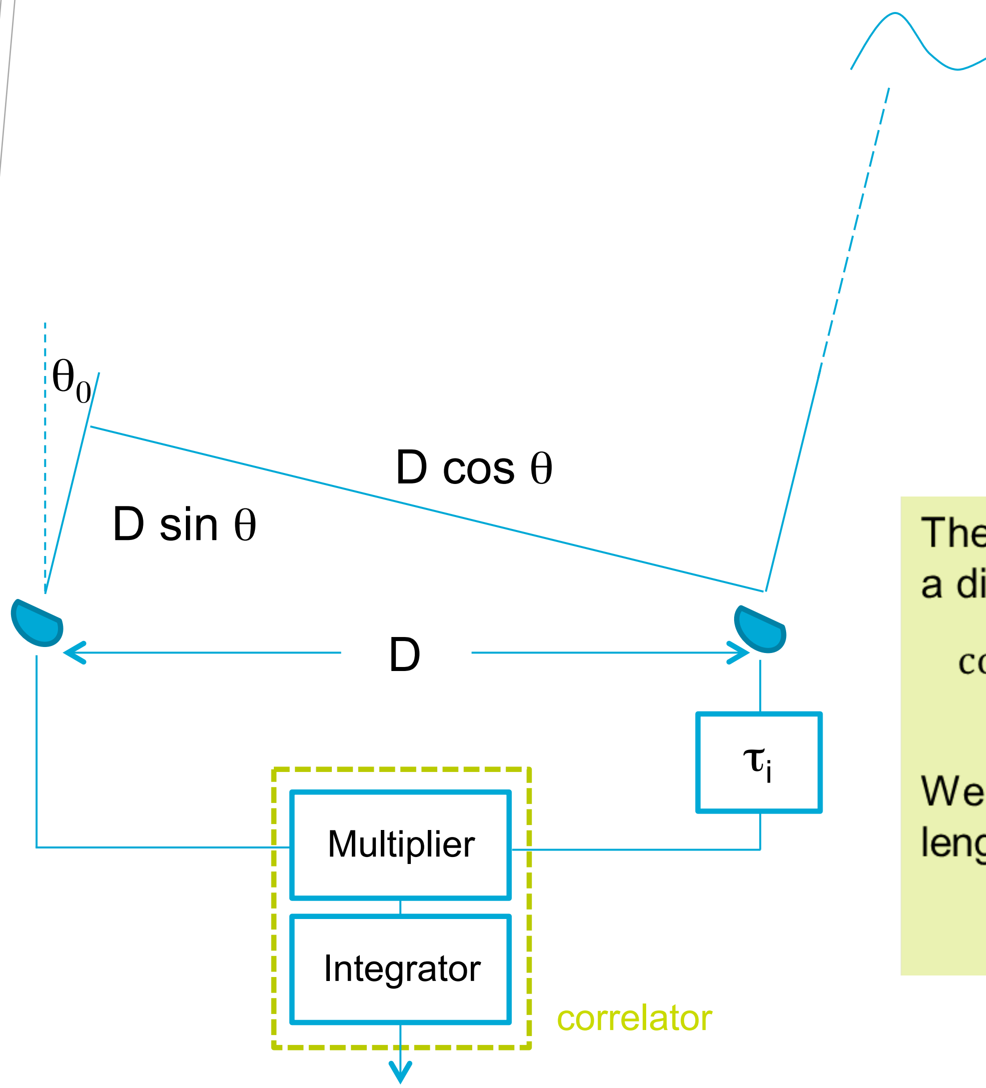


This is the fringe pattern for an interferometer with $D/\lambda = 3$

One Dimensional Interferometry

- We now consider a very simple interferometer to derive some of its important properties.
  - Two (mostly) independent antenna, separated by distance D
  - The antennas have no special position or orientation upon the Earth, but the baseline is stationary with respect to position and orientation
    - the antennas themselves may move around to look at different regions of the sky
  - We consider only a quasi-monochromatic receiver system sensitive to a small range of frequencies centred on ν_0
  - The receiver only sees one of the two orthogonal polarisations
  - There is no frequency mixing – we are considering an "RF" interferometer

One Dimensional Interferometry



Interferometers generally track the target source as the Earth rotates. We call the angle the source makes with the baseline (θ_0) the "phase reference position".

We adjust τ_i to match the geometric delay due to the baseline length:

$$\tau = \tau_g(\theta_0) - \tau_i = 0$$

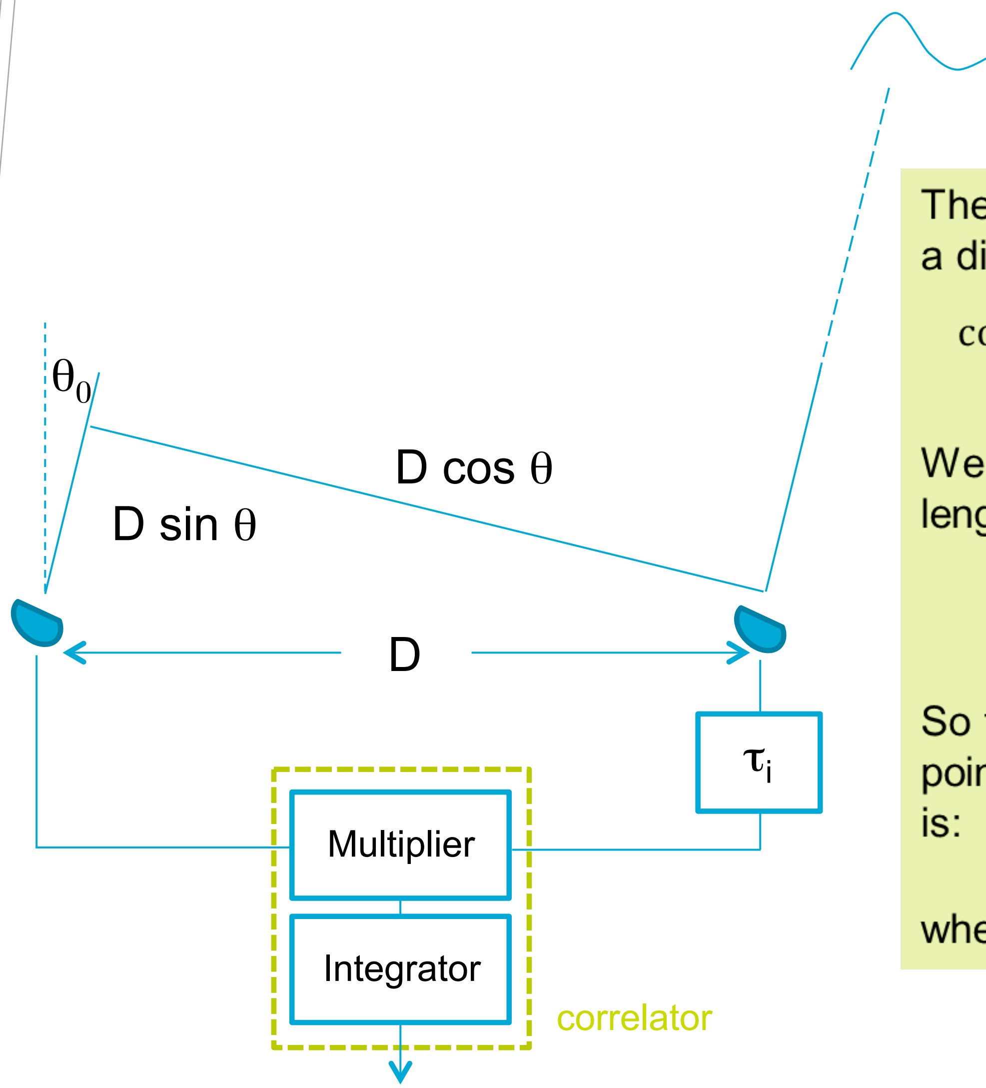
The fringe response term for radiation from a direction $(\theta_0 - \Delta\theta)$ (small angle) is:

$$\cos(2\pi\nu_0\tau) \approx \cos[2\pi\nu_0 \left(\frac{D}{c}\right) \sin\Delta\theta \cos\theta_0]$$

We identify the component of the baseline length normal to the reference position:

$$u = \frac{\nu_0 D \cos\theta_0}{c}$$

One Dimensional Interferometry



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We identify the component of the baseline length normal to the reference position:

$$u = \frac{\nu_0 D\cos\theta_0}{c}$$

So the interferometer fringe response to a point source at some position $\theta = \theta_0 + \Delta\theta$ is:

$$F(l) = \cos(2\pi\nu_0\tau) = \cos(2\pi ul)$$

where $l = \sin\Delta\theta$

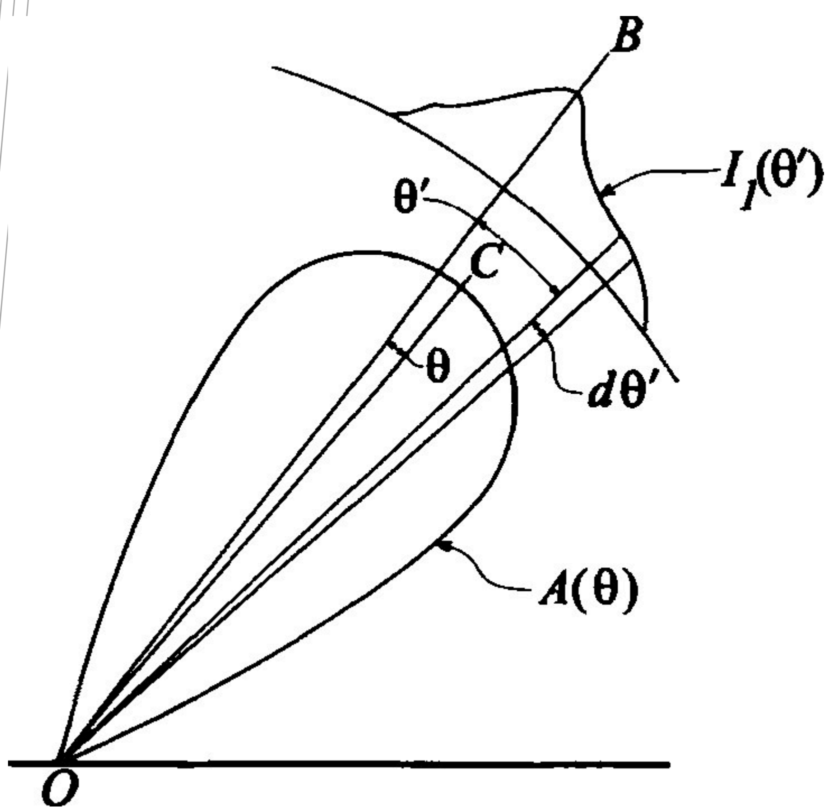
Some Important Assumptions

- This relation only holds if the emission we observe in the sky is **spatially incoherent.**
  - This holds only if we can think of the objects in the sky as collections of independent point sources with varying intensities and phases.
  - In most scenarios, this assumption is valid.

- The time delay between the two antennas is D/c sin θ.
  - This holds only if the incoming waves are planar.
  - We call this the "far field".

The Interferometer Response

- To go beyond just the fringe response, we make the interferometer look at some source in the sky.



The total output power from each antenna is a cross-correlation of the antenna reception pattern $A(\theta)$ with the intensity distribution of the source $I_1(\theta')$. This is proportional to:

$$\int_{source} A(\theta - \theta') I_1(\theta') d\theta'$$

Moving to an interferometer, the $A(\theta)$ gets replaced with the interferometer response:

$$R(l) = \int_{source} \cos[2\pi u(l - l')] A(l') I_1(l') dl'$$

This is a *convolution* integral, so:

$$R(l) = \cos(2\pi ul) * [A(l) I_1(l)]$$

The Fourier Transform

- We'll get to more complex analysis of Fourier transforms in a little while, but for now, all we need to know is that a convolution of two functions f and g is equivalent to the multiplication of the two quantities F and G, where F is the Fourier transform of f, and similarly for G and g.

$$f * g \Leftrightarrow FG$$

- Fourier transforms reveal the frequency of the components in a function
  - The interferometer response: $R(l) \Leftrightarrow r(u)$
  - The source intensity distribution: $I_1(l) \Leftrightarrow V(u)$
    - Here, $V(u)$ is the visibility function

The Interferometer Response

- Let's transform the interferometer response:

$$R(l) = \cos(2\pi ul) * [A(l)I_1(l)]$$

- The Fourier transform equation is:

$$F(u) = \int_{-\infty}^{\infty} f(x)e^{-i2\pi xu}\, dx$$

- The fringe response FT can be looked up in a table (as can most FTs!) for a value $u = u_0$

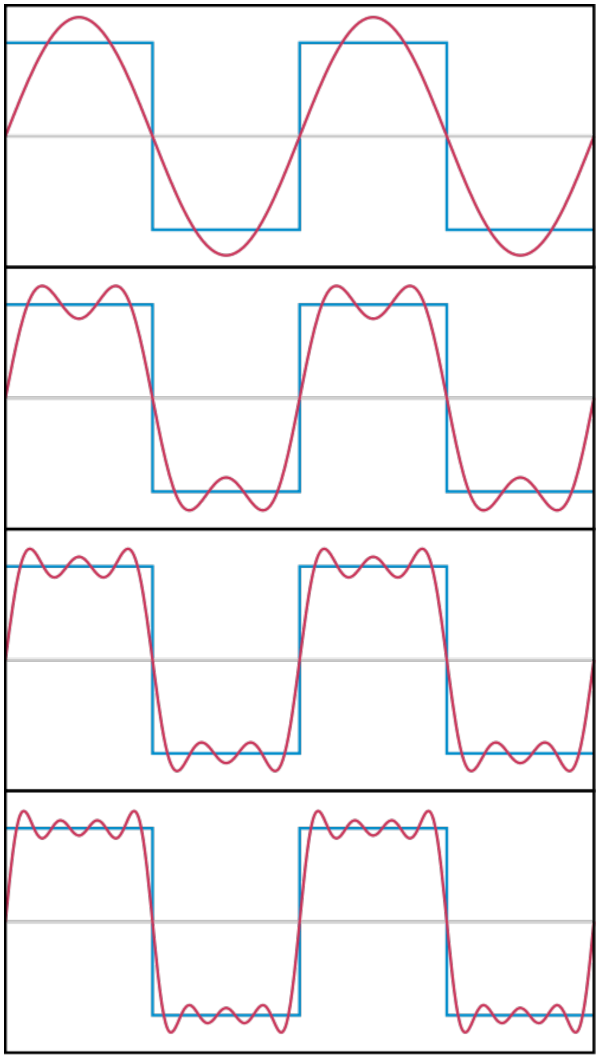
$$\cos(2\pi u_0 l) \Leftrightarrow \frac{1}{2}\,[\delta(u + u_0) + \delta(u - u_0)]$$

- We disregard $A(l)$ by considering a source that is small compared to the antenna beamsize, so:

$$R(l) \Leftrightarrow r(u) = \frac{1}{2}[V(-u_0)\delta(u + u_0) + V(u_0)\delta(u - u_0)]$$

$$\delta(x) = \begin{cases} 0, & x \neq 0 \\ 1, & x = 0 \end{cases}$$ is the Dirac delta function

The Fourier Series



- In general, any function can be generated from a number of sinusoidal components with different periods, amplitudes and phases; this is the Fourier series.
    - The Fourier transform takes some function and tells you the amplitudes and phases of each sinusoidal component as a function of period.

