

Wide-Band Imaging



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Outline :

- What is wideband imaging ?
- Two Algorithms
- Many Examples

Wide-band Imaging Sensitivity

Frequency Range :	(1 –2 GHz)	(4 –8 GHz)	(8 –12 GHz)
Bandwidth : $\nu_{max} - \nu_{min}$	1 GHz	4 GHz	4 GHz
Bandwidth Ratio : $\nu_{max} : \nu_{min}$	2 : 1	2 : 1	1.5 : 1
Fractional Bandwidth : $(\nu_{max} - \nu_{min}) / \nu_{mid}$	66%	66%	40%

Broad-band receivers increase the 'instantaneous' imaging sensitivity of an instrument

Continuum sensitivity : $\sigma_{cont} \propto \frac{T_{sys}}{\sqrt{N_{ant} (N_{ant} - 1)} \delta \tau \delta \nu}$
 (at field-center)

50 MHz \rightarrow 2 GHz Theoretical sensitivity improvement : $\sqrt{\frac{2 \text{ GHz}}{50 \text{ MHz}}} \approx 6$ times.

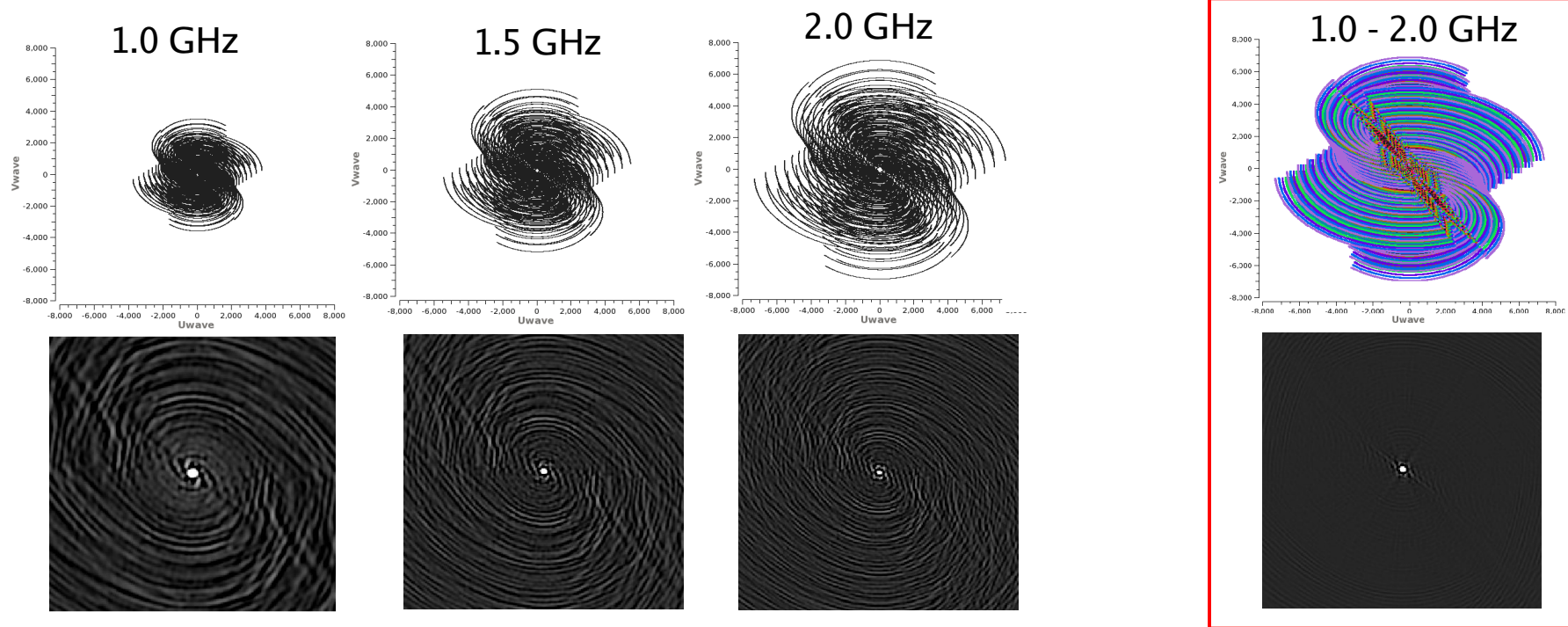
In practice, effective broadband sensitivity for imaging depends on bandpass shape, data weights, and regions of the spectrum flagged due to RFI (radio-frequency interference).

Frequency-dependent UV-coverages and PSFs

Spatial-frequency coverage and imaging properties change with frequency :

- Angular-resolution increases at higher frequencies
- Sensitivity to large scales decreases at higher frequencies
- Wideband UV-coverage has fewer gaps => lower Psf sidelobe levels

$$S(u, v)_\nu = \frac{\vec{b}}{\lambda} = \frac{\vec{b} \nu}{c}$$



Broad-band signal → Channels : Voltages measured by a broad-band receiver are split into several narrow-band channels to measure visibilities at multiple frequencies, and to allow the correct uv-coverage to be used.

The maximum channel-width for imaging, is controlled by the size of the uv grid-cells (desired field-of-view).

Frequency-dependent Sky Brightness

When the source intensity varies with frequency, different channels measure the visibility function of different sky-brightness distributions

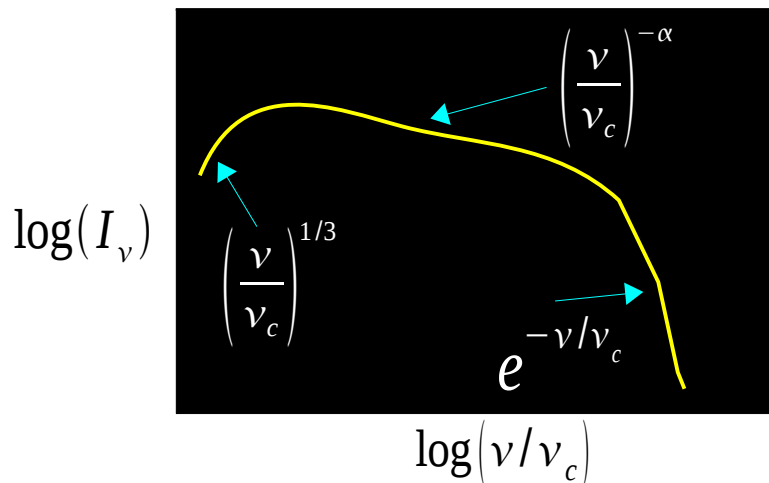
$$V(u_\nu, v_\nu) = \iint I(l, m, \nu) e^{2\pi i(u_\nu l + v_\nu m)} dl dm$$

=> Cannot apply standard imaging techniques to the combined visibilities.

(1) Each point on the source has an intrinsic spectrum :

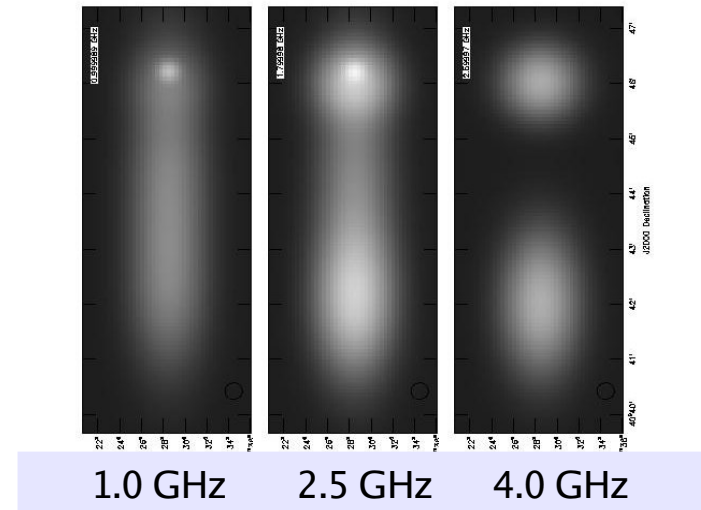
- The radio synchrotron spectrum is often a power law with varying spectral-index (spectral curvature)

$$I_\nu = I_{\nu_0} \left(\frac{\nu}{\nu_0} \right)^{\alpha + \beta \log(\nu/\nu_0)}$$

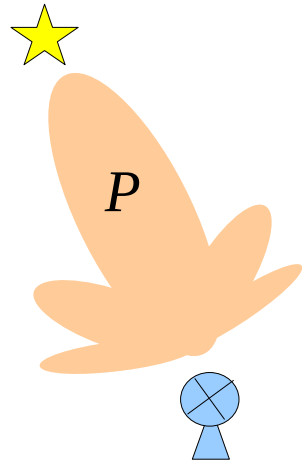


(2) Frequency probes source structure :

- Spectrum traces velocity structure (doppler-shifted line emission)
- Frequency probes depth in a 3D volume (solar flares/loops)



Frequency Dependent Antenna response (Primary-Beam)



Primary-beam scales with frequency

$$HPBW_{\nu} = \frac{\lambda}{D} = \frac{c}{\nu D}$$

Bandpass calibration does **not** correct for off-axis gains or their frequency-dependence.

The average effect in the image-domain is a multiplication by an artificial PB-spectrum

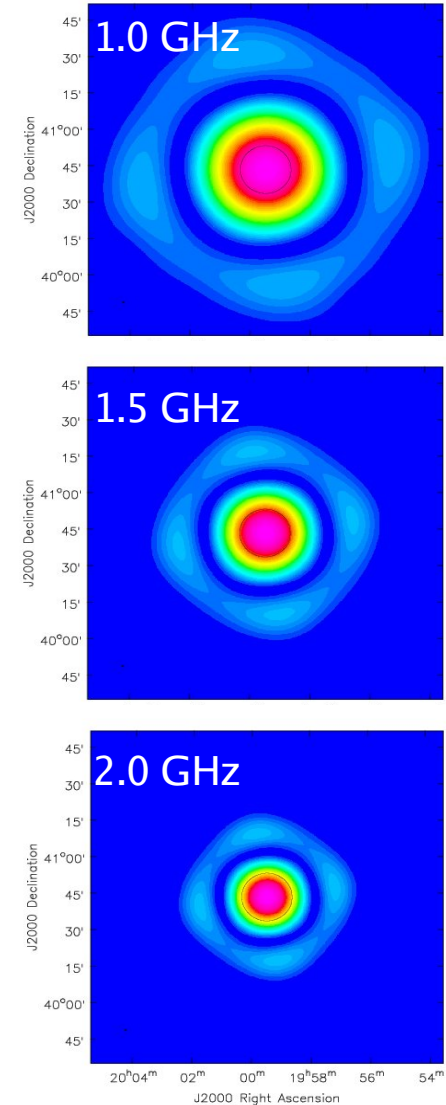
=> Away from the pointing center, the Primary Beam introduces an artificial 'spectral index' on the measured sky : $\alpha_{observed} = \alpha_{sky} + \alpha_{PB}$

About -0.4 at the PB=0.8 (6 arcmin from the center for VLA 1.5 GHz)
 About -1.4 at the HPBW (15 arcmin from the center for VLA 1.5 GHz)

Wide-field sensitivity depends on frequency.

Continuum sensitivity to a source with with a non-flat spectrum $\propto \frac{\sum_{\nu} w_{\nu} \cdot P_{\nu} \cdot I_{\nu}}{\sum_{\nu} I_{\nu}}$

EVLA Primary Beams

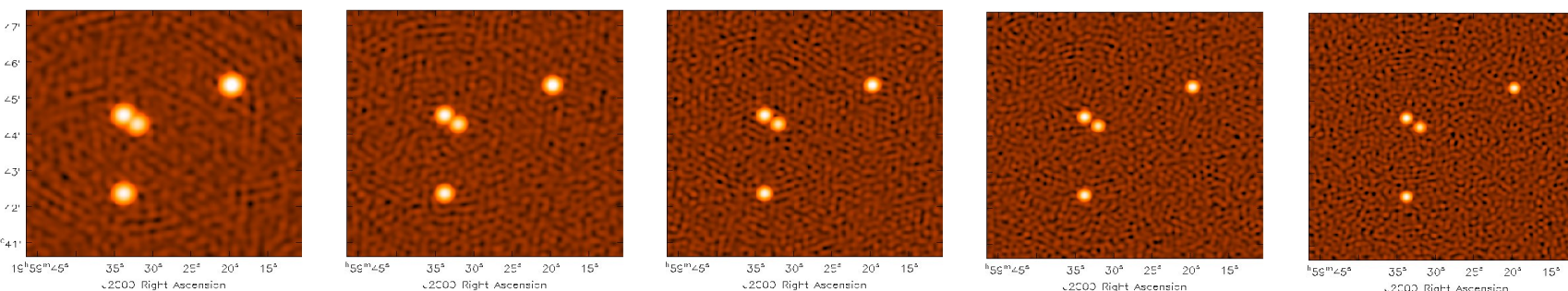


What is wide-band Imaging ?

- Use broad-band receivers to increase instantaneous continuum sensitivity
- Measure visibilities in many narrow-band channels to avoid bandwidth-smearing
- Use multi-frequency-synthesis
 - to increase the uv-coverage used in deconvolution and image-fidelity
 - to make images at the angular-resolution allowed by the highest frequency
- Account for the sky spectrum
 - by modeling and reconstructing the spectrum as well as the intensity
 - by flattening it out (bandpass self-calibration) –if there is only one source...
- Account for the frequency-dependent off-axis gains of the antennas
 - by including the PB-spectrum in the sky-spectrum model
 - by applying wide-field imaging techniques to eliminate the PB frequency dependence during imaging (gridding).

Cube (Spectral-Line) Imaging

- (1) Image and deconvolve each channel separately (add them to form a continuum image).
- (2) During image-restoration, convolve mages from all channels with a common 'restoring beam' derived from the angular-resolution allowed by the lowest-frequency.
- (3) Source spectra can be derived from the smoothed restored images (at low angular resolution)
- (4) Imaging-fidelity is limited to the single-frequency UV-coverage
 - Reconstructions may not be consistent across frequency
- (5) Imaging sensitivity is limited to the single-channel sensitivity $\sigma_{chan} = \sigma_{continuum} \sqrt{N_{chan}}$
 - Will not deconvolve sources that are below σ_{chan} but above $\sigma_{continuum}$



Example images at 5 frequencies between 1 and 2 GHz

- varying angular resolution

- good for a quick-look to assess data-quality.
- It can handle arbitrary spectra and has no spectral-model dependence.

For telescopes with good single-frequency uv-coverage, it may suffice. For sparse arrays like the ATCA, it may not..... but you can often do better (use all data together + model the spectrum....)