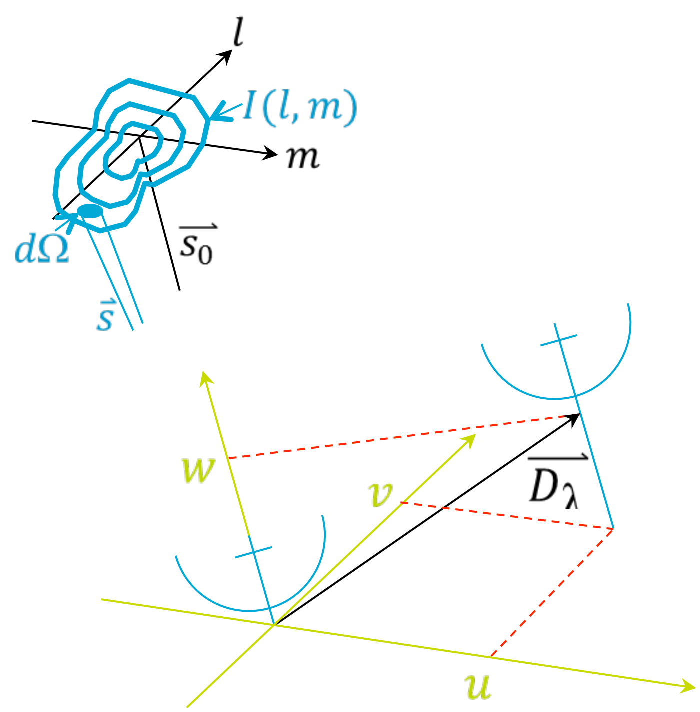
The Interferometer Response

- $$R(l) \Leftrightarrow r(u) = \frac{1}{2}[V(-u_0)\delta(u+u_0) + V(u_0)\delta(u-u_0)]$$
- What does this mean?
  - The visibility function $V(u)$ is the spatial frequency representation of the intensity distribution $I(l)$; it gives the amplitude and phase of each sinusoidal component in the distribution
    - While the radiation pattern $I(l)$ is a real quantity, its Fourier transform to $V(u)$ creates a complex visibility
  - At any particular instant, the interferometer has only a single value of $u = u_0$
  - This interferometer is thus only sensitive to spatial frequencies $\pm u_0$

The interferometer is a spatial frequency filter.

Another Dimension

- The sky has two dimensions, so we need to expand our theory!
  - We introduce the uv plane, which is always normal to the direction of the interferometer's phase tracking centre
  - We introduce a third coordinate w that is perpendicular to this plane and points along the phase tracking vector



Phase tracking centre = $\vec{s_0}$

Component direction = $\vec{s}$

Baseline vector = $\overrightarrow{D_\lambda}$

$$\overrightarrow{D_\lambda} \cdot \vec{s_0} = w$$

$$\overrightarrow{D_\lambda} \cdot \vec{s} = (ul + vm + w\sqrt{1 - l^2 - m^2})$$

$$d\Omega = \frac{dl\, dm}{\sqrt{1 - l^2 - m^2}}$$

van Cittert-Zernike Theorem

- The interferometry visibility function:

$$V(u,v,w) = \iint_{-\infty}^{\infty} A_N(l,m) I(l,m) e^{-i2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} \frac{dl\, dm}{\sqrt{1-l^2-m^2}}$$

If you can satisfy (by keeping the synthesized field small)

$$\left(\sqrt{1-l^2-m^2}-1\right)w \approx -\frac{1}{2}(l^2+m^2)w$$

This reduces to

$$V(u,v,w) \approx V(u,v,0) = \iint_{-\infty}^{\infty} \frac{A_N(l,m) I(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm)} dl\, dm$$

van Cittert-Zernike Theorem

- $$V(u,v,w) \approx V(u,v,0) = \iint_{-\infty}^{\infty} \frac{A_N(l,m)I(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm)} dl\, dm$$

Taking the inverse transform:

$$\frac{A_N(l,m)I(l,m)}{\sqrt{1-l^2-m^2}} = \iint_{-\infty}^{\infty} V(u,v) e^{i2\pi(ul+vm)} du\, dv$$

This is the van Cittert-Zernike Theorem, and it states that the intensity distribution of the incoming radiation (which is what we wish to know) is the Fourier transform of the "spatial coherence" function (which we can approximate with measurements by interferometers).