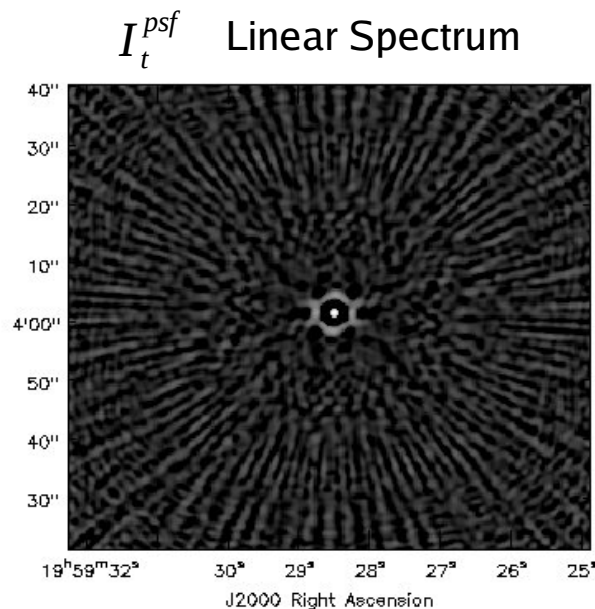
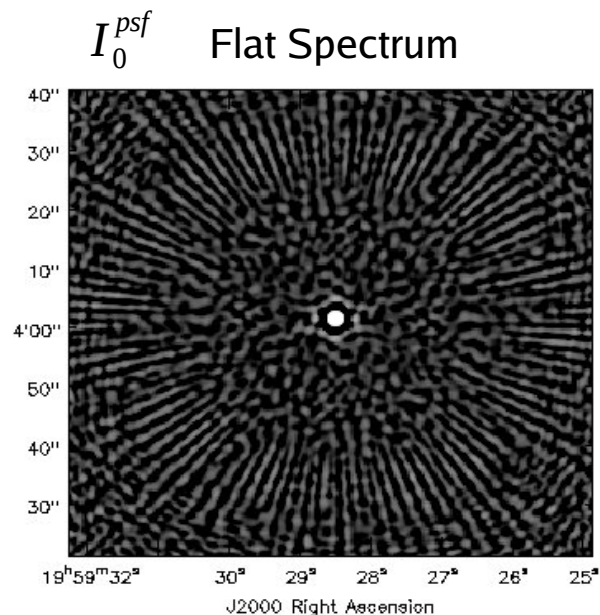
Continuum Imaging : (multi-scale) multi-frequency-synthesis

Sky Model : Collection of multi-scale flux components whose amplitudes follow a polynomial in frequency

$$I_\nu^{sky} = \sum_t I_t \left(\frac{\nu - \nu_0}{\nu_0} \right)^t \quad \text{where } I_t = \sum_s [I_s^{shp} * I_{s,t}]$$

(1) Define the instrument's response

to each term of a Taylor polynomial in frequency : $I_t^{psf} = \sum_\nu \left(\frac{\nu - \nu_0}{\nu_0} \right)^t I_\nu^{psf}$



The observed image is a sum of convolutions....

$$I^{obs} = \sum_t I_t^{psf} * I_t^{obs}$$

Follow the math for basic polynomial-fitting.....

(2) Do a joint deconvolution of ALL Taylor-PSFs (spectral PSFs) from a series of dirty-images formed as Taylor-weighted averages of individual-frequency images.

Continuum Imaging : (multi-scale) multi-frequency-synthesis

Sky Model : Collection of multi-scale flux components whose amplitudes follow a polynomial in frequency

$$I_\nu^{sky} = \sum_t I_t \left(\frac{\nu - \nu_0}{\nu_0} \right)^t \quad \text{where } I_t = \sum_s [I_s^{shp} * I_{s,t}]$$

Algorithm : Linear least squares + deconvolution

Data Products : Taylor-Coefficient images $I_0^m, I_1^m, I_2^m, \dots$ that represent the sky spectrum

Interpretation :

- As a power-law (spectral index and curvature) $I_\nu = I_{\nu_0} \left(\frac{\nu}{\nu_0} \right)^{\alpha + \beta \log(\nu/\nu_0)}$

$$I_0^m = I_{\nu_0} \quad I_1^m = I_{\nu_0} \alpha \quad I_2^m = I_{\nu_0} \left(\frac{\alpha(\alpha-1)}{2} + \beta \right)$$

- PB-correction : Model the average PB-spectrum with a Taylor-polynomial, and do a post-deconvolution Polynomial-Division

$$\frac{(I_0^m, I_1^m, I_2^m, \dots)}{(P_0, P_1, P_2, \dots)} = (I_0^{sky}, I_1^{sky}, I_2^{sky}, \dots)$$

Using Wide-Band Models for other processing....

WideBand Model : $I_0^m, I_1^m, I_2^m, \dots$

Evaluate spectrum $I_\nu^{sky} = \sum_t I_t \left(\frac{\nu - \nu_0}{\nu_0} \right)^t$

(1) Wide-Band Self-Calibration

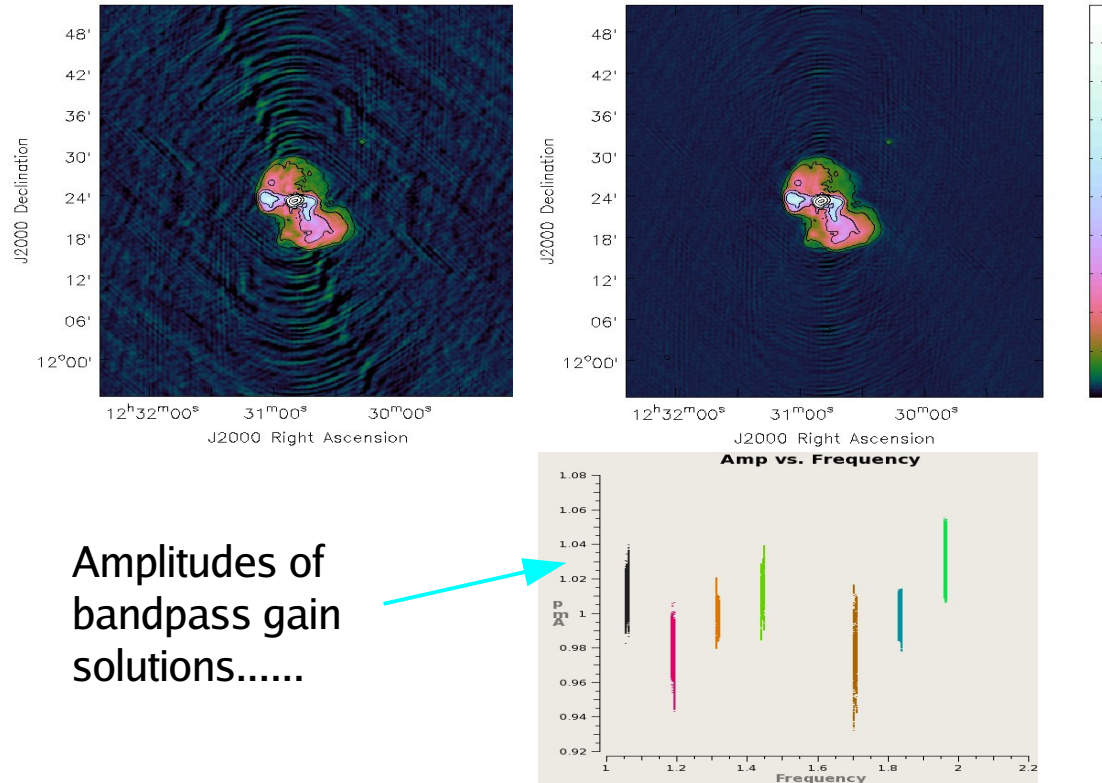
- Can be used on target source, after initial calibration per spw.
- Can use it on the calibrator itself to bootstrap the model.

(2) Continuum Subtraction

- De-select frequency channels in which your spectral-lines exist.
- Make a wide-band image model of the continuum intensity and spectra
- Predict model-visibilityes over **all** channels
- Subtract these model visibilityes from the data

(3) Combination with single-dish data

- Use Taylor-coefficient images made from single-dish images, as a starting model

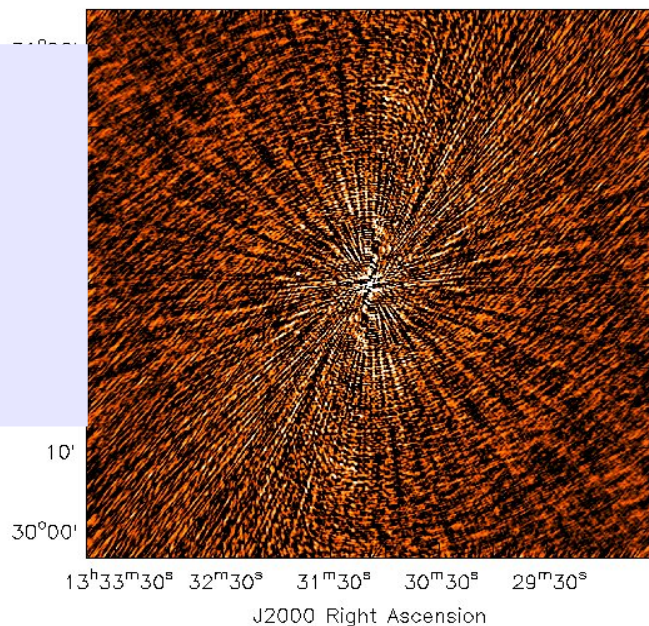


Dynamic-range with MS-MFS : 3C286 example : $N_t=1,2,3,4$

NTERMS = 1

Rms :
9 mJy -- 1 mJy

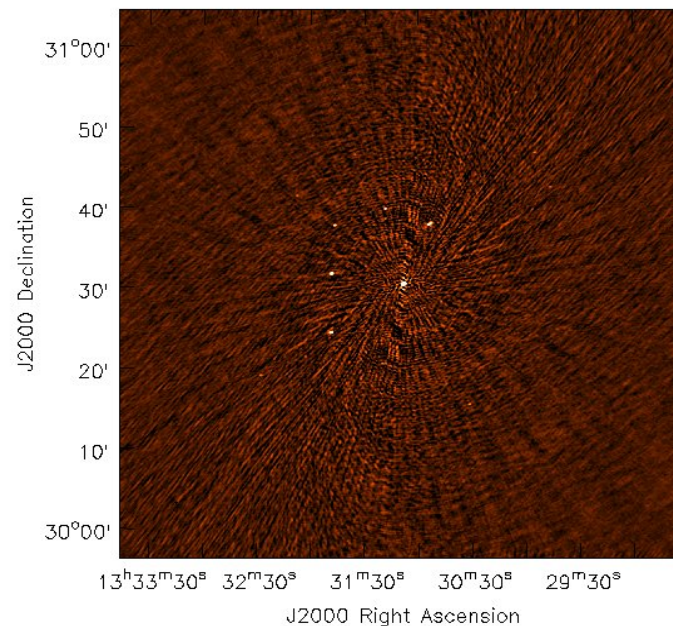
DR :
1600 -- 13000



NTERMS = 2

Rms :
1 mJy -- 0.2 mJy

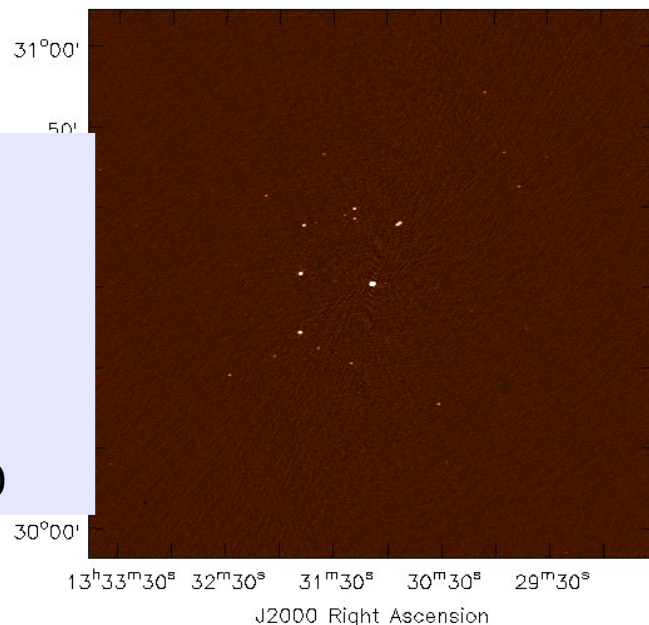
DR :
10,000 -- 17,000



NTERMS = 3

Rms :
0.2 mJy -- 85 μ Jy

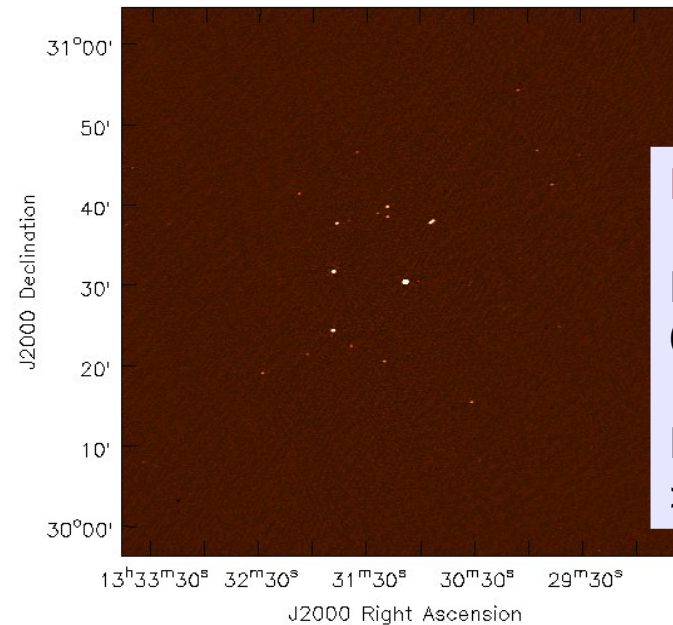
DR :
65,000 -- 170,000



NTERMS = 4

Rms
0.14 mJy -- 80 μ Jy

DR :
>110,000 -- 180,000



Errors in polynomial fitting + Imaging (empirical)

For a 1 Jy point source with spectral index of -1.0 ...

- If spectra are ignored during MFS imaging => Errors increase with bandwidth.

Dynamic-range limits for VLA uv-coverage (natural)

1-2 GHz => ~ 1000

1-3 GHz => few 100

- If spectra are modeled + High signal-to-noise => Need higher-order polynomials to fit a power-law

1 term (flat spectrum) => peak intensity error of 0.1 (on 1 Jy)

2 terms (linear spectrum) => peak intensity error of 0.02, spectral index error of 0.1

3 terms (quadratic spectrum) => intensity error of 0.0001, spectral index error of 0.05

- If spectra are modeled + Low signal-to-noise => Higher-order polynomials give more errors

The following situations give similar error on spectral index (~ 0.1) for a point source....