Correlator Theory

- An interferometer is usually hooked up to a correlator, which is there to split the data up into frequency bins (for spectral line work) and to integrate the data (to reduce the data volume). Let's look at how the correlator works.
- The response of the correlator can be written as:

$$r(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} V_1(t) V_2^*(t - \tau) dt$$

where:

T is integration time (much longer than the frequency and 1/bandwidth)

V_n is the voltage coming from the antenna n

- This is a "cross-correlation" of the power coming from the two antennas in the interferometer

Correlator Theory

- $$V_1(t) \star V_2(t) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} V_1(t) V_2^*(t-\tau)\,dt$$

To turn this into a convolution integral, we need to define

$$V_{2-}(t) = V_2(-t)$$

Then:

$$V_1(t) \star V_2(t) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} V_1(t) V_{2-}^*(\tau - t)\,dt = V_1(t) * V_{2-}^*(t)$$

From the convolution theorem:

$$V_1(t) \star V_2(t) \Leftrightarrow \widehat{V_1}(\nu)\widehat{V_2^*}(\nu)$$

This "cross power spectrum" is a function of frequency.

By using the correlator to integrate the product of two antenna voltages, we can create a cross-correlation spectrum as a function of frequency with Fourier transforms. This is how XF correlators (or lag correlators) work.

Back to reality

- One interferometer is not enough to go backwards from the spatial coherence function to the incoming radiation intensity distribution.
  - We need some ways to increase the number of uv samples; the more information we have about the spatial frequencies in the field, the better we can approximate reality.
- First improvement: build more antennas!
  - The number of baselines (or separate interferometers if you like) is

$$n_A(n_A - 1)/2,$$

where n_A is the number of antenna
  - It is best to randomly space the antenna to make it less likely that the same uv spacing will be sampled twice

And the world keeps turning

- Perhaps the most useful way of getting more uv coverage is to observe for a long time while the Earth rotates.
  - Since u and v are functions of the phase reference position, they will change as the Earth rotates.
  - Both u and v are spatial frequencies, and as we saw before, an interferometer is a filter that selects out both $\pm u$ and $\pm v$ in a single "snapshot". Because of this relationship, we need only to get an observation for half the rotation, ie. 12 hours.

- Extra bandwidth also helps
  - Since u and v are measured in wavelengths, the same real baseline length will give different uv points for different frequencies.
  - More bandwidth and the shape of the bandpass have an effect on the visibilities (FTs respond to edges), but we won't go into that now – see a textbook!

Array Design

- Let's look at some real arrays now, and their properties
- The Australia Telescope Compact Array (ATCA) near Narrabri is a six-element configurable interferometer array

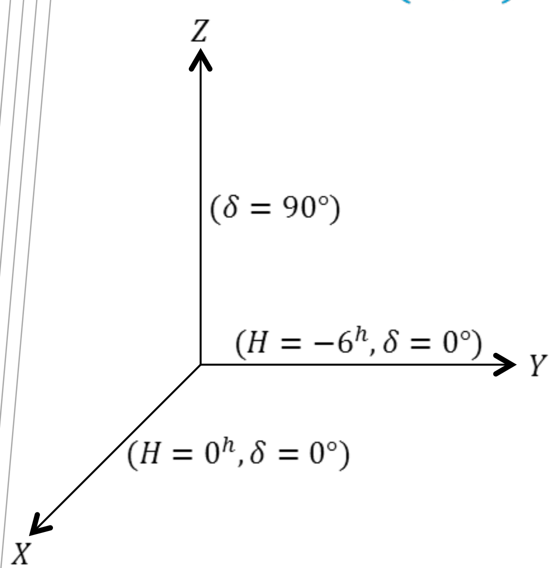
- In most configurations it has baselines that have only East-West alignments, with no components North-South – why is this?
  - We saw before that to keep w out of the visibility equation, we needed to restrict the synthesized field size; for EW arrays, there is another way to do this without such a restriction

East-West Arrays

- There is another restriction that affects EW arrays.
- Let's look at the uvw coordinate system more closely:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\cos H & \cos\delta \\ \cos\delta & -\cos\delta\sin H & \sin\delta \end{bmatrix} \begin{bmatrix} X_\lambda \\ Y_\lambda \\ Z_\lambda \end{bmatrix},$$

where (H, δ) are the hour angle and declination of the phase reference position.



Rotating the coordinate system to keep $w = 0$, we get:

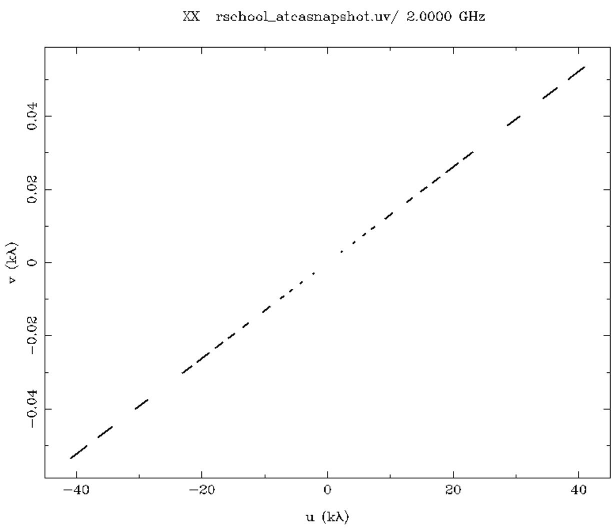
$$u' = u \,,\, v' = v \; cosec \; \delta$$