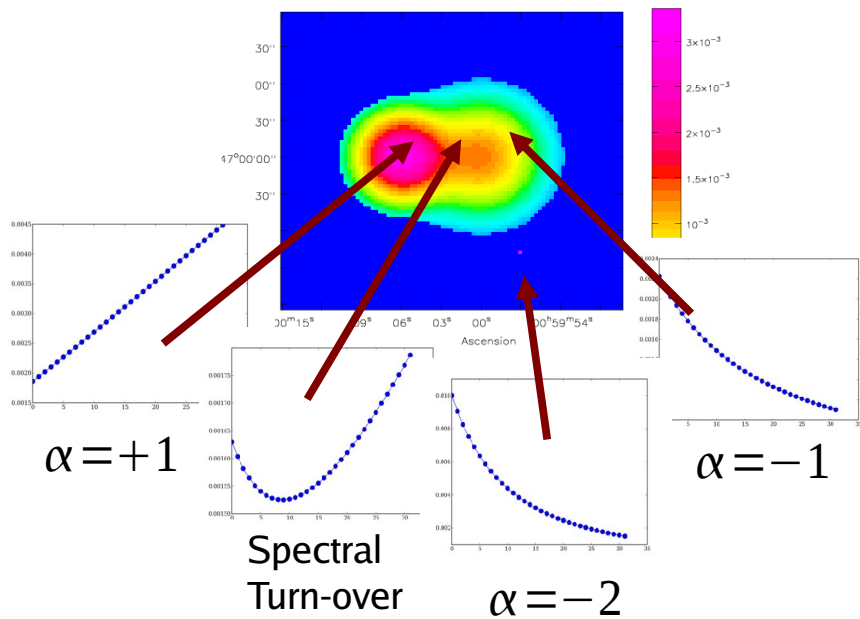
L-Band + C-Band : 1-8 GHz : Sources with signal-to-noise ratio of 10~20

L-Band only (1-2 GHz) or C-Band only (4-8 GHz) : Sources with SNR ~ 40

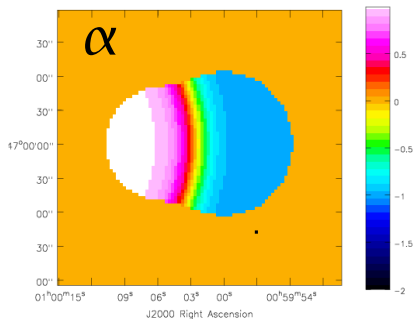
For extended emission, spectral index errors ≤ 0.2 only for SNR > 100.....

Example of wideband-imaging on extended-emission

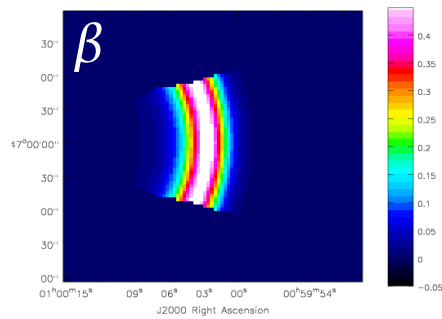
Intensity Image



Average Spectral Index

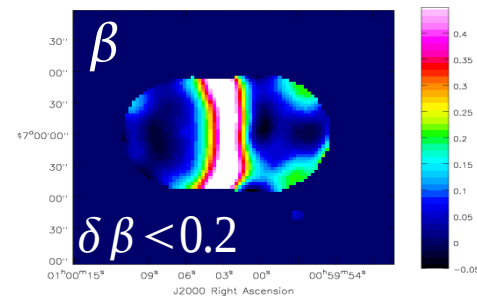
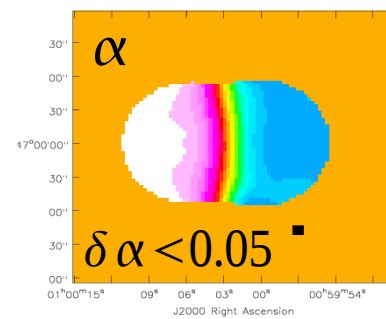
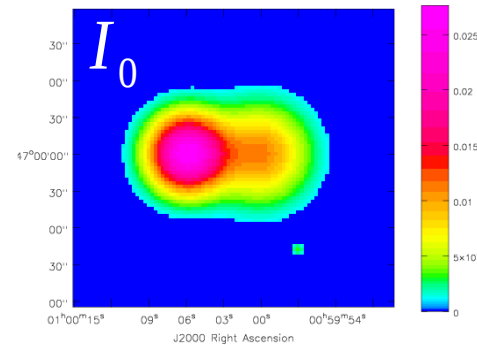


Gradient in Spectral Index

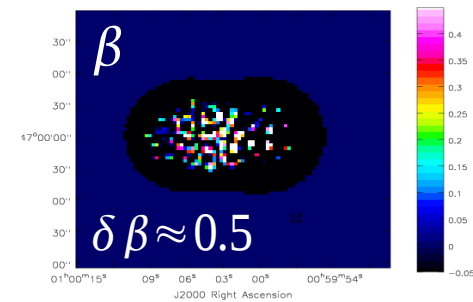
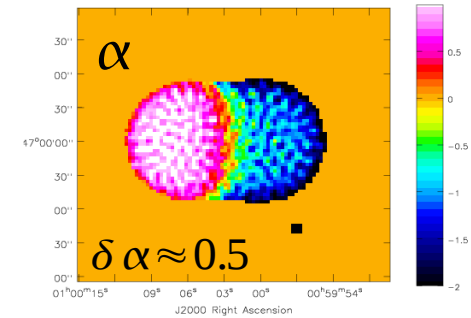
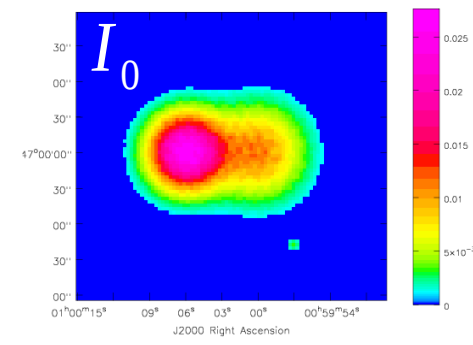


MFS

multi-scale

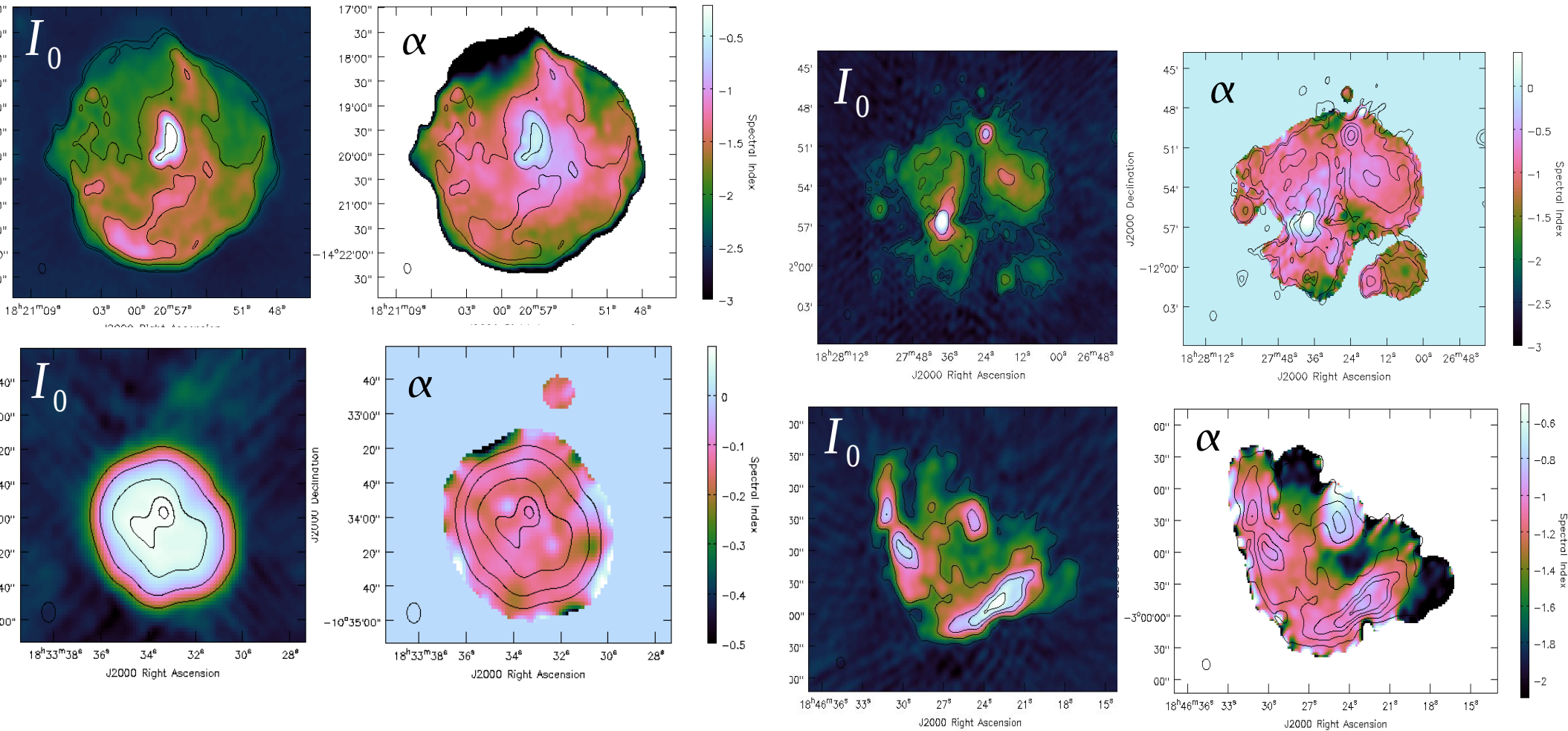


point-source



=> For extended emission - spectral-index error is dominated by 'division between noisy images'
 - a multi-scale model gives better spectral index and curvature maps

Extended emission – SNR example (a realistic expectation)



These examples used $n_{\text{terms}}=2$, and about 5 scales.

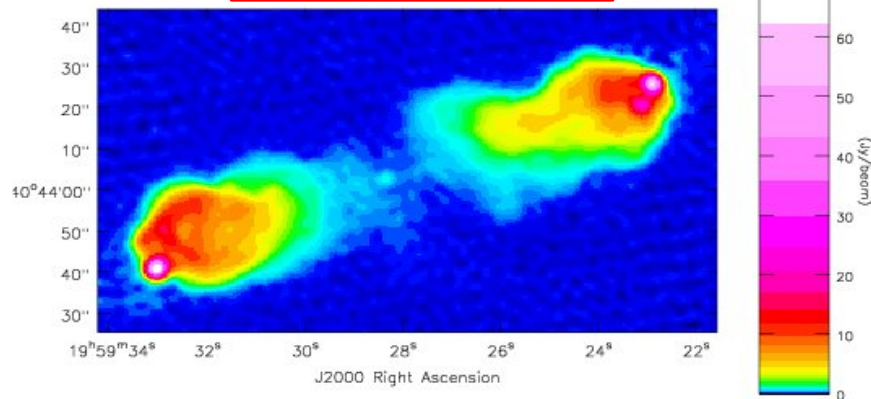
=> Within 1-2 GHz and 4-8 GHz, can tell-apart regions by their spectral-index (± 0.2) if $\text{SNR} > 100$.
 (this accuracy will increase with wider bandwidths –1-3 GHz CABB)

=> These images have a dynamic-range limit of few x 1000 ---> residuals are artifact-dominated

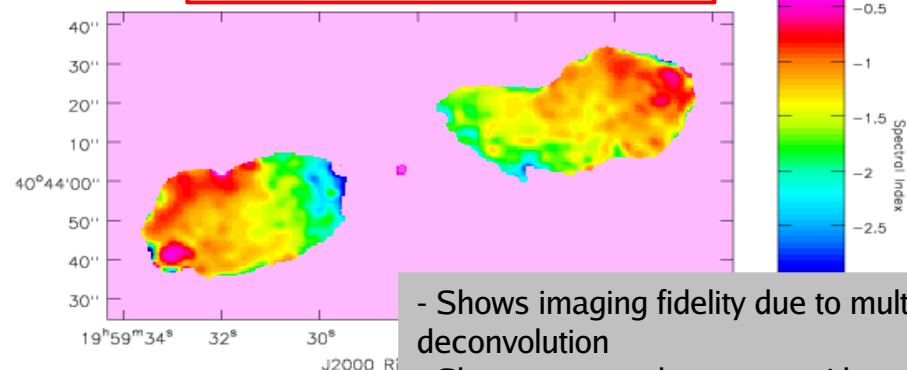
Single channel vs MFS imaging – Image Fidelity

Data : 20 VLA snapshots at 9 frequencies between 1.1 and 1.8 GHz + wide-band self-calibration

Intensity Image

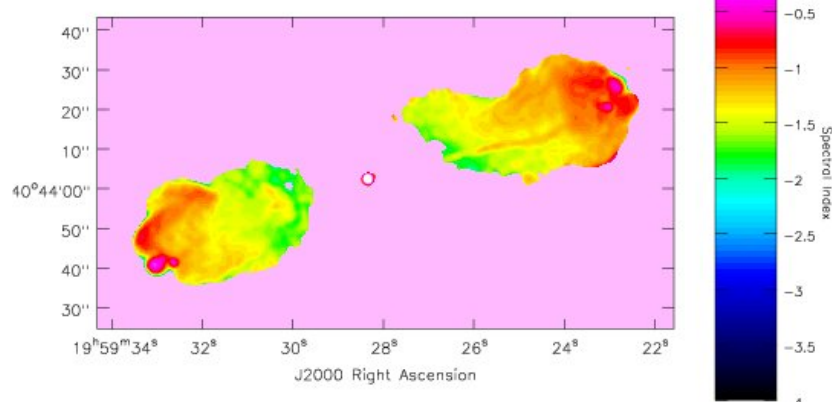


MS-MFS Spectral Index



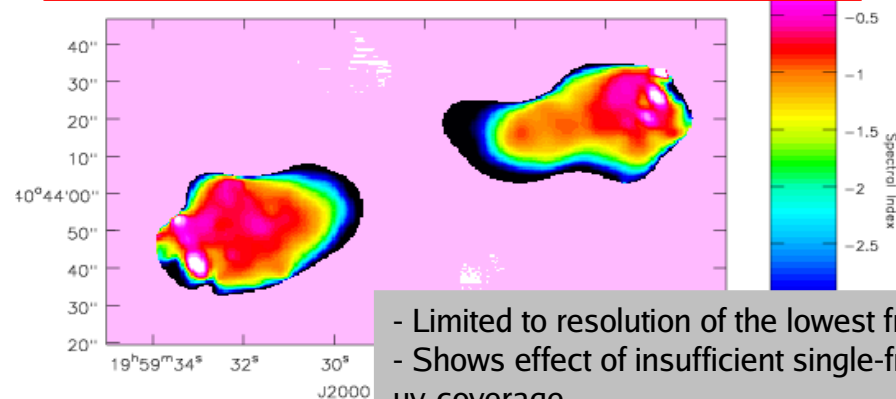
- Shows imaging fidelity due to multi-scale deconvolution
- Shows expected structure with errors < 0.2

Two-point spectrum (1.4 –4.8 GHz)



C.Carilli et al, Ap.J. 1991.
(Made from VLA A,B,C,D Config, full-synthesis runs at L and C band)