Which, after Fourier transform to (l', m') becomes:

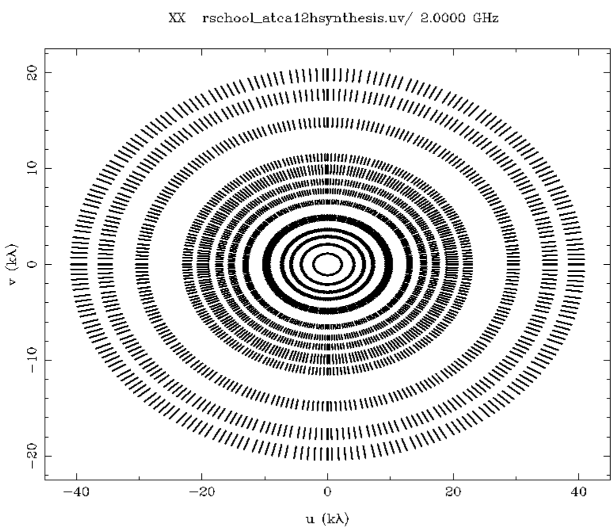
$$l' = l, m' = m \sin\delta_0$$

This means that East-West arrays have a constant response to sources where $\delta = 0$ (ie. on the celestial equator) – so Earth rotation does not give us more information!

ATCA uv coverage examples

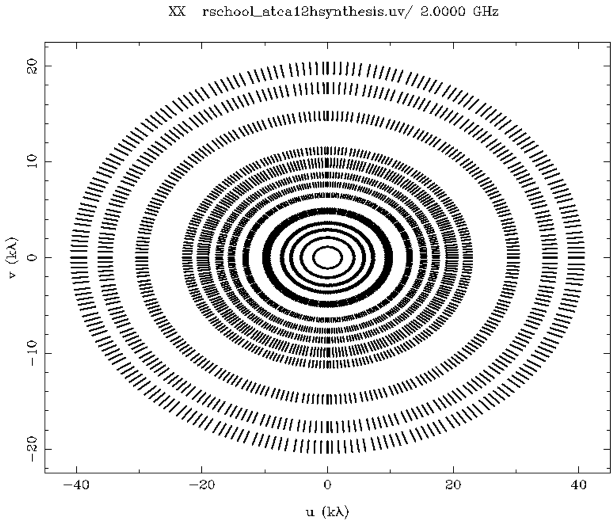


This is an observation over a small time range with the 6km array configuration for a source at $\delta = -30°$, central frequency 2.0 GHz, bandwidth 128 MHz.



The same parameters as above, but with 12 hours Earth rotation, fills in the uv plane.

ATCA uv coverage examples



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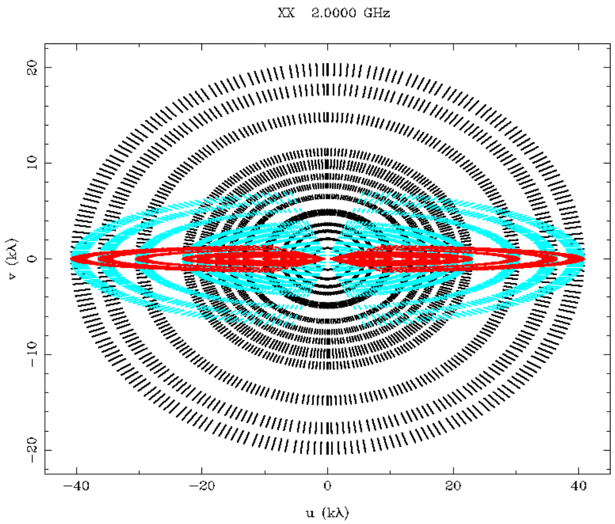
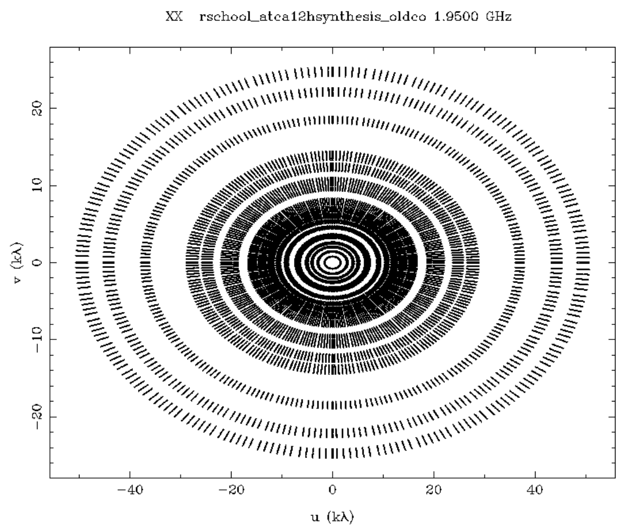
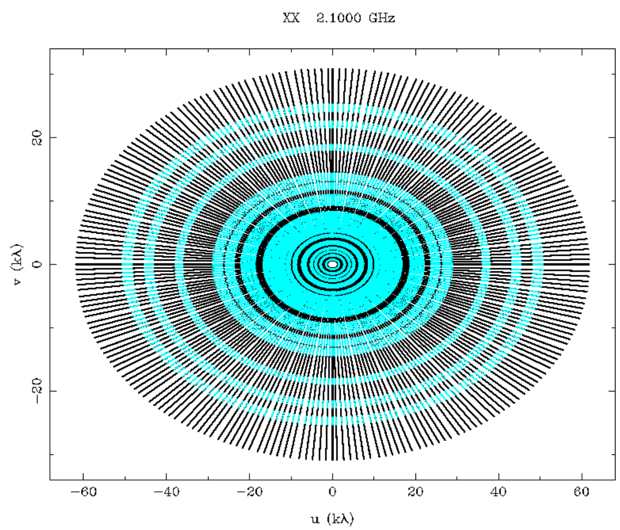