Spectral Index from single-SPW images

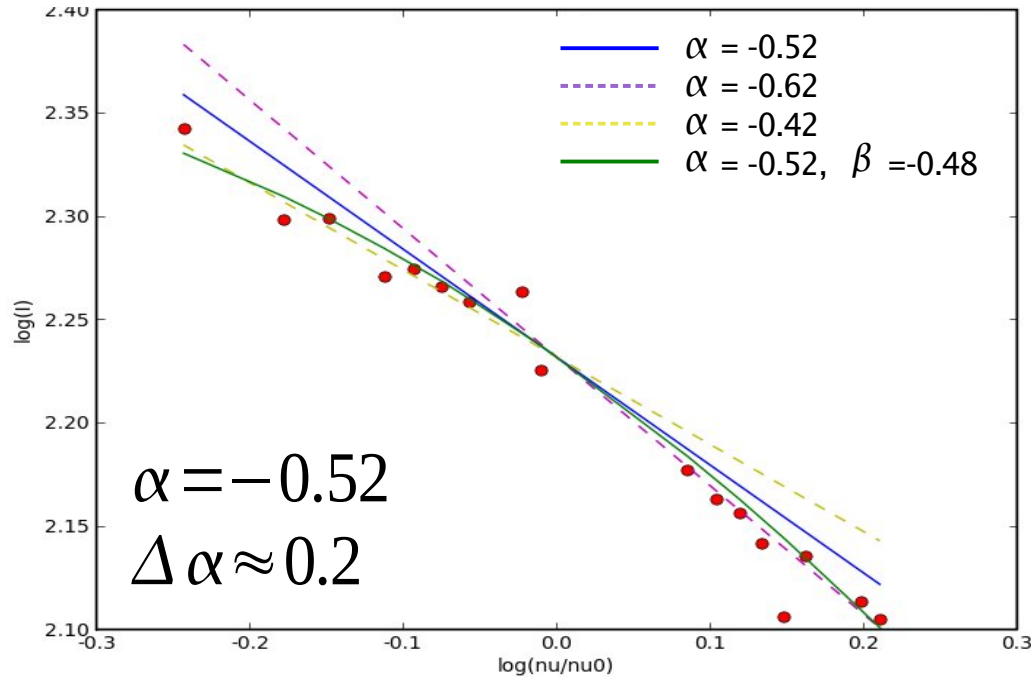


- Limited to resolution of the lowest frequency
- Shows effect of insufficient single-frequency uv-coverage

=> It helps to use the combined uv-coverage

Spectral Curvature

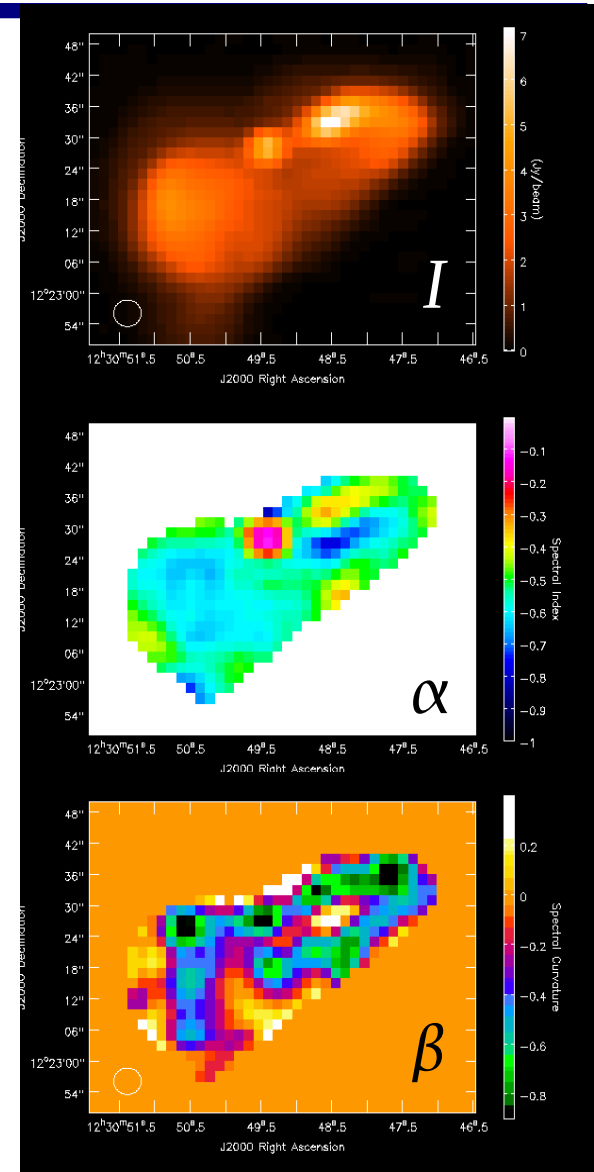
Data : 10 VLA snapshots at 16 frequencies (1.2 -2.1 GHz)



From existing P-band (327 MHz), L-band(1.42 GHz) and C-band (5.0 GHz) images of the core/jet

P-L spectral index : -0.36 ~ -0.45

L-C spectral index : -0.5 ~ -0.7

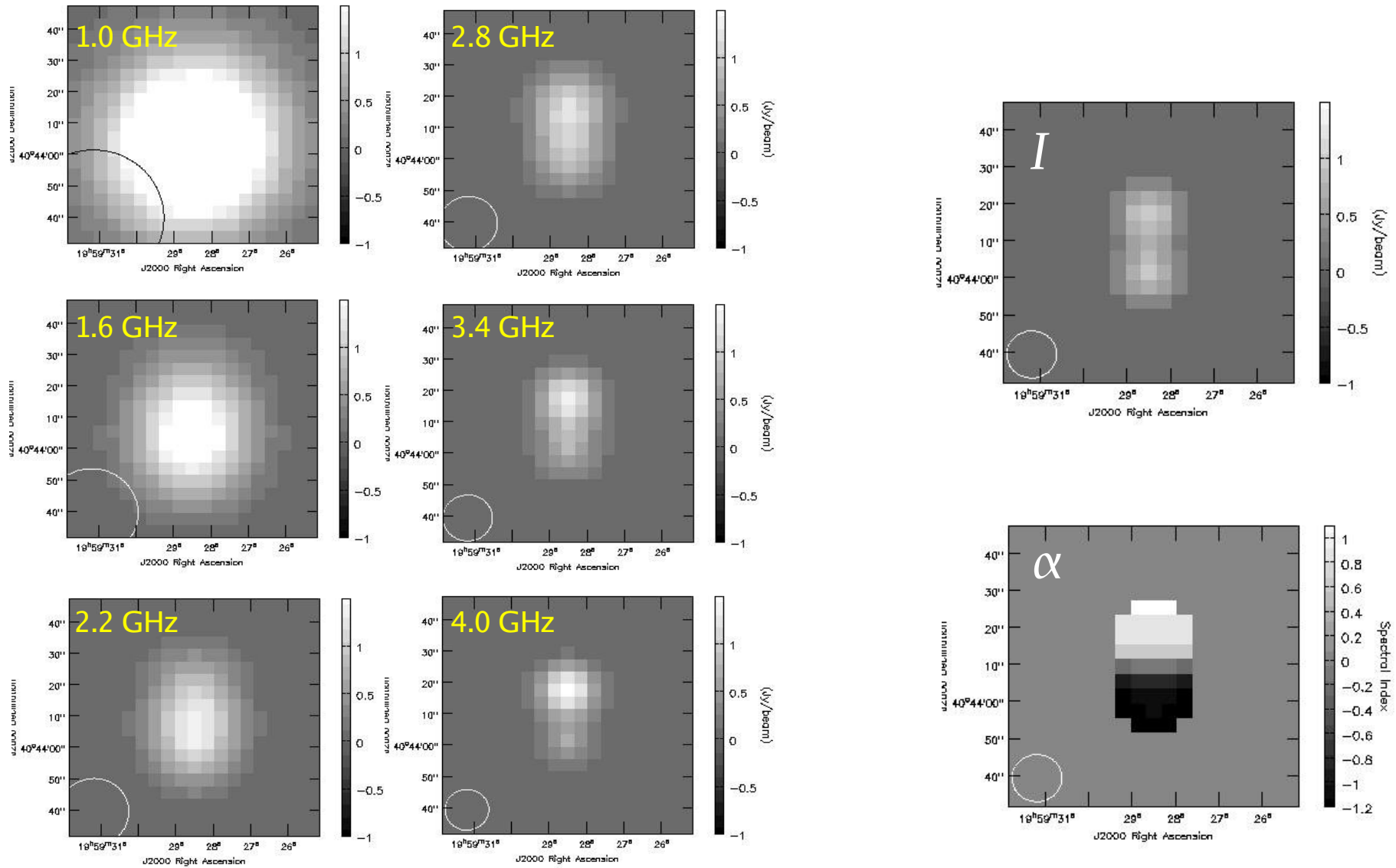


=> Need SNR > 100 to fit spectral index variation ~ 0.2 (at the 1-sigma level ...)

=> Be very careful about interpreting β

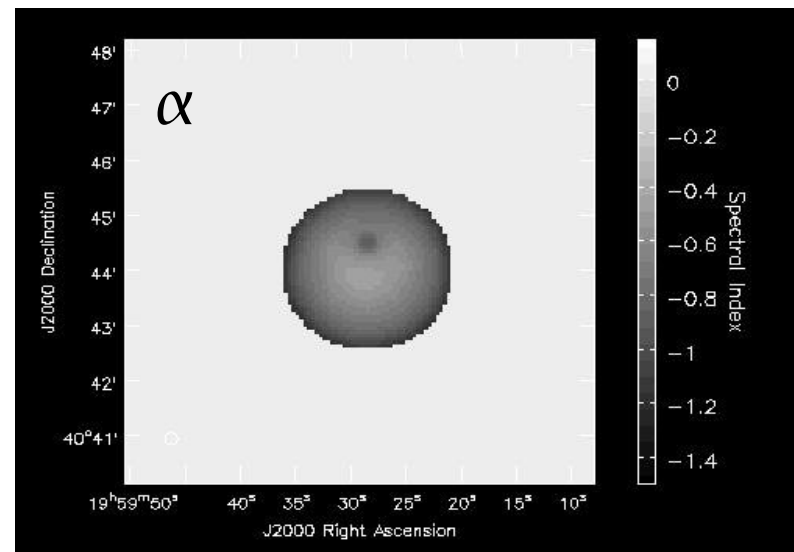
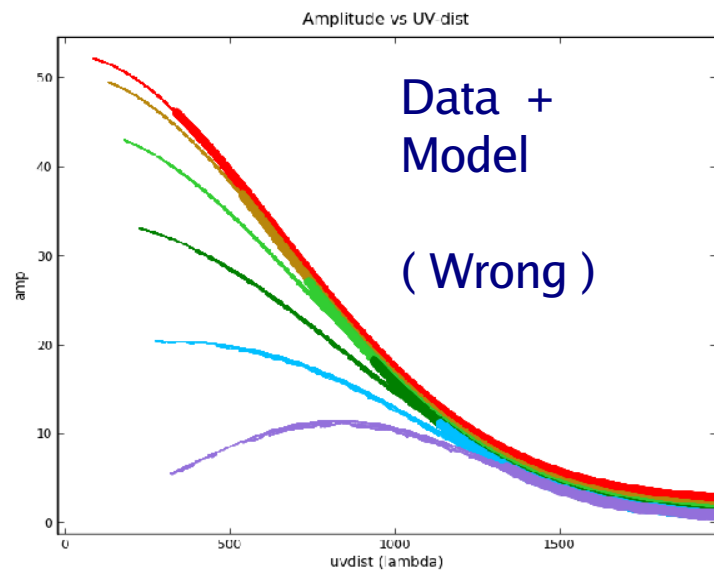
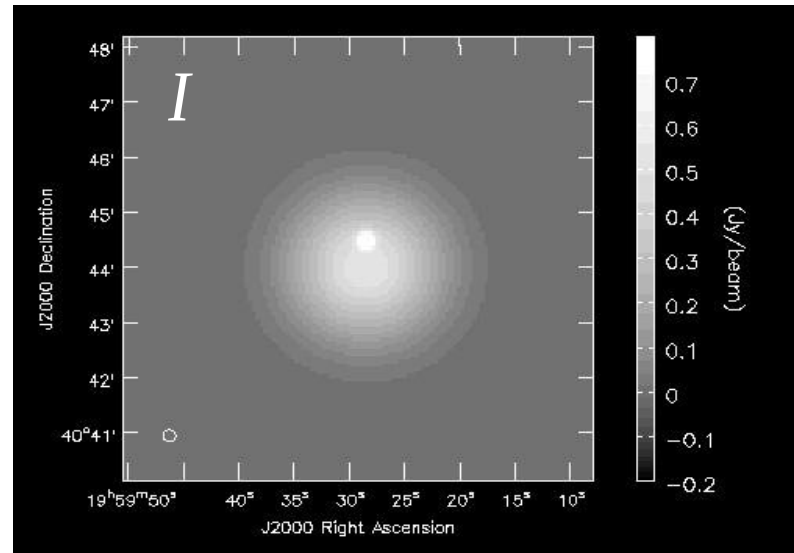
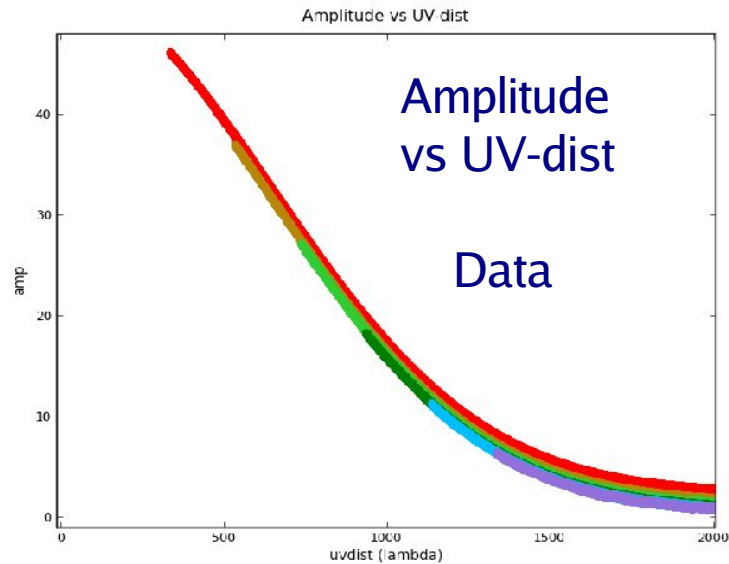
Moderately Resolved Sources + High SNR

Can reconstruct the spectrum at the angular resolution of the highest frequency (only high SNR)



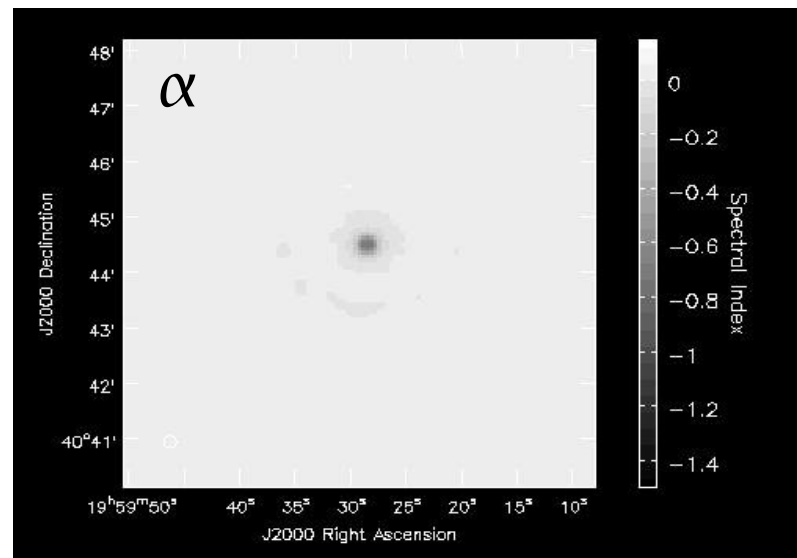
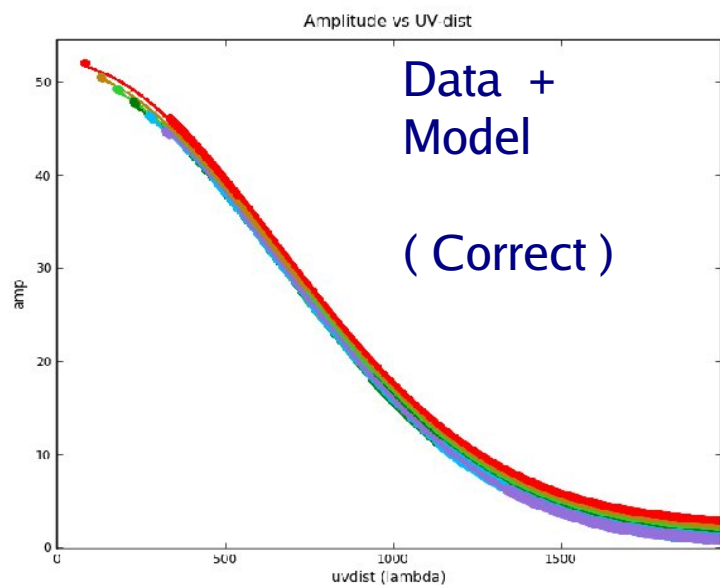
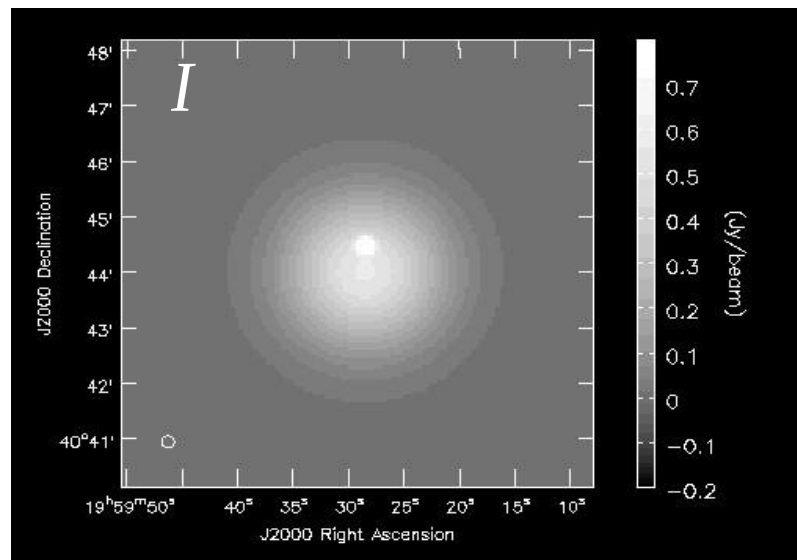
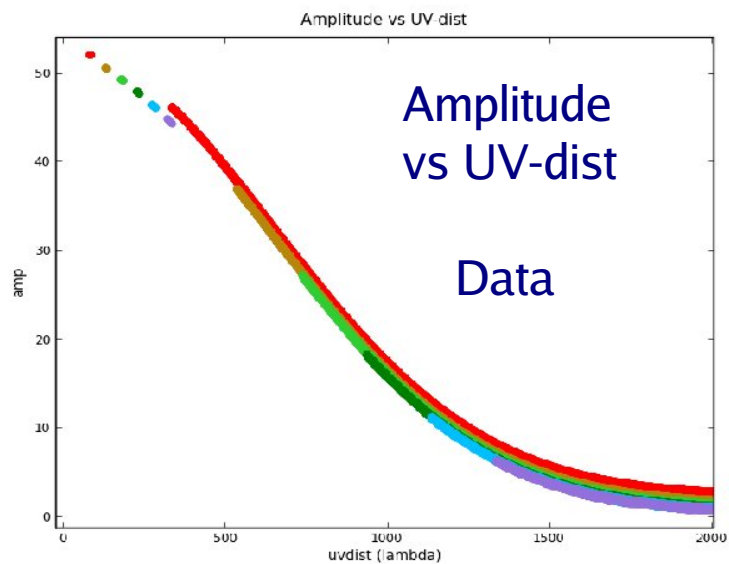
Very large spatial scales – Unconstrained spectrum

The spectrum at the largest spatial scales is NOT constrained by the data



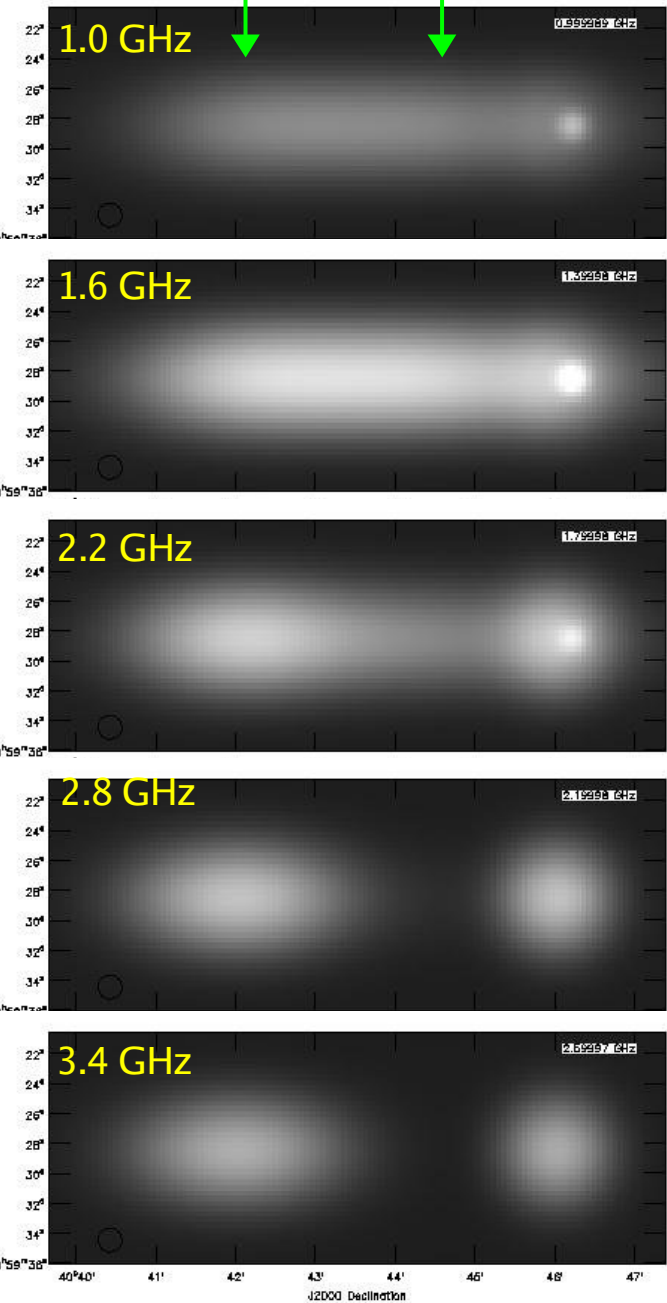
Very large spatial scales – Need additional information

External short-spacing constraints help (visibility data, or starting image model)

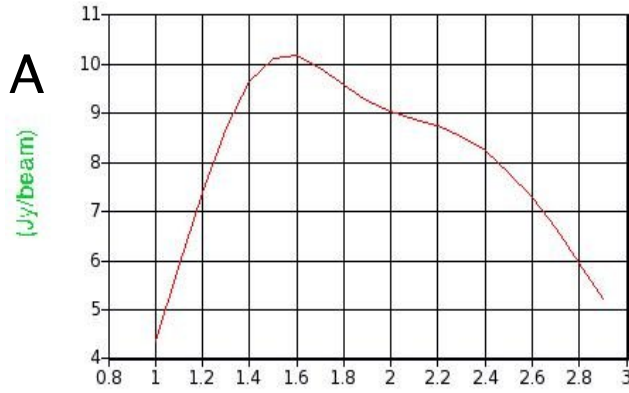


Non power-law spectra : Polynomial Spectral Fit

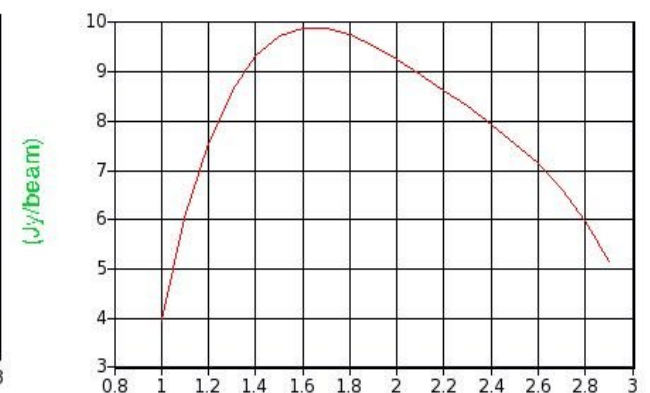
A B



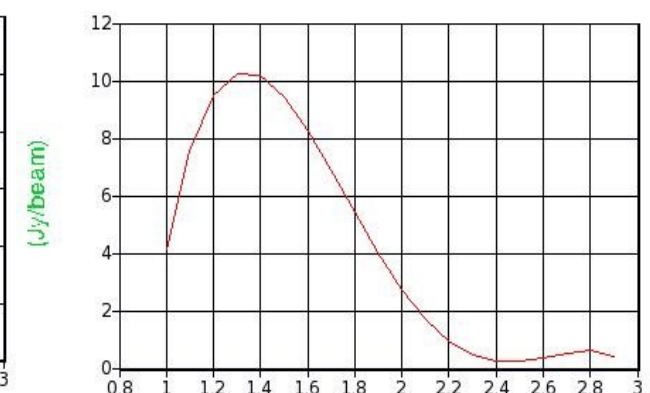
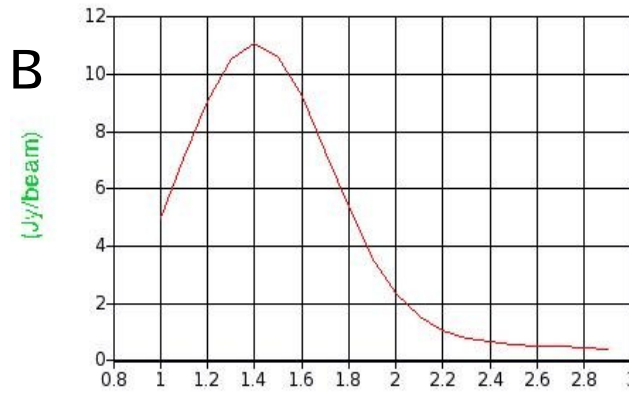
True Spectrum



Reconstructed Spectrum



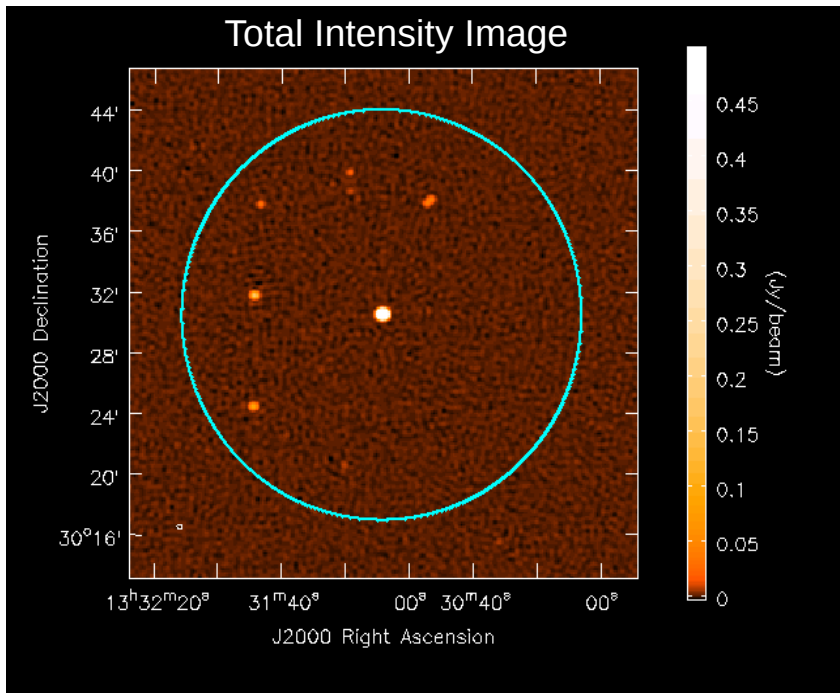
B



Angular resolution depends on the highest sampled frequency at which the emission exists.

Example of Imaging with wide-band PB (artificial spectrum)

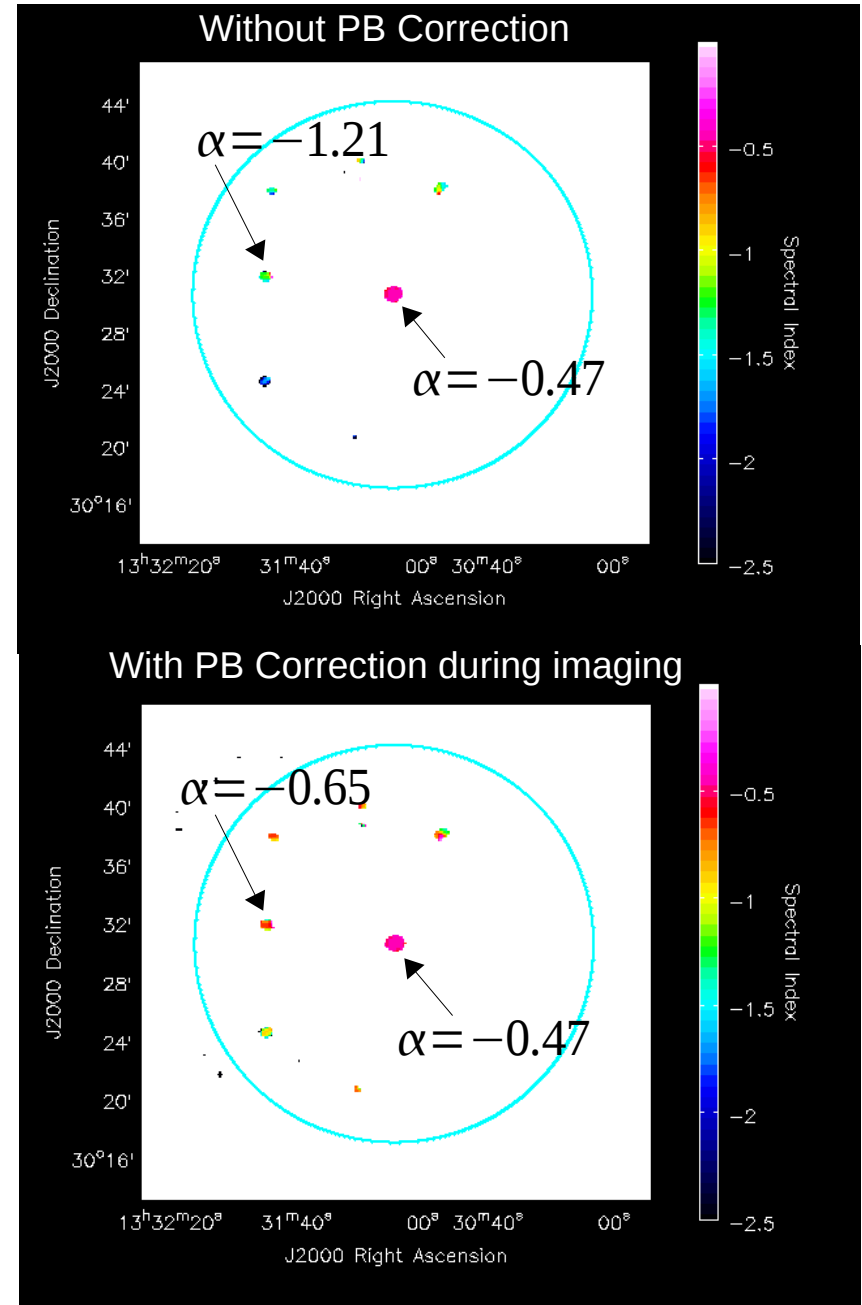
3C286 field , C-config , L-band (30min)