The same parameters as above, but now the blue tracks are for a source at $\delta = -10°$, and the red tracks are for a source at $\delta = -2°$. Note the foreshortening of the tracks in the v direction, while u is unaffected.

ATCA uv coverage examples



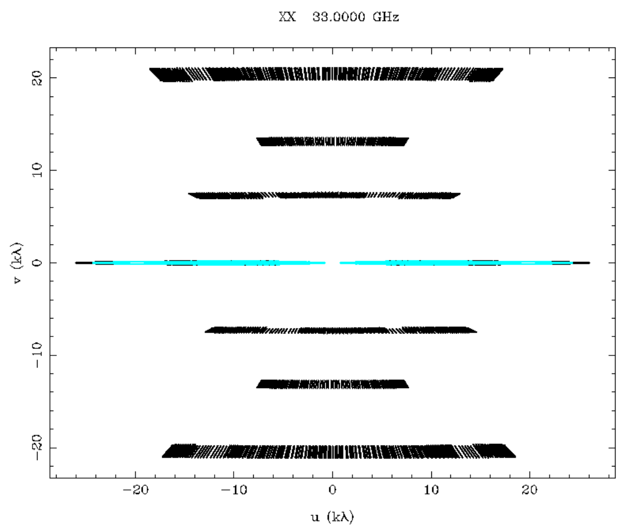
This is an observation over a 12 hour range with the 6km array configuration for a source at $\delta = -30°$, using a typical 21cm/13cm setup with the "old" ATCA correlator with a total of 256 MHz of bandwidth.



The old correlator tracks are now shown in blue, while the tracks possible at 16cm with CABB are shown in black: a total of 2 GHz of bandwidth makes a big difference!

2-dimensional arrays

- When we need to look at a source on the equator, we need a 2D array, such as an ATCA Hybrid array.
  - In the hybrid arrays, a couple of antenna are moved on the North spur.



For a source at $\delta = -0.5°$, we compare an EW214 array (blue tracks) to an H214 array (black tracks). We use a high frequency of 33 GHz and CABB bandwidths.

More antenna

- To get better coverage both instantaneously and over time, we can add more antenna.
  - VLA has 27 antennas