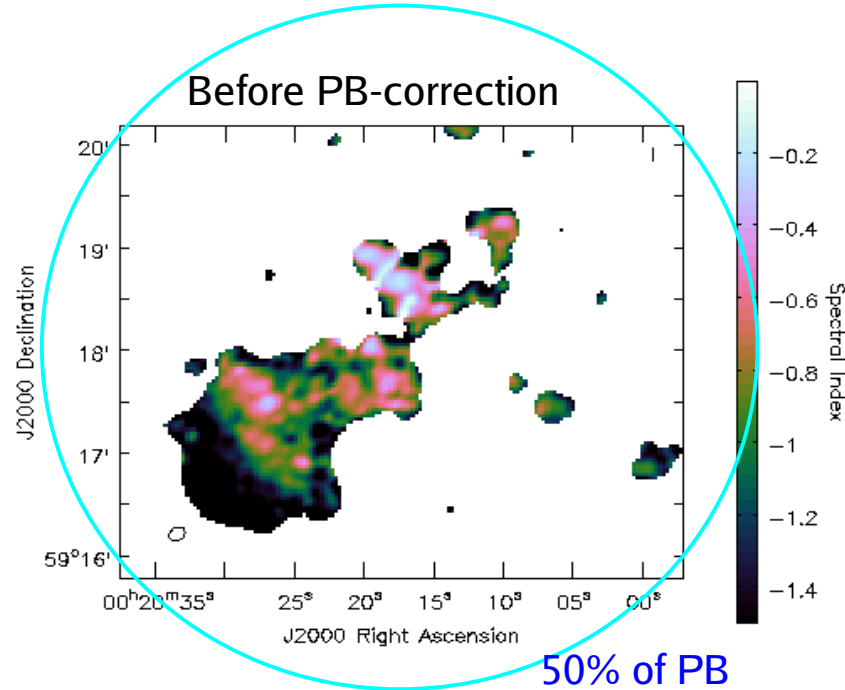
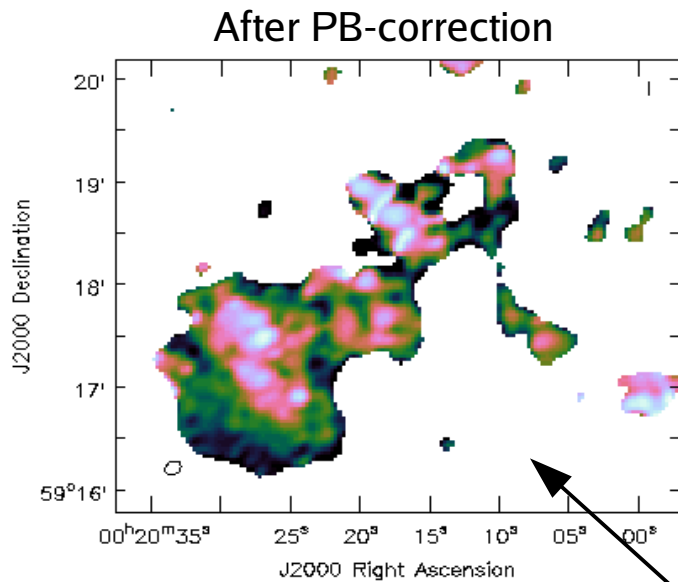
Post-deconvolution polynomial-division of the model spectrum by the PB-spectrum

Verified spectral-indices by pointing directly at one background source.

Obtained $\delta \alpha = 0.05$ to 0.1 for SNR of 1000 to 20



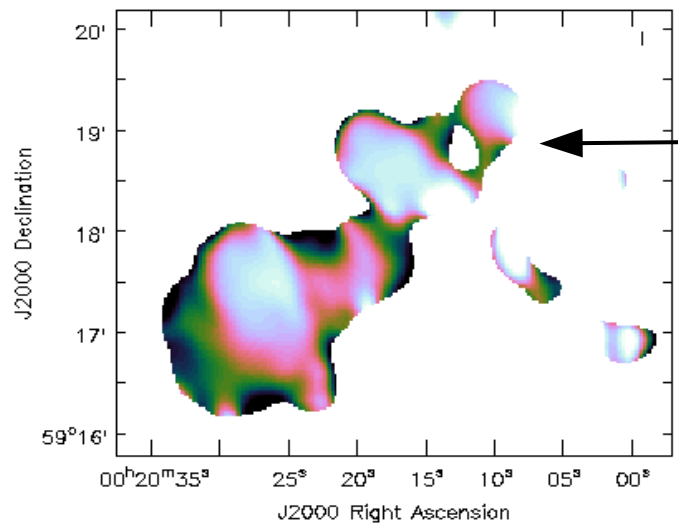
Continuum (MS-MFS) vs Cube Imaging (with PB-correction)



IC10 Dwarf Galaxy :

Spectral Index across C-Band.

Dynamic-range ~ 2000 (~ noise-limited image obtained)



MS-MFS : Result of wide-band PB-correction after MT-MS-MFS.

Cube : Spectral-index map made by PB-correcting single-SPW images smoothed to the lowest resolution.

Note that any post-deconvolution PB-correction assumes that the primary-beam does not vary / rotate during the observation.

=> Dynamic range limit of $10^4 \sim 10^5$

=> Valid within ~HPBW (depends on dynamic-range)

Example : SNR G55.7+3.4

7 hour synthesis, L-Band, 8 spws x 64 chans x 2 MHz, 1sec integrations

Due to RFI, only 4 SPWs were used for initial imaging (1256, 1384, 1648, 1776 MHz)

(All flagging and calibration done by D.Green)

J2000 Declination

30'

15'

22°00'

45'

30'

15'

21°00'

45'

Imaging Algorithms applied : MS-MFS with W-Projection

(nterms=2, multiscale=[0, 6, 10, 18, 26, 40, 60, 80])

Peak Brightness : 6.8 mJy

Extended Emission : ~ 500 micro Jy

Peak residual : 65 micro Jy

Off-source RMS : 10 micro Jy (theoretical = 6 micro Jy)

19^h26^m

24^m

23^m

22^m

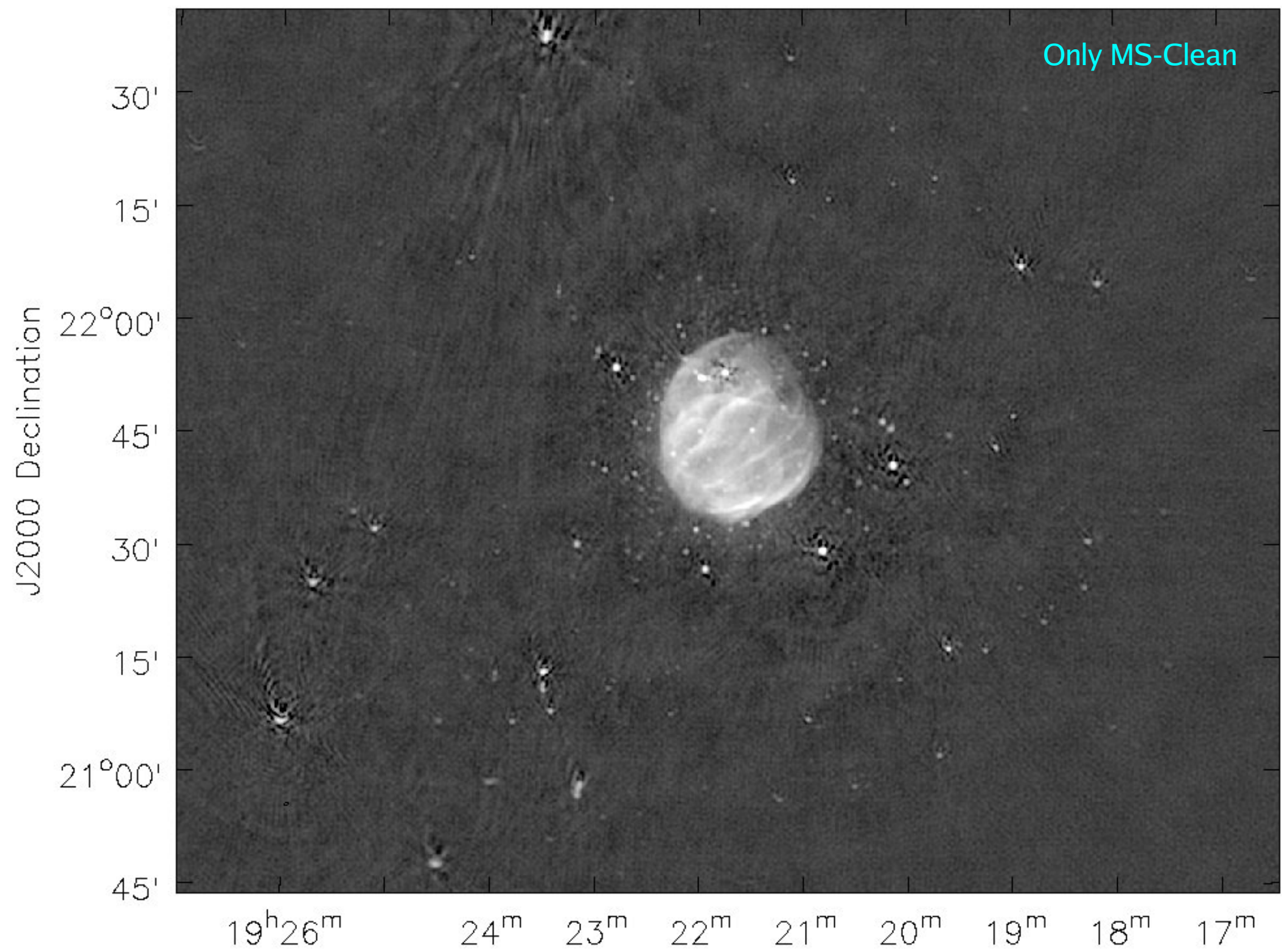
21^m

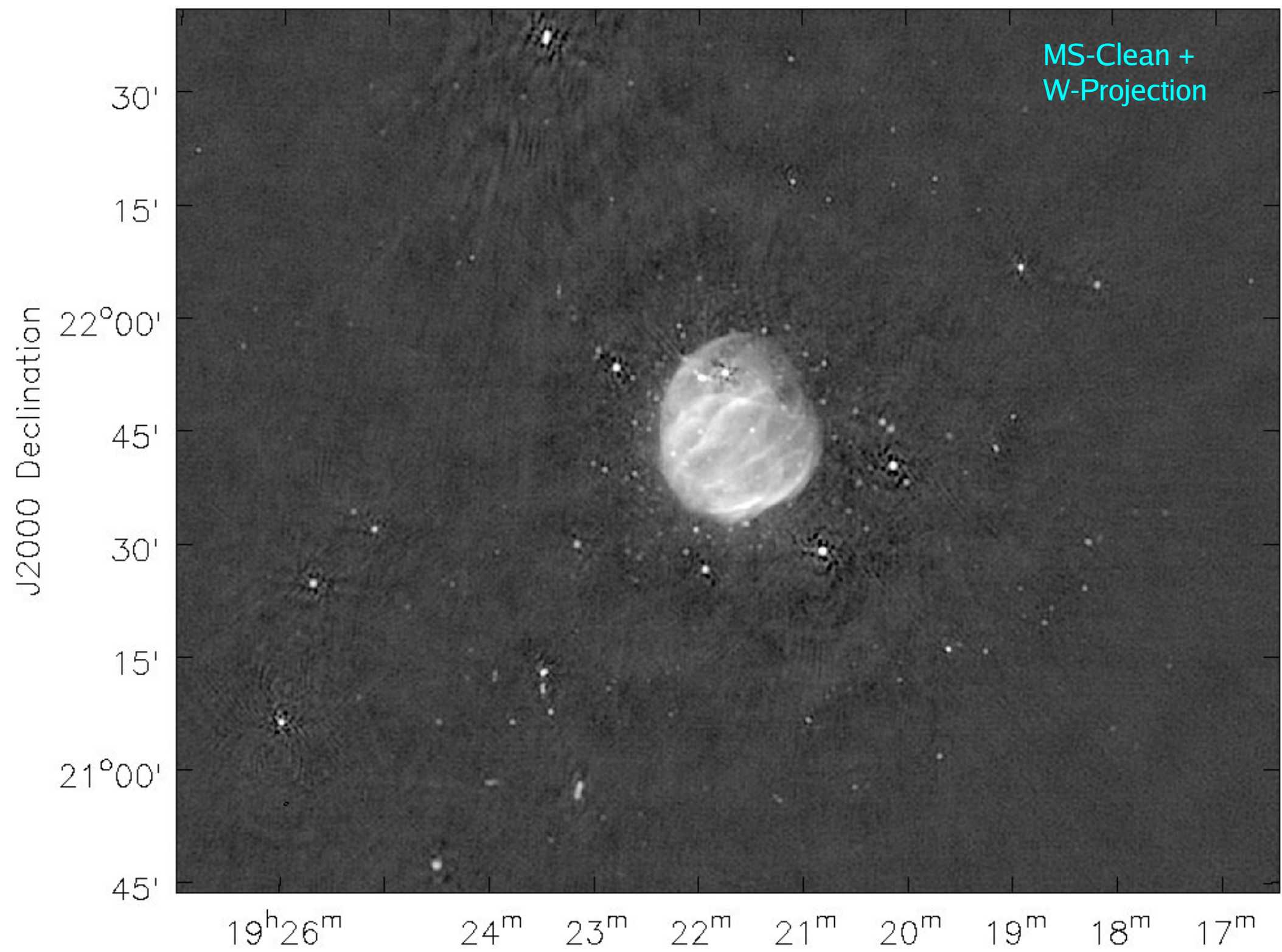
20^m

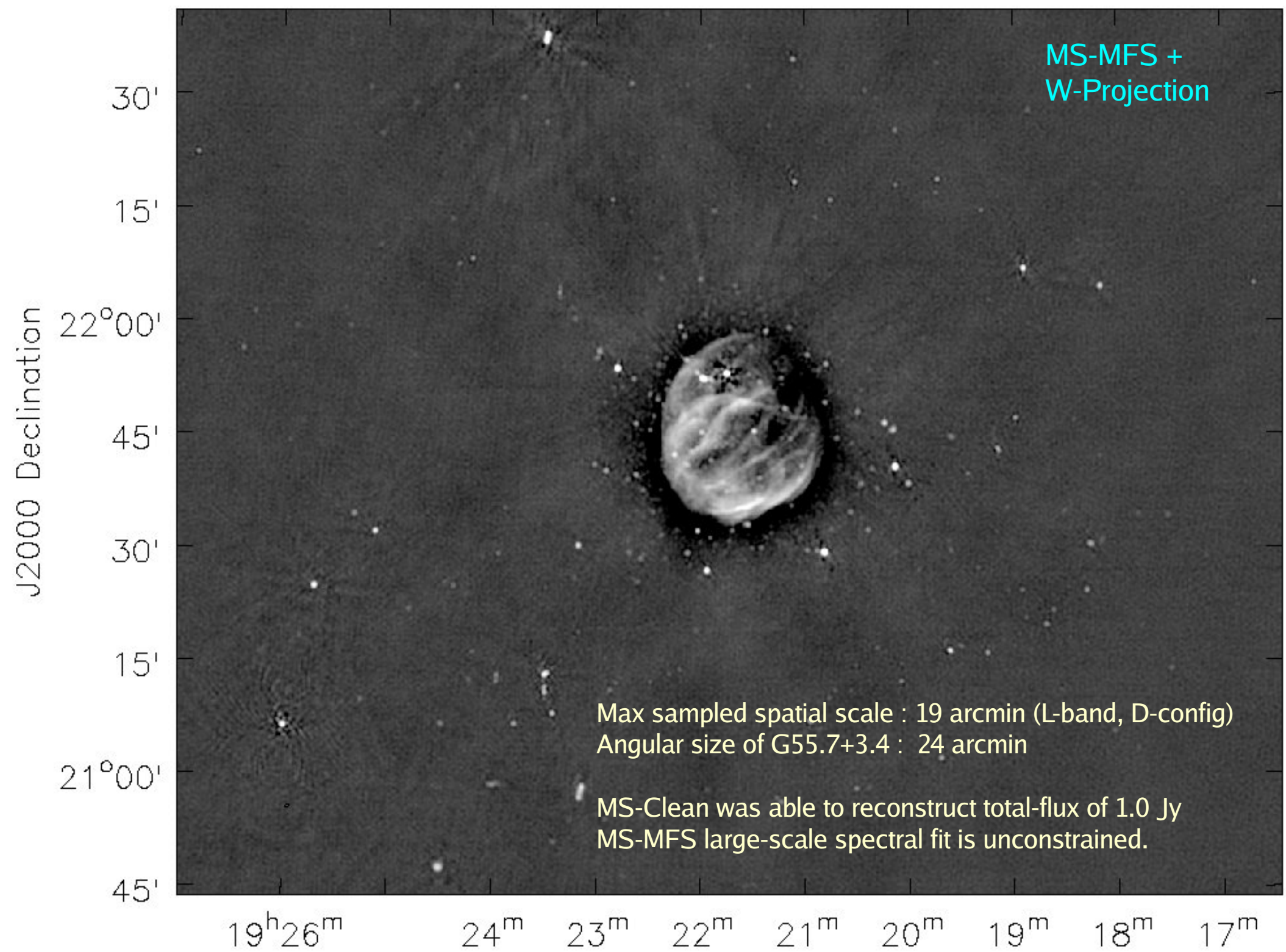
19^m

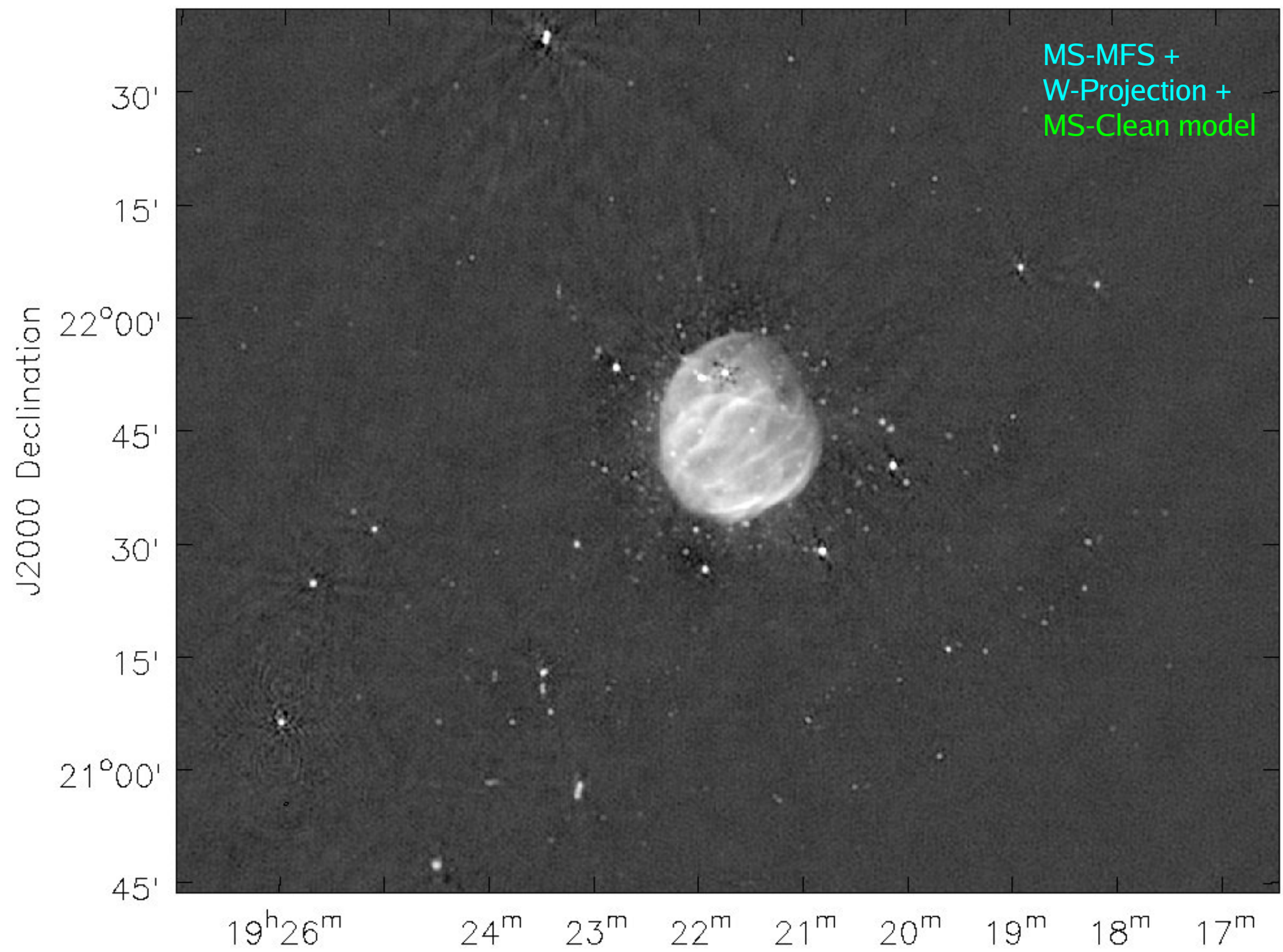
18^m

17^m



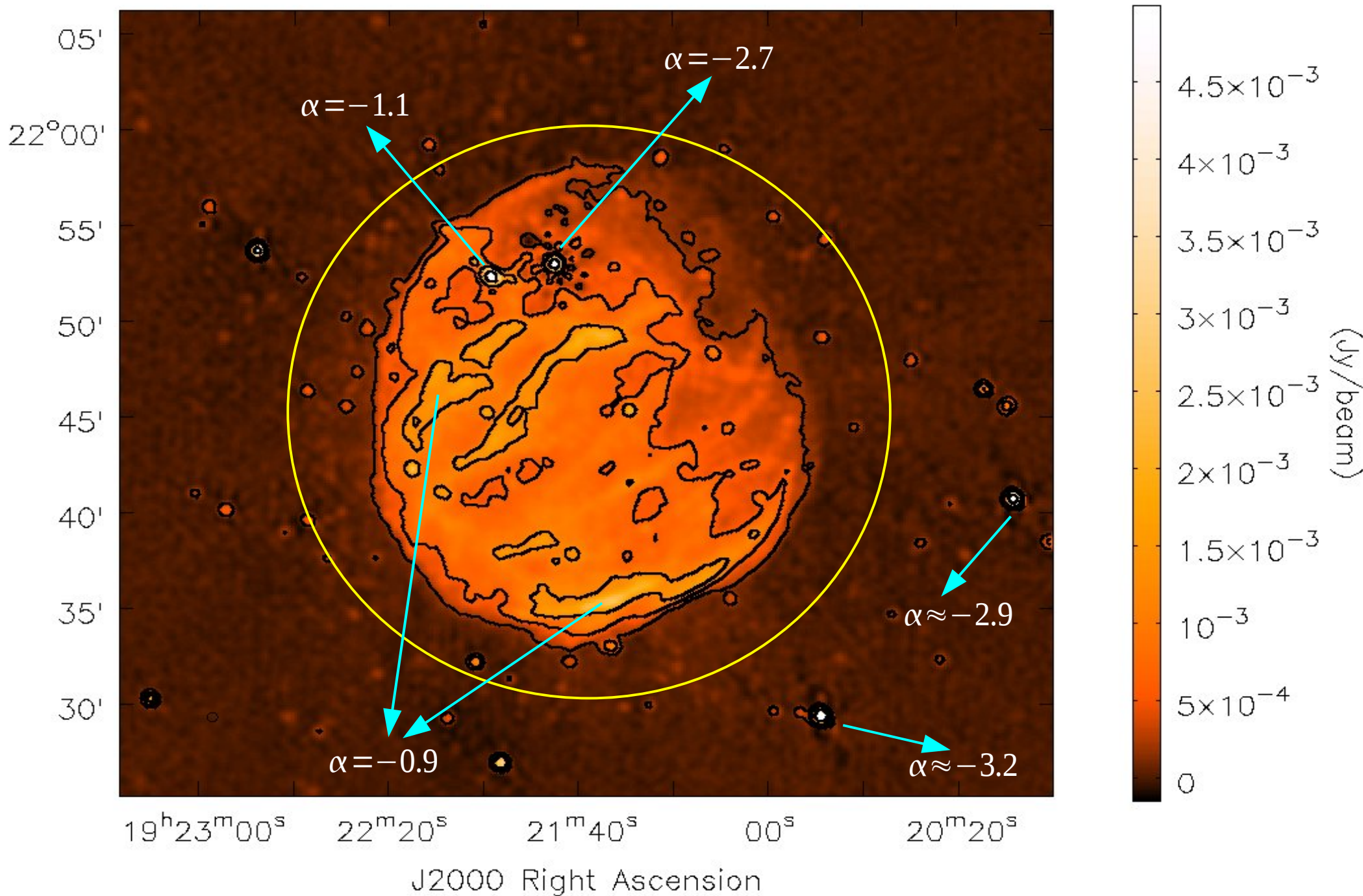






G55.7+3.4 : Supernova-Remnant + Pulsar

Spectral Indices are artificially-steepened by the Primary Beam



Wide-field sensitivity because of wide-bandwidths

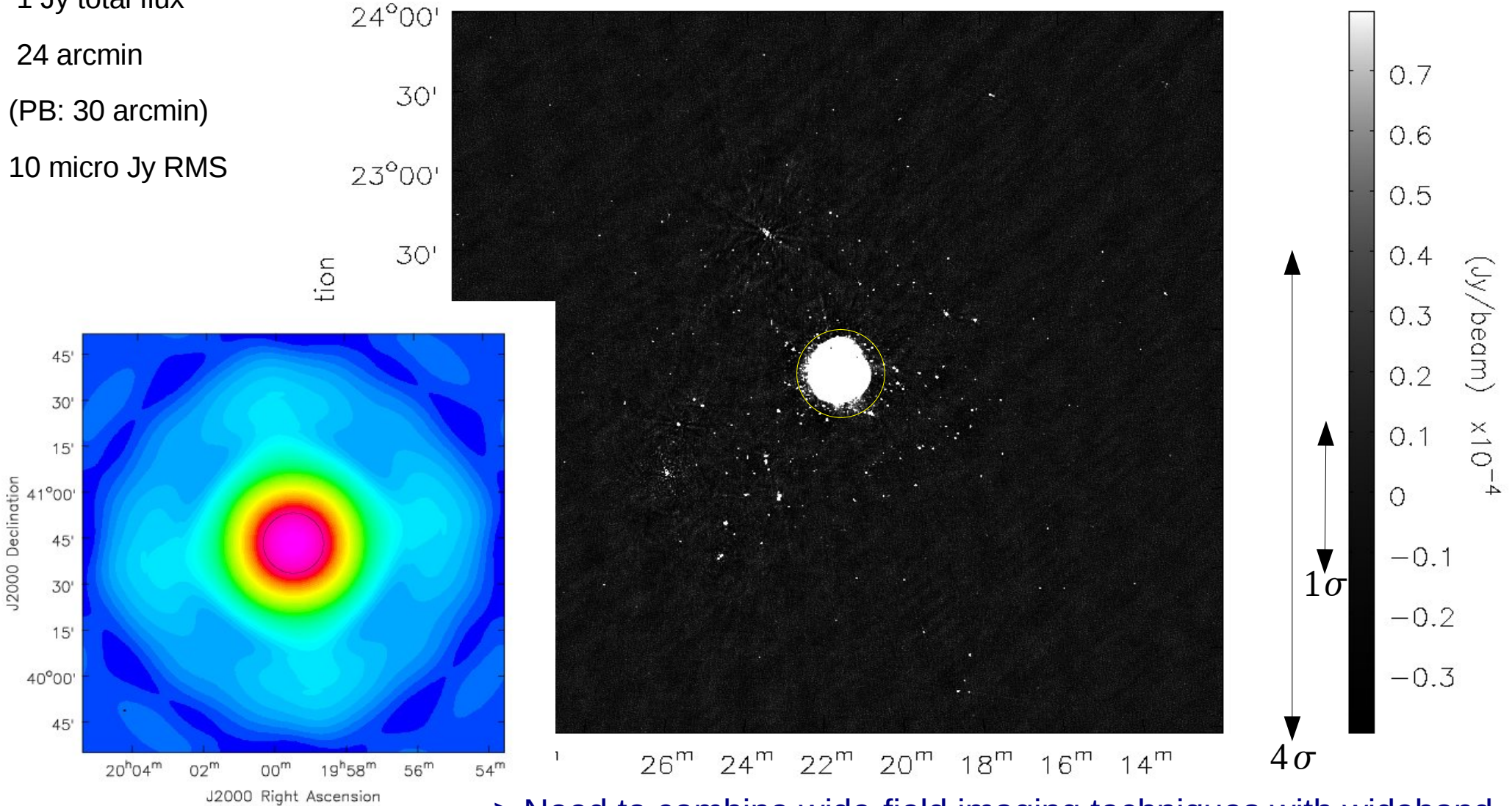
G55.7+3.4 : Galactic supernova remnant : 4 x 4 degree field-of-view from one EVLA pointing