- ASKAP will have 36 antenna

More antenna

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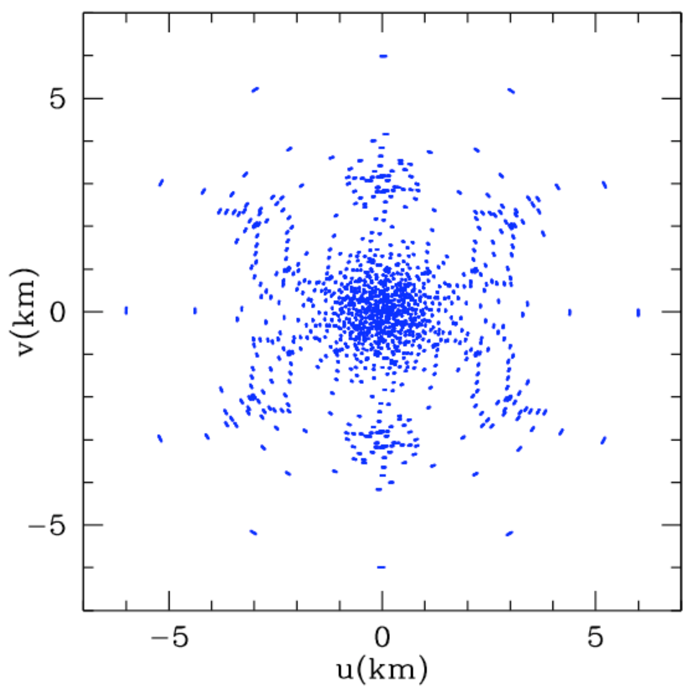
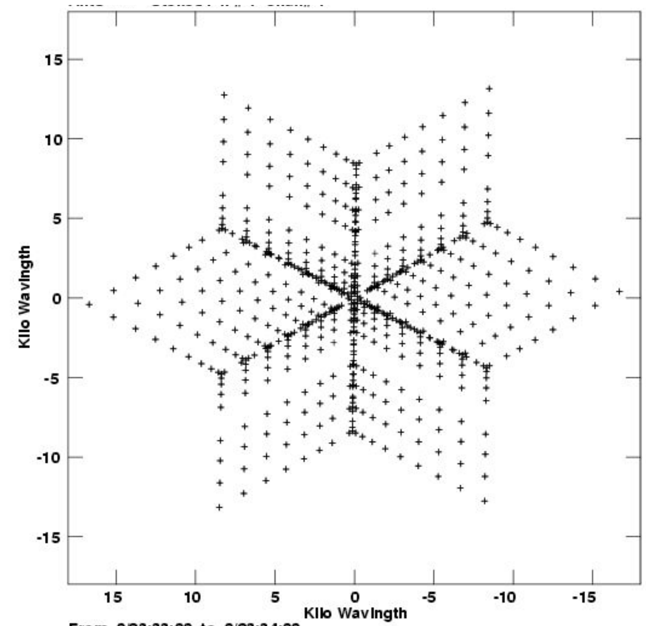
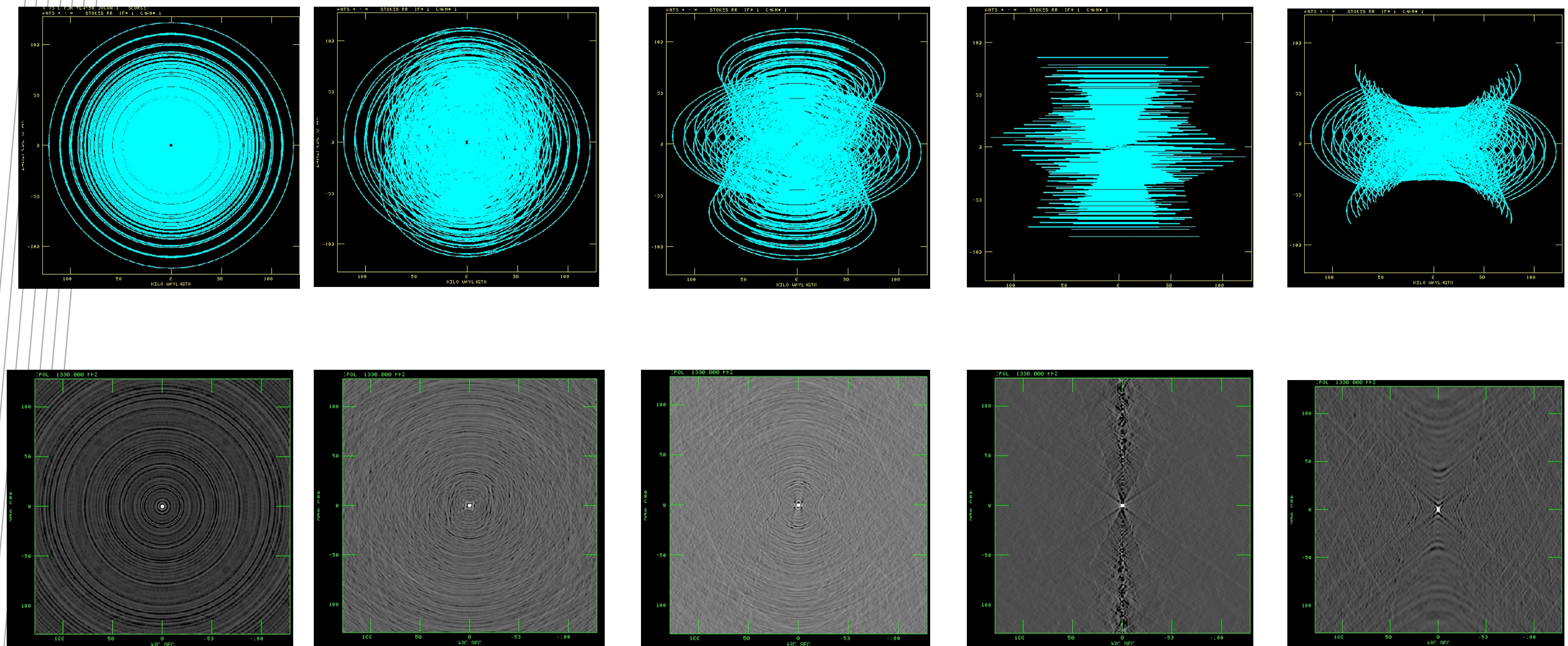
  - VLA has 27 antennas





- ASKAP will have 36 antenna

VLA Coverage and Beams



$\delta=90$ $\delta=60$ $\delta=30$ $\delta=0$ $\delta=-30$

There is a lot more!

Things left out of this talk

- Integration time limits in the correlator
- The (u', v') plane for EW arrays
- Most of the mathematical properties of the visibility
- How bandwidth and the receiving system bound the fringe function
- Frequency mixing and the phase tracking centre

To get information about these, read the textbook or ask questions!

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Thank you