1 Jy total flux

24 arcmin

(PB: 30 arcmin)

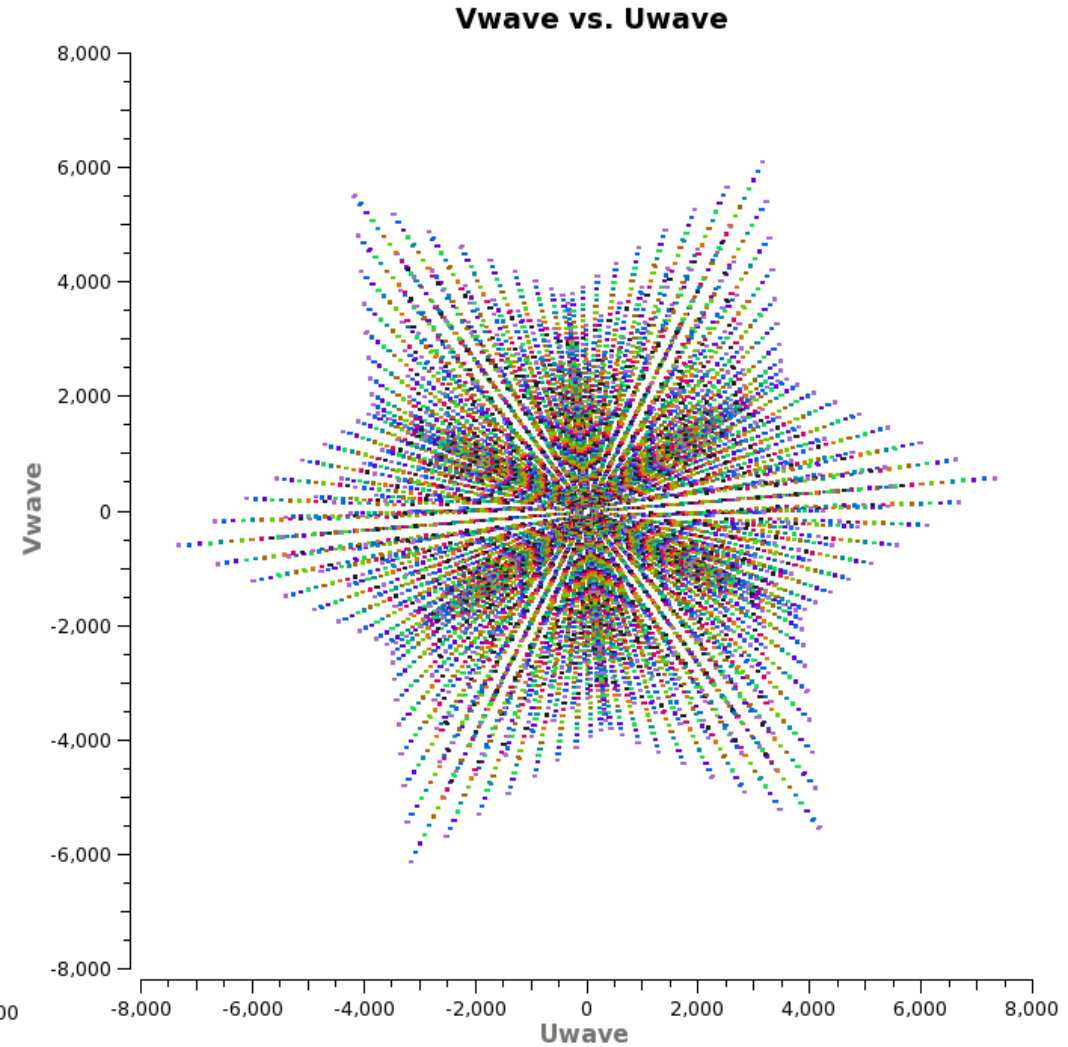
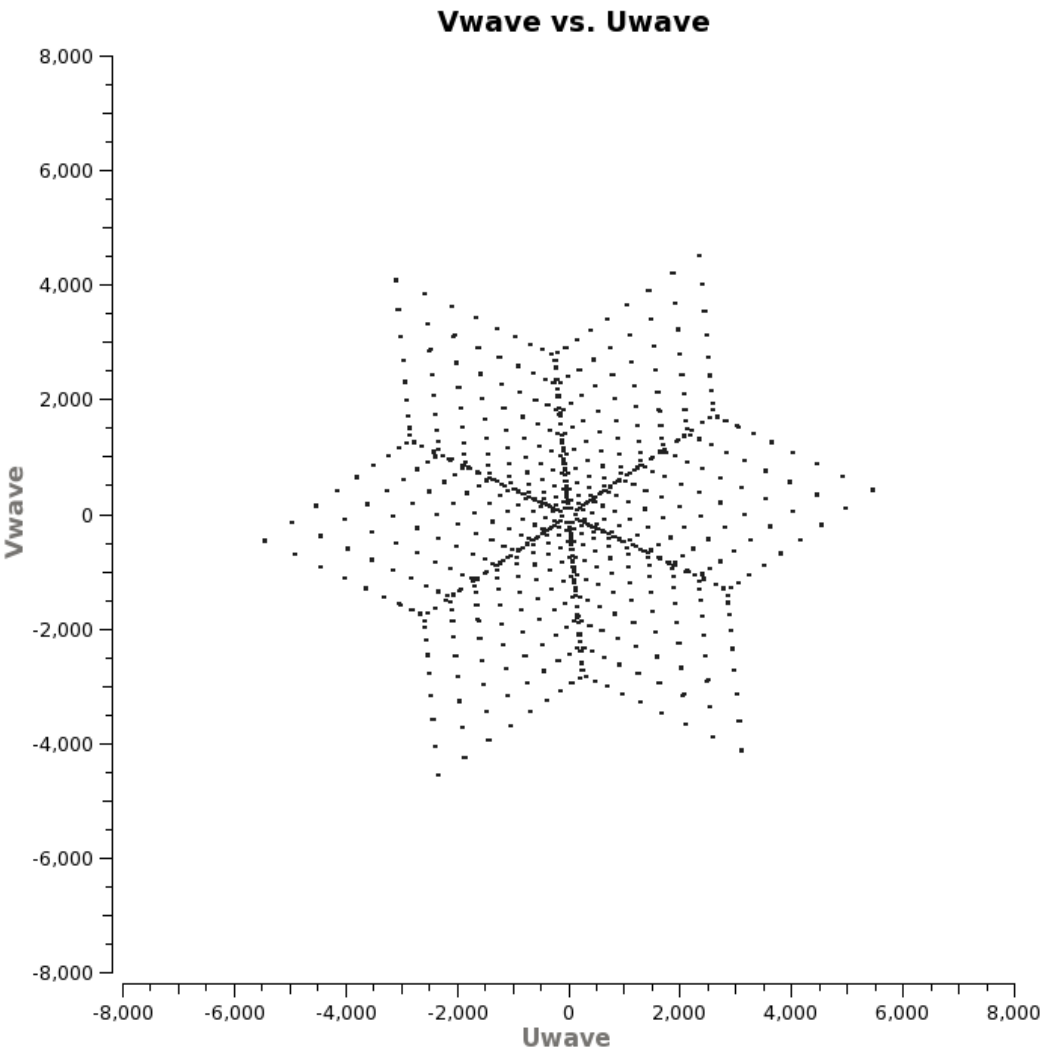
10 micro Jy RMS



=> Need to combine wide-field imaging techniques with wideband..

Multi-Frequency-Synthesis : Snapshot

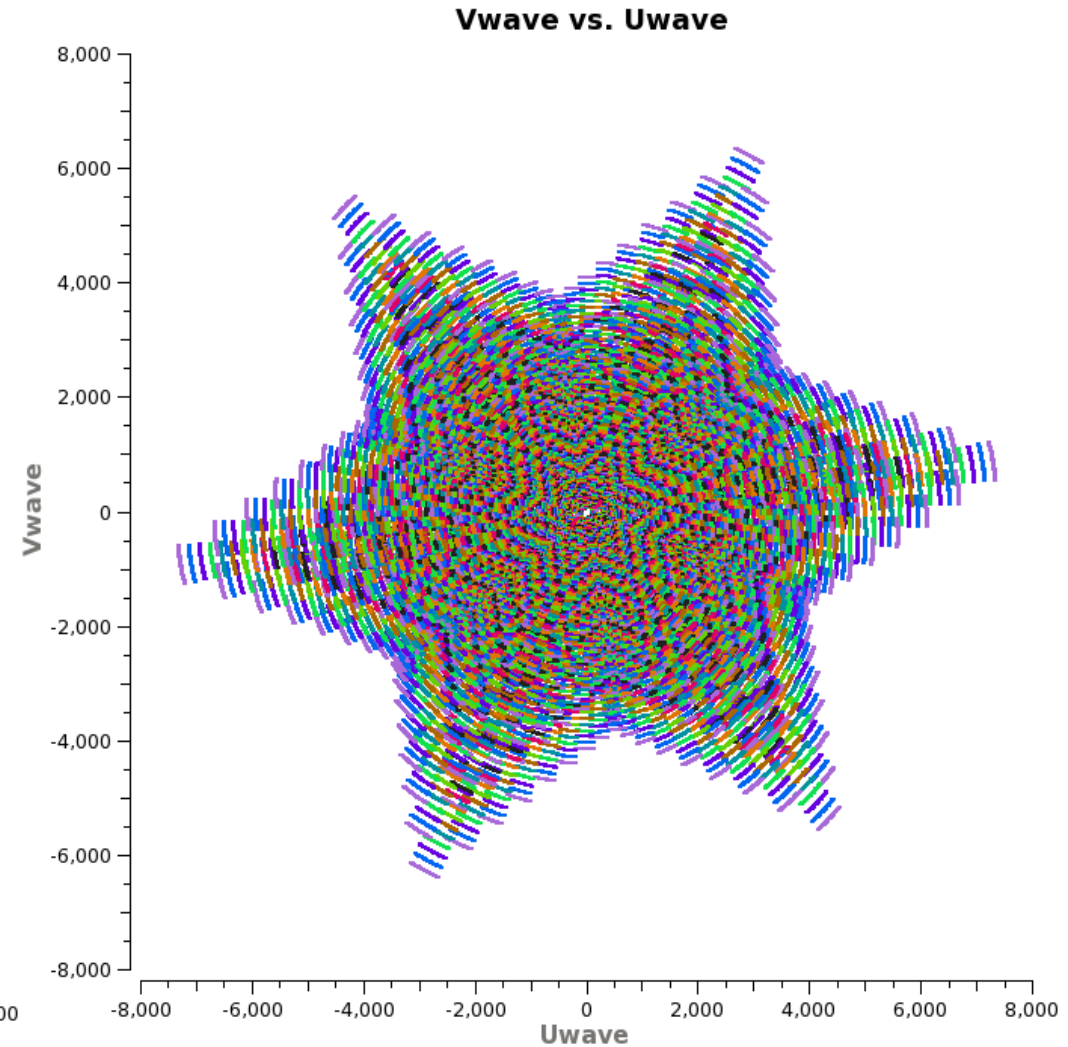
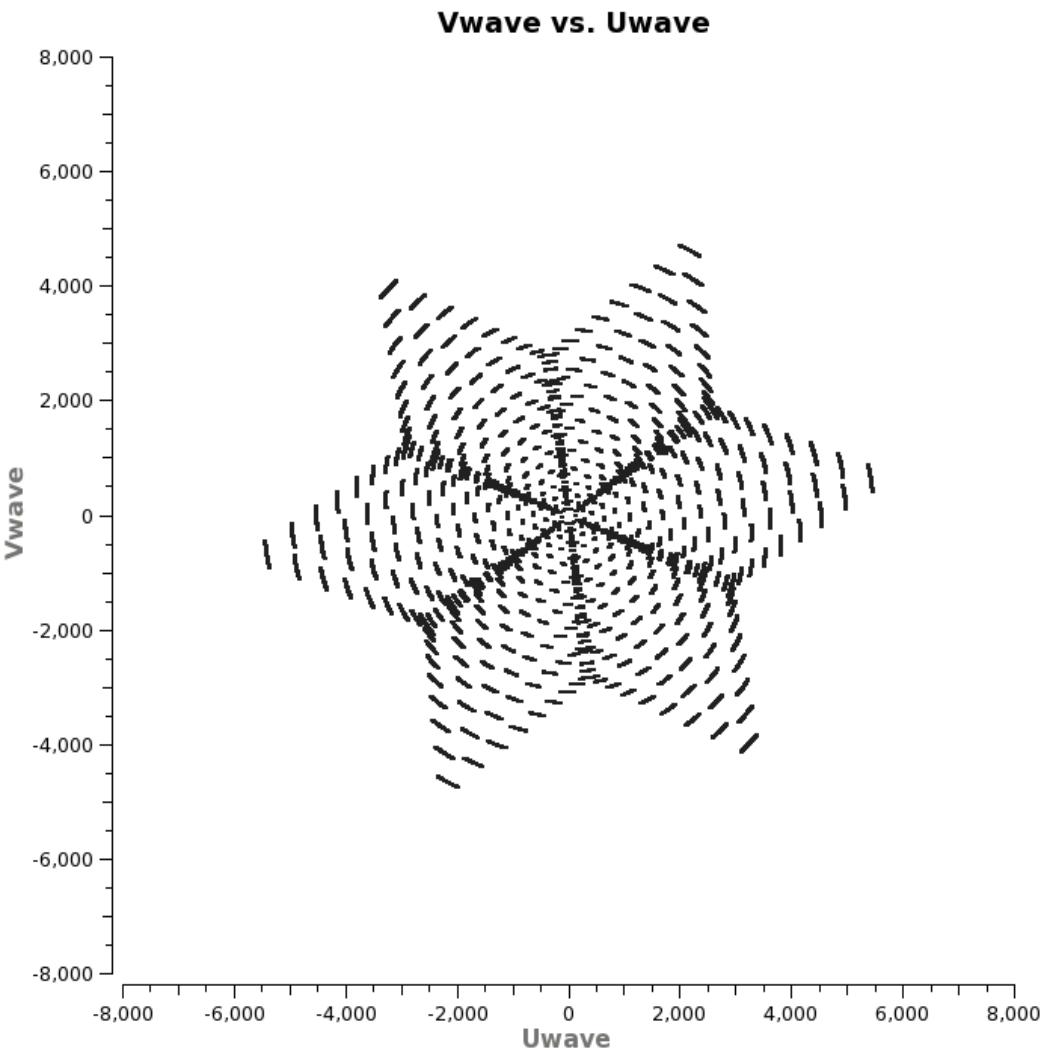
Observing tip.....



Wideband UV-coverage fills the UV-plane radially.....

Multi-Frequency-Synthesis : 30 min

Observing tip.....



Small time-increments generate good uv-filling => Plan wideband observations in small time-chunks, spread out in time to cover more spatial-frequencies at-least once.

Summary

Broad-Band Receivers



Cube-Imaging (or per SPW) will suffice for a quick-look.



Multi-Frequency-Synthesis for better sensitivity



Reconstruct Intensity and Spectrum during Imaging



Pay attention to the **many** sources of error in the model-fitting process.



If this is done correctly, you could get increased imaging sensitivity (over wide fields), high-fidelity high dynamic-range reconstructions of both spatial and spectral structure, all from a single wide-band observation.

